Symmetric and anti-symmetric Landau parameters and magnetic properties of dense quark matter

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We calculate the dimensionless Fermi liquid parameters (FLPs), $F_{0,1}^{\text{sym}}$ and $F_{0,1}^{\text{asym}}$, for spin asymmetric dense quark matter. In general, the FLPs are infrared divergent due to the exchange of massless gluons. To remove such divergences, the hard density loop (HDL) corrected gluon propagator is used. The FLPs so determined are then invoked to calculate magnetic properties such as magnetization $\langle M \rangle$ and magnetic susceptibility χ_M of spin polarized quark matter. Finally, we investigate the possibility of magnetic instability by studying the density dependence of $\langle M \rangle$ and χ_M.

I. INTRODUCTION

The study of strongly interacting matter has been an area of contemporary research for quite some time now. Such studies are usually made in the extreme condition of temperature and/or density. The high temperature (T) studies are more relevant to the ultrarelativistic heavy ion collisions while the investigations involving high chemical potential (μ) or extreme case of cold matter are more appropriate to astrophysics [1]. It should, however, be noted that efforts are being directed recently to also study the properties of a very dense system in the laboratory where matter with predominantly large chemical potential might be formed [2]. We here restrict ourselves to zero temperature and investigate some of the properties of quark matter in presence of a weak magnetic field.

It has been shown recently that the degenerate quark matter can show para-ferro phase transition below a critical density [3]. To examine this possibility, in Ref. [3], a variational calculation was performed. Subsequently, various other calculations were also performed in different formalisms to investigate such a possibility with varied conclusions [4–11].

The issue of spontaneous phase transition in dense quark system at zero temperature was also examined in [12] by invoking relativistic Fermi liquid theory (RFLT). In particular, this was accomplished by calculating the chemical potential (μ) and energy density of degenerate quark matter in terms of the Landau parameters (LPs). The RFLT was first developed by Baym and Chin [13,14] to study the properties of high density nuclear matter. However, the formalism developed in Ref. [13] is valid for unpolarized matter and LPs calculated there are spin averaged. Here, on the other hand, we deal with polarized quark matter which requires evaluation of the LPs with explicit spin dependencies.

Recently, in [15,16] the authors have studied the magnetic properties of degenerate quark matter in presence of weak uniform external magnetic field B. Similar investigation was also made in Ref. [11] by evaluating the effective potential and employing quark magnetic moment as an order parameter. These calculations were, however, restricted to the case of unpolarized matter. On the contrary, our concern here is the

magnetic properties of polarized quark system. Consequently, we first determine various spin combination of LPs such as spin symmetric ($F_{0,1}^{+(-),\text{sym}}$) and spin anti-symmetric ($F_{0,1}^{+(-),\text{asym}}$) parameters and express quantities such as magnetization and magnetic susceptibility in terms of these parameters. It is needless to mention that unlike [11,15,16], the expressions for χ_M and $\langle M \rangle$, as presented here, depend on the spin polarization parameter $\xi = (n_q^+ - n_q^-)/(n_q^+ + n_q^-)$, where n_q^+ and n_q^- correspond to densities of spin-up and spin-down quarks, respectively.

It is well known that the calculations of LPs require evaluation of the forward scattering amplitudes which are plagued with infrared divergences arising out of the exchange of massless gluons. Formally, such divergences can be removed by using HDL corrected gluon propagator. This can also be achieved by introducing screening mass for the gluons. Such regularizations are necessary for the evaluation of individual LPs. On the other hand, in various physical quantities like the ones we calculate here, the LPs appear in particular combinations where such divergences cancel at least to the order with which we are presently concerned.

The plan of the article is as follows. In Sec. II we derive the expressions of LPs for polarized quark matter. In Sec. III, we calculate magnetic susceptibility in terms of LPs with explicit spin dependencies both with bare and HDL corrected gluon propagator. In Sec. IV we summarize and conclude.

II. SYMMETRIC AND ANTI-SYMMETRIC LANDAU PARAMETERS

In this section we calculate LPs for spin polarized quark matter. We are dealing with quasiparticles whose spins are all eigenstates of the spin along a given direction, viz. z. The quasiparticle interaction can be written as the sum of two parts, viz. spin symmetric ($f_{pp'}^{\text{sym}}$) and anti-symmetric ($f_{pp'}^{\text{asym}}$) parameters [14,16]:

$$f_{pp'}^{ss'} = f_{pp'}^{\text{sym}} + (s \cdot s') f_{pp'}^{\text{asym}}. \tag{1}$$

Assuming that the spins are randomly oriented with respect to the momentum, we take average over the angles θ_1 and θ_2 corresponding to spins s and s'. The angular averaged interaction parameter is given by [12][1]

$$\overline{f_{pp'}^{ss'}}\Big|_{p=p_f^s,\, p'=p_f^{s'}} = \int \frac{d\Omega_1}{4\pi} \int \frac{d\Omega_2}{4\pi} f_{pp'}^{ss'}\Big|_{p=p_f^s,\, p'=p_f^{s'}}. \quad (2)$$

Here the spin may be either parallel ($s = s'$) or antiparallel ($s = -s'$) [3,12]. Thus the scattering possibilities are denoted by $(+, +), (+, -), (-, -)$, etc. The interaction parameters can now be redefined as

$$\begin{aligned} f_{pp'}^{++} &= f_{pp'}^{\text{sym}} + f_{pp'}^{\text{asym}} = f_{pp'}^{--}, \\ f_{pp'}^{+-} &= f_{pp'}^{\text{sym}} - f_{pp'}^{\text{asym}} = f_{pp'}^{-+}. \end{aligned} \quad (3)$$

Once these interaction parameters are known, the FLPs can be determined by expanding $f_{pp'}^{ss'}$ into the Legendre polynomial:

$$f_l^{ss'} = (2l + 1) \int \frac{d\Omega}{4\pi} P_l(\cos\theta) f_{pp'}^{ss'}, \quad (4)$$

where $\cos\theta = \hat{p} \cdot \hat{p}'$. We define symmetric and antisymmetric part of LPs $f_l^{s,\text{sym(asym)}}$ what one does to dealing with the isospins in nuclear matter [12,14]:

$$\begin{aligned} f_l^{+(-),\text{sym}} &= \tfrac{1}{2}(f_l^{++(--)} + f_l^{+-(-+)}), \\ f_l^{+(-),\text{asym}} &= \tfrac{1}{2}(f_l^{++(--)} - f_l^{+-(-+)}). \end{aligned} \quad (5)$$

It should be noted here that, $f_{pp'}^{+-} = f_{pp'}^{-+}$.

The dimensionless LPs are defined as $F_l^{s,\text{sym(asym)}} = N^s(0) f_l^{s,\text{sym(asym)}}$ [12], where $N^s(0)$ is the density of states at the Fermi surface, which can be written as

$$\begin{aligned} N^s(0) &= \int \frac{d^3p}{(2\pi)^3} \delta(\varepsilon_{ps} - \mu^s) \\ &= \frac{N_c p_f^{s2}}{2\pi^2} \left(\frac{\partial p}{\partial \varepsilon_{ps}} \right)\Big|_{p=p_f^s}. \end{aligned} \quad (6)$$

Here, N_c is the color factor, ε_{ps} and μ^s are the spin dependent quasiparticle energy and chemical potential, respectively. It is evident from Eq. (6) that for spin polarized matter, the density of states is spin dependent. This, as we shall see, makes the calculation cumbersome. In the above expression $(\partial p/\partial \varepsilon_{ps})|_{p=p_f^s}$ is the inverse Fermi velocity ($1/v_f^s$), where v_f^s is given by [12,15]

$$v_f^s = \frac{p_f^s}{\mu^s} - \frac{N_c p_f^{s2}}{2\pi^2} \frac{f_1^{s,\text{sym}}}{3}. \quad (7)$$

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[1]Denoted hereafter as $\overline{f_{pp'}^{ss'}} = f_{pp'}^{ss'}$.

With the bare propagator, the angular averaged spin dependent interaction parameter yields [12]

$$\begin{aligned} &f_{pp'}^{++}\Big|_{p=p'=p_f^+} \\ &= -\frac{g^2}{9\varepsilon_f^{+2} p_f^{+2}(1 - \cos\theta)} \\ &\quad \times \left[2m_q^2 - p_f^{+2}(1 - \cos\theta) + \frac{2m_q p_f^{+2}}{3(\varepsilon_f^+ + m_q)} \right], \quad (8) \end{aligned}$$

$$\begin{aligned} &f_{pp'}^{+-}\Big|_{p=p_f^+,\, p'=p_f^-} \\ &= \frac{g^2}{9\varepsilon_f^+ \varepsilon_f^-} \Bigg\{ 1 - \left[\frac{m_q p_f^{+2}}{3(\varepsilon_f^+ + m_q)} + \frac{m_q p_f^{-2}}{3(\varepsilon_f^- + m_q)} \right] \\ &\quad \times \frac{1}{(m_q^2 - \varepsilon_f^+ \varepsilon_f^- + p_f^+ p_f^- \cos\theta)} \Bigg\}. \quad (9) \end{aligned}$$

Here, m_q is the quark mass, $p_f^\pm = p_f(1 \pm \xi)^{1/3}$, $\varepsilon_f^\pm = (p_f^{\pm2} + m_q^2)^{1/2}$, and p_f is the Fermi momentum of the unpolarized matter ($\xi = 0$). Similarly, $f_{pp'}^{--}$ can be obtained by replacing p_f^+ with p_f^- and ε_f^+ with ε_f^- in Eq. (8). One can find dimensionless LPs, $F_{0,1}^{\text{sym}}$ and $F_{0,1}^{\text{asym}}$ (suppressing spin indices) by considering the OGE interaction. But both of these ($F_{0,1}^{\text{sym(asym)}}$) exhibit infrared divergences because of the term $(1 - \cos\theta)$ that appears in the denominator of the interaction parameter [see Eq. (8)]. This divergence disappears if one uses HDL corrected gluon propagator to evaluate the scattering amplitudes [17].

To construct HDL corrected gluon propagator with explicit spin dependence one needs to evaluate the expressions for longitudinal (Π_L) and transverse (Π_T) polarization which have been derived in [9]. We borrow the results directly:

$$\begin{aligned} \Pi_L(k_0, k) &= \frac{g^2}{4\pi^2}(C_0^2 - 1) \sum_{s=\pm} p_f^s \varepsilon_f^s \\ &\quad \times \left[-1 + \frac{C_0}{2v_f^s} \ln\left(\frac{C_0 + v_f^s}{C_0 - v_f^s} \right) \right], \quad (10) \end{aligned}$$

$$\begin{aligned} \Pi_T(k_0, k) &= \frac{g^2}{16\pi^2} C_0 \sum_{s=\pm} p_f^{s2} \left[\frac{2C_0}{v_f^s} + \left(1 - \frac{C_0^2}{v_f^{s2}} \right) \right. \\ &\quad \left. \times \ln\left(\frac{C_0 + v_f^s}{C_0 - v_f^s} \right) \right]. \quad (11) \end{aligned}$$

Here, $C_0 = k_0/|k|$, is the dimensionless variable and $v_f^\pm = p_f^\pm/\varepsilon_f^\pm$. It might be noted here, that the expressions for Π_L and Π_T look rather similar to what one obtains in the case of unpolarized matter ($\xi = 0$) [18] with only a difference in $v_f^\pm$. In the static limit, i.e., $C_0 \to 0$, the spin dependent Debye mass (m_D) is given by

$$\Pi_L = m_D^2 = \frac{g^2}{4\pi^2} \sum_{s=\pm} p_f^s \varepsilon_f^s. \quad (12)$$

It is to be mentioned here, that the screening mass of the gluon is spin dependent and the transverse gluons are screened only dynamically [15,16]. With these, the symmetric combination

of dimensionless LPs are found to be

$$F_0^{+,\mathrm{sym}} = \frac{g^2 p_f^+}{144\pi^2} \Bigg\{ \frac{1}{\varepsilon_f^+} \Bigg[12 - \frac{12m_q^3 + 12m_q^2\varepsilon_f^+ + 3m_q m_D^2 + 4p_f^{+2}m_q + 3m_D^2\varepsilon_f^+}{p_f^{+2}(m_q + \varepsilon_f^+)} \ln\left(1 + \frac{4p_f^{+2}}{m_D^2}\right) \Bigg]$$

$$+ \frac{1}{\varepsilon_f^-} \Bigg[12 + \frac{1}{p_f^+ p_f^-(m_q + \varepsilon_f^+)(m_q + \varepsilon_f^-)} \{ m_q^2[3m_D^2 - 2(p_f^{+2} + p_f^{-2})]$$

$$+ m_q[3m_D^2(\varepsilon_f^+ + \varepsilon_f^-) - 2(\varepsilon_f^+ p_f^{-2} + \varepsilon_f^- p_f^{+2})] + 3m_D^2\varepsilon_f^+\varepsilon_f^- \} \ln\left(\frac{2m_q^2 - m_D^2 + 2p_f^+ p_f^- - 2\varepsilon_f^+\varepsilon_f^-}{2m_q^2 - m_D^2 - 2p_f^+ p_f^- - 2\varepsilon_f^+\varepsilon_f^-}\right) \Bigg] \Bigg\}, \quad (13)$$

$$F_1^{+,\mathrm{sym}} = \frac{g^2 p_f^+}{48\pi^2} \Bigg\{ \frac{1}{\varepsilon_f^+} \Bigg[\frac{12m_q^3 + 12m_q^2\varepsilon_f^+ + 3m_q m_D^2 + 4p_f^{+2}m_q + 3m_D^2\varepsilon_f^+}{p_f^{+2}(m_q + \varepsilon_f^+)} \Bigg[2 - \left(1 + \frac{m_D^2}{2p_f^{+2}}\right) \ln\left(1 + \frac{4p_f^{+2}}{m_D^2}\right) \Bigg] \Bigg]$$

$$+ \frac{1}{\varepsilon_f^-} \Bigg[\frac{m_q^2[3m_D^2 - 2(p_f^{+2} + p_f^{-2})] + m_q[3m_D^2(\varepsilon_f^+ + \varepsilon_f^-) - 2(\varepsilon_f^+ p_f^{-2} + \varepsilon_f^- p_f^{+2})] + 3m_D^2\varepsilon_f^+\varepsilon_f^-}{p_f^+ p_f^-(m_q + \varepsilon_f^+)(m_q + \varepsilon_f^-)} \Bigg]$$

$$\times \Bigg[2 - \left(\frac{2m_q^2 - m_D^2 - 2\varepsilon_f^+\varepsilon_f^-}{2p_f^+ p_f^-}\right) \ln\left(\frac{2m_q^2 - m_D^2 + 2p_f^+ p_f^- - 2\varepsilon_f^+\varepsilon_f^-}{2m_q^2 - m_D^2 - 2p_f^+ p_f^- - 2\varepsilon_f^+\varepsilon_f^-}\right) \Bigg] \Bigg\}. \quad (14)$$

In deriving Eqs. (13) and (14), we consider exchange of longitudinal gluons only. In Eqs. (13) and (14), the term in the first square bracket arises due to the scattering of like-spin states $(++)$, while the latter comes from the scattering of unlike-spin states $(+-)$. Similarly one may determine other combination of LPs like $F_{0,1}^{-,\mathrm{sym}}$, $F_{0,1}^{+,\mathrm{asym}}$, $F_{0,1}^{-,\mathrm{asym}}$, etc. In Fig. 1, the density dependence of symmetric and anti-symmetric combination of dimensionless LPs is shown. Similar plots for the LPs in isospin asymmetric nuclear matter can be found in [19]. There, however, the calculated LPs are finite, as the nucleon-nucleon interactions involve exchanges of massive mesons such as σ, ω, δ, and ρ, etc. It is interesting to note that the results of isospin asymmetric nuclear matter for the LPs are qualitatively same as those of dense quark system.

III. MAGNETIC SUSCEPTIBILITY

Now, we proceed to calculate the magnetic susceptibility for which an uniform magnetic field B is applied along the z axis. The magnetic susceptibility is defined as [15,16]

$$\chi_M = \sum_f \frac{\partial \langle M \rangle_f}{\partial B} \Bigg|_{B=0}, \quad (15)$$

where $\langle M \rangle_f$ is the magnetization for each flavor. Here, the magnetic field is considered to be significantly weak for which the spinors remain unaffected and only modification enters through the single particle energy. Here, we consider one flavor quark matter and suppress the flavor indices.

In presence of constant magnetic field B, the magnetization depends on the difference of the number densities $\delta n_p^{\mathrm{asym}} = \delta n_{p,s=1} - \delta n_{p,s=-1}$, where

$$n_{ps} = \left[1 + \exp \beta \left(\varepsilon_{ps} - \mu - \tfrac{1}{2} g_D(p)\mu_q s B \right) \right]^{-1}. \quad (16)$$

In the last equation, μ_q denotes the Dirac magnetic moment and $g_D(p)$ is the gyromagnetic ratio. The magnetization is given by [15]

$$\langle M \rangle = \frac{\mu_q}{2} N_c \int \frac{d^3 p}{(2\pi)^3} g_D(p)\delta n_p^{\mathrm{asym}}. \quad (17)$$

For constant magnetic field, the variation of the distribution function yields [15,16]

$$\delta n_{ps} = \frac{\delta n_{ps}}{\delta \varepsilon_{ps}} \Bigg[-\frac{1}{2} g_D(p)\mu_q s B + N_c \sum_{s'} \int \frac{d^3 p}{(2\pi)^3} f_{pp'}^{ss'} \delta n_{p's'} \Bigg] \quad (18)$$
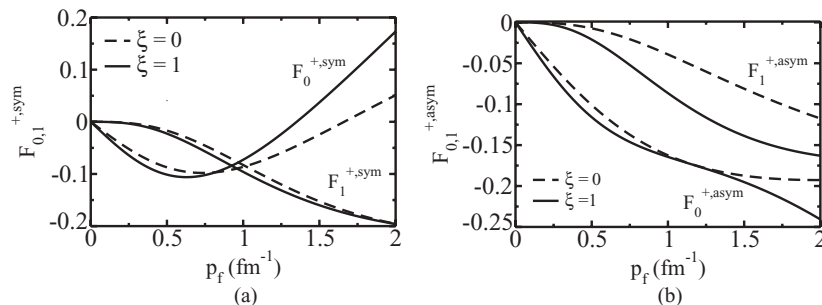

FIG. 1. Dimensionless LPs as a function of Fermi momentum for unpolarized and polarized quark matter. Symmetric and anti-symmetric combination of LPs are plotted in (a) and (b), respectively.

and δn_p^{asym} is therefore given by

$$
\delta n_p^{\text{asym}} = -\frac{1}{2} g_D(p) \mu_q B \left(\frac{\partial n_p^+}{\partial \varepsilon_p^+} + \frac{\partial n_p^-}{\partial \varepsilon_p^-} \right)
$$
$$
+ N_c \frac{\partial n_p^+}{\partial \varepsilon_p^+} (f_0^{++} \delta n^+ + f_0^{+-} \delta n^-)
$$
$$
- N_c \frac{\partial n_p^-}{\partial \varepsilon_p^-} (f_0^{-+} \delta n^+ + f_0^{--} \delta n^-). \qquad (19)
$$

With the help of Eqs. (17) and (19) the average magnetization becomes

$$
\langle M \rangle = \frac{\frac{1}{4} \overline{g}_D^2 \mu_q^2 B [N^+(0) + N^-(0)]}{1 + [N^+(0) + N^-(0)] f_0^{\text{asym}}}, \qquad (20)
$$

where we have suppressed the spin indices for $f_l^{s,\text{asym(sym)}}$. The expression of $\langle M \rangle$ may be compared with the one presented in [15,16] to see the difference between the unpolarized and polarized matter. Likewise, the magnetic susceptibility is found to be

$$
\chi_M = \left(\frac{\overline{g}_D \mu_q}{2} \right)^2 \frac{[N^+(0) + N^-(0)]}{1 + [N^+(0) + N^-(0)] f_0^{\text{asym}}}, \qquad (21)
$$

where $\overline{g}_D$ is the angular averaged gyromagnetic ratio [15,16].

With the help of Eqs. (6) and (21) we express the magnetic susceptibility in terms of LPs as

$$
\chi_M = \chi_P \left[1 + \frac{N_c (p_f^+ \mu^+ + p_f^- \mu^-)}{2\pi^2} \left(f_0^{\text{asym}} - \frac{1}{3} f_1^{\text{sym}} \right) \right]^{-1}. \qquad (22)
$$

Here, $\chi_P = \overline{g}_D^2 \mu_q^2 N_c (p_f^+ \mu^+ + p_f^- \mu^-)/(8\pi^2)$ is the Pauli susceptibility [15,16]. For unpolarized matter $\xi = 0$, implying $p_f^+ = p_f^-$, $\mu^+ = \mu^-$, and $N^+(0) = N^-(0)$. From Eq. (22) we get the well-known result for magnetic susceptibility [15,16]

$$
\chi_M = \chi_P \left[1 + \frac{N_c p_f \mu}{\pi^2} \left(f_0^{\text{asym}} - \frac{1}{3} f_1^{\text{sym}} \right) \right]^{-1}. \qquad (23)
$$

A. Susceptibility with bare propagator

We have already mentioned that the individual LPs are infrared divergent when evaluated with the bare gluon propagator. But the combination $(f_0^{\text{asym}} - \frac{1}{3} f_1^{\text{sym}})$ is always finite and turns out to be

$$
f_0^{\text{asym}} - \frac{1}{3} f_1^{\text{sym}} = \frac{1}{8} \left[\int_{-1}^{+1} d(\cos\theta)(1 - \cos\theta)(f_{pp'}^{++} + f_{pp'}^{--}) \right.
$$
$$
\left. - \int_{-1}^{+1} d(\cos\theta)(1 + \cos\theta)(f_{pp'}^{+-} + f_{pp'}^{-+}) \right]
$$
$$
= I_1 - I_2. \qquad (24)
$$

Using Eqs. (8), (9), and (24) we have

$$
I_1 = -\frac{g^2}{36} \left\{ \frac{1}{p_f^{+2} \varepsilon_f^{+2}} \left[2m_q^2 - p_f^{+2} + \frac{2m_q p_f^{+2}}{3(\varepsilon_f^+ + m_q)} \right] + [p_f^+ \to p_f^-, \varepsilon_f^+ \to \varepsilon_f^-] \right\}, \qquad (25)
$$

$$
I_2 = \frac{g^2}{36 \varepsilon_f^+ \varepsilon_f^-} \times \frac{1}{3 p_f^{+2} p_f^{-2} (m_q + \varepsilon_f^+)(m_q + \varepsilon_f^-)}
$$
$$
\times \left\{ -2p_f^+ p_f^- \left[p_f^- \varepsilon_f^+ (m_q p_f^- - 3m_q p_f^+ - 3p_f^+ \varepsilon_f^-) + m_q^2 (p_f^{+2} - 3p_f^+ p_f^- + p_f^{-2}) \right. \right.
$$
$$
\left. + m_q p_f^+ \varepsilon_f^- (p_f^+ - 3p_f^-) \right] + m_q \left[\varepsilon_f^+ (m_q^2 p_f^{-2} - p_f^+ p_f^{-3} - p_f^{+2} \varepsilon_f^{-2} - m_q \varepsilon_f^- [p_f^{+2} + p_f^{-2}]) \right.
$$
$$
\left. \left. - p_f^{-2} \varepsilon_f^- \varepsilon_f^{+2} + (m_q^2 - p_f^+ p_f^-)(\varepsilon_f^- p_f^{+2} + m_q [p_f^{+2} + p_f^{-2}]) \right] \ln \left(\frac{m_q^2 + p_f^+ p_f^- - \varepsilon_f^+ \varepsilon_f^-}{m_q^2 - p_f^+ p_f^- - \varepsilon_f^+ \varepsilon_f^-} \right) \right\}. \qquad (26)
$$

To determine χ_M for various ξ, we insert Eq. (24) in Eq. (22), where I_1 and I_2 are given by Eqs. (25) and (26).

B. Susceptibility with HDL corrected propagator

In this section we consider the screening effects due to HDL corrected propagator of the gauge field [17]. The scattering

amplitude can be written as [15]

$$
\mathcal{M}_{ps,p's'} = -\frac{4g^2}{9} [\mathcal{T}^{00}(Ps, P's') D_{00} + \mathcal{T}^{ij}(Ps, P's') D_{ij}]. \qquad (27)
$$

In the Coulomb gauge, we have $D_{00} = \Delta_l$ and $D_{ij} = (\delta_{ij} - q_i q_j / q^2) \Delta_t$, where $q = p - p'$. Δ_l and Δ_t denote the longitudinal and transverse gluon propagators given by [20]

$$
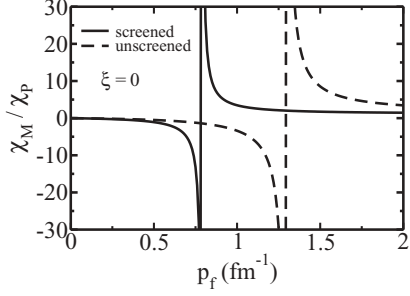\Delta_l = \frac{1}{q^2 + m_D^2}, \qquad \Delta_t = \frac{1}{q_0^2 - q^2}. \qquad (28)
$$

FIG. 2. Density dependence of magnetic susceptibility. Screening effects (solid line) are compare with the simple OGE case (dashed line) for unpolarized quark matter.

For spin parallel ($s = s'$) and antiparallel ($s = -s'$) interaction, Δ_l and Δ_t have the following form:

$$\Delta_l(s = s')|_{p=p'=p_f^{\pm}} = \frac{1}{2p_f^{\pm 2}(1 - \cos\theta) + m_D^2},$$

$$\Delta_l(s = -s')|_{p=p_f^+, p'=p_f^-} \qquad (29)$$

$$= \frac{1}{p_f^{+2} + p_f^{-2} - 2p_f^+ p_f^-(1 - \cos\theta) + m_D^2},$$

and

$$\Delta_t(s = s')|_{p=p'=p_f^{\pm}} = -\frac{1}{2p_f^{\pm 2}(1 - \cos\theta)},$$

$$\Delta_t(s = -s')|_{p=p_f^+, p'=p_f^-} = \frac{1}{2(m_q^2 - \varepsilon_f^+ \varepsilon_f^- + p_f^+ p_f^- \cos\theta)}. \qquad (30)$$

The matrix element given by Eq. (27) can be calculated easily with OGE. We find that [16]

$$\mathcal{T}^{00}(Ps, P's') = \text{Tr}[\gamma^0 \rho(P, s)\gamma^0 \rho(P', s')]$$

$$= \frac{1}{4m_q^2}\big[2p_0 p_0' - P \cdot P' + m_q^2$$

$$+ (m_q^2 - P \cdot P')(2a_0 b_0 - a \cdot b)$$

$$+ 2a_0 p_0'(P \cdot b) - 2p_0 p_0'(a \cdot b)$$

$$+ 2p_0 b_0(a \cdot P') - (P \cdot b)(P' \cdot a)\big] \qquad (31)$$

and

$$\mathcal{T}^{ij}(Ps, P's')$$

$$= \text{Tr}[\gamma^i \rho(P, s)\gamma^j \rho(P', s')]$$

$$= \frac{1}{4m_q^2}\big\{(1 - a \cdot b)\widehat{p^i p'^j} + (m_q^2 - P \cdot P')\widehat{a^i b^j}$$

$$+ (a \cdot P')\widehat{p^i b^j} + (b \cdot P)\widehat{p'^i a^j} + g^{ij}\big[(m_q^2 - P \cdot P')$$

$$\times (1 - a \cdot b) - (P \cdot b)(P' \cdot a)\big]\big\}, \qquad (32)$$

with a symbol $\widehat{a^i b^j} = a^i b^j + b^i a^j$. Here, the spin vector a_μ and b_μ are given by

$$a = s + \frac{p(p \cdot s)}{m_q(\varepsilon_p + m_q)}; \quad a^0 = \frac{p \cdot s}{m_q}, \qquad (33)$$

$$b = s' + \frac{p'(p' \cdot s')}{m_q(\varepsilon_{p'} + m_q)}; \quad b^0 = \frac{p' \cdot s'}{m_q}. \qquad (34)$$

To evaluate the spin symmetric and spin anti-symmetric combination of LPs, we need to calculate the scattering amplitudes both for spin nonflip ($s = s'$) and spin flip ($s = -s'$) interactions. The traces relevant for the longitudinal gluon exchange are given by

$$\mathcal{T}_{00}^{++} = \frac{1}{6m_q^2(m_q + \varepsilon_f^+)^2}\big[12m_q^4 + 12m_q^3 \varepsilon_f^+$$

$$+ 6m_q \varepsilon_f^+ p_f^{+2}(1 + \cos\theta) + 6m_q^2 p_f^{+2}(2 + \cos\theta)$$

$$+ p_f^{+4}(2 + 3\cos\theta)\big], \qquad (35)$$

$$\mathcal{T}_{00}^{+-} = \frac{p_f^{+2} p_f^{-2}}{6m_q^2(m_q + \varepsilon_f^+)(m_q + \varepsilon_f^-)}. \qquad (36)$$

Similarly, the coefficient of Δ_t turns out to be

$$\left[\mathcal{T}_{ij} \times \left(\delta^{ij} - \frac{q^i q^j}{q^2}\right)\right]^{++}$$

$$= -\frac{p_f^{+2}}{6m_q^3(m_q + \varepsilon_f^+)^2}\big[6m_q p_f^{+2} + 2p_f^{+2}\varepsilon_f^+$$

$$+ 2m_q^2 \varepsilon_f^+(4 + 3\cos\theta) + m_q^3(8 + 3\cos\theta)\big], \qquad (37)$$

$$\left[\mathcal{T}_{ij} \times \left(\delta^{ij} - \frac{q^i q^j}{q^2}\right)\right]^{+-} = -\frac{1}{6m_q^3(m_q + \varepsilon_f^+)(m_q + \varepsilon_f^-)(p_f^{+2} + p_f^{-2} - 2p_f^+ p_f^- \cos\theta)}$$

$$\times \big\{-p_f^{+2}\big[m_q(p_f^{+2} + p_f^{-2}) + p_f^{-2}\varepsilon_f^+ + p_f^{+2}\varepsilon_f^-\big][2p_f^{-2} + m_q(m_q + \varepsilon_f^-)]$$

$$+ m_q(m_q + \varepsilon_f^+)\big[-p_f^{-2}\big(m_q[p_f^{+2} + p_f^{-2}] + p_f^{-2}\varepsilon_f^+ + p_f^{+2}\varepsilon_f^-\big)$$

$$+ 2(p_f^{+2} - 3p_f^+ p_f^- + p_f^{-2})(m_q + \varepsilon_f^-)(m_q^2 - \varepsilon_f^+ \varepsilon_f^-)\big]$$

$$+ p_f^+ p_f^- \cos\theta\big[2p_f^{-2}\big(m_q[p_f^{+2} + p_f^{-2}] + p_f^{-2}\varepsilon_f^+ + p_f^{+2}\varepsilon_f^-\big)$$

$$+ m_q\big(2m_q^2[2p_f^{+2} - 3p_f^+ p_f^- + 2p_f^{-2}] + p_f^{-2}\varepsilon_f^{+2} + p_f^{+2}\varepsilon_f^{-2}$$

$$+ m_q \varepsilon_f^-[5p_f^{+2} - 6p_f^+ p_f^- + 3p_f^{-2}] + m_q \varepsilon_f^+[3p_f^{+2} - 6p_f^+ p_f^- + 5p_f^{-2}]$$

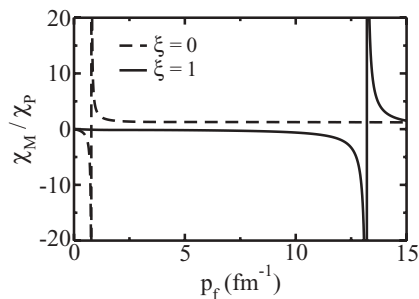$$+ 3\varepsilon_f^+ \varepsilon_f^-[p_f^{+2} + p_f^{-2} - 2p_f^+ p_f^- \cos\theta]\big)\big]\big\}. \qquad (38)$$

FIG. 3. Magnetic susceptibility vs Fermi momentum using screened gluon mass for unpolarized and complete polarized quark matter.

FIG. 4. Variation of magnetization with ξ for a magnetic field $B = 10^7$ G. Solid, dashed and dash-dotted lines represent 0.1 fm^{-3}, 0.15 fm^{-3}, and 0.25 fm^{-3}, respectively.

Using Eqs. (27)–(30) and (35)–(38) one can easily calculate the required combination $(f_0^{\text{asym}} - \frac{1}{3} f_1^{\text{sym}})$ to evaluate the magnetic susceptibility. Inserting this particular combination of f_0 and f_1 in Eq. (22) we get χ_M. To determine χ_M, we need to evaluate first μ^+ and μ^-. This can be done by adopting the procedure outlined in Ref. [12]. With these, we can estimate χ_M numerically for the polarized and unpolarized matter at various densities. The corresponding results are discussed below.

In Fig. 2 we plot the magnetic susceptibility of cold and dense unpolarized quark matter as a function of Fermi momentum. It is observed that, upon inclusion of the screening effects, the divergence move toward lower densities. This is consistent with what one obtains for unpolarized matter [15,16]. Such shifts toward lower density are expected, as we know, that the screening effect weakens the Fock exchange interaction (see Refs. [15,16]). Moreover, this divergence is related to the magnetic phase transition of quark matter which shows up when the square bracketed term in Eq. (22) vanishes. As noted in [3] and also in [12], this density approximately corresponds to the critical density for para-ferro phase transition. For the numerical estimation, we take $\alpha_c = g^2/4\pi = 2.2$ and $m_q = 300$ MeV [3,12,15,16].

In Fig. 3, the density dependence of magnetic susceptibility both for unpolarized and polarized matter has been shown. We see that the magnetic susceptibility diverges at some critical density which increases with increasing ξ. It is apparent from the figure that, if the critical density for para-ferro phase transition becomes lower than the critical density for the magnetic transition, the latter cannot take place. Thus, we conclude, that the magnetic transition depends on the critical density of para-ferro phase transition.
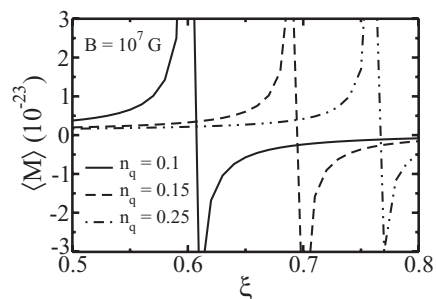
In Fig. 4, we show ξ dependence of the magnetization for various densities. Note that the divergences appear at higher ξ for larger density. Here the magnetic dipole moments of the quarks are taken to be $\mu_u = 1.852\mu_N$, $\mu_d = -0.972\mu_N$, and $\mu_s = -0.581\mu_N$, where the nuclear magneton $\mu_N = 3.152 \times 10^{-14}$ MeV/Tesla [21].

IV. SUMMARY AND CONCLUSION

In this work, we calculate dimensionless LPs $F_{0,1}^{\text{sym}}$ and $F_{0,1}^{\text{asym}}$ for dense quark matter. These are then used to calculate magnetic susceptibility and magnetization of degenerate quark matter and the results are found to be consistent with previous calculations in the appropriate limits. The qualitative behavior of the FLPs as a function of density is also found to be very similar to those of nuclear matter having isospin asymmetry.

We observe that χ_M is free of all the infrared divergences even in the massless gluon limit. It is, however, numerically sensitive to the Debye mass. It is shown that the critical density for the magnetic transition in polarized matter is higher than that of the unpolarized one. The divergence and sign change of the magnetic susceptibility signal the magnetic instability of the ferromagnetic phase.

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