Elastic electron-deuteron scattering beyond one-photon exchange

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We discuss elastic electron-deuteron (ed) scattering beyond the Born approximation. The reaction amplitude contains six generalized form factors, but only three linearly independent combinations of them (generalized charge, quadrupole, and magnetic form factors) contribute to the reaction cross section in second-order perturbation theory. We examine the two-photon exchange and find that it includes two types of diagrams, where two virtual photons are interacting with the same nucleon and where the photons are interacting with different nucleons. It is shown that the two-photon-exchange amplitude is strongly connected with the deuteron wave function at short distances.

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I. INTRODUCTION

The study of electron scattering on the nucleon and the light nuclei provides a convenient tool to study the structure of strongly interacting systems. Due to the smallness of the fine structure constant $\alpha \approx \frac{1}{137}$, one may expect that the Born approximation (one-photon exchange, OPE) should describe such processes with an accuracy of a few percent. Nevertheless, Thomas Jefferson National Accelerator Facility (JLab) polarization measurements of $G_E^p(Q^2)/G_M^p(Q^2)$ [1–3], together with their theoretical analysis [4–7], show that higher order perturbative effects, such as two-photon exchange (TPE), can strongly affect some observables of the elastic electron-nucleon scattering.

Also for more complicated hadronic systems, like the deuteron, ^{3}He, ^{4}He, etc., TPE should contribute. Thus for precise studies of these nuclei a quantitative theoretical investigation of TPE effects is important; until now only a few estimates have been done of the contribution of TPE [8–13].

The aim of this article is to estimate the TPE amplitude for electron-deuteron (ed) scattering in the framework of semirelativistic calculations, with deuteron wave functions from "realistic" NN potentials.

The article is organized as follows. In Sec. II we study the general structure of the reaction amplitude beyond OPE and define six independent generalized form factors that determine the amplitude. We show that only three linearly independent combinations of these generalized form factors contribute to the cross section in second-order perturbation theory. We call the corresponding combinations of the form factors generalized charge, quadrupole, and magnetic form factors. These generalized form factors are computed in Sec. III. Section IV contains numerical results and a brief discussion.

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II. KINEMATICS AND DEFINITIONS

The electron and deuteron momenta in the initial and final states of elastic ed scattering are denoted by k, k' and d, d', respectively; $q = k - k'$ is the transferred momentum; M and m are deuteron and nucleon masses; and actual calculations are done with $m \approx \frac{1}{2}M$.

All calculations are done in the Breit frame, where the deuteron has the same energy E_d in the initial and final state and moves along the z direction (Fig. 1). We get

$$\begin{aligned} &d_0 = d_0' = E_d = \sqrt{M^2 + Q^2/4},\\ &\vec{d}_\perp = \vec{d}\,'_\perp = 0, \quad d_3 = -d_3' = -Q/2,\\ &q_0 = q_1 = q_2 = 0, \quad q_3 = Q,\\ &k_0 = k_0' \equiv E_e, \quad \vec{k}_\perp = \vec{k}\,'_\perp, \quad k_3 = -k_3' = Q/2, \end{aligned} \tag{1}$$

where Q is the modulus of the transferred momentum. For definiteness we assume that the transverse momentum of the electron is directed along the x axis:

$$k_1 = E_e \cos\frac{\theta}{2}, \quad k_2 = 0, \quad k_3 = \frac{1}{2}Q = E_e \sin\frac{\theta}{2}. \tag{2}$$

In this frame the commonly used polarization parameter ϵ can be expressed in terms of the electron scattering angle θ by

$$\epsilon = \frac{\cos^2\frac{\theta}{2}}{1 + \sin^2\frac{\theta}{2}}; \tag{3}$$

note that $\mathrm{tg}^2\frac{\theta}{2} = (1+\eta)\mathrm{tg}^2\frac{\theta_{\mathrm{lab}}}{2}$, where $\eta = \frac{Q^2}{4M^2}$.

The polarization vectors for the incoming and outgoing deuterons with spin z-projection λ are denoted by $\epsilon_{(\lambda)}(d)$ and

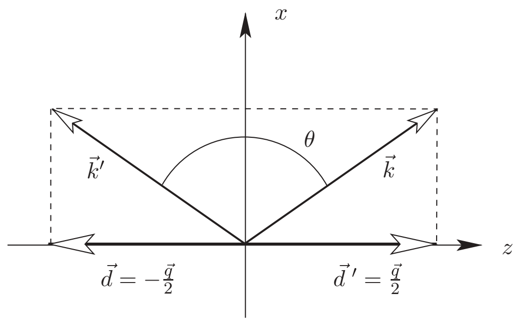

FIG. 1. The electron and deuteron three-momenta in the Breit frame.

$\epsilon_{(\lambda)}(d')$, respectively,

$$\begin{aligned} \epsilon_{(\pm1)}(d) &= \epsilon_{(\pm1)}(d') = -\sqrt{\frac{1}{2}}(0, \pm1, i, 0), \\ \epsilon_{(0)}(d) &= \frac{1}{M}\left(-\frac{Q}{2}, 0, 0, E_d\right), \\ \epsilon_{(0)}(d') &= \frac{1}{M}\left(\frac{Q}{2}, 0, 0, E_d\right). \end{aligned} \tag{4}$$

The required electron electromagnetic current $j^\mu = \bar{u}_h(k')\gamma^\mu u_h(k)$ is written as

$$\begin{aligned} j_0 &= 2E_e \cos\frac{\theta}{2}, \quad j_1 = -2E_e, \\ j_2 &= -2ihE_e \sin\frac{\theta}{2}, \quad j_3 = 0. \end{aligned} \tag{5}$$

Because the electrons are ultrarelativistic, the helicities of the incoming and outgoing electrons are the same; their signs are specified by h.

Instead of the usual reaction amplitude $\mathcal{M}$ it is useful to introduce the reduced amplitude $T_{\lambda'\lambda,h}$ by

$$\mathcal{M} = \frac{16\pi\alpha}{Q^2} E_e E_d T_{\lambda'\lambda,h}. \tag{6}$$

It follows from P and T invariance that, in the Breit frame, this amplitude must have the following properties

$$\begin{aligned} T_{\lambda'\lambda;h} &= (-1)^{\lambda-\lambda'} T_{-\lambda'-\lambda;-h}, \quad \text{from } P \text{ invariance}, \\ T_{\lambda'\lambda;h} &= T_{-\lambda-\lambda';h}, \quad \text{from } T \text{ invariance}. \end{aligned} \tag{7}$$

The reaction amplitude is determined by six independent invariant amplitudes (form factors), which are specified by the following parametrization:

$$T_{\lambda'\lambda;h} = \begin{pmatrix} \mathcal{G}_{11}\cos\frac{\theta}{2} & -\sqrt{\frac{\eta}{2}}\mathcal{G}_{10}^h & \mathcal{G}_{1,-1}^h \\ \sqrt{\frac{\eta}{2}}\mathcal{G}_{10}^{-h} & \mathcal{G}_{00}\cos\frac{\theta}{2} & -\sqrt{\frac{\eta}{2}}\mathcal{G}_{10}^h \\ \mathcal{G}_{1,-1}^{-h} & \sqrt{\frac{\eta}{2}}\mathcal{G}_{10}^{-h} & \mathcal{G}_{11}\cos\frac{\theta}{2} \end{pmatrix}, \tag{8}$$

where lines and columns correspond to the allowing order, $(\lambda', \lambda) = +1, 0, -1$, and

$$\mathcal{G}_{10}^h = f_1 + h\sin\frac{\theta}{2} f_2, \quad \mathcal{G}_{1,-1}^h = f_3 + h\sin\frac{\theta}{2} f_4. \tag{9}$$

The form factors $\mathcal{G}_{11}$, $\mathcal{G}_{00}$, and $f_1, \ldots, f_4$ are complex functions of the two independent kinematical variables, for example, Q^2 and θ.

The relation of the amplitude $T_{\lambda'\lambda;h}$ to the invariant amplitudes $G_1, \ldots, G_6$ used by other authors [14,15] is given in Appendix A.

Later on the amplitude (8) is expanded in α, and only terms of order zero and one are kept. As shown in Eq. (A8), at zero order (OPE approximation) the amplitude written in terms of the charge, magnetic, and quadruple form factors (G_C, G_M, and G_Q) becomes

$$T^{(0)}_{\lambda'\lambda;h} = \begin{pmatrix} (G_C - \frac{2}{3}\eta G_Q)\cos\frac{\theta}{2} & -\sqrt{\frac{\eta}{2}}G_M(1 + h\sin\frac{\theta}{2}) & 0 \\ \sqrt{\frac{\eta}{2}}G_M(1 - h\sin\frac{\theta}{2}) & (G_C + \frac{4}{3}\eta G_Q)\cos\frac{\theta}{2} & -\sqrt{\frac{\eta}{2}}G_M(1 + h\sin\frac{\theta}{2}) \\ 0 & \sqrt{\frac{\eta}{2}}G_M(1 - h\sin\frac{\theta}{2}) & (G_C - \frac{2}{3}\eta G_Q)\cos\frac{\theta}{2} \end{pmatrix}, \tag{10}$$

where the form factors $G_C(Q^2)$, $G_Q(Q^2)$, and $G_M(Q^2)$ are real and depend upon Q^2 only.

Next the generalized electric, quadrupole, and magnetic from factors $\mathcal{G}_C(Q^2, \theta)$, $\mathcal{G}_Q(Q^2, \theta)$, and $\mathcal{G}_M(Q^2, \theta)$, which reproduce the spin structure of Eq. (10), and the additional form factors $g_1(Q^2, \theta)$, $g_2(Q^2, \theta)$, and $g_3(Q^2, \theta)$ are introduced as follows:

$$\begin{aligned} \mathcal{G}_{11} &= \mathcal{G}_C - \frac{2}{3}\eta\mathcal{G}_Q, \quad \mathcal{G}_{00} = \mathcal{G}_C + \frac{4}{3}\eta\mathcal{G}_Q, \\ f_1 &= \mathcal{G}_M + g_1\sin^2\frac{\theta}{2}, \quad f_2 = \mathcal{G}_M - g_1, \\ f_3 &= g_2, \quad f_4 = g_3. \end{aligned} \tag{11}$$

In OPE + TPE approximation the form factors can be written as

$$\begin{aligned} \mathcal{G}_C &= G_C + \delta\mathcal{G}_C, \quad \mathcal{G}_Q = G_Q + \delta\mathcal{G}_Q, \\ \mathcal{G}_M &= G_M + \delta\mathcal{G}_M, \end{aligned} \tag{12}$$

where δ stands for the terms of order α; likewise, the form factors g_1, g_2, and g_3 are also proportional to α.

By standard calculation one derives the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\rm M}}{\cos^2\frac{\theta}{2}}\overline{|T|^2}, \tag{13}$$

where $\sigma_{\rm M}$ is the Mott cross section and

$$\overline{|T|^2} = \frac{1}{6}\sum_{\lambda,\lambda',h}|T_{\lambda'\lambda;h}|^2 = \cos^2\frac{\theta}{2}\left[\mathcal{A}(Q^2,\theta) + \mathrm{tg}^2\frac{\theta_{\rm lab}}{2}\mathcal{B}(Q^2,\theta)\right] + \mathcal{O}(\alpha^2) = \left(1+\sin^2\frac{\theta}{2}\right)\left[\epsilon|\mathcal{G}_E(Q^2,\theta)|^2 + \frac{2}{3}\eta|\mathcal{G}_M(Q^2,\theta)|^2\right] + \mathcal{O}(\alpha^2), \tag{14}$$

with

$$\mathcal{A}(Q^2,\theta) = |\mathcal{G}_C(Q^2,\theta)|^2 + \tfrac{8}{9}\eta^2|\mathcal{G}_Q(Q^2,\theta)|^2 + \tfrac{2}{3}\eta|\mathcal{G}_M(Q^2,\theta)|^2,$$
$$\mathcal{B}(Q^2,\theta) = \tfrac{4}{3}(1+\eta)\eta|\mathcal{G}_M(Q^2,\theta)|^2, \tag{15}$$
$$\mathcal{G}_E^2 = |\mathcal{G}_C(Q^2,\theta)|^2 + \tfrac{8}{9}\eta^2|\mathcal{G}_Q(Q^2,\theta)|^2.$$

The advantage of using the form factors $\mathcal{G}_C$, $\mathcal{G}_Q$, and $\mathcal{G}_M$ is that the expression for the cross section has the same form as the Rosenbluth formula; nevertheless the Rosenbluth separation of the structure functions $\mathcal{A}(Q^2,\theta)$ and $\mathcal{B}(Q^2,\theta)$ can no longer be done because they depend on two variables.

III. CALCULATION OF THE TWO-PHOTON EXCHANGE

In what follows the contribution of meson exchange currents to the TPE amplitude are neglected, and two types of TPE diagrams are considered, where the virtual photons interact directly with the nucleons

$$\mathcal{M}_2 = \mathcal{M}^{\rm I} + \mathcal{M}^{\rm II}. \tag{16}$$

One of them, $\mathcal{M}^{\rm I} = \mathcal{M}_p^{\rm I} + \mathcal{M}_n^{\rm I}$, corresponds to diagrams where both photons interact with the same nucleon (Fig. 2, top). The other type, $\mathcal{M}^{\rm II} = \mathcal{M}_{\rm P}^{\rm II} + \mathcal{M}_{\rm X}^{\rm II}$, corresponds to the diagrams where the photons interact with different nucleons (Fig. 2, bottom).

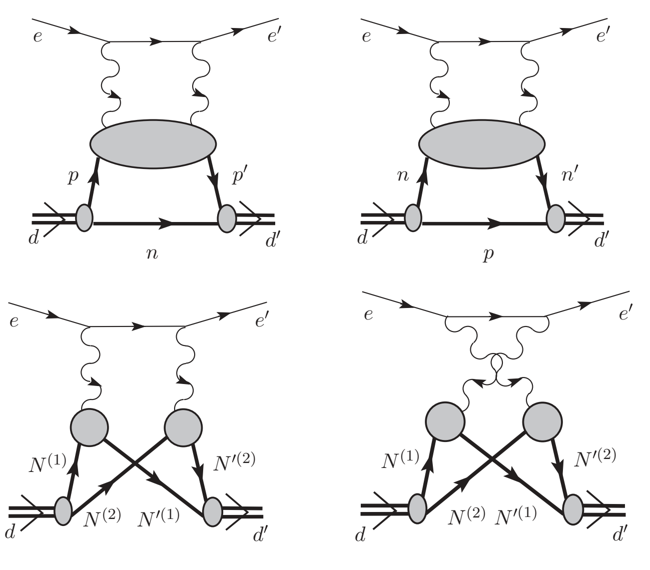

FIG. 2. Two-photon exchange diagrams. The top diagrams correspond to the amplitudes $\mathcal{M}_p^{\rm I}$ and $\mathcal{M}_n^{\rm I}$ and the bottom diagrams correspond to the amplitudes $\mathcal{M}_{\rm P}^{\rm II}$ (left) and $\mathcal{M}_{\rm X}^{\rm II}$ (right).

The deuteron structure is described by the nonrelativistic wave function

$$\Psi(\lambda,\vec p\,) = \sum_{\sigma_1,\sigma_2}\Psi_{\sigma_1\sigma_2}(\lambda,\vec p\,) = \sum_{\sigma_1,\sigma_2}\left[\sqrt{\frac{1}{4\pi}}\left\langle\frac{1}{2}\frac{1}{2}\sigma_1\sigma_2\right|1\lambda\right\rangle U_0(p) - \sum_{\xi,M}Y_{2\xi}(\widehat{p}\,)\left\langle\frac{1}{2}\frac{1}{2}\sigma_1\sigma_2\right|1M\right\rangle\langle 21\xi M|1\lambda\rangle U_2(p)\Bigg] \times|N_1\sigma_1,N_2\sigma_2\rangle, \tag{17}$$

where $\vec p$ is the internal momentum in the deuteron, $|N_1\sigma_1,N_2\sigma_2\rangle$ is the spin-isospin wave function of the two nucleons, and $\langle\cdots|\cdots\rangle$ are Clebsh-Gordan coefficients.

A. $\mathcal{M}_N^{\rm I}$ diagram

The TPE amplitude for a nucleon N has the following structure [4]

$$\mathcal{M}_{2\gamma N} = \frac{4\pi\alpha}{Q^2}\bar u_h'\gamma_\mu u_h\langle\vec p\,'_N\sigma'|\widehat{H}_N^\mu|\vec p_N\,\sigma\rangle, \tag{18}$$

where $\widehat{H}_N^\mu$ is the "effective hadron current"

$$\widehat{H}_N^\mu = \Delta\widetilde{F}_1^N\gamma^\mu - \Delta\widetilde{F}_2^N[\gamma^\mu,\gamma^\nu]\frac{q_\nu}{4m} + \widetilde{F}_3^N K_\nu\gamma^\nu\frac{P^\mu}{m^2}. \tag{19}$$

In Eqs. (18) and (19), p_N and p'_N are the nucleon momenta, σ and σ' are the nucleon spin projections, $|\vec p_N\,\sigma\rangle$ and $|\vec p\,'_N\,\sigma'\rangle$ are the nucleon spinors, $K=(k+k')/2$, $P=(p_N+p'_N)/2$; $\Delta\widetilde{F}_1^N$ and $\Delta\widetilde{F}_2^N$ may be called corrections to the Dirac and Pauli form factors, and $\widetilde{F}_3^N$ is a new form factor. All the quantities $\Delta\widetilde{F}_1^N$, $\Delta\widetilde{F}_2^N$, and $\widetilde{F}_3^N$ are of order α. They are complex functions of two kinematical variables, for example, Q^2 and $\nu = 4PK$.

Because we employ a nonrelativistic deuteron wave function we must put

$$p_p \approx \left(m,\tfrac{1}{2}\vec d + \vec p\,\right),\quad p_n \approx \left(m,\tfrac{1}{2}\vec d - \vec p\,\right),\quad p'_p \approx \left(m,\tfrac{1}{2}\vec d\,' + \vec p\,'\right),\quad p'_n \approx \left(m,\tfrac{1}{2}\vec d\,' - \vec p\,'\right), \tag{20}$$

where $\vec p$ and $\vec p\,'$ are the internal momenta in the deuteron.

From Eq. (18) it follows that the $\mathcal{M}^{\rm I}$ amplitude is given by

$$\mathcal{M}^{\rm I} = \frac{4\pi\alpha}{Q^2}\bar u'\gamma_\mu u\mathcal{D}^\mu(\lambda',\lambda), \tag{21}$$

where $\mathcal{D}^\mu(\lambda'\lambda)$ is an effective deuteron current. The latter is derived in the same way as the deuteron current J^μ in the impulse approximation [16] with the nucleon current substituted by the effective hadron current (19),

$$\mathcal{D}^\mu(\lambda',\lambda) = \frac{E_d}{m}\int d^3p\Psi^\dagger\left(\vec p + \frac{1}{2}\vec q,\lambda'\right)\left(\widehat{H}_p^\mu + \widehat{H}_n^\mu\right)\Psi(\vec p,\lambda). \tag{22}$$

Here $\Psi(\vec{p},\lambda)$ and $\Psi(\vec{p}+\frac{1}{2}\vec{q},\lambda')$ are wave functions of the deuteron in the initial and final states.[1]

Later on we will need a nonrelativistic reduction of matrix elements of the effective hadron current. Retaining the terms linear in the nucleon momentum one gets (see Appendix B)

$$\begin{aligned}
&\langle \vec{p}\,'_N\sigma'|\widehat{H}^0_N|\vec{p}_N\,\sigma\rangle \\
&\quad\approx 2m\chi^\dagger_{\sigma'}\left(\delta\mathcal{G}^N_E - \frac{iE_eQ\sigma^2}{2m^2}\cos\tfrac{\theta}{2}\widetilde{F}^N_3\right)\chi_\sigma \\
&\quad\equiv \chi^\dagger_{\sigma'}\mathcal{H}^0_N\chi_\sigma, \\
&\langle \vec{p}\,'_N\sigma'|\widehat{\vec{H}}_N|\vec{p}_N\,\sigma\rangle \\
&\quad\approx \chi^\dagger_{\sigma'}\left[i(\vec{\sigma}\times\vec{q})\left(\delta\mathcal{G}^N_M - \frac{\epsilon E_e}{m}\widetilde{F}^N_3\right) + 2\vec{P}\delta\mathcal{G}^N_E\right]\chi_m \\
&\quad\equiv \chi^\dagger_{\sigma'}\vec{\mathcal{H}}_N\chi_\sigma,
\end{aligned} \tag{23}$$

where $\vec{\sigma}$ are Pauli matrices and $\chi_{\sigma'}$ and χ_σ are Pauli spinors.

The generalized nucleon electric and magnetic form factors are defined by (see Ref. [7])

$$\begin{aligned}
\delta\mathcal{G}^N_E &= \Delta\widetilde{F}^N_1 - \tau\Delta\widetilde{F}^N_2 + \frac{\nu}{4m^2}\widetilde{F}^N_3, \\
\delta\mathcal{G}^N_M &= \Delta\widetilde{F}^N_1 + \Delta\widetilde{F}^N_2 + \frac{\epsilon\nu}{4m^2}\widetilde{F}^N_3.
\end{aligned} \tag{24}$$

Here $\tau = \frac{Q^2}{4m^2} \approx 4\eta$ and $\nu \approx mE_e$. After substitution of Eqs. (23) and (24) into Eq. (22) the matrix elements of the effective deuteron current between the initial and final deuteron states become

$$\mathcal{D}^0(\lambda',\lambda) = \begin{cases}
2E_d\left(\delta\mathcal{G}^{\rm I}_C - \frac{2}{3}\eta\delta\mathcal{G}^{\rm I}_Q\right), & \text{if}\quad \lambda=\lambda'=\pm1, \\
2E_d\left(\delta\mathcal{G}^{\rm I}_C + \frac{4}{3}\eta\delta\mathcal{G}^{\rm I}_Q\right), & \text{if}\quad \lambda=\lambda'=0, \\
-2i\frac{E_eE_d}{m} & \\
\times\sqrt{\eta}\cos\frac{\theta}{2}\langle\lambda'|J_2|\lambda\rangle\mathcal{F}_3, & \text{if}\quad \lambda'-\lambda=\pm1, \\
0, & \text{if}\quad \lambda'-\lambda=\pm2,
\end{cases}$$

$$\vec{\mathcal{D}}(\lambda',\lambda) = i\langle\lambda'|\vec{J}\times\vec{q}\,|\lambda\rangle\frac{E_d}{M}\left(\delta\mathcal{G}^{\rm I}_M - \frac{\epsilon E_e}{m}\mathcal{F}_3\right), \tag{25}$$

where $\vec{J} = (J_1, J_2, J_3)$ is an operator of the deuteron total angular momentum, and

$$\begin{aligned}
\delta\mathcal{G}^{\rm I}_C &= 2\delta\mathcal{G}^S_E\left[I^0_{00}(Q) + I^0_{22}(Q)\right], \\
\delta\mathcal{G}^{\rm I}_Q &= \frac{3\sqrt{2}}{\eta}\delta\mathcal{G}^S_E\left[I^2_{20}(Q) - \frac{1}{2\sqrt{2}}I^2_{22}(Q)\right], \\
\delta\mathcal{G}^{\rm I}_M &= \frac{M}{m}\left\{\frac{3}{2}\delta\mathcal{G}^S_E\left[I^0_{22}(Q) + I^2_{22}(Q)\right] + 2\delta\mathcal{G}^S_M\right. \\
&\quad\left.\times\left[I^0_{00}(Q) - \frac{1}{2}I^0_{22}(Q) + \sqrt{\frac{1}{2}}I^2_{20}(Q) + \frac{1}{2}I^2_{22}(Q)\right]\right\}, \\
\mathcal{F}_3 &= 2\frac{M}{m}\widetilde{F}^S_3\left[I^0_{00}(Q) - \frac{1}{2}I^0_{22}(Q) + \sqrt{\frac{1}{2}}I^2_{20}(Q) + \frac{1}{2}I^2_{22}(Q)\right].
\end{aligned} \tag{26}$$

[1]Here the additional multiplier $(2m)^{-1}$ appears because of the additional multiplier $2m$ in the effective current [see Eq. (23)] in comparison with the nucleon currents (3) and (4) of Ref. [16].

In these expressions the notation $I^L_{\ell'\ell}(Q) = \int_0^\infty dr\, j_L(\frac{1}{2}Qr)u_{\ell'}(r)u_\ell(r)$ was used, where $j_L(x)$ is a spherical Bessel function, $u_\ell(r)$ is the radial deuteron wave function for orbital momentum ℓ, and $\delta\mathcal{G}^S_E = \frac{1}{2}(\delta\mathcal{G}^p_E + \delta\mathcal{G}^n_E)$, etc. Contracting the effective deuteron current with the electron current j_μ one arrives at

$$g^{\rm I}_1 = -\epsilon\frac{E_e}{m}\mathcal{F}_3. \tag{27}$$

The corrections g_2 and g_3 are obviously vanishing for the first type of diagram, $g^{\rm I}_2 = g^{\rm I}_3 = 0$.

B. $\mathcal{M}^{\rm II}$ diagrams

In Ref. [12] the contribution of the $\mathcal{M}^{\rm II}$ diagram was estimated within a nonrelativistic approach with a Gaussian deuteron wave function. The present calculations are similar, but use a deuteron wave function extracted from a "realistic" NN potential; also, a modern parametrization for the nucleon form factors has been adopted.

The appropriate amplitude is given by the sum of two diagrams displayed at the bottom of Fig. 2, $\mathcal{M}^{\rm II} = \mathcal{M}^{\rm II}_{\rm P} + \mathcal{M}^{\rm II}_{\rm X}$, where

$$i\mathcal{M}^{\rm II}_{\rm P,X} = \int\frac{d^4p}{(2\pi)^4}\frac{d^4p'}{(2\pi)^4}\tilde{t}^{\rm P,X}_{\mu\nu}G(\Delta_1,\Delta_2)\frac{\mathcal{T}^{\mu\nu(\lambda'\lambda)}_{\rm P,X}}{D}. \tag{28}$$

Here $p = \frac{1}{2}(p^{(1)} - p^{(2)})$ and $p' = \frac{1}{2}(p'^{(1)} - p'^{(2)})$ are the relative momenta in the initial and final deuteron,

$$\begin{aligned}
\tilde{t}^{\rm P}_{\mu\nu} &= \frac{\bar{u}_h(k')(-ie\gamma_\mu)i(\not{l}+\mu)(-ie\gamma_\nu)u_h(k)}{l^2-\mu^2+i0}, \\
\tilde{t}^{\rm X}_{\mu\nu} &= \tilde{t}^{\rm P}_{\nu\mu}, \\
G(\Delta_1,\Delta_2) &= \frac{-i}{\Delta_1^2-\kappa^2+i0}\cdot\frac{-i}{\Delta_2^2-\kappa^2+i0}, \\
\mathcal{T}^{\mu\nu(\lambda'\lambda)}_{\rm P}\left(\Delta_1^2,\Delta_2^2\right) &= (ie)^2{\rm Tr}\left\{id^{(\lambda')}(p'^{(1)},p'^{(2)})\,(\not{p}'^{(1)}+m)\right. \\
&\quad\times\Gamma^\mu_1\left(\Delta_1^2\right)(\not{p}^{(1)}+m)id^{(\lambda)}(p^{(1)},p^{(2)}) \\
&\quad\left.\times(\not{p}^{(2)}-m)\bar{\Gamma}^\nu_2\left(\Delta_2^2\right)(\not{p}'^{(2)}-m)\right\}, \\
\mathcal{T}^{\mu\nu(\lambda'\lambda)}_{\rm X}\left(\Delta_1^2,\Delta_2^2\right) &= \mathcal{T}^{\mu\nu(\lambda'\lambda)}_{\rm P}\left(\Delta_2^2,\Delta_1^2\right), \\
D &= [(p'^{(1)})^2-m^2+i0][(p^{(1)})^2-m^2+i0] \\
&\quad\times[(p^{(2)})^2-m^2+i0][(p'^{(2)})^2-m^2+i0].
\end{aligned} \tag{29}$$

where P and X superscripts (subscripts) mean appropriate quantities related to diagrams with "parallel" photons (left bottom, Fig. 2) and "crossed" photons (right bottom, Fig. 2); $\not{A} \equiv A_\mu\gamma^\mu$. In Eq. (29) we use the following notations: l is the four-momentum of the intermediate electron, $\Delta_1 = k - l$ and $\Delta_2 = l - k'$ are the four-momenta of the virtual photons, and κ is an infinitesimal photon mass introduced in the photon propagators to regulate the infrared divergences; $\Gamma^\mu_1(\Delta_1^2)$ and $\bar{\Gamma}^\mu_2(\Delta_2^2)$ are electromagnetic currents for the nucleon and antinucleon, in which the form factors are functions of Δ_1^2 and Δ_2^2, respectively; and $d^{(\lambda)}(p^{(1)},p^{(2)})$ and $d^{(\lambda')}(p'^{(1)},p'^{(2)})$ are dpn vertex functions for the initial and final deuteron.

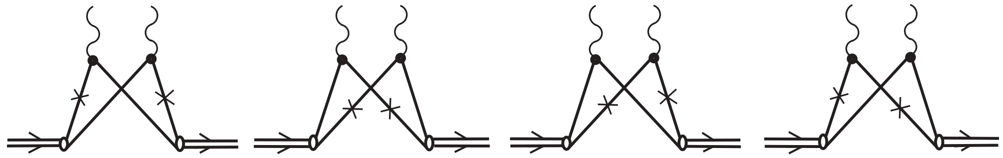

FIG. 3. Four types of poles taken into account in integration over dp_0 and dp'_0.

In expression (29) for $\mathcal{T}^{\mu\nu(\lambda'\lambda)}_{\rm P,X}$ moving along the nucleon loop (bold lines in the bottom diagrams in Fig. 2), a line with an arrow opposite to the motion corresponds to a fermion propagator and a line with an arrow along the motion corresponds to an antifermion propagator.

An infrared divergent term appears in $\mathcal{M}^{\rm II}$ when one photon is soft, $\Delta_1 \to 0$, $\Delta_2 \to q$ or $\Delta_1 \to q$, $\Delta_2 \to 0$. It is canceled by radiative corrections that are not of interest in this article. The configuration where each intermediate photon carries about half of the transferred momentum (hard-photon approximation) is emphasized,

$$\Delta_1 \sim \Delta_2 \sim \frac{q}{2}. \tag{30}$$

In this case there is no infrared divergent term.

To relate the dpn vertex to the deuteron wave function with one of the nucleons on the mass shell one must integrate over dp_0 and dp'_0. Four types of poles contribute to this integral (see Fig. 3). What follows is a discussion of the contribution coming from the poles of the first diagram of Fig. 3,

$$dT^{\mu\nu}_{\rm P,X} \equiv \frac{d^4p d^4p'}{(2\pi)^8}\frac{\mathcal{T}^{\mu\nu(\lambda'\lambda)}_{\rm P,X}}{D} \to -\frac{1}{4}\frac{d^3pd^3p'}{2E_1 2E'_2(2\pi)^6} \times \frac{\mathcal{T}^{\mu\nu(\lambda'\lambda)}_{\rm P,X}}{\left(p'^{(1)2}-m^2+i0\right)\left(p^{(2)2}-m^2+i0\right)}, \tag{31}$$

where $E_1 = \sqrt{m^2+(\vec{p}-\frac{1}{4}\vec{q}\,)^2}$ and $E'_2 = \sqrt{m^2+(\vec{p}\,'-\frac{1}{4}\vec{q}\,)^2}$.

One can use the expansion

$$\not{p}^{(1)}+m = \sum_{\sigma_1}|\vec{p}^{\,(1)}\sigma_1\rangle\langle\vec{p}^{\,(1)}\sigma_1|, \qquad \not{p}'^{(2)}-m = \sum_{\sigma'_2}|\vec{p}^{\,\prime(2)}\sigma'_2;c\rangle\langle\vec{p}^{\,\prime(2)}\sigma'_2;c| \tag{32}$$

(in the last equation c means charge conjugated spinor) and define the wave functions of the initial and final deuteron by

$$\phi^{(\lambda)}(p^{(1)},p^{(2)}) = \frac{d^{(\lambda)}(p^{(1)},p^{(2)})}{p^{(2)2}-m^2+i0}, \qquad \phi^{(\lambda')}(p'^{(1)},p'^{(2)}) = \frac{d^{(\lambda')}(p'^{(1)},p'^{(2)})}{p'^{(1)2}-m^2+i0}. \tag{33}$$

These wave functions are normalized by the condition

$$\int\frac{d^3p}{2E_1(2\pi)^3}{\rm Tr}\,\phi^{\dagger(\lambda)}(p^{(1)},p^{(2)})\phi^{(\lambda)}(p^{(1)},p^{(2)}) = 1 \tag{34}$$

[and similarly for $\phi^{(\lambda')}(p'^{(1)},p'^{(2)})$], which comes from the requirement $G_C(0)=1$.

Note that in general the nucleons $N'^{(1)}$ and $N^{(2)}$ are not on-shell and at this step one cannot use expansions similar to Eq. (32) for $\not{p}'^{(1)}+m$ and $\not{p}^{(2)}-m$. Nevertheless we assume that the relative momenta in the initial and final deuteron are restricted by

$$|\vec{p}\,| \sim |\vec{p}\,'| \ll Q. \tag{35}$$

This means that in all expansions one must keep terms linear in $\vec{p}$ and $\vec{p}\,'$ only and

$$E_{1,2} \approx \frac{1}{2}E_d \pm \frac{(\vec{d}\cdot\vec{p}\,)}{E_d}, \quad E'_{1,2} \approx \frac{1}{2}E_d \pm \frac{(\vec{d}\,'\cdot\vec{p}\,')}{E_d}, \tag{36}$$

$$\vec{p}^{\,(1,2)} = -\frac{1}{4}\vec{q} \pm \vec{p}, \quad \vec{p}^{\,\prime(1,2)} = \frac{1}{4}\vec{q} \pm \vec{p}\,', \tag{37}$$

$$\Delta_{1,2} = \frac{1}{2}q \pm \delta, \quad \delta = \left(-\frac{Q}{2E_d}(p'_3+p_3),\ \vec{p}\,'-\vec{p}\right). \tag{38}$$

One sees that in the framework of our approximation all nucleons become on-shell and one can use an expansion similar to Eq. (32) for $\not{p}'^{(1)}+m$ and $\not{p}^{(2)}-m$. As a result all diagrams in Fig. 3 give the same contribution and

$$dT^{\mu\nu}_{\rm P} \approx -\frac{e^2d^3pd^3p'}{(2\pi)^6 2E_1 2E'_2}\sum_{\sigma_1,\sigma_2,\sigma'_1,\sigma'_2}\langle\vec{p}^{\,\prime(2)}\sigma'_2;c|\phi^{\lambda'}|\vec{p}^{\,\prime(1)}\sigma'_1\rangle \times\langle\vec{p}^{\,\prime(1)}\sigma'_1|\Gamma^{\mu}_1(\Delta_1^2)|\vec{p}^{\,(1)}\sigma_1\rangle\langle\vec{p}^{\,(1)}\sigma_1|\phi^{\lambda}|\vec{p}^{\,(2)}\sigma_2;c\rangle \times\langle\vec{p}^{\,(2)}\sigma_2;c|\bar{\Gamma}^{\mu}_2(\Delta_2^2)|\vec{p}^{\,\prime(2)}\sigma'_2;c\rangle. \tag{39}$$

$dT^{\mu\nu}_{\rm X}$ is obtained by the exchange $\Delta_1^2 \leftrightarrow \Delta_2^2$ in Eq. (39). Using the fact that $\langle\vec{p}^{\,(1)}\sigma_1|\phi^{\lambda}|\vec{p}^{\,(2)}\sigma_2;c\rangle$ and $\langle\vec{p}^{\,\prime(2)}\sigma'_2;c|\phi^{\lambda'}|\vec{p}^{\,\prime(1)}\sigma'_1\rangle$ are Lorentz invariants, one can substitute the nonrelativistic deuteron wave functions (17) instead of these wave functions:

$$\frac{1}{\sqrt{E_d}}\langle\vec{p}^{\,(1)}\sigma_1|\phi^{\lambda}|\vec{p}^{\,(2)}\sigma_2;c\rangle \to (2\pi)^{3/2}\Psi_{\sigma_1\sigma_2}(\lambda,\vec{\tilde{p}}), \qquad \frac{1}{\sqrt{E_d}}\langle\vec{p}^{\,\prime(2)}\sigma'_2;c|\phi^{\lambda'}|\vec{p}^{\,\prime(1)}\sigma'_1\rangle \to (2\pi)^{3/2}\Psi^{\dagger}_{\sigma'_1\sigma'_2}(\lambda,\vec{\tilde{p}}\,'). \tag{40}$$

This substitution must be completed by the transformation of the current $\bar{\Gamma}^{\nu}_2 \to \Gamma^{\nu}_2$. In Eq. (40)

$$\vec{\tilde{p}} = \left(p_1,\, p_2,\, \frac{M}{E_d}p_3\right) = \left(\vec{p}_\perp,\, \frac{M}{E_d}p_3\right) \quad \text{and} \quad \vec{\tilde{p}}\,' = \left(p_1{}',\, p_2{}',\, \frac{M}{E_d}p_3{}'\right) = \left(\vec{p}\,'_\perp,\, \frac{M}{E_d}p_3{}'\right) \tag{41}$$

are the internal momenta in the deuteron rest frame.

Expanding the current matrix elements $\langle\vec{p}^{\,\prime(i)}\sigma'|\Gamma^{\mu}_i(\Delta_i^2)|\vec{p}^{\,(i)}\sigma\rangle = 2m\chi^{\dagger}_{\sigma'}\widetilde{\Gamma}^{\mu}_i(\Delta_i^2)\chi_\sigma$ in terms of

Pauli spinors one gets

$$\mathcal{M}^{\rm II} = -\frac{64\alpha^2(4\pi)^2 E_d}{Q^6}\int \frac{d^3\tilde{p}\, d^3\tilde{p}'}{(2\pi)^3} \times \Psi^\dagger(\lambda',\vec{\tilde{p}}')\left[\tau_h^{\mu\nu}\tilde{\Gamma}_{1\mu}\left(\frac{1}{4}Q^2\right)\tilde{\Gamma}_{2\nu}\left(\frac{1}{4}Q^2\right)+\vec{\tilde{p}}\vec{A} + \vec{\tilde{p}}'\vec{B} + \mathcal{O}(\tilde{p}^2,\tilde{p}'^2)\right]\Psi(\lambda,\vec{\tilde{p}}), \quad (42)$$

where $\vec{A}$ and $\vec{B}$ are some vectors and

$$\tau_h^{\mu\nu} = \bar{u}_h(k')[\gamma^\mu(\not{k}-\tfrac{1}{2}\not{q})\gamma^\nu + \gamma^\nu(\not{k}'+\tfrac{1}{2}\not{q})\gamma^\mu]u_h(k) = j^\nu(k+k')^\mu + j^\mu(k+k')^\nu. \quad (43)$$

The integrals $\int d^3\tilde{p}\;\vec{\tilde{p}}\Psi(\lambda,\vec{\tilde{p}})$ and $\int d^3\tilde{p}'\;\vec{\tilde{p}}'\Psi(\lambda',\vec{\tilde{p}}')$ obviously vanish after angular integration and one arrives at

$$\mathcal{M}^{\rm II} \approx -\frac{64\alpha^2(4\pi)^2 E_d}{Q^6}\,\tau_h^{\mu\nu}\mathfrak{M}_{\mu\nu}^{\lambda'\lambda}, \quad (44)$$

where $\mathfrak{M}_{\mu\nu}^{\lambda'\lambda} = \psi^*_{\lambda'}(0)\tilde{\Gamma}_{1\mu}(\frac{1}{4}Q^2)\tilde{\Gamma}_{2\nu}(\frac{1}{4}Q^2)\psi_\lambda(0)$; $\psi_{\lambda'}(0)$, $\psi_\lambda(0)$, and $\psi_{\lambda'}(0)$ are the deuteron wave functions in coordinate space at $\vec{r}=0$; and

$$\tilde{\Gamma}_k^0\left(\frac{1}{4}Q^2\right) = G_E^k\left(\frac{1}{4}Q^2\right),\qquad \vec{\tilde{\Gamma}}_k\left(\frac{1}{4}Q^2\right) = \frac{i}{2M}(\vec{\sigma}\times\vec{q}\,)G_M^k\left(\frac{1}{4}Q^2\right) \quad (45)$$

(suffix $k=1,2$ enumerates the nucleons).

For further calculations it is useful to introduce "plus" and "minus" components of the tensors according to $A_\pm = \sqrt{\frac{1}{2}}(A_1 \pm iA_2)$. The contraction of the lepton and deuteron tensors becomes

$$\tau^{\mu\nu}\mathfrak{M}_{\mu\nu}^{\lambda'\lambda} = \mathfrak{M}_{00}^{\lambda'\lambda}\tau_{00} - 2\big(\mathfrak{M}_{0+}^{\lambda'\lambda}\tau_{0-} + \mathfrak{M}_{0-}^{\lambda'\lambda}\tau_{0+}\big) + \big(\mathfrak{M}_{++}^{\lambda'\lambda}\tau_{--} + 2\mathfrak{M}_{+-}^{\lambda'\lambda}\tau_{-+} + \mathfrak{M}_{--}^{\lambda'\lambda}\tau_{++}\big), \quad (46)$$

where

$$\begin{aligned}
\tau_{00} &= 8E_e^2\cos\frac{\theta}{2},\\
\tau_{0+} &= -2\sqrt{2}E_e^2\left(2-\sin^2\frac{\theta}{2} - h\sin\frac{\theta}{2}\right),\\
\tau_{0-} &= -2\sqrt{2}E_e^2\left(2-\sin^2\frac{\theta}{2} + h\sin\frac{\theta}{2}\right),\\
\tau_{++} &= 4E_e^2\cos\frac{\theta}{2}\left(1 - h\sin\frac{\theta}{2}\right),\\
\tau_{--} &= 4E_e^2\cos\frac{\theta}{2}\left(1 + h\sin\frac{\theta}{2}\right),\quad \tau_{-+} = 4E_e^2\cos\frac{\theta}{2}
\end{aligned} \quad (47)$$

and

$$\begin{aligned}
\mathfrak{M}_{00}^{11} &= \mathfrak{M}_{00}^{-1-1} = \mathfrak{M}_{00}^{00} = \frac{\mathcal{C}}{4\pi}G_{EE},\\
\mathfrak{M}_{0+}^{10} &= \mathfrak{M}_{0+}^{0-1} = -\mathfrak{M}_{0-}^{01} = -\mathfrak{M}_{0-}^{-10}\\
&= -\frac{\mathcal{C}}{4\pi}\sqrt{\eta}\,G_{EM}, \quad (48)\\
\mathfrak{M}_{++}^{1-1} &= \mathfrak{M}_{--}^{-11} = \frac{\mathcal{C}}{2\pi}\eta\,G_{MM},\quad \mathfrak{M}_{\pm\mp}^{00} = -\frac{\mathcal{C}}{4\pi}\eta\,G_{MM},
\end{aligned}$$

with the abbreviations

$$\begin{aligned}
\mathcal{C} &= [u_0'(r)]^2|_{r=0},\quad G_{EE} = G_E^p\big(\tfrac{1}{4}Q^2\big)G_E^n\big(\tfrac{1}{4}Q^2\big),\\
G_{MM} &= G_M^p\big(\tfrac{1}{4}Q^2\big)G_M^n\big(\tfrac{1}{4}Q^2\big),\\
G_{EM} &= \tfrac{1}{2}\big[G_E^p\big(\tfrac{1}{4}Q^2\big)G_M^n\big(\tfrac{1}{4}Q^2\big) + G_M^p\big(\tfrac{1}{4}Q^2\big)G_E^n\big(\tfrac{1}{4}Q^2\big)\big].
\end{aligned}$$

Finally we get the following amplitudes,

$$\begin{aligned}
T_{11}^{\rm II} &= \varkappa\cos\frac{\theta}{2}G_{EE},\quad T_{00}^{\rm II} = \varkappa\cos\frac{\theta}{2}(G_{EE}-\eta G_{MM}),\\
T_{10,h}^{\rm II} &= -\varkappa\sqrt{\frac{\eta}{2}}G_{EM}\left(2-\sin^2\frac{\theta}{2}+h\sin\frac{\theta}{2}\right), \quad (49)\\
T_{1-1,h}^{\rm II} &= \varkappa\eta G_{MM}\cos\frac{\theta}{2}\left(1+h\sin\frac{\theta}{2}\right),
\end{aligned}$$

where

$$\varkappa = -\frac{128\alpha\mathcal{C}E_e}{Q^4}, \quad (50)$$

and one arrives at

$$\begin{aligned}
\delta\mathcal{G}_C^{\rm II} &= \varkappa\left(G_{EE} - \frac{1}{3}\eta G_{MM}\right),\quad \delta\mathcal{G}_Q^{\rm II} = -\frac{\varkappa}{2}G_{MM},\\
\delta\mathcal{G}_M^{\rm II} &= \frac{2\varkappa G_{EM}}{1+\sin^2\frac{\theta}{2}},\quad g_1^{\rm II} = \frac{\varkappa G_{EM}\cos^2\frac{\theta}{2}}{1+\sin^2\frac{\theta}{2}}, \quad (51)\\
g_2^{\rm II} &= g_3^{\rm II} = \varkappa\eta\cos\frac{\theta}{2}G_{MM}.
\end{aligned}$$

One should note that Eq. (42) is not valid at $\theta\to 0$ (or, equivalently, $\epsilon\to 1$). Indeed, the denominator of the electron propagator

$$\frac{1}{4}Q^2 + \frac{E_eQ}{M}(\tilde{p}_z + \tilde{p}'_z) - 2\cos\frac{\theta}{2}E_e(\tilde{p}_x - \tilde{p}'_x)Q^2$$

contains products of E_e and components of the internal momenta. From Eq. (2) it follows that $E_e\to\infty$ when $\theta\to 0$ and the factor $\mathcal{C}$ should be changed to

$$\mathcal{C}\to\mathcal{S} = \frac{1}{(2\pi)^3} \times \int\frac{d^3\tilde{p}d^3\tilde{p}'U_0(\tilde{p})U_0(\tilde{p}')}{1+\frac{4E_e}{QM}(\tilde{p}_z+\tilde{p}'_z) - 8\cos\frac{\theta}{2}\frac{E_e(\tilde{p}_x-\tilde{p}'_x)}{Q^2}+i0}. \quad (52)$$

The amplitudes in Eq. (49) with the substitution of Eq. (52) in Eq. (50) coincide with the results of Ref. [12].

To evaluate the integral in Eq. (52) one can use the integral representation for the denominator

$$\frac{1}{\alpha+i0} = -i\int_0^\infty d\tau e^{i(\alpha+i0)\tau} \quad (53)$$

and reduce Eq. (52) to the one-dimensional integral

$$\mathcal{S} = -\frac{i}{4}Q^2\int_0^\infty\frac{d\tau}{y^2}e^{\frac{i}{4}Q^2\tau}u_0^2(y), \quad (54)$$

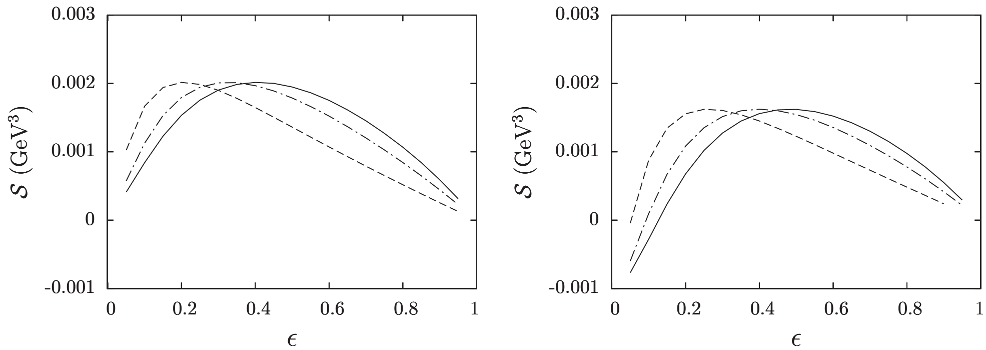

FIG. 4. $\mathcal{S}$ factor calculated for the CD-Bonn potential [17] (left) and the Paris potential [18] (right). Dashed, dot-dashed and solid curves are for Q^2 =1, 2, and 3 GeV^2, respectively.

where $y = \tau E_e\sqrt{4\cos^2\frac{\theta}{2} + \frac{Q^2}{M^2}}$. By changing the variable in Eq. (54) one gets

$$\mathcal{S} = -if\int_0^\infty \frac{dy}{y^2}e^{ify}u_0^2(y), \tag{55}$$

where

$$f = \frac{Q^2}{4E_e\sqrt{4\cos^2\frac{\theta}{2} + \frac{Q^2}{M^2}}}. \tag{56}$$

For the standard parametrization of the wave function

$$u_0(y) = \sum_n c_n e^{-\alpha_n y}, \quad \text{with} \quad \sum_n c_n = 0, \tag{57}$$

we obtain (Appendix C)

$$\mathcal{S} = -if\sum_n\sum_m c_n c_m(\alpha_n + \alpha_m - if)\ln(\alpha_n + \alpha_m - if). \tag{58}$$

In Eq. (55) the exponent reduces to 1 in the limit $E_e \to \infty$ for fixed Q and

$$\Re\mathrm{e}T^{\mathrm{II}}_{\lambda'\lambda} \sim \sin\frac{\theta}{2}, \quad \Im\mathrm{m}T^{\mathrm{II}}_{\lambda'\lambda} \to \mathrm{const}; \tag{59}$$

that is, at the limit $\theta \to 0$ the TPE does not contribute to the cross section in the next order of the α expansion.

The ϵ and Q^2 dependence of $\mathcal{S}$ is displayed in Fig. 4. One sees that the $\mathcal{S}$ factor depends strongly on the NN potential

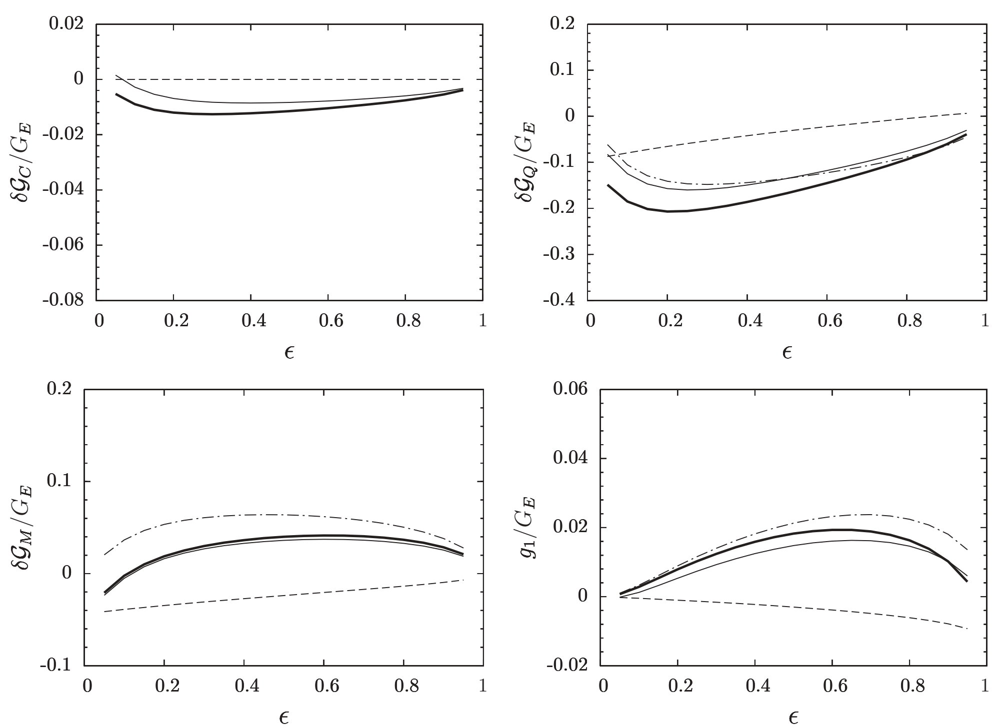

FIG. 5. Two-photon-exchange corrections $\delta\mathcal{G}_C/G_E$, $\delta\mathcal{G}_Q/G_E$, $\delta\mathcal{G}_M/G_E$, and g_1/G_E at $Q^2 = 1$ GeV^2. Dashed, dot-dashed, and solid (bold) curves are for $\mathcal{M}^{\mathrm{I}}$, $\mathcal{M}^{\mathrm{II}}$, and $\mathcal{M}^{\mathrm{I}} + \mathcal{M}^{\mathrm{II}}$, respectively, calculated with the CD-Bonn potential. The solid (thin) curves depict $\mathcal{M}^{\mathrm{I}} + \mathcal{M}^{\mathrm{II}}$ calculated with the Paris potential.

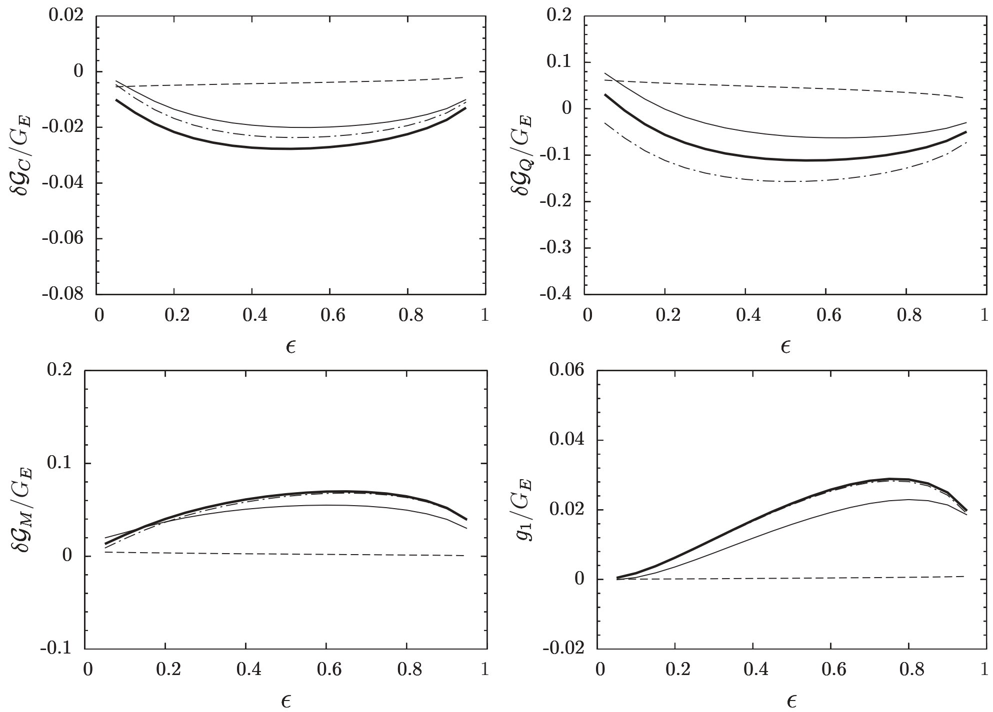

FIG. 6. Same as Fig. 5 for $Q^2 = 2$ GeV^2.

and in any case it is very different from the constant value $\mathcal{C} = [u_0'(0)]^2$. The reason is as follows: from Eq. (58) one gets that $\mathcal{S} \to \mathcal{C}$ at the formal limit

$$f \gg \alpha_n + \alpha_m. \tag{60}$$

But from expression (56) it follows that $f \to \frac{1}{2} M \sin\frac{\theta}{2}$ when $Q^2 \to \infty$ and the condition (60) cannot be fulfilled at any Q^2. Note that a similar situation takes place in the evaluation of the so-called triangle diagram in pd backward scattering [19].

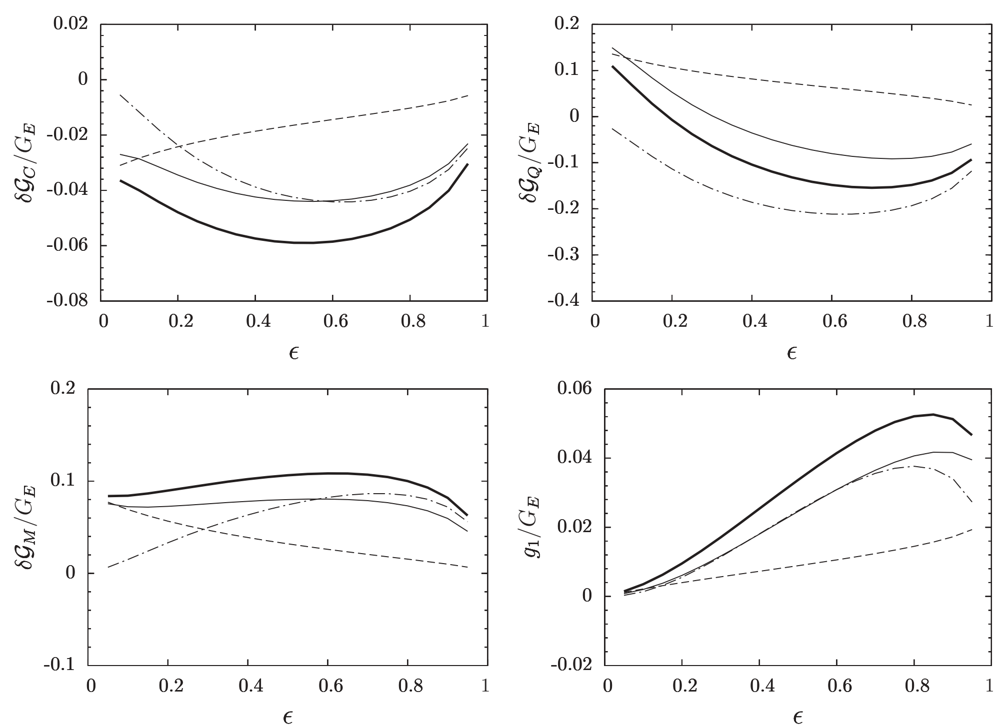

FIG. 7. Same as Fig. 5 for $Q^2 = 3$ GeV^2.

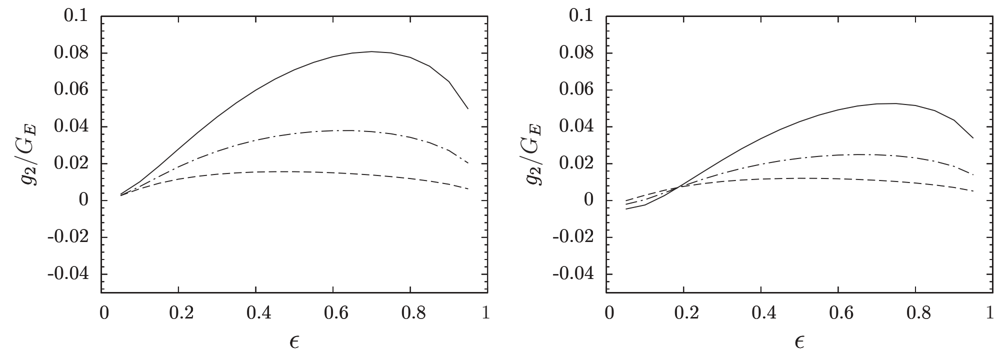

FIG. 8. Ratio of $g_2 = g_3$ to G_E. Dashed, dot-dashed and solid curves are for $Q^2 = 1$, 2, and 3 GeV^2, respectively. Left and right panels are for the CD-Bonn and Paris potentials, respectively.

We have also studied the deuteron D-wave contribution to the $\mathcal{S}$ factor and found that it contributes less than 10%.

IV. NUMERICAL RESULTS AND DISCUSSION

Figures 5–8 display the ϵ behavior of the TPE corrections $\delta\mathcal{G}_C/G_E$, $\delta\mathcal{G}_Q/G_E$, $\delta\mathcal{G}_M/G_E$, and $g_{1,2}/G_E$ calculated with the deuteron wave function for the CD-Bonn and the Paris potentials. The form factor $G_E(Q^2)$ was calculated in the framework of the impulse approximation.

In the present calculations of the TPE correction in $\mathcal{M}^{\rm I}$ the amplitudes $\Delta\widetilde{F}^N_{1,2}$ and $\widetilde{F}_3$ from the theoretical calculations of Ref. [6] are used. At $Q^2 < 6$ GeV^2 they are practically independent of the parametrization of the nucleon form factor. For $\mathcal{M}^{\rm II}$ we use the following parametrization of the nucleon form factor:

(i) For the magnetic form factors of the proton and neutron, we use the dipole parametrizations

$$G^p_M(Q^2) = \mu_p G_D(Q^2), \quad G^n_M(Q^2) = \mu_n G_D(Q^2), \tag{61}$$

where $G_D(Q^2) = (1 + Q^2/0.71)^{-2}$.

(ii) The electric form factors of the proton and neutron were taken from the parametrization of the JLab data (see [21]),

$$G^p_E(Q^2) = (1.0587 - 0.14265Q^2)G_D(Q^2), \tag{62}$$

and so-called Galster parametrization [22], respectively.

One sees that two-photon exchange may give a large contribution to the elastic ed scattering, although caution is required because these estimates have large uncertainties. The most important source of uncertainty comes from the $\mathcal{S}$ factor, which is determined by the short-range part of the deuteron wave function. The last quantity is very poorly known (see, e.g., Table I). Of course, besides NN degrees of freedom, non-nucleon (quark) degrees of freedom should also be taken into account in this region and one may expect that in the framework of more realistic estimates the two-photon corrections may be smaller. The implication is that the experimental study of two-photon exchange in elastic ed scattering at $Q^2 \sim$ few GeV^2 can give important information about the deuteron structure at short distances.

TABLE I. $u'_0(0)$ for some popular potentials.

$u'(0)$, fm$^{-3/2}$	Potential	Ref.
1.1978×10^{-1}	Paris	[18]
3.1035×10^{-1}	CD-Bonn	[17]
2.6860×10^{-1}	Nijm I	[20]
2.6730×10^{-2}	Nijm II	[20]
3.1571×10^{-1}	Nijm 93	[20]
5.8334×10^{-2}	Reid 93	[20]

In summary, we estimated the two-photon-exchange amplitude in elastic ed scattering. There are six independent form factors that determine this amplitude, but only three of them contribute to the cross section in second-order perturbation theory.

There are two types of two-photon-exchange diagrams. For the first type two intermediate photons interact with the same nucleon. For the second type the intermediate photons interact with different nucleons.

We show that the two-photon-exchange amplitude is strongly connected with the deuteron structure at short distances.

ACKNOWLEDGMENTS

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APPENDIX A: GENERAL EXPRESSION FOR THE $ed \to ed$ AMPLITUDE IN THE BREIT FRAME

From the invariance under Lorentz transformations and space and time inversions it follows that the amplitude of the elastic scattering of a spin-$\frac{1}{2}$ particle (the electron) off a spin-1 particle (the deuteron) has nine invariant amplitudes (form factors). Usually for ultrarelativistic electrons the mass can be neglected and the electron helicity is conserved. In this case the number of form factors is reduced to six and the amplitude has the general form [14,15]

$$\mathcal{M} = -\frac{4\pi\alpha}{q^2} j^\mu J_\mu \equiv \frac{4\pi\alpha}{Q^2} T_{\lambda'\lambda,h}, \tag{A1}$$

where j_μ is the electromagnetic current for the electron and J^μ is an "effective current" for the deuteron

$$\begin{aligned} J_\mu = -\Big\{ & G_1(\epsilon'^*\epsilon)(d+d')_\mu + G_2[(\epsilon'^*q)\epsilon_\mu - \epsilon'^*_\mu(\epsilon q)] \\ & - G_3\frac{(\epsilon'^*q)(\epsilon q)}{2M^2}(d+d')_\mu + G_4\frac{(\epsilon'^*K)(\epsilon K)}{2M^2}(d+d')_\mu \\ & + G_5[(\epsilon'^*K)\epsilon_\mu + \epsilon'^*_\mu(\epsilon K)] \\ & + G_6\frac{(\epsilon'^*K)(\epsilon q)-(\epsilon'^*q)(\epsilon K)}{2M^2}(d+d')_\mu\Big\}. \end{aligned} \tag{A2}$$

Here $K = k + k'$. The form factors $G_1, \ldots, G_6$ are complex functions of two variables, for example, Q^2 and θ.

In the Breit frame one simply finds

$$\begin{aligned} \epsilon_{(\pm)}q &= \epsilon'_{(\pm)}q = 0, \quad \epsilon_{(0)}q = \epsilon'_{(0)}q = -\frac{E_d Q}{M}, \\ \epsilon'^*_{(\pm)}\epsilon_{(\pm)} &= -1, \quad \epsilon'^*_{(\mp)}\epsilon_{(\pm)} = 0, \quad \epsilon_{(\pm)}\epsilon_{(0)} = \epsilon_{(\pm)}\epsilon'_{(0)} = 0, \\ \epsilon'^*_{(0)}\epsilon_{(0)} &= -\left(1+\frac{Q^2}{2M^2}\right), \\ \epsilon_{(\pm)}K &= \epsilon'_{(\pm)}K = \pm\sqrt{2}E_e\cos\frac{\theta}{2}, \\ \epsilon_{(0)}K &= -\epsilon'_{(0)}K = -\frac{E_e Q}{M} \end{aligned} \tag{A3}$$

and

$$\begin{aligned} \epsilon_{(0)}j &= -\frac{E_e Q}{M}\cos\frac{\theta}{2}, \quad \epsilon'_{(0)}j = \frac{E_e Q}{M}\cos\frac{\theta}{2}, \\ \epsilon_{(\pm)}j &= \sqrt{2E_e}\left(\pm 1 - h\sin\frac{\theta}{2}\right), \\ \epsilon'_{(\pm)}j &= \sqrt{2E_e}\left(\pm 1 + h\sin\frac{\theta}{2}\right), \\ (d+d')j &= 4E_e E_d\cos\frac{\theta}{2}. \end{aligned} \tag{A4}$$

From Eqs. (A1)–(A4) it follows that

$$\begin{aligned} T_{11,h} &= T_{-1-1,h} \\ &= \left[G_1 - \left(\frac{E_e}{M}\right)^2\cos^2\frac{\theta}{2}G_4 - \frac{E_e}{E_d}G_5\right]\cos\frac{\theta}{2}, \\ T_{00,h} &= \Bigg[(1+2\eta)G_1 - 2\eta G_2 + 2(1+\eta)\eta G_3 \\ &\quad - 2\left(\frac{E_e}{M}\right)^2\eta G_4 - 4\frac{E_e E_d}{M^2}G_6\Bigg]\cos\frac{\theta}{2}, \\ T_{10,h} &= -T_{01,-h} = T_{0-1,h} = -T_{-10,-h} \\ &= \sqrt{2}\frac{Q}{4E_d M}\Bigg[-E_d G_2 + 2\frac{E_d E_e^2}{M^2}\cos^2\frac{\theta}{2}G_4 \\ &\quad + E_e\left(1+\cos\frac{\theta}{2}\right)G_5 + 2\frac{E_d^2 E_e}{M^2}\cos\frac{\theta}{2}G_6 \\ &\quad + h\sin\frac{\theta}{2}(-E_d G_2 + E_e G_5)\Bigg], \\ T_{1-1,h} &= T_{-11,-h} \\ &= \frac{E_e}{E_d}\cos\frac{\theta}{2}\left[\frac{E_e E_d}{M^2}\cos\frac{\theta}{2}G_4 + \left(1+h\sin\frac{\theta}{2}\right)G_5\right]. \end{aligned} \tag{A5}$$

In the Born approximation the form factors G_4, G_5, and G_6 vanish and the form factors G_1, G_2, and G_3 become the real functions $G_1^{(0)}$, $G_2^{(0)}$, and $G_3^{(0)}$ of one variable Q^2; that is,

$$\begin{aligned} G_1 &= G_1^{(0)}(Q^2) + \mathcal{O}(\alpha), \quad G_2 = G_2^{(0)}(Q^2) + \mathcal{O}(\alpha), \\ G_3 &= G_3^{(0)}(Q^2) + \mathcal{O}(\alpha), \quad G_4 \sim G_5 \sim G_6 \sim \alpha. \end{aligned} \tag{A6}$$

Commonly the charge, magnetic, and quadrupole form factors, $G_C(Q^2)$, $G_M(Q^2)$, and $G_Q(Q^2)$ are used instead of the form factors $G_1^{(0)}$, $G_2^{(0)}$, and $G_3^{(0)}$. They are connected by

$$\begin{aligned} G_1^{(0)}(Q^2) &= G_C(Q^2) - \frac{2}{3}\eta G_Q(Q^2), \\ G_2^{(0)}(Q^2) &= G_M(Q^2), \\ G_3^{(0)}(Q^2) &= \frac{1}{1+\eta}\Bigg[-G_C(Q^2) + G_M(Q^2) \\ &\quad + \left(1+\frac{2}{3}\eta\right)G_Q(Q^2)\Bigg], \end{aligned} \tag{A7}$$

and Eqs. (A5) are reduced to

$$\begin{aligned} T^{(0)}_{11,h} &= T^{(0)}_{-1-1,h} = \left[G_C(Q^2) - \frac{2}{3}\eta G_Q(Q^2)\right]\cos\frac{\theta}{2}, \\ T^{(0)}_{00,h} &= \left[G_C(Q^2) + \frac{4}{3}\eta G_Q(Q^2)\right]\cos\frac{\theta}{2}, \\ T^{(0)}_{10,h} &= -T^{(0)}_{01,-h} = T^{(0)}_{0-1,h} = -T^{(0)}_{-10,-h} \\ &= -\sqrt{\frac{\eta}{2}}G_M(Q^2)\left(1+h\sin\frac{\theta}{2}\right), \\ T^{(0)}_{1-1,h} &= T^{(0)}_{-11,-h} = 0. \end{aligned} \tag{A8}$$

APPENDIX B: NONREALATIVISTIC REDUCTION OF THE EFFECTIVE HADRON CURRENT

In the Breit frame $K = E_e(1, \cos\frac{\theta}{2}, 0, 0)$ and

$$\begin{aligned} &\langle \vec{p}\,'^{(N)}\sigma'|H_N^0|\vec{p}^{\,(N)}\sigma\rangle \\ &\quad \approx \chi^\dagger_{\sigma'}\Bigg\{2m\Delta\widetilde{F}_{1N} + \frac{Q}{2m}(-Q + 2i\epsilon^{3nm}p^n\sigma^m)\Delta\widetilde{F}_{2N} \\ &\quad + \frac{E_e}{m}\left[2m - \cos\frac{\theta}{2}(2p^1 + iQ\sigma^2)\right]\widetilde{F}_{3N}\Bigg\}\chi_\sigma, \end{aligned} \tag{B1}$$

$$\begin{aligned} &\langle \vec{p}\,'^{(N)}\sigma'|H_N^a|\vec{p}^{\,(N)}\sigma\rangle \\ &\quad \approx 2\chi^\dagger_{\sigma'}[(p^a - iQ\epsilon^{a3n}\sigma^n)\Delta\widetilde{F}_{1N} + iQ\epsilon^{3an}\sigma^n\Delta\widetilde{F}_{2N}]\chi_\sigma, \end{aligned} \tag{B2}$$

$$\langle \vec{p}\,'^{(N)}\sigma'|H_N^3|\vec{p}\,^{(N)}\sigma\rangle \approx (p^{(N)}+p'^{(N)})^3\Delta\widetilde{F}_{1N}\chi_{\sigma'}^\dagger\chi_\sigma, \tag{B3}$$

where $a = 1, 2$.

Note that up to terms of order $\mathcal{O}(\frac{p^1}{m}) \sim \frac{50\,\mathrm{MeV}/c}{m}$, the amplitudes $\Delta\widetilde{F}_{1N}$, $\Delta\widetilde{F}_{2N}$, and $\widetilde{F}_{3N}$ are independent on the nucleon momenta. In this approximation the terms proportional to $p_\perp$ will vanish after integration in Eq. (22) and Eqs. (B1) and (B3) become

$$\begin{aligned}\langle \vec{p}\,'^{(N)}\sigma'|H_N^0|\vec{p}\,^{(N)}\sigma\rangle &\approx \chi_{\sigma'}^\dagger\left[2m\Delta\widetilde{F}_{1N} - \frac{Q^2}{2m}\Delta\widetilde{F}_{2N} + \frac{E_e}{m}\left(2m - iQ\cos\frac{\theta}{2}\sigma^2\right)\widetilde{F}_{3N}\right]\chi_\sigma \\ &= \chi_{\sigma'}^\dagger\left(2m\Delta\mathcal{G}_E - i\frac{E_eQ}{m}\cos\frac{\theta}{2}\sigma^2\widetilde{F}_{3N}\right)\chi_\sigma,\end{aligned} \tag{B4}$$

$$\langle \vec{p}\,'^{(N)}\sigma'|H_N^a|\vec{p}\,^{(N)}\sigma\rangle \approx 2iQ\epsilon^{3an}(\Delta\widetilde{F}_{1N}+\Delta\widetilde{F}_{2N})\chi_{\sigma'}^\dagger\sigma^n\chi_\sigma. \tag{B5}$$

APPENDIX C

Let us consider the integral

$$I = \int_0^\infty \frac{dy}{y^2}\, e^{ify}[u_0(y)]^2, \tag{C1}$$

where f is a real constant. With Eq. (57) for $u_0(y)$ the integral becomes a series,

$$I = \sum_n\sum_m c_nc_m\int_0^\infty \frac{dy}{y^2}\, e^{(if-\alpha_n-\alpha_m)y}, \tag{C2}$$

with all terms divergent. We regularize the expressions by defining

$$I = \lim_{\varepsilon\to 0} I_\varepsilon, \tag{C3}$$

where

$$\begin{aligned}I_\varepsilon &= \int_0^\infty \frac{dy}{y^{2-\varepsilon}}\, e^{ify}[u_0(y)]^2 \\ &= \sum_n\sum_m c_nc_m(\alpha_n+\alpha_m-if)^{1-\varepsilon}\Gamma(-1+\varepsilon).\end{aligned} \tag{C4}$$

By expanding the Γ function near the pole

$$\Gamma(-1+\varepsilon) = -\frac{1}{\varepsilon}+\gamma-1-\mathcal{O}(\varepsilon) \tag{C5}$$

and taking into account the constraint $\sum_n c_n = 0$, one gets

$$\begin{aligned}I &= \lim_{\varepsilon\to 0}\left[\sum_n\sum_m c_nc_m(\alpha_n+\alpha_m-if) \times \ln(\alpha_n+\alpha_m-if)+\mathcal{O}(\varepsilon)\right] \\ &= \sum_n\sum_m c_nc_m(\alpha_n+\alpha_m-if)\ln(\alpha_n+\alpha_m-if).\end{aligned} \tag{C6}$$

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