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# Time modulation of $K$-electron capture decay of hydrogen-like ions with multiphoton resonance transitions

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Multiphoton resonance transitions between ground hyperfine states are used for time modulation of the electron capture decay of hydrogen-like ions with the Gamow-Teller transition $1^+ \to 0^+$. The proposed mechanism offers a time oscillating decay with a frequency of up to 0.1 Hz. An experiment to observe the modulation is proposed for ions stored in a Penning trap. An attempt to understand the Gesellschaft für Schwerionenforschung mbH (GSI) anomaly with multiple photon transitions is made.

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Long-standing attempts have been made to change the decay of radioactive nuclei artificially using different external factors. It was suggested by Segré [1] that orbital electron capture (EC) decay rates depend on the density of atomic electrons within a nucleus. Thus, environmental conditions may alter electron densities and affect EC decay rates. However, this effect is small and does not exceed the fractions of percentage point.

The introduction of Penning traps and storage rings into nuclear physics has allowed to study radioactive hydrogen-like (H-like) ions. Recently a single-ion spectroscopy technique with a time resolution of <1 s has been developed at Gesellschaft für Schwerionenforschung mbH (GSI) Darmstadt. With this elegant method, the oscillatory modulation of the exponential EC decay for $^{140}Pr^{58+}$ and $^{142}Pm^{60+}$ ions was found in Ref. [2]. The modulation period of 7.1 s is incommensurable with all the energy intervals involved in the experiment with the exclusion of the neutrino mass difference. Perhaps this was the main motivation for the letter [3] relating the anomaly to neutrino mixing.

Our goal in this work is to show that there is a mechanism related to Rabi oscillations that offers such a short period. To observe the time modulation of the EC decay, we propose to apply Penning trap mass spectrometry with ions in the intermediate mass region having either of two "sterile" relative to the EC decay hyperfine state. We also analyze the effect of the magnetic fields of dipole and quadrupole magnets in the Experimental Storage Ring (ESR) on the GSI oscillations.

The $1s_{1/2}$ state of an H-like ion has two hyperfine levels with the total angular momentum $F = I \pm 1/2$, where $I$ is the nuclear spin. In the case of a positive nuclear magnetic moment $\mu$, the ground state of an ion has $F = I - 1/2$, whereas for negative $\mu$, the order of these energy levels is reversed. When $I = 1$, the hyperfine splitting of the $1s_{1/2}$ state is given by [4]

$$\Delta E = \frac{2\alpha(\alpha Z)^3(\mu/\mu_N)m^2c^2}{M(2\sqrt{1-(\alpha Z)^2}-1)\sqrt{1-(\alpha Z)^2}}, \quad (1)$$

where $m$, $M$, $\mu_N$, and $\alpha$ are the masses of the electron and proton, the nuclear magneton, and the fine-structure constant, respectively. For nuclei with $I^\pi = 1^+$, EC decay from the $F = 3/2$ state is forbidden because fully ionized daughter nuclei have $I^\pi = 0^+$. This feature of such ions may be used to observe the time modulation of EC decay.

The oscillatory behavior of the EC decay of such ions can be obtained with the Rabi resonance method used in nuclear magnetic spectroscopy. We consider an ion in a static magnetic field in the $z$ direction, resulting in Zeeman splitting of both hyperfine levels. Transitions between the Zeeman components of these two states are driven by the transverse oscillating field along the $x$ direction at frequency $\omega$. When the frequency $\omega$ is tuned to $\Delta E/\hbar$, the oscillation of populations for the states $F = 1/2$, $M$, and $F = 3/2$, $-M$, where $M = 1/2$ or $-1/2$, is modulated by the Rabi frequency, which is proportional to the amplitude of the driving field. Because the EC decay probability is proportional to the occupation of the $F = 1/2$ state, the Rabi frequency may be observed as the time modulation of the EC decay rates. Time modulation of EC decay is also possible with resonance transitions involving several field quanta.

*Penning trap.* A single H-like ion in a Penning trap is an ideal object for observing the modulation of EC decay. The ion having a charge-to-mass ratio $q/M$ is confined in a strong magnetic field $\mathbf{B}(0, 0, B_0)$ superimposed on a weak electrostatic quadrupole field [5]. Solution of the motion equations yields the cyclotron, axial, and magnetron harmonic oscillations. The ion mass is determined via the determination of the free-space cyclotron frequency $\omega_c = qB_z/Mc$, which is derived from three measured harmonic frequencies. The single-ion spectroscopy developed in the ESR experiments can be installed in a Penning trap [6]. Using this technique, it is possible to realize a fully controllable experiment on the time modulation of EC decay by single- and multiphoton resonance.

We are interested in the time evolution of the two hyperfine states of an ion exposed by the trap magnetic field $\mathbf{B}$ and irradiated by the in-plane elliptically polarized driving field $\mathbf{b}(b_x \sin \omega t, 0, b_z \cos \omega t)$. The strength of the latter is restricted by the frequency resolution of the single-ion spectroscopy. It is necessary for a shift in the cyclotron frequency $\omega_c$ caused by the influence of field $\mathbf{b}$ to be small compared to the change in $\omega_c$ owing to the transition from a parent ion to a daughter nucleus. As a rough approximation, this gives $b < B_0 Q_{EC}/(Mc^2)$, where $Q_{EC}$ is the decay energy.

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The total Hamiltonian is $H = H_0 + U(t) + V(t)$, where $H_0 = H_D + (e/2)\mathbf{B}\cdot[\boldsymbol{\alpha}\mathbf{r}]$ describes the $1s_{1/2}$ state of an H-like ion in a Penning trap. Here $H_D$ is the Dirac Hamiltonian with hyperfine interaction and $\boldsymbol{\alpha}$ is the Dirac matrix. The eigenfunctions and eigenvalues of $H_0$ are $|+\rangle = |3/2, M_+\rangle$, $E_{3/2} + (1/3)g\mu_B B_0 M_+ = \hbar\omega_3$, and $|-\rangle = |1/2, M\rangle$, $E_{1/2} - (1/3)g\mu_B B_0 M = \hbar\omega_1$. Here $\mu_B$ is the Bohr magneton and $g$ is the gyromagnetic ratio of the bound electron in an H-like ion. The terms $U(t) = (e/2)[\boldsymbol{\alpha}\mathbf{r}]_z b_z \cos\omega t$ and $V(t) = (e/2)[\boldsymbol{\alpha}\mathbf{r}]_x b_x \sin\omega t$ represent the diagonal and off-diagonal parts of the interaction between an ion and the driving field.

There are three independent transitions between the lower and the upper Zeeman sub-states that can be used for modulation of EC decay:

(i) Transitions between state $|3/2, +1/2\rangle$ (or $|3/2, -1/2\rangle$) and state $|3/2, -1/2\rangle$ (or $|1/2, +1/2\rangle$) with energy $\Delta E = E_{3/2} - E_{1/2} = \hbar\omega_0$ regardless of the sign of $\mu$.

(ii) Transitions between state $|3/2, \mp 3/2\rangle$ and state $|1/2, \mp 1/2\rangle$ for $\mu > 0$ (upper sign) and $\mu < 0$ (lower sign) with energy $\Delta E - (2/3)g\mu_B B_0$.

(iii) Transitions between state $|3/2, \pm 3/2\rangle$ and state $|1/2, \pm 1/2\rangle$ for $\mu > 0$ (upper sign) and $\mu < 0$ (lower sign) with energy $\Delta E + (2/3)g\mu_B B_0$.

Thus, we have the two discrete states $|+\rangle$ and $|-\rangle$ with the energies $\epsilon_3 = \hbar\omega_3$ and $\epsilon_1 = \hbar(\omega_1 - i\lambda/2)$, where $\lambda$ is the EC decay constant. Let the amplitudes for an ion in these states be $a_+(t)$ and $a_-(t)$. Then the system evolves according to the equations

$$\begin{aligned} i\hbar\dot{a}_- &= (\epsilon_1 + U_- \cos\omega t)a_- + V_{-+}a_+ \sin\omega t, \\ i\hbar\dot{a}_+ &= (\epsilon_3 + U_+ \cos\omega t)a_+ + V_{+-}a_- \sin\omega t, \end{aligned} \tag{2}$$

where $U_+ = g\mu_B b_z M_+/3$, $U_- = -g\mu_B b_z M/3$, and

$$V_{+-} = \begin{cases} \hbar V = \frac{1}{3\sqrt{2}} g\mu_B b_x & \text{for} \quad \text{(i)}, \\ \hbar\tilde{V} = -\frac{M_+}{\sqrt{3}|M_+|} g\mu_B b_x & \text{for} \quad \text{(ii) and (iii).} \end{cases} \tag{3}$$

To eliminate the diagonal oscillating terms in Eqs. (2), we perform the transformation

$$a_\pm = A_\pm \exp[-i(\omega_1 + \omega_3)t/2 - i(U_\pm/\hbar\omega)\sin\omega t] \tag{4}$$

which reduces the equations for transitions (i) to the form

$$\begin{aligned} i\dot{A}_- &= -\tfrac{1}{2}(\omega_0 + i\lambda)A_- + VA_+ \sin\omega t, \\ i\dot{A}_+ &= \tfrac{1}{2}\omega_0 A_+ + VA_- \sin\omega t. \end{aligned} \tag{5}$$

The other two transitions are described by the equations

$$\begin{aligned} i\dot{A}_- &= -\frac{1}{2}(\tilde{\omega}_0 + i\lambda)A_- + \tilde{V}A_+ \exp\left(i\frac{U}{\omega}\sin\omega t\right)\sin\omega t, \\ i\dot{A}_+ &= \frac{1}{2}\tilde{\omega}_0 A_+ + \tilde{V}A_- \exp\left(-i\frac{U}{\omega}\sin\omega t\right)\sin\omega t, \end{aligned} \tag{6}$$

where $\tilde{\omega}_0 = \omega_3 - \omega_1$, $U = (U_- - U_+)/\hbar$. Through the use of the expansion of an exponent in the Bessel functions, these equations are converted to

$$\begin{aligned} i\dot{A}_- &= -\frac{1}{2}(\tilde{\omega}_0 + i\lambda)A_- - i\tilde{V}A_+ \sum_{m=-\infty}^{\infty} J'_m\left(\frac{U}{\omega}\right)e^{im\omega t}, \\ i\dot{A}_+ &= \frac{1}{2}\tilde{\omega}_0 A_+ + i\tilde{V}A_- \sum_{m=-\infty}^{\infty} J'_m\left(\frac{U}{\omega}\right)e^{-im\omega t}, \end{aligned} \tag{7}$$

where $J'_m$ is the derivative of the Bessel function. Equations (5) and (7) describe an ion with a positive nuclear magnetic moment $\mu$. When $\mu < 0$, it is necessary to change the sign of $\omega_0$ and $\tilde{\omega}_0$. Up to this point all of the equations are exact.

Equations (5) for $\lambda = 0$ are the basic ones in the theory of the resonance interaction of a two-level system with a periodic field. When $\omega \approx \omega_0$, we get the usual Rabi formula for amplitudes replacing the sine by one exponent [7]. For $K$-photon resonance when $\omega_0 \approx K\omega$, Shirley [8] found the approximate solution for a weak driving field. In the adiabatic approximation, $K \gg 1$, the solution of Eqs. (5) was first obtained by Zaretskii and Krainov [9] and then by Duvall *et al.* [10]. In the weak-field regime, $b_x \ll B_0$, which is adequate for the Penning trap experiment, we use Sambe's perturbation approach [11] provided that $\lambda \ll \omega$. In the $K$th order of the perturbation theory in the quasienergy representation, Eqs. (5) can be approximated by

$$\begin{aligned} i\dot{A}_- &= -\tfrac{1}{2}(\omega_{\rm res} - K\omega + i\lambda)A_- + \tfrac{1}{2}\Gamma_K A_+, \\ i\dot{A}_+ &= \tfrac{1}{2}(\omega_{\rm res} - K\omega)A_+ + \tfrac{1}{2}\Gamma_K A_-, \end{aligned} \tag{8}$$

where

$$\omega_{\rm res} = \omega_0\left[1 + \frac{K^2}{K^2 - 1}\left(\frac{g\mu_B b_x}{3\sqrt{2}\hbar\omega_0}\right)^2\right], \quad K > 1, \tag{9}$$

is the shift in the resonance frequency related to the dynamic Zeeman effect, which is referred to as the Bloch-Siegert shift. The shift is of the order $V^2/\omega_0(b_x/B_0)$. For $K = 1$ the coefficient before the parentheses is equal to $1/4$. The half-width of $K$-photon resonance is

$$\Gamma_K = \frac{2\omega}{[(K-1)!!]^2}\left(\frac{g\mu_B b_x}{6\sqrt{2}\hbar\omega}\right)^K, \tag{10}$$

the photon number $K$ being odd for these transitions.

We now turn to a close examination of Eqs. (7) for transitions (ii) and (iii). When the frequency $K\omega$ ($K$ is an even or odd integer) is nearly resonant with the energy separation $\tilde{\omega}_0$ of the two states, terms with $m = K$ represent a resonance driving field, whereas those with $m \neq K$ result in the resonance frequency being shifted. In the second-order perturbation theory, we obtain

$$\omega_{\rm res} = \tilde{\omega}_0\left\{1 + 2\left(\frac{g\mu_B b_x}{\sqrt{3}\hbar\tilde{\omega}_0}\right)^2 \sum_{\substack{m=-\infty \\ m\neq K}}^{\infty} \frac{\left[J'_m\left(\frac{2g\mu_B b_z}{3\hbar\omega}\right)\right]^2}{1 - \frac{m}{K}}\right\}. \tag{11}$$

Substitution of $A_\pm$ with $A_\pm \exp\{\mp i[K\omega t + (\pi/2)]/2\}$ leads us to Eqs. (8), with $\omega_{\rm res}$ given by Eq. (11) and the half-width

$$\Gamma_K = \frac{2g\mu_B b_x}{\sqrt{3}\hbar}\left|J'_K\left(\frac{2g\mu_B b_z}{3\hbar\omega}\right)\right|. \tag{12}$$

We assume that an ion is in the lower hyperfine state when it is injected into a Penning trap at $t = 0$. With this initial condition, Eqs. (8) can be easily solved for $\lambda \ll \Gamma_K$. The probability of finding an ion in the decaying state $|-\rangle$ is given by

$$P_F(t) = \delta_{FF_-}\left[\cosh\left(\frac{\lambda\Delta}{2\Omega}t\right) - \frac{\Delta}{\Omega}\sinh\left(\frac{\lambda\Delta}{2\Omega}t\right)\right]e^{-\lambda t/2} + \frac{(\delta_{FF_+} - \delta_{FF_-})\Gamma^2_{\text{res}}}{\Delta^2 + \Gamma^2_{\text{res}}}\left[\sinh^2\left(\frac{\lambda\Delta}{4\Omega}t\right) + \sin^2\left(\frac{1}{2}\Omega t\right)\right]e^{-\lambda t/2}, \quad (13)$$

where $\Omega = \sqrt{\Delta^2 + \Gamma_K^2}$ is the Rabi frequency and $\Delta = \omega_{\text{res}} - K\omega$ is the detuning. The probability depends on the lower-state angular momentum $F$ taking the values $F_\pm = I \pm 1/2$. Expression (13) describes damped oscillations with frequency $\Omega$ decaying on the scale of *half* the EC decay constant $\lambda$. Nevertheless, the total number of decays for infinite time calculated with Eq. (13) is equal to the number of radioactive ions at $t = 0$.

To elucidate the time modulation of EC decay, we utilize the concept of the dressed atom introduced by Shirley [8] as the quantum limit of the semiclassical theory. An ion with $\mu > 0$ coasted in a trap and exposed to a resonance field is described by the quantum wave function

$$\Psi(t) = a_+(t)|+\rangle|N - K\rangle + a_-(t)|-\rangle|N\rangle, \quad (14)$$

where $|N\rangle$ and $|N - K\rangle$ are wave functions with a fixed number of photons having energy $\hbar\omega$. Thus, the dressed ion has two hyperfine states entangled with photons. EC decay at time $t$ disentangles the state (14) by projecting $\Psi(t)$ on state $|-\rangle$ with probability $P_{F_-}(t)$, Eq. (13), to which the decay rate is proportional.

For the time modulation to be observed, an H-like radioactive ion must have the following properties.

(i) Its half-life $T_{1/2}$ is longer than its preparation and cooling time.
(ii) It has a large fraction of EC decay, implying a large $Z$.
(iii) The lifetime $\tau$ of the upper hyperfine state is longer than $T_{1/2}$, implying a small $Z$.
(iv) The frequency $\Omega$ of the time modulation of EC decay is greater than the EC decay constant $\lambda$.

Table I reports H-like ions that may be considered candidates for experimental observation of time modulation. The hyperfine splitting $\Delta E$ is calculated by Eq. (1). The mean time $\tau$ for the magnetic dipole transition between the upper and the lower hyperfine states is estimated with the expression [12]

$$\tau = \frac{3\lambda_e}{2\alpha c}\left(\frac{mc^2}{\Delta E}\right)^3 \frac{2I + 1}{2F + 1}, \quad (15)$$

where $\lambda_e$ is the electron Compton wavelength and $F$ is the angular momentum of the ground state. The H-like ion $^{68}$Ga seems to be the best candidate. In Table II we compare the amplitudes of the two fields with different polarizations calculated from Eqs. (10) and (12) for different photon numbers $K$ and a fixed width $\Gamma_K$. The trap field is $B_0 = 1.0$ T.

TABLE I. Hydrogen-like radioactive ions with allowed Gamow-Teller transition $1^+ \to 0^+$ appropriate for observation of the time modulation of electron capture (EC) decay. The half-life $T_{1/2}$ of the nuclear ground state is given for neutral atoms.

| H-like ion | $\mu$ ($\mu_N$) | $\Delta E$ (meV) | $\tau$ | $T_{1/2}$ | $Q_{\text{EC}}$ (keV) | EC/$\beta^+$ (%) |
|---|---|---|---|---|---|---|
| $^{18}$F | +0.58[a] | 0.672 | 5.55 yr | 109.8 min | 1655 | 3.3/96.7 |
| $^{64}$Cu | −0.217[b] | 8.95 | 10.3 h | 12.7 h | 1673 | 43/18 |
| $^{68}$Ga | 0.0117[b] | 0.596 | 7.95 yr | 67.6 min | 2921 | 8.9/88.0 |
| $^{78}$Br | 0.13[b] | 9.75 | 15.9 h | 6.46 min | 3754 | 8/92 |
| $^{82}$Rb | +0.554[b] | 48.0 | 7.20 min | 1.27 min | 4378 | 4/96 |

[a]Calculated with Schmidt's factors $g_j^p$ and $g_j^n$.
[b]Data from Ref. [13]. No sign is given if it is unknown.

It is shown that the circularly polarized field is more suitable for experimental use.

*Storage ring ESR.* We now try to use multiple transition mechanism for interpretation of the GSI oscillations. It should be emphasized, however, that this problem is a more complicated one, owing to the extremely complex configuration of the ring magnetic field designed to store ions. Moreover, the available experimental information is still incomplete.

The ESR lattice has a periodic structure with spatial period $L = C/2$, where $C = 108.36$ m is the circumference of the ring. Thus, an ion circulating in the ESR is exposed to the periodic magnetic fields of bending dipole and quadrupole magnets. We consider the two fields that are in the direction of the $x$ axis tangent to the main orbit and the $z$ axis perpendicular to its plane. In the reference frame of a moving ion they can be expressed by the Fourier series

$$b_x(t) = \sum_{n=1}^{\infty} b_{xn}\sin(2\pi n\gamma vt/L), \quad B_z(t) = B_{z0} + \sum_{n=1}^{\infty} b_{zn}\cos(2\pi n\gamma vt/L), \quad (16)$$

where $t$ is the proper time. For the ions under consideration, the Lorentz factor is $\gamma = 1.43$. The frequency $\omega_n$ of the $n$th harmonic is equal to $n\omega$, where $\omega/2\pi = \gamma v/L = 5.65$ MHz is the frequency of the first harmonic and the interval between nearby harmonics. The latter is much greater than the width of the multiphoton resonance at frequency $\omega_n$. Therefore, resonances with different $n$ values do not overlap and can be considered to be independent.

First, we consider the transition, induced by the fields of the $n$th harmonic, between hyperfine states $F = 1/2$ and $F = 3/2$ with energy $\Delta E \sim 1$ eV. In principle, a resonance transition with a Rabi frequency of 0.1 Hz is possible for very high $n$ and low $K$. However, this is a fallacious result, because the mean lifetime of the upper $F = 3/2$ state, $\tau = 0.03$ s, is much shorter than the modulation period. Hence, the "sterile" state cannot be the source of the observed oscillations.

Next let us consider the transitions between the Zeeman components of the lower hyperfine state. For modulation to be possible, the weak-interaction operator of EC decay has to have the different matrix elements for states with different

TABLE II. Strength of linearly polarized ($b_l = b_x$ for transition (i) with $\omega_0/2\pi = 144$ GHz) and circularly polarized ($b_c = b_x = b_z$ for transition (ii) with $\tilde{\omega}_0/2\pi = 126$ GHz) harmonic fields required to observe $K$-photon resonance with half-width $\Gamma_K/2\pi = 0.1$ Hz in the $^{68}Ga^{30+}$ ion.

| | $K = 1$ | $K = 2$ | $K = 3$ | $K = 4$ | $K = 5$ |
|---|---|---|---|---|---|
| $b_l$ (T) | $1.54 \times 10^{-11}$ | – | $2.38 \times 10^{-3}$ | – | $9.07 \times 10^{-2}$ |
| $b_c$ (T) | $1.00 \times 10^{-12}$ | $2.62 \times 10^{-6}$ | $3.47 \times 10^{-4}$ | $3.94 \times 10^{-3}$ | $1.69 \times 10^{-2}$ |

$M$ values. We assume that this is the case. Then the two quasistationary states exposed to the fields of the $n$th harmonic evolve according to (2), where $a_+$ and $a_-$ are the amplitudes of the lower $|1/2, +1/2\rangle$ and upper $|1/2, -1/2\rangle$ states at energies $\epsilon_1 = \hbar(\omega_+ - i\lambda/2)$ and $\epsilon_3 = \hbar(\omega_- - i\lambda/2)$, respectively.

Suppose the ion $^{140}Pr^{58+}$ (or $^{142}Pm^{60+}$) circulating in the ESR moves along the main orbit. Then it is exposed to the periodic magnetic field of dipole magnets only. One period $L$ contains three identical dipoles arranged symmetrically around the center of the central dipole magnet. The dipole magnet of length $d = 6.5$ m involves a strong bending field $\mathbf{B}(0, 0, B_0)$, $B_0 = 1.197$ T, and a fringe field $\mathbf{b}(b_x, b_y, b_z)$. The latter is calculated by the code OPERA [14]. It is localized in the small region $\Delta x = 29$ cm on both sides of each dipole. Component $b_z$ decreases monotonically from $B_0$ to 0. Component $b_x$ has a pulse-shaped form with the maximal value $b_0 = 36$ mT, and component $b_y$ is too small to be taken into account. The Fourier coefficients in Eqs. (16) are easily found for these components, and the matrix elements in Eqs. (2) are $U_- = -U_+ = pf_n/n$, $V_{+-} = V_{-+} = -qg_n/n$, where

$$p = \frac{g\mu_B\gamma B_0}{\pi}, \quad q = \frac{g\mu_B b_0}{\pi}, \quad \varpi = \frac{g\mu_B\gamma B_0 d}{\hbar L}, \quad (17)$$

$\varpi = \omega_- - \omega_+$ being the Zeeman splitting. The periodic in $n$ functions $f_n \sim g_n \lesssim 1$ depend on the lattice parameters. The splitting $\varpi/2\pi = 5.65$ GHz is well above the frequency of the first harmonic. This means that the harmonics with a high $n$ are involved in the resonance transitions. Hence, the driving field is weak and we can use the approximate solution of Eqs. (7). Table III reports the half-width and detuning for $K$-photon resonance calculated according to the formulas

$$\Gamma_K = \left|\frac{qg_n}{\pi\hbar n} J'_K\left(\frac{2pf_n}{n\hbar\omega_n}\right)\right|, \quad \Delta = \varpi - K\omega_n, \quad (18)$$

for the harmonics, which are closest to the resonance.

TABLE III. Half-width and detuning for harmonics close to the $K$-photon resonance for a $^{140}Pr^{58+}$ or $^{142}Pm^{60+}$ ion moving along the main orbit. Here $K$ is the integer value of $\varpi/\omega_n$. All frequencies are in the laboratory frame.

| $n$ | $K$ | $\Gamma_K$ (Hz) | $\Delta$ (MHz) | $n$ | $K$ | $\Gamma_K$ (Hz) | $\Delta$ (MHz) |
|---|---|---|---|---|---|---|---|
| 143 | 7 | $1.48 \times 10^{-10}$ | −6.92 | 250 | 4 | 24.4 | −2.97 |
| 166 | 6 | $2.88 \times 10^{-5}$ | 12.9 | 251 | 4 | 0.304 | −18.8 |
| 167 | 6 | $5.22 \times 10^{-4}$ | −10.9 | 333 | 3 | 18.9 | 0.989 |
| 168 | 6 | 0.328 | −34.6 | 500 | 2 | 854 | −2.79 |
| 201 | 5 | 0.782 | −22.7 | 999 | 1 | 2420 | 0.989 |

The large values of $\Delta$ in Table III may be attributed to the inaccurate trajectory used. In a storage ring, the periodic motion of an ion in the axial direction (rotation) is coupled with its periodic motion in the transverse direction (betatron oscillations). The latter leads to an ion being exposed to the magnetic field of quadrupole lenses that change $\Gamma_K$ and $\Delta$. However, for the resonance condition $\Delta \sim \Gamma_K$ to be realized, an ion has to move along a particular, "resonance" orbit. Decays of ions stored on nonresonance orbits generate a background with the conventional exponential law.

The plausible explanation for the observed oscillation amplitude is related to the coupling of the $F = 1/2$ and $F = 3/2$ states. The Hamiltonian corresponding to Eqs. (7) offers the coupling owing to virtual transitions in states $F = 1/2$ and $F = 3/2$ with different $M$ values and nonzero photon numbers. The transitions change the energy (like the Lamb shift) and the wave functions of states $|1/2, \pm 1/2\rangle$. However, these changes are small, owing to the weak driving field of the $n$th harmonic and cannot explain the observed amplitude.

It is not inconceivable also that the coupling of these states is effected by the static or periodic potential $W$ associated with the ESR fields. Solution of the Schrödinger equation for the two hyperfine states gives, in the limit $W^{1/2,\pm1/2}_{3/2,M}/\Delta E \ll 1$ (matrix elements $W$ are assumed to be real), the new eigenfunctions

$$\begin{bmatrix} |1\rangle \\ |2\rangle \end{bmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{bmatrix} \left|\frac{1}{2}, \frac{1}{2}\right\rangle \\ \left|\frac{1}{2}, -\frac{1}{2}\right\rangle \end{bmatrix},$$

$$\tan\theta = \frac{2(W\cdot W)^{\frac{1}{2},\frac{1}{2}}_{\frac{1}{2},-\frac{1}{2}}}{\hbar\varpi\Delta E}. \quad (19)$$

With this basis, we find, following the method of Ref. [11], the quasienergy function

$$\Psi(t) = (b_1|1\rangle + b_2|2\rangle e^{-iK\omega_n t})e^{-\frac{i}{2}(\omega_- + \omega_+ - i\lambda)t}, \quad (20)$$

satisfying the initial condition $\Psi(0) = |1\rangle$. Here the coefficients

$$b_1 = \cos(\Omega t/2) - i\frac{\Delta}{\Omega}\sin(\Omega t/2), \quad b_2 = -i\frac{\Gamma_K}{\Omega}\sin(\Omega t/2)$$

represent Rabi oscillations (slow motion). Function (20) describes the evolution of the system and allows us to calculate the amplitude of the transition of the mother ion from state $1s_{1/2}$, $F = 1/2$, into the daughter nucleus. The probability of EC decay is proportional to the square of the amplitude averaged over the period of the $n$th harmonic (fast motion)

$$P_{\mathrm{EC}} \sim |\mathcal{M}|^2 \left[1 - \sin\theta + \sin\theta \frac{2\Gamma_K^2}{\Omega^2} \sin^2\left(\frac{\Omega}{2} t\right)\right] e^{-\lambda t}, \quad (21)$$

where $|\mathcal{M}|^2 = |\mathcal{M}_{+\frac{1}{2}}|^2 = |\mathcal{M}_{-\frac{1}{2}}|^2$ are the squared amplitudes of EC decay calculated with the unperturbed states. Equation (21) describes damped Rabi oscillations with EC decay constant $\lambda = \lambda_{\mathrm{EC}}$ and amplitude $\sin\theta \sim W^2/(\hbar\varpi\Delta E)$. Comparing the latter with observed amplitude 0.2, we find $W \sim 0.001$ eV. The electric quadrupole field required to achieve this value is of the order of atomic fields and cannot exist in the ESR. The required magnetic field $\mathbf{B}(0, B_y, B_z)$ should be of the order of 20 T, which is greater by a factor of 20 than the ESR fields.

*Conclusion.* The effect of atomic electron on the EC decay of H-like ions is very important. We have shown for the first time that the EC decay rates of such ions can be modulated by using the Rabi resonance method with single-photon or multiphoton transitions. For a long time, multiphoton resonance has been the subject of intense experimental and theoretical studies in different fields, except for nuclear physics. We have proposed an experiment with intermediate-mass ions stored in a Penning trap that can demonstrate this phenomenon.

In an attempt to understand the GSI oscillations, we have proposed an alternative mechanism involving resonance multiphoton transitions, induced by the periodic magnetic field of the ESR, between the magnetic substates of the ground $F = 1/2$ state of the radioactive ions $^{140}\mathrm{Pr}^{58+}$ and $^{142}\mathrm{Pm}^{60+}$. The mechanism includes the coupling of $F = 1/2$ and $F = 3/2$ hyperfine states to explain the oscillation amplitude. The important point of this scenario is that the predicted EC decay constant does not violate the experimentally and theoretically established ratio $\lambda_{\mathrm{EC}}^{\mathrm{H-like}}/\lambda_{\mathrm{EC}}^{\mathrm{He-like}} \approx 3/2$ [15]. We find that the periodic magnetic field of the ESR can generate oscillations with a frequency of about 0.1 Hz in transitions involving several photons. However, the field required to explain the observed amplitude should be many times higher than that of the ESR. Moreover, to tune to the resonance a stored ion should move along specific, "resonance" orbits. This cast some doubts on the involvement of the hyperfine structure in the observed oscillations. Therefore, it is necessary to perform a single-ion experiment with $^{123,5}\mathrm{Cs}^{54+}$ ions having the ground state $1s_{1/2}F = 0$ or with He-like ions without a hyperfine structure. There is a fundamental difference between experiments using these two types of ions. If an experiment with He-like ions shows no oscillations, it will support the suggested mechanism of Rabi oscillations with hyperfine level mixing. The same result in an experiment with ions having the ground state $F = 0$ will support the suggested mechanism and kill the neutrino mixing hypothesis.

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