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Narrow states in the three-proton emitter ^{17}Na

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Based on a microscopic cluster model that reproduces well the ^{17}C spectrum below the neutron threshold, we study the spectrum of its mirror nucleus ^{17}Na. We find that the ^{17}Na ground state should be located at 2.4 MeV above the $^{16}\mathrm{Ne}(0^+_1) + p$ threshold, being a broad $1/2^+$ resonance with a width of 1.4 MeV. However, there should exist at least four narrow excited states in ^{17}Na; three of them, $3/2^+_1$, $7/2^+_1$, and $9/2^+_1$, decay into the $^{16}\mathrm{Ne}(2^+_1) + p$ channel, and the other, $5/2^+_1$, decays mainly into the $^{16}\mathrm{Ne}(0^+_1) + p$ channel. Because the daughter nucleus ^{16}Ne is a two-proton emitter, the narrow ^{17}Na states must undergo a sequential three-proton decay via intermediate states in ^{16}Ne.

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Introduction. The development of experimental techniques that allow the fragments of in-flight-decay to be tracked and identified makes it possible to study nuclear structure beyond the proton drip line, where nuclei exist only as resonances in the continuum. With these techniques, one-proton (1*p*) and two-proton (2*p*) emitters have been identified [1–5]. In the medium and heavy mass regions, the lifetime of 1*p* and 2*p* emitters is long, compared to typical nuclear lifetimes, because the strong Coulomb barrier prevents their decay. In the light mass region, however, the Coulomb barrier is lower, the lifetimes are shorter, and, as a consequence, the resonance widths are larger. For example, the ground states of ^{15}F or ^{10,11}N are observed in reaction cross sections as broad *s*-wave resonances. It has been predicted, however, that, even in this region, proton emission from some excited states may be suppressed by structural reasons and that such states can be narrow. For example, three negative-parity states in ^{15}F, $1/2^-$, $5/2^-$, and $3/2^-$, located between 4.5 and 7.6 MeV, are predicted to have a width of only a few kilo–electron volts [6,7]. Two of these states have recently been identified using in-flight decay tracking techniques [1]. Their estimated widths are less than 400 keV. In addition, a state in ^{15}F with a similarly small width has been seen at 7.6 MeV. It was pointed out in Ref. [1] that all these states have a peculiar cluster structure based on excited core states that are in turn 1*p* emitters.

In this paper, we show that narrow states can also exist in the spectrum of another proton-rich nucleus, ^{17}Na, which is the mirror analog of ^{17}C famous for its peculiar structure. The neutron binding energy in the ground state $^{17}\mathrm{C}(3/2^+)$ is only 728 keV, typical of halo nuclei. However, knockout and Coulomb breakup experiments have shown that the weakly bound $^{16}\mathrm{C}(0^+) + n$ configuration is suppressed in $^{17}\mathrm{C}(3/2^+)$ and that this state is mainly based on the $^{16}\mathrm{C}(2^+) + n$ configuration [8,9], where the neutron binding energy is 2.5 MeV. A similar structure should be expected in the mirror nucleus $^{17}\mathrm{Na}(3/2^+)$. Therefore, the decay branch $^{17}\mathrm{Na}(3/2^+) \to {}^{16}\mathrm{Ne}(0^+) + p$ could be suppressed, and if energetically allowed, the main decay mode would be $^{17}\mathrm{Na}(3/2^+) \to {}^{16}\mathrm{Ne}(2^+) + p$. If its decay energy is below the Coulomb barrier, then its width may be small. Because the decay product ^{16}Ne is unstable with respect to 2*p* emission, ^{17}Na should be a three-proton (3*p*) emitter.

At present, nothing is known about ^{17}Na. The only theoretical calculation, performed with a deformed Hartree-Fock model, suggests that the last proton orbital may be bound [10]. However, this is unlikely because ^{19}Na is unbound [11] and ^{17}Na is obtained from ^{19}Na by further removal of two deeply bound neutrons. To study ^{17}Na, we use a two-center microscopic cluster model (MCM) in which excited states of the ^{16}Ne core are included. First, we show that this model solves the long-standing problem of reproducing the spectrum of its mirror analog ^{17}C below the neutron decay threshold. Then we use the same model to predict positions and decay widths of the ^{17}Na states.

^{17}C properties in the MCM. Experimental spectrum of ^{17}C and previous theoretical calculations: Experimentally, three levels are known below the neutron threshold in ^{17}C (see Fig. 1). Their spin parities have been established from the analysis of the γ-ray spectrum observed in the $p + {}^{17}\mathrm{C}$ inelastic scattering [12] and from the study of one-neutron removal reaction from ^{18}C [13]. Unbound excited states have been observed in the β decay of ^{17}B [14] and in the three-neutron transfer $^{14}\mathrm{C}(^{12}\mathrm{C},^{9}\mathrm{C})^{17}\mathrm{C}$ reaction [15]. Three unbound excited states have also been recently identified at $E_x = 2.20(3)$, 3.05(3), and 6.13(9) MeV from inelastic proton scattering by detecting neutrons in coincidence with ^{16}C [16] emitted in the $^{17}\mathrm{C}^* \to {}^{16}\mathrm{C} + n$ decay. It was suggested in Ref. [16] that these levels have spin parities of $7/2^+$, $9/2^+$, and $5/2^+$ respectively. For the last two levels, this assignment agrees with the one based on the three-nucleon transfer study. However, the spin-parity assignment of $7/2^+$ for $E_x = 2.20(3)$ MeV contradicts the observed width of this state equal to 0.53(4) MeV. At this energy, the $l = 2$ decay channel $^{16}\mathrm{C}(2^+) + n$ is closed, while the $l = 4$ decay into the $^{16}\mathrm{C}(0^+) + n$ channel should have a width smaller than 1 keV. Also, the state at 2.20(3) MeV identified in the $^{17}\mathrm{C}(p, p')^{17}\mathrm{C}$ reaction coincides in energy with the state at 2.25(2) MeV seen in the β decay of ^{17}B. There exist strong arguments for ^{17}B to

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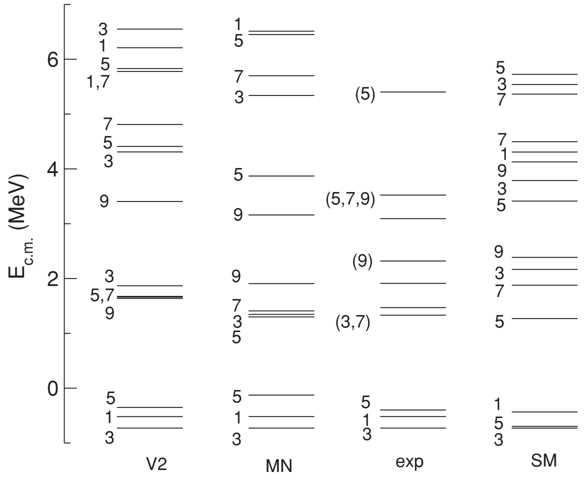

FIG. 1. ^{17}C spectra calculated in the MCM with the V2 and MN interactions in comparison to the experimental spectrum (exp) and to the shell model (SM) WBP predictions. Labels correspond to $2J$. The spin-parity assignment for observed unbound levels corresponds to that suggested by $3n$ transfer [15].

have the spin parity of $3/2^-$, as it is obtained from ^{13}B($3/2^-$) by adding four neutrons, coupled to spin state 0^+, into the sd shell. Because no $7/2^+$ states could be populated in the ^{17}B β decay, the spin-parity assignment $7/2^+$ suggested for the 2.20(3) MeV state in Ref. [16] is most likely erroneous, even though the ^{17}C$(p,p')^{17}$C* cross sections for this state can be explained by the $L = 2$ excitation to the $7/2^+$ state in the distorted-wave Born approximation calculations [16,17]. The 2.20(3) MeV state is more likely either $3/2^+_2$ or $5/2^+_2$.

The $spsdpf$ shell model with the WBT interaction cannot reproduce the excitation energies and ordering of the two first excited states (see Fig. 1). The modification of this interaction made in Ref. [18] does not improve the situation. The bound ^{17}C spectrum cannot be understood in the two-body potential model with deformation and the 2^+ excitation of the ^{16}C core either, if standard sets of potentials are used [19,20]. An l-dependent $n + {}^{16}$C potential and a nonstandard spin-orbit interaction should be used for these purposes, but these lead to difficulties in explaining the spectrum of ^{18}C [20]. Also, no success has been made in explaining the ^{17}C spectrum in the Multi-Channel Algebraic Scattering theory [17], in which only the first 2^+ state of the ^{16}C core was taken into account. This suggests that ^{16}C(4^+) excitations can be important in ^{17}C, consistent with a strong population of the ^{16}C excitation energies around 4 MeV in nucleon knockout from ^{17}C [8], where the 2^+_2, 3^+, and 4^+ states have been seen.

In Ref. [21], an attempt was made to understand the ^{17}C spectrum within a two-cluster MCM. In this model, the valence neutrons in the ^{16}C core were allowed to occupy only the $0d_{5/2}$ subshell. This gives 0^+, 2^+, and 4^+ excitations of the core, all of which were included in the calculations. This model was able to reproduce the positions and separation energies of the $3/2^+$ and $1/2^+$ states by tuning the strength of the spin-orbit force. However, the $5/2^+$ state was still unbound by about 300 keV. This can be a consequence of excluding the $1s_{1/2}$ and $0d_{3/2}$ orbitals from the model space used for the ^{16}C core.

New MCM study of ^{17}C: Here we extend the multichannel MCM study of ^{17}C performed in Ref. [21]. In a microscopic theory, the Hamiltonian of the system reads

$$H = \sum_{i=1}^{A} T_i + \sum_{j>i=1}^{A} V_{ij}, \qquad (1)$$

where T_i is the kinetic energy of nucleon i, and V_{ij} a nucleon-nucleon interaction. For large nucleon numbers, the Schrödinger equation associated with this Hamiltonian cannot be solved exactly. We used here the cluster approximation, where the wave function in partial wave J^π with channel spin I reads

$$\Psi^{JM\pi} = \sum_{I_C l I} \mathcal{A}\left[Y_l(\Omega_\rho)\otimes\left[\phi_C^{I_C}\otimes\phi_n\right]^I\right]^{JM} g^{J\pi}_{I_C l I}(\rho). \qquad (2)$$

Here I_C is the spin of ^{16}C and l is the relative angular momentum, taken as 0, 2, and 4. In Eq. (2), $\phi_C^{I_C}$ and ϕ_n are the internal wave functions of the clusters, $g^{J\pi}_{I_C l I}(\rho)$ is the radial function depending on the relative coordinate ρ, and $\mathcal{A}$ is the antisymmetrizer that permutes the last nucleon with the nucleon of the core. A cluster model is well adapted to exotic weakly bound (or unbound) nuclei, as the asymptotic behavior at large ρ, crucial for them, is exactly taken into account through the R-matrix method. In this method, the total width Γ of unbound states is determined as the imaginary part of a complex eigenvalue (see Ref. [22] for details). The partial widths Γ_l in each channel are obtained from the associated eigenvectors and provide the reduced widths γ_l^2 as

$$\Gamma_l = 2ka\gamma_l^2/|O_l|^2, \qquad (3)$$

where k is the wave number, a is the channel radius, and $O_l(ka)$ is an outgoing Coulomb function. The reduced widths γ_l^2 do not depend on the energy of the resonance and are proportional to the wave function at $r = a$.

We use two effective nucleon-nucleon (NN) forces, Volkov V2 [23] and Minnesota (MN) [24], complemented by a zero-range spin-orbit force [25]. Both forces have one adjustable parameter (m and u, respectively) that gives the strength of the NN potentials in odd partial waves.

Unlike in Ref. [21], the two ^{16}C valence neutrons occupy the $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$ orbitals, which gives many excitations in ^{16}C. We consider only the $2^+_{1,2}$, 3^+_1, and 4^+_1 states, motivated by neutron knockout experiments where they are strongly populated. The single-particle wave functions in ^{16}C were taken from the oscillator shell model with the oscillator radius of 1.6 fm.

The theoretical and experimental spectra of ^{16}C are shown in Fig. 2. The V2 potential gives a correct excitation energy for ^{16}C(2^+_1), which is very important for predicting the widths of the ^{17}Na decay modes into two different channels. With MN, the excitation energy of the 2^+_1 state is only 1.39 MeV, which is about 300 keV lower than the experimental value of 1.766 MeV.

Let us now discuss the MCM ^{17}C spectrum shown in Fig. 1 and compared to experiment. The parameter m (or u) and the spin-orbit amplitude have been chosen to reproduce both the ^{16}C $+ n$ threshold in ^{17}C and the excitation energy of ^{17}C($1/2^+_1$). These values, $m = 0.64922$

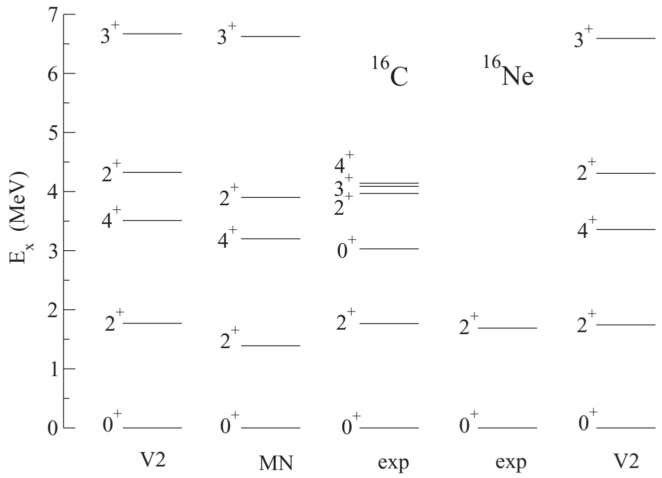

FIG. 2. Theoretical and experimental ^{16}C and ^{16}Ne spectra.

and $S_0 = 31.28$ MeV.fm^5 for V2 ($u = 0.76706$ and $S_0 = 38.95$ MeV.fm^5 for MN), are in the range of typical values used previously in the MCM for other cluster systems. With these parameters, the energies of the three low-lying states are in very good agreement with experiment. In particular, the level ordering is correctly reproduced by the MCM. For comparison, the shell model spectrum calculated with the WBP interaction [26] using the NuShell code [27] is also shown in Fig. 1.

To understand the structure of ^{17}C bound states in terms of core excitations, we have calculated the spectroscopic factors for the $\langle ^{17}\mathrm{C}|^{16}\mathrm{C}\rangle$ overlap in MCM. We list them in Table I in comparison to shell model predictions. We confirm that the ground state $^{17}\mathrm{C}(3/2^+)$ is based on the $^{16}\mathrm{C}(2_1^+) + n$ configuration, which has a spectroscopic factor similar to that obtained in the shell model. We also confirm the shell model results for the strong 2_2^+ and 4^+ core excitations in $^{17}\mathrm{C}(3/2_1^+)$, but we do not get significant strength for $^{16}\mathrm{C}(3_1^+) + n$.

The first $1/2_1^+$ excited state is mainly based on the $^{16}\mathrm{C}(0^+) + n$ and $^{16}\mathrm{C}(2_2^+) + n$ configurations. Unlike in the shell model, the contributions from the 2_1^+ and 3_1^+ states are small. As for the second excited state, $^{17}\mathrm{C}(5/2_1^+)$, its structure in terms of core excitations is similar to the shell model one, except for the s-wave $^{16}\mathrm{C}(3^+) + n$ and d-wave $^{16}\mathrm{C}(2_2^+) + n$ configurations, where the spectroscopic factors are close to zero.

The MCM predicts that the lowest unbound $3/2_2^+$, $5/2_2^+$, $7/2_1^+$, and $9/2_1^+$ levels form a group in a narrow region around 2 MeV above the $^{16}\mathrm{C} + n$ threshold (see Fig. 1). In the shell model these levels are also around 2 MeV but with a larger spread. For V2, the $7/2_1^+$ and $9/2_1^+$ resonances have a width of the order of 10^{-12} and 10^{-6} MeV, respectively, as their decay to the only open $l = 4$ channel $^{16}\mathrm{C}(0_1^+) + n$, which is suppressed by the strong centrifugal barrier. The $5/2_2^+$ state is below the $^{16}\mathrm{C}(2_1^+) + n$ threshold and has a width of 15 keV, while the $3/2_2^+$ state is 80 keV above the $^{16}\mathrm{C}(2_1^+) + n$ threshold and has the width of 265 keV owing to the presence of the s-wave component in this channel. For MN, all these levels have widths of less than 20 keV.

The first four unbound levels in the experimental spectrum are indeed concentrated around 2 MeV above the threshold and they should probably correspond to the theoretical levels $3/2_2^+$, $5/2_2^+$, $7/2_1^+$, and $9/2_1^+$. Because the states at 2.06 and 3.10 MeV belonging to this group have not been seen in the β decay of ^{17}B, they probably have spin parities of $7/2_1^+$ and $9/2_1^+$. The spin assignment of $9/2_1^+$ for the state at 3.05(3) MeV agrees well with both the (p, p') and the $3n$ transfer experiments. The state at 2.06 MeV could be $7/2_1^+$ according to the $3n$ transfer study. Therefore, the two remaining states, at 2.25 and 2.64 MeV, should be the $3/2_2^+$ and $5/2_2^+$ states. It is very important to reproduce the positions of the ^{17}C states for the ^{17}Na study. For these purposes, we refitted the parameter m of V2 individually for each of these states to reproduce their positions precisely. In these new calculations, the $7/2_1^+$ has a width of 30 meV, as it is still below the $^{16}\mathrm{C}(2_1^+) + n$ threshold, while the $9/2_1^+$ state, which is now above this threshold, has a width of 18 keV, which is smaller than the width of 0.10(5) MeV observed in the $3n$ transfer. The 2.25-MeV state has a width of either 8 or 40 keV, depending on whether it has spin parity $5/2_2^+$ or $3/2_2^+$. In both cases it disagrees with the width of 0.53(4) MeV deduced from inelastic proton scattering. The 2.64-MeV state has a width of 368 or 595 keV for the $3/2_2^+$ or $5/2_2^+$ prescription, respectively. No experimental width for this state is available.

TABLE I. Spectroscopic factors $S_l = S_{l,j=l-1/2} + S_{l,j=l+1/2}$ for the $\langle ^{17}\mathrm{C}|^{16}\mathrm{C}\rangle$ overlap calculated in the MCM with the V2 and MN potentials in comparison with the shell model (SM) values.

$I_C(^{16}\mathrm{C})$	l	$^{17}\mathrm{C}(3/2^+)$			$^{17}\mathrm{C}(1/2^+)$			$^{17}\mathrm{C}(5/2^+)$		
		V2	MN	SM	V2	MN	SM	V2	MN	SM
0_1^+	0				0.828	0.841	0.644			
	2	0.010	0.010	0.035				0.558	0.537	0.701
2_1^+	0	0.328	0.383	0.163				0.037	0.066	0.096
	2	1.260	1.243	1.445	0.034	0.016	0.415	0.520	0.593	0.226
2_2^+	0	0.030	0.033	0.225				0.050	0.030	0.014
	2	0.127	0.152	0.090	0.366	0.408	0.372	0	0.002	0.631
4_1^+	2	0.372	0.389	0.381				0.969	0.965	0.916
3_1^+	0							0	0	0.301
	2	0.026	0.002	0.285	0.091	0.254	1.027	0.060	0.128	0.003

TABLE II. Spins, energies, and widths in various $^{16}\mathrm{Ne}+p$ channels of $^{17}\mathrm{Na}$ states with the V2 interaction. All energies and widths are given as mega–electron volts.

J^π	E	E_x	$\Gamma(0_1^+)$	$\Gamma(2_1^+)$
$1/2_1^+$	2.40	0	1.36	
$3/2_1^+$	2.57	0.17	0.001	0.024
$5/2_1^+$	2.97	0.57	0.123	0.021
$7/2_1^+$	4.35	1.95	8×10^{-8}	0.025
$(5/2_2^+)$[a]	4.38	1.98	0.032	1.561
$(3/2_2^+)$[a]	5.27	2.87	0.146	1.237
$(3/2_2^+)$[b]	4.63	2.23	0.108	0.828
$(5/2_2^+)$[b]	5.32	2.92	0.059	2.509
$9/2_1^+$	5.41	3.01	6×10^{-6}	0.210

[a] Assuming that the $^{17}\mathrm{C}$ states at 2.25 and 2.64 MeV are $5/2_2^+$ and $3/2_2^+$, respectively.

[b] Assuming that the $^{17}\mathrm{C}$ states at 2.25 and 2.64 MeV are $3/2_2^+$ and $5/2_2^+$, respectively.

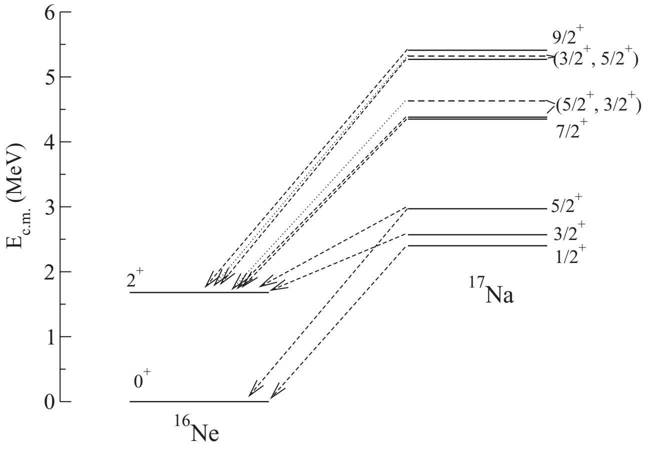

FIG. 3. $^{17}\mathrm{Na}$ decay scheme with the V2 interaction.

All levels above 2 MeV, calculated for spin less than $9/2^+$, have a width much larger than 1 MeV for both V2 and MN. No level with a width typical of those observed in the $^{14}\mathrm{C}(^{12}\mathrm{C},^{9}\mathrm{C})^{17}\mathrm{C}$ reaction, $0.10(5) \leqslant \Gamma \leqslant 0.66(20)$ MeV, is predicted in this region. In particular, we find for the $5/2_4^+$ level a width of 2.5 MeV, which is mainly caused by the s-wave decay into the $^{16}\mathrm{C}(2_1^+)+n$ channel. This spin parity has been assigned to the level 6.2 MeV both in $3n$ transfer and in proton inelastic scattering. However, the observed widths of this state, 0.35(15) MeV from $3n$ transfer and $0.26^{+0.4}_{-0.26}$ MeV from (p, p'), are much smaller than the theoretical predictions.

The MCM spectrum of ^{17}Na *and its decay properties*. Based on MCM with the V2 potential that reproduces the positions of the $^{17}\mathrm{C}$ bound states, we have calculated the energies and widths of the $1/2_1^+$, $3/2_1^+$, and $5/2_1^+$ states in $^{17}\mathrm{Na}$. They are reported in Table II. These states lie above both the $^{16}\mathrm{Ne}(0_1^+)+p$ and $^{16}\mathrm{Ne}(2_1^+)+p$ decay thresholds, so we show the partial widths $\Gamma(0_1^+)$ and $\Gamma(2_1^+)$ for decay to these individual channels as well. The $1/2_1^+$ state becomes the ground state. It mainly decays to the s-wave $^{16}\mathrm{Ne}(0_1^+)+p$ channel and has the large width of 1.36 MeV. The $3/2_1^+$ state is located just above $1/2_1^+$ but it has a different decay mode, $^{16}\mathrm{Ne}(2_1^+)+p$, with the small width of only 24 keV. The $5/2_1^+$ state is also narrow, with $\Gamma(0_1^+)=125$ and $\Gamma(2_1^+)=20$ keV. Because the MCM overpredicts the location of the $5/2_1^+$ state by 46 keV, we can expect that the energy and the width of the $5/2_1^+$ in $^{17}\mathrm{Na}$ are also slightly overestimated.

For the analogs of the first four unbound states in $^{17}\mathrm{C}$ we used the modified V2 potentials discussed in the previous section. The decay scheme of the lowest part of the $^{17}\mathrm{Na}$ spectrum is shown in Fig. 3. We find that the $^{17}\mathrm{Na}(7/2_1^+)$ level should be very narrow. It decays into the d-wave $^{16}\mathrm{Ne}(2_1^+)+p$ and s-wave $^{16}\mathrm{Ne}(4_1^+)+p$ channels with partial widths of $\Gamma(2_1^+)=25$ keV and $\Gamma(4_1^+)=98$ keV, respectively. It should be noted that the theoretical value of the latter threshold is underestimated by 630 keV, and therefore, the energy in this channel is too high. Decreasing this energy by tuning the Majorana parameter m, we obtain a partial width $\Gamma(4_1^+)=4$ keV. Thus, the $^{17}\mathrm{Na}(7/2_1^+)$ state should be as narrow as $^{17}\mathrm{Na}(3/2_1^+)$. A similar situation occurs for the $^{17}\mathrm{Na}(9/2_1^+)$ state. The partial width for the decay into $^{16}\mathrm{Ne}(2_1^+)+p$ is predicted to be 211 keV. A similar width is expected for the decay into the $^{16}\mathrm{Ne}(4_1^+)+p$ channel. Tuning the energy of this channel to reproduce the position of the $^{17}\mathrm{C}(9/2_1^+)$ state with respect to the $^{16}\mathrm{Ne}(4_1^+)+p$ threshold, similarly to what has been done in the case of $7/2_1^+$, we get a partial width $\Gamma(4_1^+)=78$ keV.

For the analogs of the $5/2_2^+$ and $3/2_2^+$ states in $^{17}\mathrm{C}$, we made two predictions on the assumption that (a) the 2.25- and 2.64-MeV states in $^{17}\mathrm{C}$ are $5/2_2^+$ and $3/2_2^+$, and (b) the 2.25- and 2.64-MeV states in $^{17}\mathrm{C}$ are $3/2_2^+$ and $5/2_2^+$. In both cases these states are broad and decay into the s-wave $^{16}\mathrm{Ne}(2_1^+)+p$ channel with a width between 0.8 and 2.5 MeV. As for all other excited states, the MCM predicts widths of 2 MeV and higher. Unless our model strongly overestimates the contribution from the $^{16}\mathrm{Ne}(0_1^+)$ and $^{16}\mathrm{Ne}(2_1^+)$ states (which is possible, as it gives overestimated widths for excited states above 3 MeV in the mirror nucleus $^{17}\mathrm{C}$), no narrow states can be expected in this area.

Conclusions. Based on the MCM with the V2 potential that reproduces the $^{17}\mathrm{C}$ bound spectrum, we predict that the $^{17}\mathrm{Na}$ ground state should be a broad $1/2^+$, $l=0$ resonance. However, there should be at least four narrow states, $3/2_1^+$, $5/2_1^+$, $7/2_1^+$, and $9/2_1^+$, in the $^{17}\mathrm{Na}$ spectrum. The decay product of these states, $^{16}\mathrm{Ne}$, is unstable with respect to the two-proton emission. Therefore, $^{17}\mathrm{Na}$ is in fact a three-proton emitter with a decay path $^{17}\mathrm{Na}\rightarrow{}^{16}\mathrm{Ne}^*+p\rightarrow{}^{14}\mathrm{O}+2p+p$. Consequently, $^{17}\mathrm{Na}$ states can be identified by detecting $^{14}\mathrm{O}+p+p+p$ events in coincidence.

The narrow $^{17}\mathrm{Na}$ states can be populated using the $^{14}\mathrm{O}(^{12}\mathrm{C},^{9}\mathrm{Li})^{17}\mathrm{Na}$ reaction in inverse kinematics with the $^{14}\mathrm{O}$ radioactive beam. It is a mirror analog of the $3n$ transfer $^{14}\mathrm{C}(^{12}\mathrm{C},^{9}\mathrm{C})^{17}\mathrm{C}$ used in Ref. [15] to study the unbound spectrum of $^{17}\mathrm{C}$. This reaction strongly populates the $^{17}\mathrm{C}(5/2_1^+)$ and $^{17}\mathrm{C}(9/2_1^+)$ states at 0.31 and 3.10 MeV, the proton-rich analogs of which should be narrow. It also populates the state at 2.06 MeV, which we think should be assigned to the $7/2_1^+$ state. Its mirror analog should also be narrow. No states have been observed in the ^{14}C(^{12}C,^{9}C)^{17}C reaction at 0.21, 2.25, or 2.64 MeV. Therefore, we can expect that their mirror analogs (which are broad) will not be populated in the ^{14}O(^{12}C,^{9}Li)^{17}Na reaction either, so that three distinctive narrow peaks would be seen in the ^{17}Na spectrum populated by the $3p$ transfer. The widths of these peaks can be studied both by analysis of the missing mass spectra and by detection in coincidence of their decay products. The ^{17}Na spectrum can be also studied in the charge exchange reaction (^{17}Ne,^{17}Na). The Borromean nucleus ^{17}Ne has two protons orbiting the ^{15}O core. The charge exchange in this core would create the proton unstable nucleus $^{15}\mathrm{F} = {}^{14}\mathrm{O} + p$. With two additional protons around it, one gets the $3p$ emitter ^{17}Na. The knowledge of its energy levels and widths will test the accuracy of available theories beyond the proton drip line.

Finally, we have made predictions only for positive-parity states. No negative-parity states have yet been seen in ^{17}C. In other $A = 17$ nuclei, states of the opposite to the ground-state parity are located near 3 MeV in ^{17}O and ^{17}F and near 2 MeV in ^{17}N. From the shell model point of view, they are intruders and their description requires taking into account many major shells. In the $^{17}\mathrm{Na} = {}^{16}\mathrm{Ne} + p$ system, intruder states can be expected at proton energies $E_p > 4$ MeV, assuming that they are 2 MeV above the ^{17}Na ground state. Although such energies are above the Coulomb barrier, these states can be narrow because of small spectroscopic factors typical of intruder states. To predict reliably the positions and the spectroscopic factors of such states in the MCM is a challenging task for the future.

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