

Role of surface energy coefficients and nuclear surface diffuseness in the fusion of heavy-ions

Ishwar Dutt and Rajeev K. Puri*

Department of Physics, Panjab University, Chandigarh 160 014, India

(Received 27 November 2009; revised manuscript received 16 February 2010; published 5 April 2010)

We discuss the effect of surface energy coefficients as well as nuclear surface diffuseness in the proximity potential and ultimately in the fusion of heavy-ions. Here we employ different versions of surface energy coefficients. Our analysis reveals that these technical parameters can influence the fusion barriers by a significant amount. A best set of these parameters is also given that explains the experimental data nicely.

DOI: [10.1103/PhysRevC.81.047601](https://doi.org/10.1103/PhysRevC.81.047601)

PACS number(s): 25.70.Jj, 24.10.-i

It is now well accepted that Coulomb interactions alone cannot define a fusion barrier. Nuclear interactions play an equally important role in deciding the fate of a reaction [1–5]. This is perhaps the compelling cause of so many new potentials one sees in the literature [1–5]. Among various nuclear potentials, one has the potentials within the proximity concept [1–3], as well as within the energy density formalism [4]. As many as two dozen potentials and their different versions are being used in the literature. It is also evident from the literature that every author has tried to justify the validity of the potential by showing that it reproduces the proximity values [4]. At the same time, it is also interesting to note that several improvements were proposed over the original proximity potential in recent times [2,3]. In the original version of the proximity potential (labeled as Prox 77) [1], $V_N(r)$ can be written as

$$V_N(r) = 4\pi\bar{R}\gamma\Phi(r - C_1 - C_2) \text{ MeV}, \quad (1)$$

where $\Phi(r - C_1 - C_2)$ is the universal function that was derived by several authors [1,3,4], \bar{R} is the reduced radius, and γ is the surface energy coefficient.

The strength of the nuclear potential depends on the relative neutron excess of the target/projectile through surface energy coefficient γ and on the mass and surface diffuseness through the reduced radius \bar{R} . Though (in-depth) attention was paid in the literature to pin down the universal function $\Phi(r - C_1 - C_2)$ accurately [3], one takes a very casual approach toward the surface energy coefficients γ and nuclear surface diffuseness. Almost all models [1–5] used different terms and/or values for these coefficients. One wonders how much these parameters can alter the results of fusion barriers and cross sections. Furthermore, it was reported that the original proximity potential overestimates the barriers by an appreciable amount [3]. We are here interested in studying the impact of various surface energy coefficients and nuclear surface diffuseness on the fusion process and shall present a modified version of the proximity potential based on a new set of surface energy coefficients.

In the original proximity potential [Eq. (1)], C_1 and C_2 denote the radii of the spherical target/projectile and are known as Süßmann's central radius. The surface energy coefficient γ was taken from the work of Myers and Świątecki [6], which

reads as

$$\gamma = \gamma_o [1 - k_s A_s^2]. \quad (2)$$

Here, $A_s = (\frac{N-Z}{A})$ where N and Z refer to the total neutron and proton content. In the above formula, γ_o is the surface energy constant and k_s is the surface-asymmetry constant. Both constants were first parameterized by Myers and Świątecki [6] by fitting the experimental binding energies. The first set of these constants yielded values $\gamma_o = 1.01734 \text{ MeV/fm}^2$ and $k_s = 1.79$, respectively. Later on, these values were revised to $\gamma_o = 0.9517 \text{ MeV/fm}^2$ and $k_s = 1.7826$ [7]. This value of γ is referred to as γ -MS.

In another attempt, Möller and Nix [8] fitted the surface energy coefficient γ with the value $\gamma_o = 1.460734 \text{ MeV/fm}^2$ and $k_s = 4.0$ in nuclear macroscopic energy calculations. Naturally, this will lead to more attraction compared to γ -MS. This version of γ is labeled as γ -MN1976.

Later on, due to the availability of a better mass formula due to Möller *et al.* [9], γ_o and k_s were refitted to a strength of 1.25284 MeV/fm^2 and 2.345 , respectively. This particular set of values were obtained directly from a least-squares adjustment to the ground-state masses of 1654 nuclei ranging from ^{16}O to $^{263}106$ and fission-barrier heights [9]. This modified γ is labeled as γ -MN1995.

In the new proximity potential version [3], Myers and Świątecki chose γ that also depends on the neutron skin of the interacting nuclei. The expression of γ obtained from the droplet model reads as

$$\gamma = 1/(4\pi r_0^2) [18.63 \text{ (MeV)} - Q(t_1^2 + t_2^2)/2r_0^2], \quad (3)$$

where t_i is the neutron skin [3]. This version of the surface energy coefficient is labeled as γ -MSNew.

As we see, all the previous four versions of γ have different strengths. In fact, in the work of Möller and Nix [10] as many as five different sets of γ parameters were listed. This will, of course, lead to different values of surface energy coefficients as well as potentials. This study [10] was based on the calculations of fission-barrier heights of 28 nuclei and ground-state masses of 1323 nuclei. Royer and Remaud [11] used the γ_o value the same as γ -MS with a different k_s value (=2.6). Recently, Pomorski and Dudek [12] obtained different surface energy coefficients by including different curvature effects in the liquid drop model. Definitely the original proximity potential [1] used the value of γ that was proposed four decades ago.

* rkpuri@pu.ac.in; drkpuri@gmail.com

As earlier stated, the nuclear surface diffuseness that enters via the reduced radius was also taken in the literature arbitrarily. For example, proximity potentials 1977 [1] and 1988 [2] use the equivalent sharp radius as

$$R = 1.28 A^{1/3} - 0.76 + 0.8 A^{-1/3} \text{ fm.} \quad (4)$$

This formula is a semi-empirical expression supported by assuming the finite compressibility of nuclei. As a result, lighter nuclei squeeze more by the surface-tension forces, whereas heavy nuclei dilate more strongly due to the Coulomb repulsion. In both potentials, the nuclear surface diffuseness enter via reduced radius $\bar{R} = \frac{C_1 C_2}{C_1 + C_2}$ that is used in Eq. (1). The central radius C is calculated from the relation

$$C-77 = R[1 - (b/R)^2 + \dots]. \quad (5)$$

For the present study, we also used the radius due to Aage Winther in Eq. (5), which reads as [5]

$$R = 1.20 A^{1/3} - 0.09 \text{ fm,} \quad (6)$$

and the corresponding central radius [Eq. (5)] is denoted as $C-AW95$.

The newer version of the proximity potential uses a different form of the radius [3]

$$R = 1.240 A^{1/3} \{1 + 1.646 A^{-1} - 0.191 A_s\} \text{ fm.} \quad (7)$$

This formula indicates that radius depends not only on the mass number, it also has a dependence on the relative neutron excess. Actually this formula is valid for even-even nuclei with $Z \geq 8$ [13]. To calculate the matter central radius C , the neutron skin is also added in Ref. [3] using the relation

$$C-00 = c + (N/A)t, \quad (8)$$

where c denotes the half-density radii of the charge distribution given by

$$c = R[1 - (7/2)b^2/R^2 - (49/8)b^4/R^4 + \dots]. \quad (9)$$

Recently, a new form of Eq. (7) with slightly different constants is also reported [14]

$$R = 1.2332 A^{1/3} + 2.8961 A^{-2/3} - 0.18688 A^{1/3} A_s. \quad (10)$$

By using this form of the radius in Eqs. (8) and (9), we can again calculate the central radius C denoted by $C-00N$.

Our calculations are made for 390 reactions involving both the symmetric $N = Z$ and asymmetric $N \neq Z$ reactions. As noted in Refs. [1–3], the surface energy coefficient γ depends strongly on the asymmetry of the reactions.

In Fig. 1, we display the nuclear potential as a function of internuclear distance “ r ” for the reactions of $^{12}\text{C} + ^{12}\text{C}$ (in the upper panel) and $^6\text{He} + ^{238}\text{U}$ (in the lower panel) using the Prox 77 with different versions of surface energy coefficient γ . We see that γ -MS leads to a shallow potential compared to other sets of γ , whereas γ -MN1976 leads to the deepest potential. We also tested all the previously highlighted surface energy coefficients, but their value lies between these extreme limits.

In Fig. 2, we display the fusion barrier heights V_B (MeV) and fusion barrier positions R_B (fm) as a function of $Z_1 Z_2$. For the clarity of the figure, only 155 reactions are displayed. We show the results of implementing γ -MS,

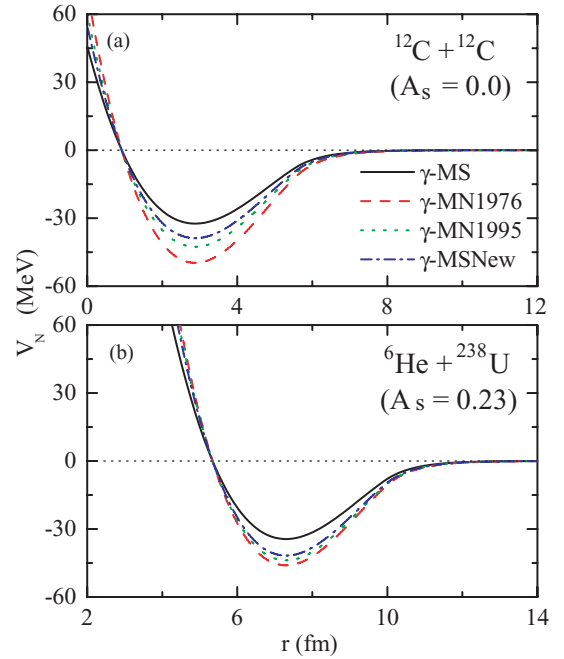


FIG. 1. (Color online) The nuclear part V_N (MeV), of the interaction potential as a function of internuclear distance “ r ” using Prox 77 with different values of surface energy coefficients γ .

γ -MN1976, γ -MN1995, and γ -MSNew as well as different surface diffuseness in the Prox 77 potential. We see some mild effects in the outcome. These effects are monotonous in nature. Due to the wide acceptability of the radius used in Prox 77 [Eq. (4)], we shall stick to the same formula. The

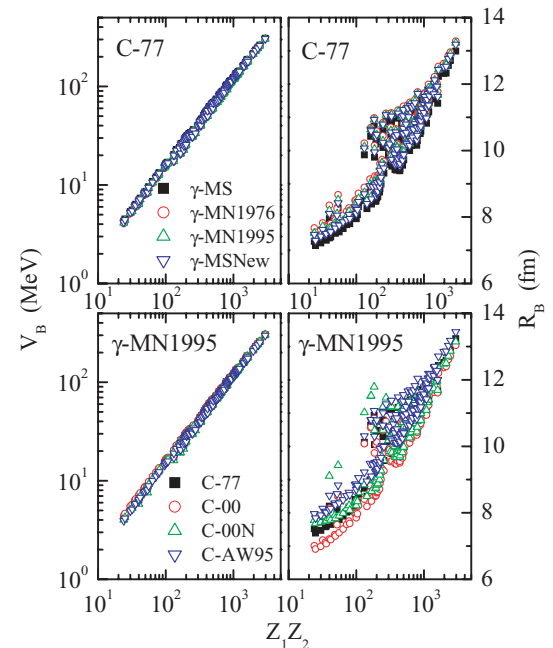


FIG. 2. (Color online) The fusion barrier heights V_B (MeV) and positions R_B (fm) as a function of $Z_1 Z_2$ using different values of surface energy coefficients γ and nuclear central radii C 's implemented in the Prox 77.

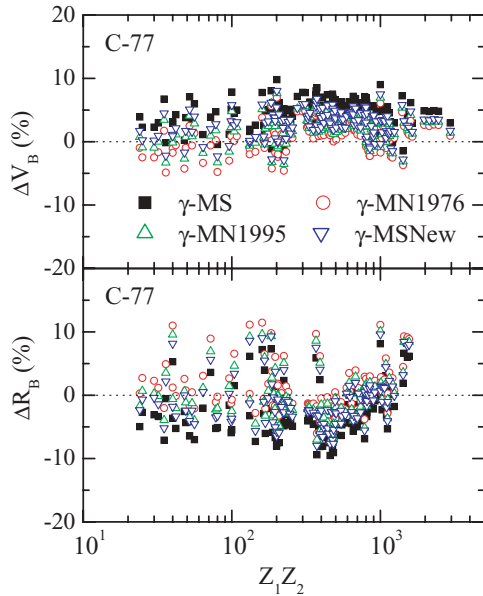


FIG. 3. (Color online) The percentage deviation $\Delta V_B(\%)$ and $\Delta R_B(\%)$ as a function of the product of charges $Z_1 Z_2$ using different versions of surface energy coefficients γ implemented in the Prox 77.

results of different γ values are quantified in Fig. 3, where we display the percentage deviation over the experimental data. The experimental data is taken from Refs. [3,15–23]. We see that the use of γ -MS, which is used in the proximity 1977, yield considerable deviations ($\pm 10\%$). Further, the use of γ -MN1976 and γ -MN1995 yield much improved results. The average deviations over 390 reactions for the fusion barrier heights are 3.99%, 0.77%, 1.77%, and 2.37% for γ -MS, γ -MN1976, γ -MN1995, and γ -MSNew, respectively. Whereas, for fusion barrier positions, its values are -1.74% , 1.95%, 0.73%, and 0.0%, respectively, over 272 reactions (barrier positions are not available for all reactions).

It is clear from the previous study that surface energy coefficients γ -MN1976 and γ -MN1995 may be better choices. To further strengthen the choice we calculate the fusion cross sections using the Wong formula [24].

In Fig. 4, we display the fusion cross sections σ_{fus} (in mb) for the reactions of $^{26}\text{Mg} + ^{30}\text{Si}$ [16], $^{28}\text{Si} + ^{28}\text{Si}$ [17–19], $^{16}\text{O} + ^{46}\text{Ti}$ [20], $^{12}\text{C} + ^{92}\text{Zr}$ [21], $^{40}\text{Ca} + ^{58}\text{Ni}$ [22], and $^{16}\text{O} + ^{144}\text{Sm}$ [23], respectively. We see that γ -MN1976/ γ -MN1995 give better results over the original proximity Prox 77. Note that both fusion barrier height and curvature affect the sub-barrier fusion probabilities. From the previous analysis, it is clear that the effect of technical

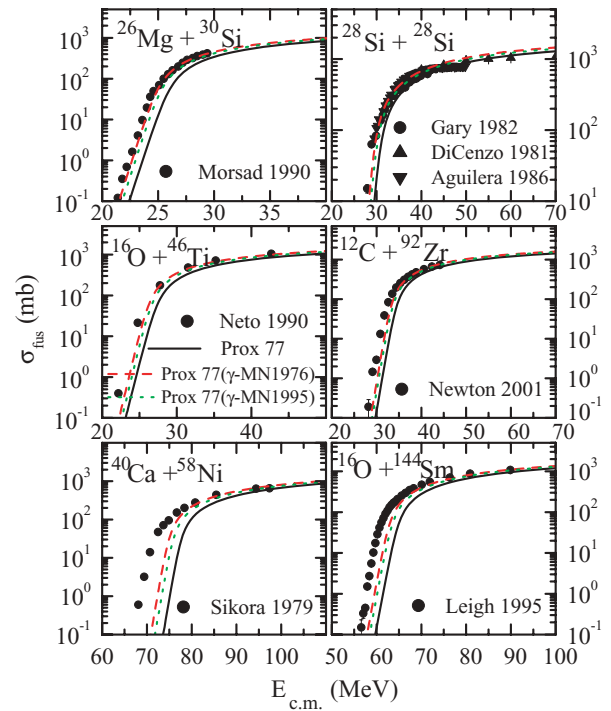


FIG. 4. (Color online) The fusion cross sections σ_{fus} (mb) as a function of center-of-mass energy $E_{\text{c.m.}}$ using different versions of γ in Prox 77. The original Prox 77 is also shown for comparison. The experimental data are from Morsad 1990 [16], Gary 1982 [17], DiCenzo 1981 [18], Aguilera 1986 [19], Neto 1990 [20], Newton 2001 [21], Sikora 1979 [22], and Leigh 1995 [23].

parameters, that is, the surface energy coefficient γ as well as surface diffuseness of the target/projectile is of the order of 10%–15%. The use of surface energy coefficient γ -MN1976/ γ -MN1995 improves the results of the Prox 77 potential considerably. This modified proximity potential is labeled as “Proximity 2010.”

In this Brief Report, we attempt to understand the role of surface energy coefficient γ as well as nuclear surface diffuseness in fusion dynamics. Our analysis reveals that these parameters can affect the nuclear potential as well as fusion barriers by the same amount as different potentials and one should be careful while choosing these technical parameters. We also propose a modified version of Prox 77 with new surface energy coefficient γ -MN1976/ γ -MN1995, which yields closer agreement with experimental data.

This work is supported by a research grant from the Department of Atomic Energy, Government of India.

- [1] J. Blocki, J. Randrup, W. J. Świątecki, and C. F. Tsang, *Ann. Phys. (NY)* **105**, 427 (1977).
 [2] W. Reisdorf, *J. Phys. G: Nucl. Part. Phys.* **20**, 1297 (1994).
 [3] W. D. Myers and W. J. Świątecki, *Phys. Rev. C* **62**, 044610 (2000).
 [4] R. K. Puri and N. K. Dhiman, *Eur. Phys. J. A* **23**, 429 (2005); R. Arora, R. K. Puri, and R. K. Gupta, *ibid.* **8**, 103 (2000);

- R. K. Puri, P. Chattopadhyay, and R. K. Gupta, *Phys. Rev. C* **43**, 315 (1991).
 [5] A. Winther, *Nucl. Phys. A* **594**, 203 (1995).
 [6] W. D. Myers and W. J. Świątecki, *Nucl. Phys.* **81**, 1 (1966).
 [7] W. D. Myers and W. J. Świątecki, *Ark. Fys.* **36**, 343 (1967).
 [8] P. Möller and J. R. Nix, *Nucl. Phys. A* **272**, 502 (1976).

- [9] P. Möller, J. R. Nix, W. D. Myers, and W. J. Świątecki, *At. Data Nucl. Data Tables* **59**, 185 (1995).
- [10] P. Möller and J. R. Nix, *Nucl. Phys. A* **361**, 117 (1981).
- [11] G. Royer and B. Remaud, *J. Phys. G: Nucl. Part. Phys.* **10**, 1057 (1984).
- [12] K. Pomorski and J. Dudek, *Phys. Rev. C* **67**, 044316 (2003).
- [13] B. Nerlo-Pomorska and K. Pomorski, *Z. Phys. A* **348**, 169 (1994).
- [14] G. Royer and R. Rousseau, *Eur. Phys. J. A* **42**, 541 (2009).
- [15] V. Tripathi *et al.*, *Phys. Rev. C* **65**, 014614 (2001); S. Sinha, M. R. Pahlavani, R. Varma, R. K. Choudhury, B. K. Nayak, and A. Saxena, *ibid.* **64**, 024607 (2001); I. Padron *et al.*, *ibid.* **66**, 044608 (2002); Z. H. Liu *et al.*, *Eur. Phys. J. A* **26**, 73 (2005); J. Skalski, *Phys. Rev. C* **76**, 044603 (2007); S. Mitsuoka, H. Ikezoe, K. Nishio, K. Tsuruta, S. C. Jeong, and Y. Watanabe, *Phys. Rev. Lett.* **99**, 182701 (2007); A. M. Stefanini *et al.*, *Phys. Rev. C* **78**, 044607 (2008); K. Washiyama and D. Lacroix, *ibid.* **78**, 024610 (2008); C. J. Lin, H. M. Jia, H. Q. Zhang, F. Yang, X. X. Xu, F. Jia, Z. H. Liu, and K. Hagino, *ibid.* **79**, 064603 (2009); A. M. Stefanini *et al.*, *Phys. Lett. B* **679**, 95 (2009).
- [16] A. Morsad, J. J. Kolata, R. J. Tighe, X. J. Kong, E. F. Aguilera, and J. J. Vega, *Phys. Rev. C* **41**, 988 (1990).
- [17] S. Gary and C. Volant, *Phys. Rev. C* **25**, 1877 (1982).
- [18] S. B. DiCenzo, J. F. Petersen, and R. R. Betts, *Phys. Rev. C* **23**, 2561 (1981).
- [19] E. F. Aguilera, J. J. Kolata, P. A. DeYoung, and J. J. Vega, *Phys. Rev. C* **33**, 1961 (1986).
- [20] R. Liguori Neto, J. C. Acquadro, P. R. S. Gomes, A. Szanto de Toledo, C. F. Tenreiro, E. Crema, N. C. Filho, and M. M. Coimbra, *Nucl. Phys. A* **512**, 333 (1990).
- [21] J. O. Newton, C. R. Morton, M. Dasgupta, J. R. Leigh, J. C. Mein, D. J. Hinde, H. Timmers, and K. Hagino, *Phys. Rev. C* **64**, 064608 (2001).
- [22] B. Sikora, J. Bisplinghoff, W. Scobel, M. Beckerman, and M. Blann, *Phys. Rev. C* **20**, 2219 (1979).
- [23] J. R. Leigh *et al.*, *Phys. Rev. C* **52**, 3151 (1995).
- [24] C. Y. Wong, *Phys. Lett. B* **42**, 186 (1972); *Phys. Rev. Lett.* **31**, 766 (1973).