Angular momentum dependence of the nuclear level density parameter

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Dependence of nuclear level density parameter on the angular momentum and temperature is investigated in a theoretical framework using the statistical theory of hot rotating nuclei. The structural effects are incorporated by including shell correction, shape, and deformation. The nuclei around $Z \approx 50$ with an excitation energy range of 30 to 40 MeV are considered. The calculations are in good agreement with the experimentally deduced inverse level density parameter values especially for ^{109}In, ^{113}Sb, ^{122}Te, ^{123}I, and ^{127}Cs nuclei.

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The nuclear level density (NLD) parameter is an important ingredient in the statistical model calculations of nuclear cross sections that are needed in many applications of fusion or fission reactor designs and in astrophysical thermonuclear rate calculation for nucleosynthesis [1]. The importance of the level density parameter in nuclear physics has been highlighted by many researchers, especially Bethe [2], Moretto [3], and Bohr and Mottleson [4]. Some of the most important concepts upon which our current understanding of the structure of low-lying nuclear shell levels is based include shell effects, pairing correlations, and collective phenomena.

The NLD parameter dependence on temperature has been investigated by various theoretical [5–14] and experimental [15–18] approaches. But the dependence of level density parameter on spin needs more attention. Most of the recent experimental efforts are limited to low excitation energy (≈10–20 MeV) and low spin of a few $\hbar$. Hence they are able to bring out the microscopic effects from shell structure that are predominant at low excitation energy but melt away at higher excitations. In our earlier works [19–21], we have reported nuclear level density and neutron emission spectra dependence on temperature and spin and it was shown that the structural transitions caused by angular momentum and temperature have influence on the level density of states and the neutron emission probability. A recent experimental work [22] has reported the angular momentum dependence of the level density parameter of the residual nuclei formed through the heavy-ion fusion reactions. In that work the residual nuclei are in the range of $Z_R = 48$–55 with excitation energy range from 30 to 40 MeV. The inverse level density parameter $K(=A/a)$ is found to be in the range of 9.0–10.5 for all the systems investigated. No microscopic interpretation of the observed results has been provided. In the present work, we investigate these residual nuclei $Z_R = 48$–55 in a theoretical framework from a microscopic point of view. As a part of this investigation, we also determine the shape and the deformation of these nuclei to study the influence of structural effects on the level density parameter.

We use the statistical theory of a hot rotating nucleus [23,24] with the grand canonical partition function of the superfluid system in terms of the single-particle eigenvalues ϵ_i and the z component of the spin projection, m_i, of the deformed oscillator potential of the Nilsson Hamiltonian

$$Q(\alpha_Z, \alpha_N, \beta, \gamma) = \sum \exp(-\beta E_i + \alpha_Z Z_i + \alpha_N N_i + \gamma M_i). \tag{1}$$

The basic ingredient in the statistical theory is a suitable shell-model level scheme for various deformations, which is generated by assuming the nucleons move in a deformed oscillator potential of the Nilsson Hamiltonian, diagonalized with cylindrical basis states [25,26] with the Hill-Wheeler [27] deformation parameter. The levels up to $N = 11$ shells of the Nilsson model with Seegar parameters [28] are used. The single-particle level schemes are different for protons and neutrons. The value of the angular deformation parameter θ ranges from -180° (oblate with symmetry axis parallel to the rotation axis) to -120° (prolate with symmetry axis perpendicular to the rotation axis) and then to -60° (oblate collective) to 0° (prolate noncollective). The axial deformation parameter δ ranges from 0 to 0.4.

Lagrangian multipliers α, β, and γ conserve the particle number, total energy, and the angular momentum of the system and are fixed by the saddle-point equations. The conservation equations in terms of the single-particle eigenvalue ϵ_i with spin projection m_i [29], at a temperature $T(=1/\beta)$, are

$$\langle Z \rangle = \sum n_i^Z = \sum \left[1 + \exp\left(-\alpha_Z + \beta\epsilon_i - \gamma m_i^Z\right)\right]^{-1}, \tag{2}$$

$$\langle N \rangle = \sum n_i^N = \sum \left[1 + \exp\left(-\alpha_N + \beta\epsilon_i - \gamma m_i^N\right)\right]^{-1}, \tag{3}$$

$$\langle E(M,T) \rangle = \sum n_i^Z \epsilon_i^Z + \sum n_i^N \epsilon_i^N, \tag{4}$$

$$\langle M \rangle = \sum n_i^Z m_i^Z + \sum n_i^N m_i^N. \tag{5}$$

Here n_i is the occupation probability.

The excitation energy of the system is derived as

$$E^*(M,T) = E(M,T) - E(0,0), \tag{6}$$

where $E(0,0)$ is the ground-state energy of the nucleus given by

$$E(0,0) = \sum \epsilon_i^Z + \sum \epsilon_i^N. \tag{7}$$

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The entropy of the system is obtained from

$$S = -\sum [n_i ln n_i + (1 - n_i) ln(1 - n_i)]. \quad (8)$$

The single-particle level density parameter $a(M, T)$ as a function of angular momentum M and temperature T is extracted using the equation

$$a(M, T) = S^2(M, T)/4E^*(M, T). \quad (9)$$

As illustrated by Moretto [3], the laboratory-fixed z axis can be made to coincide with the body-fixed z' axis and it is possible to identify and substitute M for the total angular momentum I. In the quantum-mechanical limit, the z component M of the total angular momentum is $M = M_N + M_Z \rightarrow I + 1/2$, where I is the total angular momentum.

To evaluate the shape and the deformation of the excited nuclei, we calculate the free energy (F) of the nuclear system and minimize it with respect to the deformation parameters (δ, θ) at a fixed value of temperature T and angular momentum M as done in our earlier work [30]:

$$\begin{aligned} F(Z, N, T, M, \delta, \theta) &= E_{\rm LDM}(Z, N) + \delta E_{\rm shell}(\delta, \theta) + E_{\rm def}(\delta, \theta) \\ &\quad + E^*(T, M, \delta, \theta) - TS(T, M, \delta, \theta). \end{aligned} \quad (10)$$

The symbols have their usual meanings as in Ref. [30].

The results are presented for the nuclei ^{108}Cd, ^{109}In, ^{112}Sn, ^{113}Sb, ^{122}Te, ^{123}I, and ^{127}Cs, which are investigated in Ref. [22]. The calculations are performed for a wide range of temperature ($T \approx 0.5$–2.0 MeV) and angular momentum (0–60$\hbar$) values. To compare our results with those of Ref. [22], we use $E^*(M, T) \approx 30$–40 MeV, which correspond to $T \approx 1.4$–1.5 MeV and angular momentum values $M \approx 10\hbar$–$25\hbar$, as in Ref. [22].

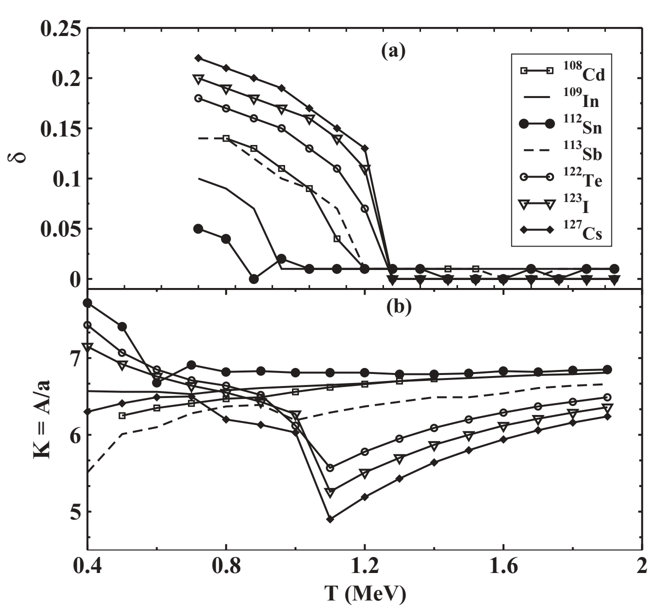

FIG. 1. (a) Axial deformation parameter δ vs temperature for ^{108}Cd, ^{109}In, ^{112}Sn, ^{113}Sb, ^{122}Te, ^{123}I, and ^{127}Cs nuclei. (b) Inverse level density parameter K variation vs temperature for the same nuclei as in Fig. 1(a).

In Figs. 1(a) and 1(b), we have shown the influence of structural transitions on the level density parameter a and inverse level density parameter $K = A/a$ for ^{108}Cd, ^{109}In, ^{112}Sn, ^{113}Sb, ^{122}Te, ^{123}I, and ^{127}Cs nuclei. The nucleus ^{112}Sn is least deformed [see Fig. 1(a)] and hence is expected to have the minimum a and maximum K. Deformation parameter δ slightly increases for nuclei below the shell closure and attains its highest values for nuclei above the shell closure. These shell effects are manifested in the plot of K versus T [Fig. 1(b)], where the level density parameter is minimum at a shell closure (as expected) and hence K is maximum at ^{112}Sn ($Z = 50$). As we deviate from a spherical shape with increasing deformation, the single-particle level density of states increases as several levels from higher oscillator shells, which get lowered in energy by deformation and contribute

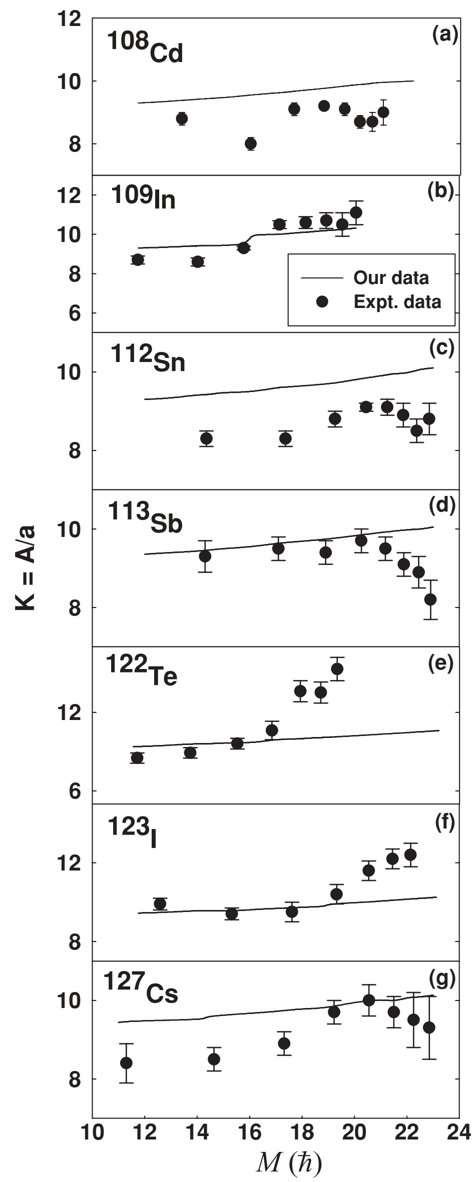

FIG. 2. Inverse level density parameter K vs angular momentum $M(\hbar)$. The solid line represents our work and points with error bars are experimental data from Ref. [22].

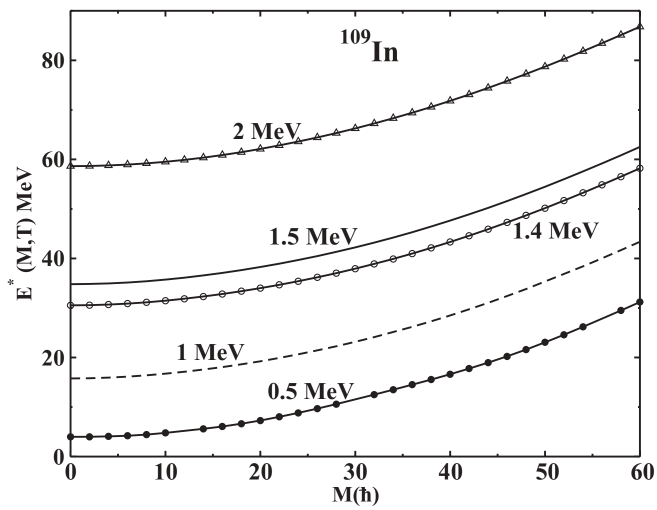

FIG. 3. Excitation energy $E^*(M, T)$ vs angular momentum $M(\hbar)$ for different temperatures for ^{109}In. Temperature values are marked on the curves.

to the level density and hence inverse level density parameter decreases and a increases. Fluctuations at smaller T imply shape and deformation changes, which get smoothed out at higher T and thus the K value remains almost constant.

The rotation of the hot nucleus deforms these nuclei that had already attained sphericity ($\delta = 0$) at $T \approx 1$ MeV. δ increases gradually with increasing M with a shape transition to oblate noncollective ($\theta = -180°$) at higher M. At $M = 60\hbar$, δ attains a high value of 0.23. Since the deformation change is gradual, the level density parameter also rises gradually without much sharp rise or fall.

In Fig. 2, we have plotted the inverse level density parameter $K = A/a(M, T)$ against the angular momentum $M(\hbar)$ for ^{108}Cd, ^{109}In, ^{112}Sn, ^{113}Sb, ^{122}Te, ^{123}I, and ^{127}Cs nuclei. The points with the error bar represent the experimental data of Ref. [22]. We vary T from 1.4 to 1.5 MeV to reproduce the excitation energy of 30 to 40 MeV for all the nuclei under investigation (with excitation energy variation versus T being shown for one of these nuclei, ^{109}In, in Fig. 3). As per the calculations, the inverse level density parameter K should increase with increasing M. This behavior is seen in our data for all the nuclear systems. On the contrary, few experimental points show a drop at larger M in the case of ^{113}Sb and ^{127}Cs. The sudden drop in K values could be either due to a sudden rise in deformation or a shape transition, which would lead to an increase in the level density parameter and decrease in K, whereas any such deformation or shape changes are not seen in our calculations. However, the nucleus ^{122}Te shows a steep rise in K values at high M. The dramatic variations in K values with M seen in the experimental data are not seen in the calculations. The origin of this discrepancy is not fully understood. In the case of ^{112}Sn, a closed-shell nucleus, the level density parameter should attain a minima and K should exhibit a maxima. However, the experimental points show a minima in K values for ^{112}Sn, which is just the opposite of what is expected and therefore our data points are much above the experimental points. The presence of shell effects is expected to be predominant at low excitation energy as is seen in Fig. 1. Shell effects melt away at T of the order of 1.4–1.5 MeV and the level density parameter varies smoothly with M in the absence of any abrupt deformation changes and shape transitions as seen in Fig. 2.

In conclusion, this work is an attempt to understand the experimentally derived level density parameter and its variation with angular momentum using a microscopic approach. The calculations are consistent with the experimental data for ^{109}In, ^{113}Sb, ^{122}Te, and ^{123}I, especially for lower values of M. The value of K varies from 9 to 10 for most of the cases experimentally as well as theoretically. The theoretical values of K increase with an increase of M whereas the corresponding K values determined from the experiment do not show a clear trend with respect to variation of angular momentum. However, both the experimental and theoretical data show the dependence of level density on the angular momentum.

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