One-dimensional hydrodynamical model including phase transition

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Analytical solution of a one-dimensional hydrodynamical model is derived, where phase transition from the quark-gluon plasma state to the hadronic state is effectively taken into account. The single-particle rapidity distribution of charged π mesons observed in relativistic heavy ion collisions is analyzed by the model. Space-time development of the fluid is also investigated.

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I. INTRODUCTION

It is widely recognized that at the initial stage of relativistic heavy ion collisions, quark-gluon plasma (QGP) is formed, and it breaks up into thousand of hadrons in the final states. It is also considered that the hydrodynamical approach is one of the effective tools [1–3] to analyze such multiple-particle production processes.

In most numerical calculations of three-dimensional hydrodynamical models, the phase transition from the QGP state to the hadronic state is taken into account [4–6]. For reviews of recent works, see Ref. [7].

Recently, exact solutions of one-dimensional hydrodynamical model have been investigated [8–11]. However, the phase transition from the QGP state is not introduced into the analytical formulations.

The hydrodynamical model consists of the energy momentum conservation, the baryon number conservation (or entropy conservation), and the equation of state. The phase transition from the QGP state to the hadronic state can be expressed by the change in the velocity of sound from that of the perfect fluid, $c_0 = 1/\sqrt{3}$, to some constant value, c_s, at the critical temperature. We assume that shear viscosity, bulk viscosity, and heat conductivity are negligibly small in the hadronic state. We also assume that the baryon number can be neglected.

The one-dimensional hydrodynamical model of the perfect fluid can be solved with an appropriate initial condition. If the change in the velocity of sound is introduced into the model, we can formulate a hydrodynamical model including the phase transition from the QGP state to the hadronic state.

In Sec. II, the one-dimensional hydrodynamical model following Landau's approach is reviewed, and a solution for potential χ is derived under an idealized initial condition. In Sec. III, a one-dimensional hydrodynamical model including the phase transition from the QGP state to the hadronic state is formulated. In Sec. IV, the single-particle distribution under the Cooper-Frye approach is shown. The single-particle rapidity distribution observed in relativistic nucleus-nucleus (AA) collisions is analyzed in Sec. V. The space-time evolution of fluid elements is also investigated there. The final section is devoted to concluding remarks.

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II. ONE-DIMENSIONAL HYDRODYNAMICAL MODEL

A. Equations for the hydrodynamical model

The hydrodynamical model proposed by Landau [1–3] is composed of the energy-momentum conservation of the perfect fluid and the equation of state. The energy-momentum conservation is given by

$$\frac{\partial}{\partial x^\nu} T^{\mu\nu} = 0, \quad \mu, \nu = 0, 1, \tag{1}$$

where $T_{\mu\nu}$ denotes the energy momentum tenser of the perfect fluid,

$$T_{\mu\nu} = (\varepsilon + p) u_\mu u_\nu - p g_{\mu\nu}. \tag{2}$$

In Eq. (2), ε denotes the energy density and p the pressure of a fluid element. The velocity of the fluid element is denoted u^μ, which satisfies $u^\mu u_\mu = 1$, and $g^{\mu\nu} = \mathrm{diag}(1, -1)$.

The equation of state is expressed as a function of the energy density ε of the pressure p. This will differ depending on whether the fluid is in the QGP state or in the hadronic state. At present we assume that the relation $dp/d\varepsilon = {c_0}^2$ holds, where c_0 is a positive constant.

In addition, thermodynamical relations,

$$\varepsilon + p = Ts, \quad d\varepsilon = T ds, \tag{3}$$

are used, where T denotes the temperature and s is the entropy density of the fluid element. Equation (3) holds whether the fluid is in the QGP state or in the hadronic state.

Projection of Eq. (1) to the direction of u_μ gives the entropy conservation,

$$\frac{\partial (s u^\nu)}{\partial x^\nu} = 0. \tag{4}$$

After Eq. (1) is projected to the direction perpendicular to u_μ, we obtain

$$-\frac{\partial T}{\partial x^\mu} + \frac{\partial (T u_\nu)}{\partial x^\mu} u^\nu = 0, \quad \mu = 0, 1. \tag{5}$$

The rapidity η of the fluid element is defined by

$$(u^0, u^1) = (\cosh\eta, \sinh\eta). \tag{6}$$

Then both Eqs. (5) reduce to

$$\frac{\partial}{\partial t}(T \sinh\eta) + \frac{\partial}{\partial x}(T\cosh\eta) = 0. \tag{7}$$

From Eq. (7), there is a function ϕ that satisfies

$$\frac{\partial\phi}{\partial t} = T\cosh\eta, \quad \frac{\partial\phi}{\partial x} = -T\sinh\eta. \tag{8}$$

By use of the Legendre transform,

$$d\chi = d(\phi - tT\cosh\eta + xT\sinh\eta),$$

we obtain the equations for the potential χ [2,3]:

$$\begin{aligned}\frac{\partial\chi}{\partial T} &= -t\cosh\eta + x\sinh\eta,\\ \frac{1}{T}\frac{\partial\chi}{\partial\eta} &= -t\sinh\eta + x\cosh\eta.\end{aligned} \tag{9}$$

Then space-time variables of the fluid elements t and x are given, respectively, as

$$\begin{aligned} t &= \frac{1}{T_0}\mathrm{e}^{\omega}\left(\frac{\partial\chi}{\partial\omega}\cosh\eta + \frac{\partial\chi}{\partial\eta}\sinh\eta\right),\\ x &= \frac{1}{T_0}\mathrm{e}^{\omega}\left(\frac{\partial\chi}{\partial\omega}\sinh\eta + \frac{\partial\chi}{\partial\eta}\cosh\eta\right),\end{aligned} \tag{10}$$

where T_0 is the initial temperature of the fluid and $\omega = \ln(T_0/T)$.

The entropy conservation, Eq. (4), is rewritten as

$$\frac{\partial}{\partial t}(s\cosh\eta) + \frac{\partial}{\partial x}(s\sinh\eta) = 0. \tag{11}$$

After changing the variables from x,t to η,T in Eq. (11), we obtain

$$\frac{T}{s}\frac{ds}{dT}\left(\frac{\partial^2\chi}{\partial\eta^2} - T\frac{\partial\chi}{\partial T}\right) - T^2\frac{\partial^2\chi}{\partial T^2} = 0.$$

B. Solution for potential χ

Using the relation $(T/s)ds/dT = {c_0}^2$ and the variable $\omega = -\ln(T/T_0)$, we obtain the partial differential equation for potential χ:

$$\begin{aligned}\frac{\partial^2\chi}{\partial\omega^2} - 2\beta_0\frac{\partial\chi}{\partial\omega} - \frac{1}{{c_0}^2}\frac{\partial^2\chi}{\partial\eta^2} &= 0,\\ \beta_0 = \frac{1-{c_0}^2}{2{c_0}^2} &= 1.\end{aligned} \tag{12}$$

Equation (12) is called the equation of telegraphy.

The initial condition for Eq. (12) is taken with a constant Q_0 as

$$\chi|_{\omega=0} = g(\eta) = 0, \quad \left.\frac{\partial\chi}{\partial\omega}\right|_{\omega=0} = G(\eta) = Q_0\delta(\eta). \tag{13}$$

The hydrodynamical model is applicable mainly to the central region [12]. As can be seen from Eq. (17), the main contribution to the fragmentation comes from the function $g(\eta)$. Therefore, we assume that $g(\eta) = 0$. At the initial stage, fluid would be formed in the very small region of rapidity space, and we set $G(\eta)$ proportional to the δ function in the rapidity space as an idealized case [13].

Introducing the new variable χ_1 as

$$\chi(\eta,\omega) = \chi_1(\eta,\omega)\mathrm{e}^{\beta_0\omega}, \tag{14}$$

we have the partial differential equation for χ_1,

$$\frac{\partial^2\chi_1}{\partial\omega^2} - {\beta_0}^2\chi_1 - \frac{1}{{c_0}^2}\frac{\partial^2\chi_1}{\partial\eta^2} = 0, \tag{15}$$

and the initial condition for Eq. (15),

$$\begin{aligned}\chi_1|_{\omega=0} &= g(\eta) = 0,\\ \left.\frac{\partial\chi_1}{\partial\omega}\right|_{\omega=0} &= G(\eta) - \beta_0 g(\eta) = Q_0\delta(\eta).\end{aligned} \tag{16}$$

The solution for χ_1 is given, in general, as [14]

$$\begin{aligned}\chi_1(\eta,\omega) = {}& \frac{1}{2}\{g(\eta+\omega/c_0) + g(\eta-\omega/c_0)\}\\ &+ \frac{c_0}{2}\int_{-\omega/c_0}^{\omega/c_0} dz\,\{G(z+\eta)\\ &- \beta_0 g(\eta)\}I_0\left(\beta_0\sqrt{\omega^2 - {c_0}^2z^2}\right)\\ &+ \frac{\beta_0 c_0\omega}{2}\int_{-\omega/c_0}^{\omega/c_0} dz\, g(z+\eta)\frac{I_1\left(\beta_0\sqrt{\omega^2-{c_0}^2z^2}\right)}{\sqrt{\omega^2-{c_0}^2z^2}}.\end{aligned} \tag{17}$$

This reduces to

$$\chi_1(\eta,\omega) = \frac{Q_0c_0}{2}I_0\left(\beta_0\sqrt{\omega^2 - {c_0}^2\eta^2}\right) \tag{18}$$

for $|\eta| < \omega/c_0$.

Then the solution for Eq. (12) under the initial condition (13) is given by

$$\chi(\eta,\omega) = \frac{Q_0c_0}{2}e^{\beta_0\omega}I_0\left(\beta_0\sqrt{\omega^2 - {c_0}^2\eta^2}\right). \tag{19}$$

III. HYDRODYNAMICAL MODEL INCLUDING A PHASE TRANSITION

A. Introduction of the phase transition

In our formulation, the phase transition from the QGP state to the hadronic state is expressed by the change in the velocity of sound. The evolution of fluid is described by the parameter $\omega = \ln(T_0/T)$, where T_0 is the initial temperature. The fluid formed just after the collision of nuclei is assumed to be in the QGP state. The velocity of sound in it is denoted c_0 ($c_0 = 1/\sqrt{3}$). It expands from $\omega = 0$ to $\omega = \omega_c - 0$, where $\omega_c = \ln(T_0/T_c) > 0$ with the critical temperature T_c. The phase transition from the QGP to the hadronic state occurs at $\omega = \omega_c$. From $\omega = \omega_c + 0$ to $\omega = \omega_f$, where $\omega_f = \ln(T_0/T_f) > \omega_c$ with the freeze-out temperature T_f, the fluid is in the hadronic state. The velocity of sound in it is assumed to be constant and is denoted c_s ($0 < c_s \leqslant c_0$).

The equation of state is assumed to be

$$p = \begin{cases} {c_0}^2\varepsilon - (1+{c_0}^2)B, & \text{for} \quad 0 < \omega < \omega_c,\\ {c_s}^2\varepsilon, & \text{for} \quad \omega_c < \omega < \omega_f,\end{cases} \tag{20}$$

where B is the bag constant of hadrons. From Eq. (20), the velocity of sound is given by

$$\sqrt{\frac{dp}{d\varepsilon}} = \begin{cases} c_0, & \text{for} \quad 0 < \omega < \omega_c,\\ c_s, & \text{for} \quad \omega_c < \omega < \omega_f.\end{cases} \tag{21}$$

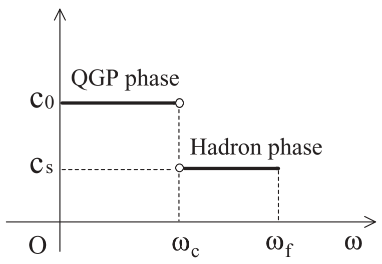

FIG. 1. Velocity of sound as a function of ω from $\omega = 0$ to $\omega = \omega_f$.

As shown in Fig. 1, the velocity of sound is discontinuous at $\omega = \omega_c$.

B. Potential $\chi(\eta,\omega)$ in quark-gluon plasma fluid

We assume that the fluid in the QGP phase, where the velocity of sound is $c_0 = 1/\sqrt{3}$, continues from $\omega = 0$ to $\omega = \omega_c - 0$. Then the partial differential equation for the potential $\chi(\eta,\omega)$ is given by Eq. (12):

$$\frac{\partial^2\chi}{\partial\omega^2} - 2\beta_0\frac{\partial\chi}{\partial\omega} - \frac{1}{c_0{}^2}\frac{\partial^2\chi}{\partial\eta^2} = 0, \tag{22}$$

where $\beta_0 = (1 - c_0{}^2)/2c_0{}^2 = 1$. The initial condition for Eq. (22) is taken as Eq. (13):

$$\chi|_{\omega=0} = g(\eta) = 0, \quad \left.\frac{\partial\chi}{\partial\omega}\right|_{\omega=0} = G(\eta) = Q_0\delta(\eta). \tag{23}$$

The solution for Eq. (22) under the initial condition, Eq. (23), is given by

$$\chi(\eta,\omega) = \frac{Q_0c_0}{2}e^{\beta_0\omega}I_0(\beta_0\sqrt{\omega^2 - c_0{}^2\eta^2}). \tag{24}$$

C. Potential $\chi(\eta,\omega)$ in hadronic fluid

In the hadronic phase, the fluid expands from $\omega = \omega_c + 0$ to $\omega = \omega_f$. During the expansion of the hadronic fluid, the velocity of sound is c_s.

For $\omega_c < \omega < \omega_f$, the partial differential equation for potential χ is given by

$$\frac{\partial^2\chi}{\partial\omega^2} - 2\beta\frac{\partial\chi}{\partial\omega} - \frac{1}{c_s{}^2}\frac{\partial^2\chi}{\partial\eta^2} = 0, \quad \beta = \frac{1 - c_s{}^2}{2c_s{}^2} \geqslant 1. \tag{25}$$

For $|\eta| < [(\omega - \omega_c)/c_s] + (\omega_c/c_0)$, the solution of Eq. (25) is given by

$$\chi(\eta,\omega) = A(\omega)I_0(\lambda), \qquad A(\omega) = \frac{Q_0c_0}{2}e^{\beta(\omega-\omega_c)+\beta_0\omega_c}, \tag{26}$$

where

$$\lambda = \beta c_s\sqrt{\eta_{\max}^2 - \eta^2}, \qquad \eta_{\max} = [(\omega - \omega_c)/c_s] + (\omega_c/c_0). \tag{27}$$

If $c_s = c_0$ ($\beta = \beta_0$), the potential $\chi(\eta,\omega)$ in Eq. (26) coincides with Eq. (24), which is the solution for Eq. (22) under the initial condition, Eq. (23).

D. Effect of phase transition on potential $\chi(\eta,\omega)$

In our formulation, the velocity of sound is changed from c_0 to c_s according to the phase transition from the QGP state to the hadronic state. This effect would appear in potential $\chi(\eta,\omega)$ in the neighborhood of $\omega = \omega_c$.

At $\omega = \omega_c - 0$, where the fluid is in the QGP state, the potential is given from Eq. (24) as

$$\chi(\eta,\omega_c - 0) = \frac{Q_0c_0}{2}e^{\beta_0\omega_c}I_0(\beta_0\sqrt{\omega_c{}^2 - c_0{}^2\eta^2}). \tag{28}$$

At $\omega = \omega_c + 0$, where the fluid is in the hadronic state, it is given from Eqs. (26) and (27) as

$$\chi(\eta,\omega_c + 0) = \frac{Q_0c_0}{2}e^{\beta_0\omega_c}I_0\left(\frac{\beta c_s}{c_0}\sqrt{\omega_c{}^2 - c_0{}^2\eta^2}\right). \tag{29}$$

From Eqs. (28) and (29), we obtain

$$\frac{\chi(\eta,\omega_c + 0)}{\chi(\eta,\omega_c - 0)} = \frac{I_0\left(\frac{\beta c_s}{c_0}\sqrt{\omega_c{}^2 - c_0{}^2\eta^2}\right)}{I_0\left(\beta_0\sqrt{\omega_c{}^2 - c_0{}^2\eta^2}\right)} \neq 1. \tag{30}$$

The potential $\chi(\eta,\omega)$ becomes discontinuous at $\omega = \omega_c$ owing to the phase transition. This fact will affect the space-time behavior of the fluid element.

IV. SINGLE-PARTICLE RAPIDITY DISTRIBUTION

To analyze the rapidity distribution of observed particles, the Cooper-Frye approach [15] is used. Then the single-particle rapidity distribution dn/dy of particles with mass m and transverse momentum p_T in nucleus-nucleus collisions is given by

$$\frac{dn}{dy} = \frac{\pi R_A{}^2}{(2\pi)^3}\int_\sigma \left.\left(\cosh y\frac{dx}{d\eta} - \sinh y\frac{dt}{d\eta}\right)\right|_{\omega=\omega_f} \times \frac{m_T}{\exp[m_T\cosh(y-\eta)/T_f] - 1}d\eta d^2p_T, \tag{31}$$

where R_A denotes the radius of colliding nuclei and $m_T = \sqrt{p_T{}^2 + m^2}$ denotes the transverse mass of observed particles. Space-time variables t and x in Eq. (31) are given by Eq. (10). Under the assumption that the Bose-Einstein distribution can be approximated by the Maxwell-Boltzmann distribution, Eq. (31) is written as

$$\frac{dn}{dy} = \frac{R_A{}^2T_f{}^2}{4\pi}\int_{-\eta_{\max}}^{\eta_{\max}} d\eta\left[-\frac{\partial}{\partial\eta}\left.\left(\chi + \frac{\partial\chi}{\partial\omega}\right)\right|_{\omega=\omega_f}\tanh(y-\eta) + c_s{}^2\frac{\partial}{\partial\omega}\left.\left(\chi + \frac{\partial\chi}{\partial\omega}\right)\right|_{\omega=\omega_f}\right]\left(\frac{m^2}{T_f{}^2} + \frac{2m}{T_f\cosh(y-\eta)} + \frac{2}{\cosh^2(y-\eta)}\right)e^{-m\cosh(y-\eta)/T_f}. \tag{32}$$

If the term multiplied by $\tanh(y-\eta)$ in the square bracket on the right-hand side of Eq. (32) is neglected, the single-particle rapidity distribution coincides with Milekhin's approach [16,17] except for the normalization factor.

By the use of Eqs. (26) and (27), the single-particle distribution is given by

$$\frac{dn}{dy} = \frac{R_A{}^2 T_f{}^2}{4\pi}(\beta c_s)^2 A(\omega_f)\int_{-\eta_{\max}}^{\eta_{\max}} d\eta \left[\frac{\beta\eta}{\lambda}\left\{\frac{\beta c_s\eta_{\max}}{\lambda} I_0(\lambda) + \left(\frac{\beta+1}{\beta} - 2\frac{\beta c_s \eta_{\max}}{\lambda^2}\right) I_1(\lambda)\right\}\tanh(y-\eta) + \left\{\left(\frac{1+\beta}{\beta} + \frac{(\beta c_s\eta_{\max})^2}{\lambda^2}\right) I_0(\lambda) + \frac{\beta}{\lambda}\left(\frac{\eta_{\max}}{c_s}+1 - 2\frac{(\beta c_s\eta_{\max})^2}{\lambda^2}\right) I_1(\lambda)\right\}\right]\left(\frac{m^2}{T_f{}^2} + \frac{2m}{T_f\cosh(y-\eta)} + \frac{2}{\cosh^2(y-\eta)}\right) e^{-m\cosh(y-\eta)/T_f}. \tag{33}$$

Space-time variables of the fluid element are written from Eqs. (10) and (26) as

$$t = \frac{1}{T_0}A(\omega)e^{\omega}\left[\beta\left\{I_0(\lambda) + \beta c_s\frac{\eta_{\max}}{\lambda}I_1(\lambda)\right\}\cosh\eta - (\beta c_s)^2\frac{\eta}{\lambda}I_1(\lambda)\sinh\eta\right], \tag{34}$$

$$x = \frac{1}{T_0}A(\omega)e^{\omega}\left[\beta\left\{I_0(\lambda) + \beta c_s\frac{\eta_{\max}}{\lambda}I_1(\lambda)\right\}\sinh\eta - (\beta c_s)^2\frac{\eta}{\lambda}I_1(\lambda)\cosh\eta\right]. \tag{35}$$

V. DATA ANALYSIS

The single-particle rapidity distribution of charged π mesons in Au-Au collisions at 200 AGeV at the RHIC [18] is analyzed by Eq. (33). In the analysis, the initial temperature T_0 is fixed at 0.95 GeV, the critical temperature T_c at 0.18 GeV, and the freeze-out temperature T_f at 0.12 GeV. The nuclear radius R_A is parametrized as $R_A = 1.2 \times A^{1/3}$, where A denotes the mass number of the nucleus and $A = 197$ for Au. Then two parameters, Q_0 and c_s, remain in our formulation. Estimated parameters are listed in Table I, and the result of the single-particle rapidity distribution of charged π mesons is shown in Fig. 2.

The space-time evolution of fluid elements at $\omega =$ constant in the x-t plane is shown in Fig. 3. The profile at $T = T_c = 0.180$ GeV is overlapped at the origin $(x,t) = (0,0)$. The dotted curve denoted BJ represents Bjorken's scaling solution, $\tau = \sqrt{t^2 - x^2} =$ constant [19].

TABLE I. Estimated values of parameters Q_0 and $c_s{}^2$ from Au + Au $\to [(\pi^+ + \pi^-)/2] + X$

Q_0	$(4.78 \pm 1.15) \times 10^{-2}$
$c_s{}^2$	0.117 ± 0.005
$\chi^2_{\min}$/n.d.f.	41.4/12

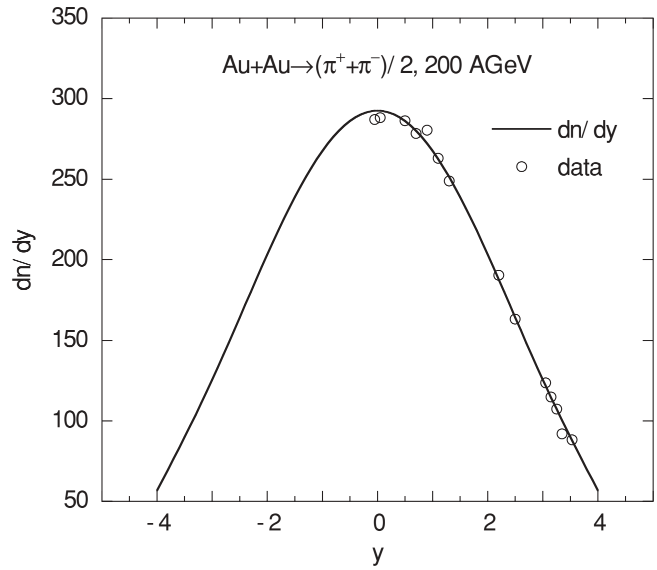

FIG. 2. Analysis of the single-particle rapidity distribution of charged pions $(\pi^+\pi^-)/2$ by Eq. (33).

To investigate the space-time evolution of fluid elements around $\omega = \omega_c$ or $T = T_c$, details of it are shown in Fig. 4. The profile at $\omega = \omega_c - 0$ or $T = T_c + 0$ is in the QGP state. It is calculated by Eq. (24). The profiles at $\omega > \omega_c$ and $T < T_c$ are in the hadronic states. These are calculated by Eqs. (26) and (27). As shown in Fig. 4, the region between the profile at $T = T_c + 0$ and that at $T = T_c - 0$ corresponds to the mixed phase where the QGP state and the hadronic state coexist.

The temperature dependence of fluid element at $\eta =$ constant is shown as a function of proper time, $\tau = \sqrt{t^2 - x^2}$, in Fig. 5. The temperature profiles of fluid elements for $T > 0.200$ GeV are abbreviated. Fluid elements with a larger rapidity in the absolute value are cooled faster, as shown in the figure. The fluid starts expansion from $T = T_0$ at $\eta = 0$ and

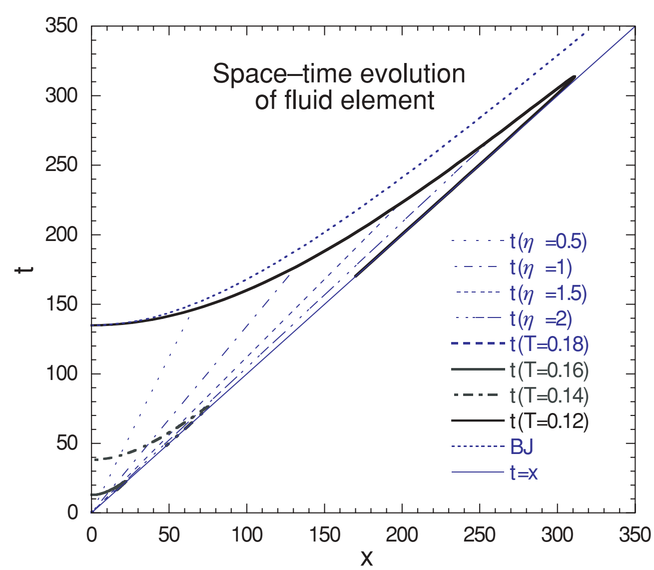

FIG. 3. (Color online) Space-time evolution of fluid elements at a fixed temperature.

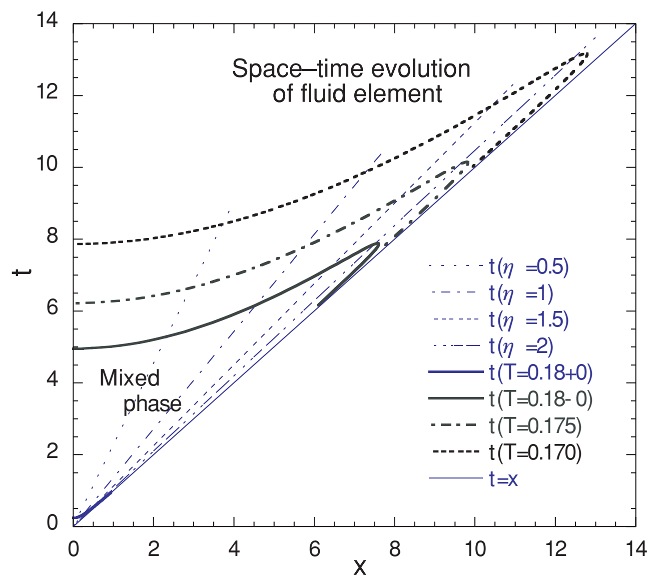

FIG. 4. (Color online) Space-time evolution of fluid elements at a fixed temperature from $T = 0.180$ GeV to $T = 0.170$ GeV.

at $\tau = 0$. For example, the fluid element at $\eta = 0$ is cooled down from $T = T_0$ at $\tau = 0$ to $T = T_c$ at $\tau = 0.238$ fm/c. It remains at $T = T_c$ from $\tau = 0.238$ fm/c to $\tau = 4.957$ fm/c. Therefore, the mixed phase in the neighborhood of $\eta = 0$ is to continue about 4.7 fm/c. The fluid element at $\eta = 0$ is cooled down to $T = T_f$ at $\tau = 134.9$ fm/c. The dotted curve denoted BJ corresponds to Bjorken's scaling solution, $T = T_c(\tau_c/\tau)^{c_s{}^2}$ with ${c_s}^2 = 0.117$, which is normalized to T_c at $(\tau, T) = (\tau_c, T_c) = (0.238, 0.18)$.

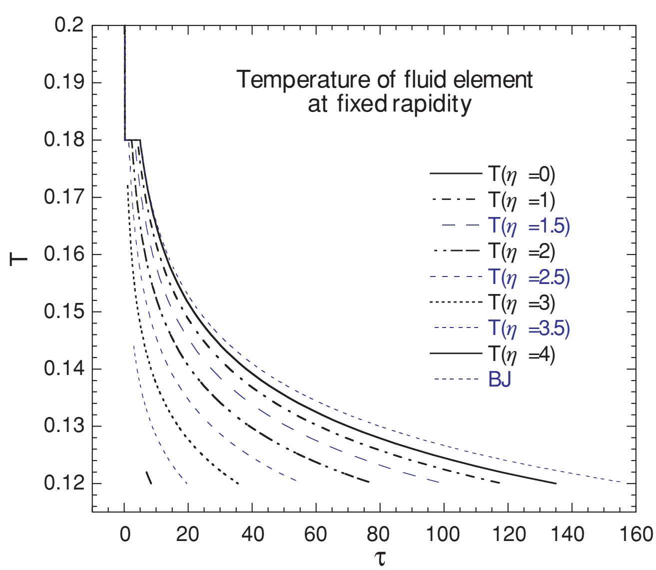

FIG. 5. (Color online) Temperature profiles of fluid elements at fixed rapidity.

VI. CONCLUDING REMARKS

We have formulated a one-dimensional hydrodynamical model including the phase transition from the QGP state to the hadronic state. At first, following Landau's hydrodynamical model, the equation of the telegraphy for potential $\chi(\eta,\omega)$ is solved under the simplified initial condition in the η-ω space with the velocity of sound, $c_0 = 1/\sqrt{3}$. Then the solution of it in the hadronic state with a constant velocity of sound, c_s ($c_s \neq c_0$), is found so as to coincide with the solution in the QGP state if $c_s = c_0$.

The space-time evolution of fluid elements from $\omega = 0$ (or $T = T_0$) to $\omega = \omega_f$ (or $T = T_f$) is calculated by our model. The nonzero finite region emerges between the profile at $\omega = \omega_c - 0$ (or $T = T_c + 0$) and that at $\omega = \omega_c + 0$ (or $T = T_c - 0$). This is caused by the discontinuity of the potential $\chi(\eta,\omega)$ at $\omega = \omega_c$ (or $T = T_c$) caused by the change in the velocity of sound from c_0 in the QGP state to c_s ($c_s < c_0$) in the hadronic state at $\omega = \omega_c$. In our calculation, the fluid element at $\eta = 0$ freezes out at $\tau = 134.9$ fm/c.

In computer simulations of the three-dimensional hydrodynamical models, the evolution of fluid elements is calculated as a function of the proper time τ. For example, the initial condition is taken as $\tau = 0.6$ fm/c and $T_0 = 0.36$ GeV at 200 AGeV [6].

In our one-dimensional hydrodynamical model, the initial condition is taken as $\tau = 0$ fm/c and $T_0 = 0.95$ GeV. The fluid element at $\eta = 0$ is cooled down to $T = 0.36$ GeV at $\tau = 0.036$ fm/c. Therefore, the temperature of the fluid in our calculation at $\tau = 0.6$ fm/c is not higher than that in the three-dimensional hydrodynamical model.

In computer simulations at 200 AGeV, the fluid element expands a few hundred femtometers along the direction of colliding nuclei before freeze-out, although the value of τ at freeze-out is about 15 fm/c [20].

The proper time of fluid elements at $\eta = 0$ at freeze-out in our calculation is 1 order larger than the proper time of fluid elements at freeze-out in the three-dimensional computer simulation. In the one-dimensional hydrodynamical model, the expansion of fluid into the transverse dimension is neglected, contrary to the three-dimensional calculations. Therefore, the temperature of the fluid element, especially in the neighborhood of $\eta = 0$, would decrease more slowly than that calculated by the three-dimensional computer simulation, where the transverse expansion of the fluid is taken into account. However, the scale of expansion of the fluid element in the space variable in our calculation shown in Fig. 3 is comparable to that along the direction of colliding nuclei in the computer simulation.

In recent lattice QCD calculations, it is shown that the crossover phase transition occurs at chemical potential $\mu = 0$ [21,22]. Let the crossover transition take place in the region $[\omega_c - (\Delta\omega/2),\ \omega_c + (\Delta\omega/2)]$. In our formulation, it would be expressed by the smooth change in the velocity of sound from c_0 to c_s. The sound of velocity $c_v(\omega)$ in $[\omega_c - (\Delta\omega/2),\ \omega_c + (\Delta\omega/2)]$ is assumed to be a continuous function with a positive value and to satisfy $c_v[\omega_c - (\Delta\omega/2)] = c_0$ and $c_v[\omega_c + (\Delta\omega/2)] = c_s$.

The potential $\chi(\omega)$ in $[0, \omega_c - (\Delta\omega/2)]$ is the same with Eq. (24). The potential $\chi(\omega)$, after the crossover transition, in $[\omega_c + (\Delta\omega/2), \omega_f]$ would be given by the equation

$$\chi(\eta,\omega) = A(\omega) I_0(\lambda), \tag{36}$$

where

$$A(\omega) = \frac{Q_0 c_0}{2} \exp\left\{[\beta[\omega - \omega_c - (\Delta\omega/2)] + \beta_0[\omega_c - (\Delta\omega/2)] + \int_{\omega_c-(\Delta\omega/2)}^{\omega_c+(\Delta\omega/2)} \beta_v(x) dx\right\},$$
$$\beta_v(x) = \frac{1 - c_v(x)^2}{2 c_v(x)^2}, \tag{37}$$

$$\lambda = \beta c_s \sqrt{\eta_{\max}^2 - \eta^2},$$
$$\eta_{\max} = \{[\omega - \omega_c - (\Delta\omega/2)]/c_s\} + \{[\omega_c - (\Delta\omega/2)]/c_0\} + \int_{\omega_c-(\Delta\omega/2)}^{\omega_c+(\Delta\omega/2)} \frac{1}{c_v(x)} dx. \tag{38}$$

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