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Examining the efficacy of isotope thermometry in the *S*-matrix approach

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Isotope thermometry, widely used to measure the temperature of a hot nuclear system formed in energetic nuclear collisions, is examined in the light of the *S*-matrix approach to the nuclear equation of state of disassembled nuclear matter. Scattering between produced light fragment pairs, hitherto neglected, is seen to have an important bearing on extraction of the system temperature and volume at freeze-out from isotope thermometry. Taking due care with the scattering effects and decay of the primary fragments, a more reliable way to extract the nuclear thermodynamic parameters, by exploiting the least-squares fit to the observed fragment multiplicities, is suggested.

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Theoretical investigations of the equation of state of infinite [1–4] and finite [5–10] nuclear matter predict the existence of a liquid-gas (LG)-type phase transition in these systems. This transition is thought to play an important role in nucleosynthesis in supernova explosions [11,12]. Laboratory experiments in collisions between energetic nuclei appear to reveal signals of an LG phase transition in hot finite nuclear systems [13–15]. Proper identification of such a transition, however, depends on the reliable measurement of the thermodynamic observables. In particular, temperature plays a pivotal role. A widely used practice to extract the temperature of hot nuclear systems is double-isotope ratio thermometry as suggested by Albergo *et al.* [16]. In an ideal scenario, the primary fragments produced in the freeze-out volume are assumed to be in their ground states. Particle and γ-decay corrections to the excited primary fragments have also been built in [17]. Generally, the feeding effect of secondary decay has been accounted for through a correction factor [13,18–20] on the measured multiplicities. The temperature is seen to increase by $\sim$10%–20% from the ideal situation.

All these analyses were done with the assumption that the fragment species produced are noninteracting within the freeze-out volume. Strong interaction corrections, appropriately taken up in the *S*-matrix approach [21], to the grand partition function of the dilute nuclear system, where, in addition to all the stable mass particles, the two-body scattering channels between them can be included systematically, are seen to modify the fragment multiplicities [22,23]. The extracted temperature as obtained in the previous analyses without strong interaction corrections may then differ from the real temperature at which the fragments were produced. In a schematic calculation [24] using the *S*-matrix approach in dilute infinite nuclear matter, neglecting secondary decay, we found that the scattering effects on the extracted temperature and volumes are not negligible. In the present communication, these ideas are incorporated to provide a realistic framework for analysis of the data in an experimental multifragmentation setup to extract the temperature and volume of a finite disassembling nucleus at freeze-out with explicit inclusion of γ and particle decay as well as effects from scattering between different fragment species.

The details of the *S*-matrix approach, as applied to nuclear systems, are given in Refs. [23–25]. For completeness, a few relevant equations are presented here highlighting the approximations. The grand partition function $\mathcal{Z}$ of a system in thermodynamic equilibrium can be written as the sum of three terms [24]:

$$\ln \mathcal{Z} = \ln \mathcal{Z}_{\rm gr} + \ln \mathcal{Z}^0_{\rm ex} + \ln \mathcal{Z}_{\rm sc}. \tag{1}$$

The first and second terms correspond to the contributions from the ground states and particle-stable excited states of all the produced fragment species behaving like an ideal quantum gas. The last term sums up the contributions from the scattering states, expressible in terms of the *S*-matrix elements. Formal expressions for these three terms are spelled out in Ref. [23].

The scattering channels, for convenience, can be separated into two parts, one containing only light particles and the other heavy ones; that is,

$$\ln \mathcal{Z}_{\rm sc} = \ln \mathcal{Z}^l_{\rm sc} + \ln \mathcal{Z}^h_{\rm sc}. \tag{2}$$

The scattering of the heavy particles is dominated by a multitude of resonances near the threshold; the *S*-matrix elements can then be approximated by resonances, which, like the excited states, are again treated as ideal-gas terms [26]. These are the particle-unstable states. Structurally, $\ln \mathcal{Z}^h_{\rm sc}$ being thus similar to $\ln \mathcal{Z}^0_{\rm ex}$, $\ln \mathcal{Z}^0_{\rm ex}$ and $\ln \mathcal{Z}^h_{\rm sc}$ are combined to give $\ln \mathcal{Z}_{\rm ex}$ ($\equiv \ln \mathcal{Z}^0_{\rm ex} + \ln \mathcal{Z}^h_{\rm sc}$), which contains contributions from particle-unstable excited states in addition to particle-stable ones.

In $\ln \mathcal{Z}^l_{\rm sc}$, only the elastic scattering channels for the pairs NN, Nt, N^3He, $N\alpha$ (N refers to the nucleon), and $\alpha\alpha$ have been included. These calculations involve virial coefficients [23,27] that are functions of only experimental entities, namely, phase shifts and binding energies. Once the partition function is obtained, total fragment multiplicities Y_i for the ith fragment species with N_i neutrons and Z_i protons

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can be evaluated as

$$Y_i = \zeta_i \left(\frac{\partial}{\partial \zeta_i} \ln \mathcal{Z} \right)_{V,T}. \tag{3}$$

Here $\zeta_i [\equiv \zeta_{Z_i,N_i}]$ is the effective fugacity, defined as $\zeta_{Z_i,N_i} = e^{\beta[\mu_{Z_i,N_i} + B(A_i,Z_i)]}$. $B(A_i, Z_i)$ is the binding energy of the fragment and μ_{Z_i,N_i} is its chemical potential, which, from chemical equilibrium, is $\mu_{Z_i,N_i} = N_i\mu_n + Z_i\mu_p$, μ_n and μ_p being the neutron and proton chemical potentials obtained from the conservation of the total neutron and proton numbers of the system, respectively, and β the inverse temperature.

For a relatively low density and not too low a temperature, the fugacity $\zeta \ll 1$ (as is the case for the density and temperature domain we have investigated). The quantal distribution can then be replaced by a classical one; expressions for the primary fragment multiplicities of the ith species are then derived as

$$Y_i = V \frac{A_i^{3/2}}{\lambda^3} e^{[\mu_n N_i + \mu_p Z_i + B(A_i,Z_i)]/T} \times \left[g_0^i + \sum_{\epsilon_j=\epsilon_0}^{\epsilon_r} g_j^i e^{-\epsilon_j^i/T} \right] + Y_{\rm sc}^i. \tag{4}$$

In Eq. (4), V is the volume of the system, $\lambda = \sqrt{2\pi/mT}$ (we use natural units $\hbar = c = 1$) is the nucleon thermal wavelength, and g_0^i and g_j^i are the degeneracies of the ground and excited states. The sum over the excited states includes both γ and particle-decay (resonance) channels. In different variants of the models of nuclear statistical equilibrium, only the first term (which also implicitly contains scattering corrections from resonances in heavy fragments) on the right-hand side of Eq. (4) has been used to obtain the nuclear thermodynamic observables. The last term $Y_{\rm sc}^i$ is the contribution to the fragment yield from scattering, it is nonzero only for the fragments in the light species set. Expressions for the multiplicity yields $Y_{\rm sc}^n$, $Y_{\rm sc}^p$, etc. (collectively written as $Y_{\rm sc}^l$), are given in Ref. [22]. Henceforth, corrections obtained with the use of $Y_{\rm sc}^l$ in the extraction of nuclear parameters are called scattering corrections.

The multiplicities of the primary excited fragments as obtained in Eq. (4) undergo changes because of subsequent particle emission. The secondary yield can be written in terms of the variables V, μ_n, μ_p, and T at freeze-out as follows. For light fragments ($A_i \leqslant 4$, $Z_i \leqslant 2$),

$$Y_i(A_i, Z_i) = V g_0^i \frac{A_i^{3/2}}{\lambda^3} e^{\{[N_i\mu_n + Z_i\mu_p + B(A_i,Z_i)]/T\}} + V \sum_j \sum_{k_j} \left\{ \frac{A_j^{3/2}}{\lambda^3} e^{\{[N_j\mu_n + Z_j\mu_p + B(A_j,Z_j)]/T\}} \times \omega_p^{k_j}(A_j, Z_j, T) x_i^{k_j}(A_j, Z_j, T) \right\} + Y_{\rm sc}^i. \tag{5}$$

The light fragments are assumed to be produced only in their ground states, their multiplicities being given by the sum of the first and last terms in Eq. (5). Their population is further fed from decay of heavier species given by the second term. The sum j runs over all species with $A_j > 4$ and $Z_j \geqslant 2$ having particle-unstable excited states and the sum k_j runs over all the particle-decaying states of the jth species. The quantity $x_i^{k_j}$ corresponds to the branching ratio of the k_jth state for emitting the ith species; it is calculated using the Weisskopf-Ewing model [28]. The quantity $\omega_p^{k_j}(A_j, Z_j, T)$ is the internal partition function for the particle-unstable states,

$$\omega_p^{k_j}(A_j, Z_j, T) = g_{k_j} e^{-\epsilon_{k_j}/T}. \tag{6}$$

For heavy particles ($A > 4$, $Z \geqslant 2$), the observed yield is

$$Y(A, Z) = V \frac{A^{3/2}}{\lambda^3} e^{[N\mu_n + Z\mu_p + B(A,Z)]/T} \left\{ g_0(A) + \omega_\gamma(A, Z, T) + \sum_{k_j} \sum_{i=1}^{6} \left(\frac{A + a_i}{A} \right)^{3/2} e^{[n_i\mu_n + z_i\mu_p + B(A+a_i, Z+z_i) - B(A,Z)]/T} \times \omega_p^{k_j}(A + a_i, Z + z_i, T) x_i^{k_j}(A + a_i, Z + z_i, T) \right\}. \tag{7}$$

In Eq. (7), ω_γ ($= \sum_k g_k e^{-\epsilon_k/T}$) is the partition function for γ-decaying states, and the sum i runs over the emitted ejectiles, for which we take only n, p, d, t, and ^{3}He, with α, a_i, and z_i being their mass and charge. Kolomiets *et al.* [17] also arrived at expressions of the type given in Eqs. (5) and (7), the important difference being the absence of the scattering correction and consideration of only the dominant decay mode. They further considered only nucleon and α-decay channels. Moreover, the feeding to the light-fragment yield was neglected. Given a set of experimental yields for four fragments, their single ratios are constructed using Eqs. (5) and (7), resulting in a system of three independent equations. The equations are solved iteratively in the Newton-Raphson method, yielding values of μ_n, μ_p, and T. The volume can then be determined with knowledge of the yield of a fragment.

To explore the effect of scattering on the extracted values of T and V of a hot fragmenting system, we resort to a numerical experiment. The primary fragment yields are calculated with the given freeze-out temperature $T_{\rm fz}$ and volume $V_{\rm fz}$ in the S-matrix approach as elucidated. The secondary yields are then calculated using Eqs. (5) and (7). These are taken to be *observed numerical data*. In Eq. (7), the first term in the curly braces corresponds to the ideal Albergo condition, yielding $T_{\rm alb}$ and $V_{\rm alb}$; addition of the second term gives the γ-decay-corrected values T_γ and V_γ; and further, addition of the last term gives $\gamma + p$ (particle)-decay-corrected values $T_{\gamma+p}$ and $V_{\gamma+p}$. Only if heavy fragments are taken for multiplicity ratios, $T_{\gamma+p} = T_{\rm fz}$ and $V_{\gamma+p} = V_{\rm fz}$. For light fragments, to arrive at the actual values of $T_{\rm fz}$ and $V_{\rm fz}$, one has to further consider contributions from scattering corrections as given by the last term in Eq. (5).

The calculations are done with ^{124}Sn as a representative system. For the fragment species, all the stable nuclei up to $A = 124$ and $Z = 50$, as well as their binding energies, are taken from Ref. [29]. All discrete levels up to an excitation energy of 20 MeV with lifetimes >200 fm/c, as well as their decay modes for $5 \leqslant A \leqslant 16$, have been taken [30,31] into consideration. For still

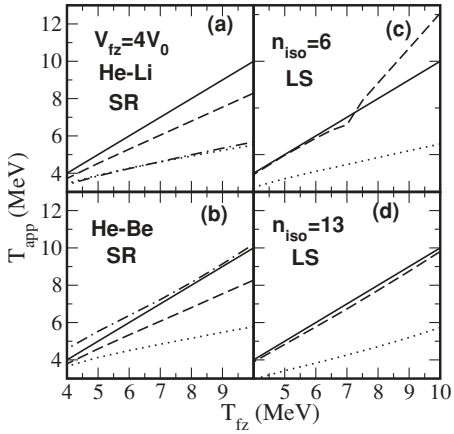

FIG. 1. (a, b) The extracted temperature T_{app} for the fragmenting system ^{124}Sn as a function of the freeze-out temperature T_{fz} shown for two different thermometers under different approximations using single ratios (SRs). (c, d) The same, shown using the least-squares method (LS) for two sets of isotopes as mentioned in the text. Dash-dotted, dotted, dashed, and solid lines correspond to T_{alb}, T_γ, $T_{\gamma+p}$, and $T_{\gamma+p+sc} = T_{fz}$, respectively.

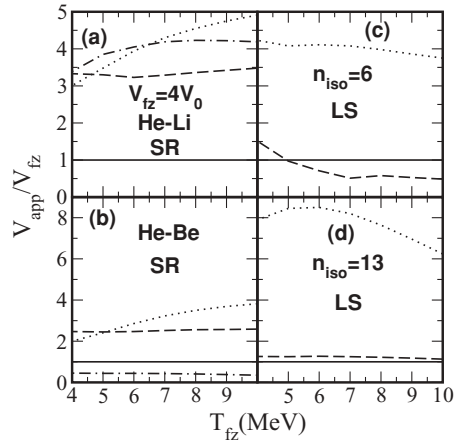

FIG. 2. The same as Fig. 1, for the extracted volume V_{app} in units of freeze-out volume V_{fz}.

heavier nuclei, the sum over excited states in Eqs. (5) and (7) is replaced by an integral convoluted with the single-particle level density $\omega(A, E)$ [23,32]. The integration limits are taken to be between 2 MeV (approximated for the first excited state) and 8 MeV (the last particle-stable state) for the γ-decaying levels; the resonance limit is taken as 20 MeV. The calculations done at different temperatures in a freeze-out volume $4V_0$ (V_0 is the normal volume of ^{124}Sn) are presented. We have chosen two sets of four fragments, namely, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li and ^{3}He, ^{4}He, ^{10}Be, ^{11}Be, which we refer to as the He-Li and He-Be thermometers. The extracted apparent temperatures T_{app} are found to be quite sensitive to the different approximations as displayed in Figs. 1(a) and 1(b). Except for T_{alb}, the other temperatures are not very sensitive to the choice of thermometer. Successive improvement of approximations is seen to bring the apparent temperature closer to the real one. With the inclusion of effects owing to $(\gamma + p)$ decay and scattering, the apparent temperature $T_{\gamma+p+sc}$, when calculated, yields the actual temperature T_{fz}. The effect of scattering is seen to be substantial.

The volume V_{app} (measured in units of V_{fz}) extracted in different approximations is displayed in Figs. 2(a) and 2(b) as a function of T_{fz} for the aforementioned thermometers. Scattering has a comparatively more significant role here than that observed in the determination of temperature. Its inclusion collapses the apparent volumes V_{app}/V_{fz} to unity.

The method so discussed suffers from two limitations. For many thermometers, there may not be convergence for the solution, as noted earlier [17]. We also found that there may be multiple solutions. We have presented those solutions that are robust in the sense that taking a considerable range of initial guess values in the iterative method, the same solutions are obtained. To overcome these limitations, we propose that the least-squares fit to the secondary multiplicities may be more fruitful for extracting the temperature and volume. Given the experimental yields for a chosen number of fragment species n_{iso}, the least-squares fit to

$$\sum_{i=1}^{n_{iso}} \left[Y_i^{exp} - Y_i(T, V, \mu_n, \mu_p)\right]^2 = \chi^2 \qquad (8)$$

has been performed. The quantities Y_i^{exp} are the experimental multiplicities, which are functions of the thermodynamic variables at freeze-out and $Y_i(T, V, \mu_n, \mu_p)$ are the yields calculated from Eqs. (5) and (7) with various approximations as explained earlier. In our calculations, Y_i^{exp} are taken from our *numerical experiment*. The extracted temperatures T_{app} in the least-squares method for the system ^{124}Sn at different given T_{fz} values and at a freeze-out volume $V_{fz} = 4V_0$ under different approximations are displayed in Figs. 1(c) and 1(d). The calculations were performed using a set of light isotopes with $n_{iso} = 6$ (n, p, d, t, ^{3}He, and ^{4}He). The calculations were repeated with a broader set ($n_{iso} = 13$) that includes, besides the light set, also the nuclei ^{6}Li, ^{7}Li, ^{9}Be, ^{10}B, ^{12}C, ^{14}N, and ^{16}O. The γ-decay-corrected temperature T_γ is found to be insensitive to the choice of fragment set and underestimates T_{fz} ($= T_{\gamma+p+sc}$) considerably. Inclusion of particle decay narrows the gap from T_{fz} significantly, particularly for the broader set of fragment species. Figures 2(c) and 2(d) display the extracted volume V_{app} as a function of the freeze-out temperature. The γ-decay-corrected volume V_γ overestimates V_{fz} significantly. Inclusion of particle decay brings it closer to V_{fz}, particularly for the larger set. The uncertainty in the $(\gamma + p)$-corrected value for the volume, with $n_{iso} = 13$, is seen to be at most 25%, and that for temperature is at most 5%. Inclusion of heavier species in the fitting procedure masks the scattering effects. The calculations have been repeated for $V_{fz} = 6V_0$ and $8V_0$; the conclusions do not change for this range of freeze-out volumes.

Along with temperature and volume, the nucleon chemical potentials μ_n and μ_p are also extracted in this method, but they are not shown here. With the knowledge of these four thermodynamic parameters, it is straightforward to determine the entropy of the disassembling system. Thus the evolution of entropy with T_{fz} can be determined, which acts as an important signature for the LG-type phase transition. This will be reported elsewhere.

In this paper, limitations of the currently used isotope thermometry for determining the temperature and volume of a hot fragmenting nuclear system have been pointed out. It is stressed that the strong interaction effects left out in such a determination leaves a sizable uncertainty. This has an important bearing on many predictions of the properties of hot finite nuclear matter. A new method, namely, the least-squares fit to the fragment multiplicities, for extracting the thermodynamic observables is proposed here. We find this more promising in a numerical experiment, and it can be readily implemented in a realistic experimental situation.

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