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Kaons in dense half-Skyrmion matter

Byung-Yoon Park,[1] Joon-Il Kim,[2] and Mannque Rho[3]
[1]*Department of Physics, Chungnam National University, Daejon 305-764, Korea*
[2]*Department of Physics, Florida State University, Tallahassee, Florida 32306, USA*
[3]*Institut de Physique Théorique, CEA Saclay, F-91191 Gif-sur-Yvette Cédex, France, and Department of Physics, Hanyang University, 133-791 Seoul, Korea*
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Dense hadronic matter at low temperatures is expected to be in a crystalline state and at high density to make a transition to a *chirally restored but color-confined* state, which is a novel phase hitherto unexplored. This phase transition is predicted in both skyrmion matter in four dimensions and instanton matter in five dimensions, the former in the form of half-skyrmions and the latter in the form of half-instantons or dyons. We predict that when K^-'s are embedded in this half-skyrmion or half-instanton (dyonic) matter, which may be reached not far past the normal density, there arises an enhanced attraction from both the soft dilaton field figuring for the trace anomaly of QCD and the Wess-Zumino term. This attraction may have relevance for a possible strong binding of antikaons in dense nuclear matter and for kaon condensation in neutron-star matter. Such kaon property in the half-skyrmion phase is highly nonperturbative and may not be accessible by low-order chiral perturbation theory. Relevance of the half-skyrmion or dyonic matter to compact stars is discussed.

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I. THE PROBLEM AND RESULTS

There is a compelling indication from baryonic matter simulated on crystal lattices that as density increases beyond the normal nuclear matter density n_0, there emerges a phase with vanishing quark condensate that is symptomatic of chiral symmetry restoration but in which colored quarks are still confined. This has been observed [1,2] with skyrmions put in an fcc-configured crystal, which turn into half-skyrmions in a cc configuration as density reaches $n = n_{1/2} > n_0$. In this case, the quark condensate $\langle \bar{q}q \rangle$ goes to zero, but the pion decay constant f_π remains nonzero, implying that hadrons have relevant degrees of freedom, although chiral symmetry is restored. This has also been shown [3] to take place with instantons in five dimensions, which figure in holographic dual QCD (hQCD) placed in an fcc crystal and which split into half-instantons in the form of dyons in a bcc crystal configuration. While the skyrmion matter is constructed either with the pion field only or with the pion field plus the lowest lying vector mesons ρ and ω, which we denote $\tilde{\rho}$, the instanton matter arises as a solitonic matter in five dimensions. When viewed in four dimensions, this contains an infinite tower of both vector and axial vector mesons. A single baryon is much better described as an instanton in hQCD [4], which is justified for both large N_c and large 't Hooft constant $\lambda = g_{YM}^2 N_c$, than is the skyrmion baryon in large N_c QCD. This is particularly true for quantities captured in quenched lattice QCD calculations. We ask a highly pertinent question: How does a meson behave in the medium consisting of a large number of these solitons?

This issue was addressed in [1] for fluctuations of pions in a dense medium. The information learned from that calculation was limited, because the pion, being nearly a genuine Goldstone boson, is largely protected by chiral symmetry, and singling out the medium effect would require great accuracy in theory (such as, e.g., high-order $1/N_c$ corrections), which the skyrmion description is not capable of providing. However, the story is quite different with the kaons. In fact, the Callan-Klebanov model [5], which describes the hyperons as bound states of fluctuating K^-'s with an $SU(2)$ soliton, has been amazingly successful, as recently reviewed in [6]. Here, the Wess-Zumino term that encodes chiral anomalies plays a singularly important role in binding of antikaons to an $SU(2)$ soliton. A similar observation was made for the two-baryon system K^-pp in [7], where it was found that there is a substantial increase in attraction between the kaon and the nucleons when the latter interact at a short distance.

In this article, we consider negatively charged kaons fluctuating in the medium described as a dense solitonic background. This involves two very important issues in physics of dense hadronic matter. One is that there is a possibility that K^- can trigger a strongly correlated mechanism to compress hadronic matter to high density [8]. Such a mechanism is thus far unavailable in the literature and may very well be inaccessible in perturbation theory.[1] The other issue is the role of kaon condensation in densely compacted star matter, which has ramifications on the minimum mass of black holes in the universe and cosmological natural selection [9]. It would be most appealing, and of great theoretical interest, to address this problem in terms of the instanton matter given in hQCD, which has the potential to also account for short-distance degrees of freedom via an infinite tower of vector and axial vector mesons. However, numerical work in this framework is unavailable. Skyrmion matter consisting of pion and $\tilde{\rho}$ has been studied [10] but has not yet been fully worked out. We therefore take the simple Skyrme model implemented with two key ingredients (viz., the Wess-Zumino term and the "soft dilaton" field χ_s, which accounts for scale symmetry tied to spontaneously

[1]As suggested by Gerry Brown, one might caricature this as an "Ice-9 effect" as depicted in Kurt Vonnegut's *Cat's Craddle*.

broken chiral symmetry) as precisely stated in [11]. Our task is to understand kaon fluctuations in the background given by the skyrmion matter described by this Lagrangian. This approach is different from others because it exploits the close connection between scale symmetry breaking encoded in the dilaton condensate, which can be restored à la Freund-Nambu [11,12], and chiral symmetry breaking encoded in the skyrmion matter, both linked to the mass of light-quark hadrons. There is fine-tuning required for the parameters of the model to achieve a quantitative comparison with nature, which we eschew in this article. We instead focus more on robust qualitative features. The basic approximation that we adopt is that the back reaction of kaon fluctuations on the background matter can be ignored, which is consistent with large N_c consideration.

Our results are summarized in Fig. 1.

In the model, we find three different regimes in properties of the kaon as density increases. At low density with chiral symmetry spontaneously broken with $\langle\bar{q}q\rangle \propto \mathrm{Tr}U \neq 0$ (where U is the chiral field defined later) and $f_\pi \neq 0$, the state is populated by skyrmions, and the mass of the kaon propagating therein drops at the rate largely controlled by chiral perturbation theory valid at low density. This behavior continues up to the density $n_{1/2}$, at which the skyrmion matter turns into half-skyrmion matter characterized by $\mathrm{Tr}U = 0$; that is, the chiral symmetry is restored but with $f_\pi \neq 0$. In this phase, the kaon mass undergoes a much more dramatic decrease until it vanishes—in the chiral limit—at the critical density n_c at which $\mathrm{Tr}U = f_\pi = 0$. The region between $n_{1/2}$ and n_c, dubbed as "hadronic freedom" regime [11]—which also played an important role in explaining dilepton processes in heavy-ion collisions [13]—is most likely inaccessible by low-order chiral perturbation theory. Given the extreme truncation of the model used here, it makes little sense to attempt to pin down precisely the onset density of the half-skyrmion phase. The best guess would be that it figures at a density between 1.3 and 3 times the normal nuclear density, $\approx 0.16\ \mathrm{fm}^{-3}$, the range indicated in Fig. 1 for the set of parameters chosen.

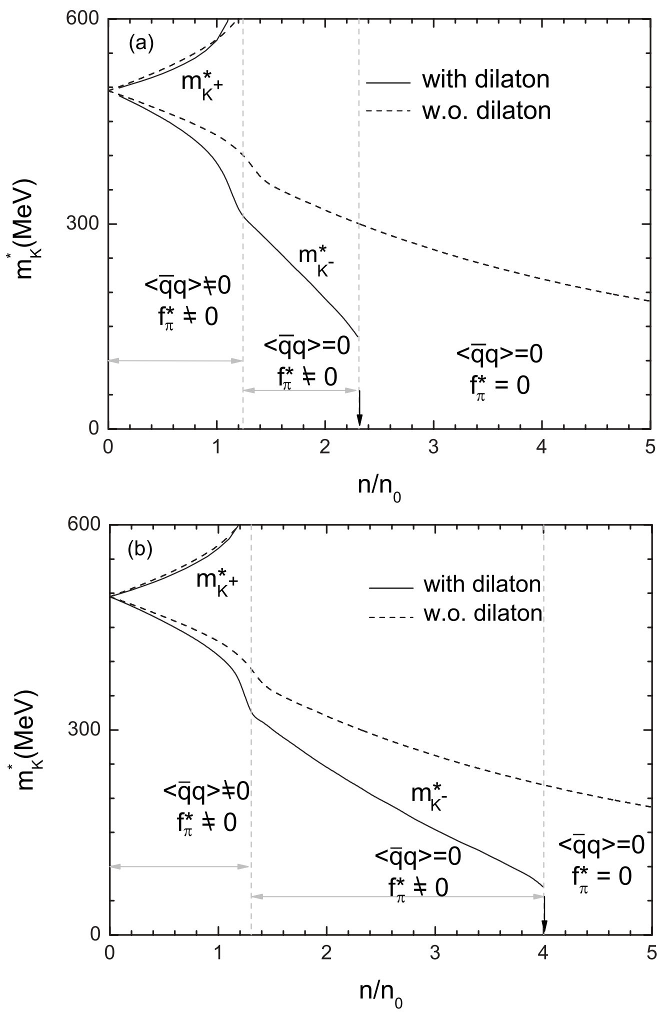

FIG. 1. $m^*_{K^\pm}$ vs. n/n_0 (where $n_0 \simeq 0.16\ \mathrm{fm}^{-3}$ is the normal nuclear matter density) in dense skyrmion matter that consists of three phases: (i) $\langle\bar{q}q\rangle \neq 0$ and $f^*_\pi \neq 0$, (ii) $\langle\bar{q}q\rangle = 0$ and $f^*_\pi \neq 0$, and (iii) $\langle\bar{q}q\rangle = 0$ and $f^*_\pi = 0$. The parameters are fixed at $\sqrt{2}ef_\pi = m_\rho = 780$ MeV and dilaton mass (a) $m_\chi = 600$ MeV (upper panel) and (b) $m_\chi = 720$ MeV (lower panel).

II. THE LAGRANGIAN

To construct dense nuclear matter into which kaons are to be embedded, we take the $SU(2)_f$ Skyrme Lagrangian given in terms of the pion field only [14] and construct a dense baryonic matter by putting the skyrmions on a crystal. This Skyrme Lagrangian can be considered as an effective low-energy Lagrangian valid at large N_c in which all other degrees of freedom are integrated out. In fact, it is seen in hQCD that the Skyrme quartic term (considered in the past *ad hoc*) does arise naturally and uniquely, capturing the physics of distances shorter than those of the lowest vector excitations (i.e., $\tilde{\rho}$). There is, however, one crucial ingredient that needs to be implemented in the Skyrme Lagrangian, namely, the dilaton field that accounts for the trace anomaly of QCD. In considering dense matter, it is essential that the scale symmetry breaking encoded in the trace anomaly be mapped to the spontaneous breaking of chiral symmetry. This point was implicit already in the 1991 proposal for scaling of hadron masses [15], but it was in [11] that the dilaton field that figures in the connection was clearly identified. There is a subtlety in distinguishing the "soft dilaton" χ_s, with which we are concerned here, and the "hard dilaton" χ_h, which is associated with the asymptotically free running of the color gauge coupling constant reflecting scale symmetry breaking in QCD. This is discussed in [11]. For our purpose, the important point is that the condensate of the soft dilaton vanishes across the chiral restoration point, whereas that of the hard dilaton remains nonzero across the critical point. The vanishing of the soft dilaton condensate is directly connected to the vanishing of the quark condensate (in the chiral limit) and hence to chiral symmetry, as shown in the case of dileptons in heavy-ion collisions [13]. It has not yet been rigorously established but is plausible [13,16] that the same holds in the case of density. In this article, we simply assume that it does and deal uniquely with the soft component, which we denote simply by χ.

Extended to three flavors and implemented with the dilaton field χ, the Skyrme Lagrangian we use takes the form [10,11]

$$\begin{aligned}\mathcal{L}_{sk} = {}& \frac{f^2}{4}\left(\frac{\chi}{f_\chi}\right)^2 \mathrm{Tr}(L_\mu L^\mu) + \frac{1}{32e^2}\mathrm{Tr}[L_\mu, L_\nu]^2 \\ &+ \frac{f^2}{4}\left(\frac{\chi}{f_\chi}\right)^3 \mathrm{Tr}\mathcal{M}(U + U^\dagger - 2) \\ &+ \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + V(\chi),\end{aligned} \tag{1}$$

where $V(\chi)$ is the potential that encodes the trace anomaly involving the soft dilaton, $L_\mu = U^\dagger\partial_\mu U$, with U being the chiral field taking values in $SU(3)$, and f_χ is the vacuum expectation value (vev) of χ. We ignore the pion mass for simplicity, so the explicit chiral symmetry-breaking mass term is given by the mass matrix $\mathcal{M} = \mathrm{diag}(0, 0, 2m_K^2)$. In $SU(3)_f$, the anomaly term (i.e., the Wess-Zumino term),

$$\mathcal{S}_{\mathrm{WZ}} = -\frac{iN_C}{240\pi^2}\int d^5x\,\varepsilon^{\mu\nu\lambda\rho\sigma}\mathrm{Tr}\,(L_\mu L_\nu L_\lambda L_\rho L_\sigma) \tag{2}$$

plays a crucial role in our approach.

III. FLUCTUATING KAONS IN THE SKYRMION MATTER

In close analogy to the Callan-Klebanov scheme [5], we consider the fluctuation of kaons in the background of the skyrmion matter u_0, following [17], as

$$U(\vec{x}, t) = \sqrt{U_K(\vec{x}, t)}U_0(\vec{x})\sqrt{U_K(\vec{x}, t)}, \tag{3}$$

$$U_K(\vec{x}, t) = e^{\frac{i}{\sqrt{2}f_\pi}\left(\begin{smallmatrix}0 & K\\ K^\dagger & 0\end{smallmatrix}\right)},$$

$$U_0(\vec{x}) = \begin{pmatrix} u_0(\vec{x}) & 0 \\ 0 & 1\end{pmatrix}. \tag{4}$$

By substituting Eq. (3) into Eq. (1) and the Wess-Zumino term, (2), we get the kaon Lagrangian in the background matter field $u_0(x)$:

$$\begin{aligned}\mathcal{L}_K = {}& \left(\frac{\chi_0}{f_\chi}\right)^2 \dot{K}^\dagger G\dot{K} - \left(\frac{\chi_0}{f_\chi}\right)^2 \partial_i K^\dagger G\partial_i K - \left(\frac{\chi_0}{f_\chi}\right)^3 m_K^2 K^\dagger K \\ &+ \frac{1}{4}\left(\frac{\chi_0}{f_\chi}\right)^2 (\partial_\mu K^\dagger V^\mu(\vec{x})K - K^\dagger V_\mu(\vec{x})\partial^\mu K) \\ &+ \frac{iN_c}{4f_\pi^2}B^0(K^\dagger G\dot{K} - \dot{K}^\dagger GK),\end{aligned} \tag{5}$$

where $\chi_0(\vec{x})$ is the classical dilaton field and

$$V_\mu(\vec{x}) = \frac{i}{2}[(\partial_\mu u_0^\dagger)u_0 - (\partial_\mu u_0)u_0^\dagger], \tag{6}$$

$$G(\vec{x}) = \frac{1}{4}(u_0 + u_0^\dagger + 2), \tag{7}$$

$$B^\mu(\vec{x}) = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\lambda\sigma}\mathrm{Tr}\,(u_0^\dagger\partial_\nu u_0 u_0^\dagger\partial_\lambda u_0 u_0^\dagger\partial_\sigma u_0). \tag{8}$$

In the spirit of mean field approximation, we take the space average on the background matter fields u_0 and χ_0 and obtain

$$\mathcal{L}_K = \alpha(\partial_\mu K^\dagger\partial^\mu K) + i\beta(K^\dagger\dot{K} - \dot{K}^\dagger K) - \gamma K^\dagger K, \tag{9}$$

where

$$\alpha = \langle\kappa^2 G\rangle, \quad \beta = \frac{N_c}{4f_\pi^2}\langle B^0 G\rangle, \quad \gamma = \langle\kappa^3 G\rangle m_K^2 \tag{10}$$

with $\kappa = \chi_0/f_\chi$. Here $\langle\cdots\rangle$ stands for the space averaging. Lagrangian (9) yields a dispersion relation for the kaon in the skyrmion matter as

$$\alpha\left(\omega_K^2 - p_K^2\right) + 2\beta\omega_K + \gamma = 0. \tag{11}$$

By solving this for ω_K and taking the limit of $p_K \to 0$, we have

$$m_K^* \equiv \lim_{p_K\to 0}\omega_K = \frac{-\beta + \sqrt{\beta^2 + \alpha\gamma}}{\alpha}. \tag{12}$$

This equation is used for evaluating the in-medium effective kaon mass.

IV. SKYRMION CRYSTAL

The key element in Eq. (9) for the kaonic fluctuation is the backfround u_0, which reflects the vacuum modified by the dense skyrmion matter. The classical dilaton field tracks the quark condensate affected by this skyrmion background u_0, which carries information on chiral symmetry of dense medium. Our approach here is to describe this background u_0 in terms of a crystal configuration. The pertinent u_0 has been worked out in detail in [1], from which we simply import the results for this work. As shown there, skyrmions put on an fcc configuration, the favored crystal configuration, make a phase transition at a density $n_{1/2}$ to a matter consisting of half-skrymions. With the parameters of the Lagrangian picked for the model, we find $n_{1/2} \sim 1.3n_0$. This is, of course, quite uncertain. As noted, our guess would range from slightly more than n_0 to $\sim 3n_0$. This is also where the instanton-to-half-instanton (or dyon) transition takes place in hQCD [3]. In this article, we do not attempt a quantitative estimate but consider the density as an approximate value.

V. VECTOR MODE

An important aspect of the half-skyrmion state is that the quark condensate $\langle\bar{q}q\rangle$, proportional to $\mathrm{Tr}(u_0 + u_0^\dagger)$, is zero in this phase, but the pion decay constant f_π, proportional to $\langle\chi_0\rangle$ in medium, could be nonzero. This means that the half-skyrmions are hadrons, not deconfined quarks. There is a resemblance to fractionized electrons in $(2 + 1)$ dimensions in condensed-matter physics [18].

What is perhaps more noteworthy is that we have here an enhanced symmetry. In hidden local symmetry theory [19], that $\langle\bar{q}q\rangle = 0$ *and* $f_\pi \neq 0$ implies that $f_\pi = f_s$ with $\langle 0|A_\mu|\pi\rangle = ip_\mu f_\pi$ and $\langle 0|V_\mu|s\rangle = ip_\mu f_s$, where s is the longitudinal component of the ρ meson. This corresponds to the vector mode conjectured by Georgi [20] to be realized in the large N_c limit of QCD. Such a mode does not exist in QCD proper with Lorentz invariance. However, one can think of it as an emergent symmetry in the presence of medium, which dense matter provides. The hadronic freedom regime, mentioned previously, encompasses this mode with $a \equiv f_s/f_\pi \approx 1$ and $g \approx 0$ (where g is the hidden gauge coupling constant) near the critical point (T_c or n_c). We expect this identification to be highly nontrivial, and it presents a novel structure at high density to which we hope to return in a future publication.

VI. EFFECTIVE KAON MASS

We now discuss the results given in Fig. 1 in some detail.

The results are given for two values of dilaton mass, $m_\chi = 600$ and 720 MeV. As explained in [11], the dilaton mass is established neither experimentally nor theoretically. We have taken two values that we consider reasonable for the soft dilaton. The former corresponds to the lowest scalar excitation seen in nature and the latter to the effective scalar needed in the Fermi-liquid description of nuclear matter implementing BR scaling [21]. In the spirit of effective field theory, the latter is preferred in our discussion. In any event, as shown in [1], the density at which the half-skyrmion matter appears is independent of the dilaton mass. This can be understood by the fact that within the approximations we are taking, $n_{1/2}$ is entirely dictated by the parameters that give the background u_0. For the skyrmion background, it is natural to pick $\sqrt{2}ef_\pi$, which is the vacuum ρ mass at tree order, to be ~ 780 MeV. This fixes e for the given $f_\pi \approx 93$ MeV [22]. The dilaton mass controls the density n_c at which the quark condensate vanishes, together with the pion decay constant and, more significantly, the rate at which the kaon mass drops as density exceeds n_0.

Our model characterizes the behavior of the kaon by three different phases, not two as in conventional approaches. In the low-density regime up to $\sim 1.3\, n_0$, the kaon interaction can be described by standard chiral symmetry treatments. For instance, the binding energy of the kaon at nuclear matter density is ~ 110 (~ 80) MeV for $m_\chi = 600$ (720) MeV. This is what one would expect in chiral perturbation theory (see [23] for review). Beyond n_0, however, as the matter enters the half-skyrmion phase with vanishing quark condensate and nonzero pion decay constant, the kaon mass starts dropping more steeply. This property is consistent with what is observed in the hadronic freedom regime in the approach to kaon condensation, which starts from the vector manifestation fixed point of hidden local symmetry [24]. Note that this form of matter, which is undoubtedly highly nonperturbative, is most likely unamenable to a chiral perturbation approach. Finally, at n_c, at which both the quark condensate and the pion decay constant vanish, the kaon mass vanishes. This happens at $n_c \approx 2.3$ (4.0) n_0 for the dilaton mass $m_\chi = 600$ (720) MeV because the dilaton condensate, $\langle\chi\rangle$, vanishes with the restoration of soft scale symmetry à la Freund-Nambu, as explained in [11]. This means that kaon condensation in symmetric nuclear matter takes place at the point at which the scale symmetry associated with the soft dilaton is restored. It is not clear whether this takes place before or coincides with deconfinement, which requires the intervention of the hard component of the gluon condensate, which is ignored in [11].

VII. COMPACT-STAR MATTER AND KAON CONDENSATION

Thus far, the kaon is treated at the semiclassical level as a quasiparticle bound to the skyrmion matter. To understand what this represents in nature, we note that when one quantizes the system in which a kaon is bound to a single skyrmion, the bound system gives rise to the hyperons, Λ and Σ, as shown by Callan and Klebanov [5]. This would suggest that multikaons bound to a skyrmion background matter, when collectively quantized, correspond to hyperonic matter. On the other hand, one can interpret the system in Ginzburg-Landau mean-field theory by focusing on the kaon fluctuation and thus identify the vanishing of the effective kaon mass in medium, $m_K^\star$, as the signal for kaon condensation. Thus, in this picture, when β equilibrium is suitably implemented in asymmetric nuclear matter in the presence of strangeness, kaon condensation and hyperon condensation would be physically equivalent. An open interesting issue here is whether one can establish a hadron-quark duality in compact stars between the kaon condensed matter depicted previously and a strange quark matter discussed in the literature. At the very least, the kaon condensed matter could be a door to the strange quark matter expected at greater densities.

What does the half-skyrmion matter do to compact stars? Here, we have subjected the kaon to the skyrmion background, which is given in the large N_c limit. In the large N_c limit, there is no distinction between symmetric matter and asymmetric matter. Compact stars have neutron excess, which typically engenders repulsion at densities greater than n_0, and hence the asymmetry effect needs to be taken into account. This effect will arise when the system is collectively quantized, giving rise to the leading $1/N_c$ correction to the energy of the bound system. Most of this correction could be translated into a correction to the effective mass of the kaon. We have no estimate of this correction, but it is unlikely to be big. The energy that arises from neutron excess, called "symmetry energy" in nuclear physics, is labeled $S(n)$ in the energy per particle E of the neutron-rich system appropriate for compact stars,

$$E(n, x) = E(n, x = 0) + E_{\rm sym}(n, x), \tag{13}$$

with

$$E_{\rm sym}(n, x) = S(n)x^2 + \cdots, \tag{14}$$

where $x = (P - N)/(N + P)$ with $N(P)$ standing for the neutron (proton) number. The ellipsis stands for higher order terms in $|x| \leqslant 1$.

In compact-star matter, kaons condense at a density less than the critical density n_c because of the electron chemical potential, which tends to increase as the matter density increases [25]. Now, the electron chemical potential is known to be predominantly, if not entirely, controlled by the symmetry energy S in charge-neutral β equilibrium systems [26]. Thus collective quantization is required to address at what density kaons will condense. Quantizing the moduli space of multiskyrmion systems (or multiinstanton systems in hQCD) has not yet been worked out fully. As mentioned, the effect of hyperons on electron chemical potential will be automatically taken into account by the moduli quantization. Even in the absence of detailed computations, however, we can say, at least within the framework of our model, that kaons will condense at a relatively low density, say, $n_K \lesssim 4n_0$.

VIII. CONCLUSION

Hidden local symmetry with vector manifestation [19], combined with a soft dilaton accounting for scale symmetry restoration à la Freund-Nambu [11], implies that the antikaon mass in dense medium falls more rapidly in the half-skyrmion phase than in the skyrmion phase in which standard chiral perturbation approach should be applicable. It is proposed that this could be relevant to an Ice-9 phenomenon in kaon-nuclear systems and kaon condensation in compact-star matter. It is also argued that the half-skyrmion phase corresponds to the hadronic freedom regime in density in parallel to that in temperature, which figures in dilepton production in heavy-ion collisions. In this article, the issue was addressed in terms of half-skyrmions in Hidden Local Symmetry (HLS). In the framework of HLS, the half-skyrmion phase could be considered to be in the vector mode of [20], which emerges as an enhanced symmetry from collective modes in the many-body system. It would be interesting to analyze the same in terms of half-instantons (or dyons) in hQCD, which in principle could account for many-body forces [27] that the skyrmion description may be unable to handle. We hope to address this problem in a future publication.

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