Differential elliptic flow of identified hadrons and constituent quark number scaling at the GSI Facility for Antiproton and Ion Research (FAIR)

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Differential elliptic flow $v_2(p_T)$ for identified hadrons is investigated in the FAIR energy regime, employing a hadronic-string transport model (UrQMD) as well as a partonic transport model (AMPT). It is observed that both models show a mass ordering of v_2 at low p_T and a switch-over resulting in a baryon-meson crossing at intermediate p_T . AMPT generates higher v_2 values compared to UrQMD. In addition, constituent quark number scaling behavior of elliptic flow is addressed. Scaling behavior in terms of the transverse momentum p_T is found to be absent for both the partonic and the hadronic model. However, UrQMD and AMPT with a string melting scenario do exhibit an NCQ scaling of v_2 to varying degrees, with respect to the transverse kinetic energy $K E_T$. But the default AMPT, where partonic scatterings are not included, does not show any considerable scaling behavior. A variable α is defined to quantify the degree of KE_T scaling. We found that UrQMD gives better scaling than AMPT at FAIR.

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I. INTRODUCTION

Statistical QCD predicts that at high temperature and/or density, hadronic matter undergoes a phase transition to a new state, where strongly interacting matter shows partonic behavior. It is generally believed that in the laboratory the only way to produce such a novel state of matter is to collide two heavy nuclei at ultrarelativistic energies. In the past two decades, relativistic heavy-ion collision experiments have been performed in different parts of the world, with the ultimate aim of mapping the QCD phase diagram and discovering this new phase of strongly interacting matter, so-called quark-gluon plasma, where the subnuclear degrees of freedom come into play over the nuclear volume rather than the nucleonic volume. In heavy-ion experiments at the CERN-SPS and Brookhaven National Laboratory (BNL) RHIC, the QCD phase diagram is studied in the region toward high temperatures and low net baryon densities. In future heavy-ion experiments at the CERN-LHC the research program will be continued in the direction of higher temperatures and lower net baryon densities. The QCD phase diagram is much less explored in the region of high baryon densities and moderate temperatures. In the upcoming Compressed Baryonic Matter (CBM) experiment [\[1\]](#page-6-0), at the future accelerator facility FAIR at GSI, heavy ions will be collided in the beam energy range between 10 and 40*A* GeV. The highest densities (6 to 12 times the normal nuclear matter density) are expected to be produced at the center of the collision zone [\[2\]](#page-6-0).

Since the idea that relativistic nuclear collisions can lead to the formation of a quark-gluon phase was perceived, lots of theoretical as well as experimental efforts have been devoted to predicting unambiguous and experimentally viable probes to indicate the production of the dense partonic medium. The collective flow of the produced particles in the transverse plane of the collision has long been predicted as a signature of the creation of a hot and dense secondary medium [\[3,4\]](#page-6-0). Of particular interest is the elliptic flow parameter v_2 , signaling strong evidence for the creation of a hot and dense system at a very early stage in noncentral collisions [\[4–6\]](#page-6-0). In RHIC experiments at BNL, a large elliptic flow has been observed [\[7\]](#page-6-0), as large as predicted by ideal hydrodynamical models [\[8\]](#page-6-0). The most striking observation, in this respect, is the number of constituent quark (NCQ) scaling of v_2 of identified hadrons.

In the present work, we have explored the p_T dependence of the elliptic flow parameter v_2 at top ($E_{\text{Lab}} = 40A \text{ GeV}$) and intermediate $(E_{Lab} = 25A \text{ GeV})$ FAIR energies for different types of strange and nonstrange hadrons. For this purpose we have employed two transport models: ultrarelativistic quantum molecular dynamics (UrQMD) [\[9\]](#page-6-0) and a multiphase transport model (AMPT) [\[10\]](#page-6-0). Apart from the differential elliptic flow $[v_2(p_T)]$, we have also investigated the constituent quark number scaling behavior of v_2 predicted by these models. Finally, we have defined a parameter *α* to quantify the degree of scaling produced by these two models.

The paper is organized as follows. In Sec. II we present the results for p_T dependence of $v₂$ for identified hadrons, as predicted by the transport models UrQMD and AMPT. The results from both models are compared and contrasted with the existing data from the NA49 experiment at 40*A* GeV. In Sec. [III](#page-3-0) we have addressed the issue of NCQ scaling of v_2 of the selected hadrons. Finally, in Sec. [IV](#page-6-0) we summarize our results and conclude.

II. DIFFERENTIAL ELLIPTIC FLOW $(v_2 \text{ VS. } p_T)$

In noncentral nuclear collisions, the extended overlap area is roughly almondlike in shape and does not possess azimuthal symmetry. The initial asymmetries in the geometry of the system give rise to anisotropic pressure gradients, which in turn can lead to anisotropies in the particle momentum

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distributions. Because the spatial asymmetries decrease rapidly with time, anisotropic flow can develop at a very early time in evolution. In this way, the properties of the hot dense matter formed during the initial stage of heavy-ion collisions can be determined by measuring the anisotropic flow. The anisotropic flow is defined as the *n*th Fourier coefficient v_n of the particle distributions in the emission azimuthal angle with respect to the reaction plane $[4,6]$, which can be written as

$$
\frac{dN}{d\phi} \propto 1 + 2 \sum v_n \cos[n(\phi - \Psi)], \tag{1}
$$

where ϕ denotes the azimuth angle of the particle, and Ψ denotes the orientation of the reaction plane, defined as the plane spanned by the impact parameter vector and incoming beam direction. The second Fourier coefficient v_2 represents the elliptic flow that characterizes the eccentricity of the particle distributions in momentum space. In a given rapidity window, it reads as

$$
v_2 = \langle \cos[2(\phi - \Psi)] \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle. \tag{2}
$$

In this paper, we have estimated v_2 as a function of transverse momentum p_T for midcentral ($b = 5-9$ fm) Au + Au collisions, at top ($E_{\text{Lab}} = 40A \text{ GeV}$) and intermediate ($E_{\text{Lab}} =$ 25*A* GeV) FAIR energies, in the midrapidity region ($-1 \leq$ $y_{c.m.} \leq 1$). In the present model calculations, the reaction plane angle is taken as zero, so v_2 is calculated directly from its definition.

UrQMD is a hadronic-string transport model based on the same principles as quantum molecular dynamics (QMD) [\[11\]](#page-6-0) and relativistic quantum molecular dynamics (RQMD) [\[12\]](#page-6-0). It incorporates a vastly extended collision term with full baryon-antibaryon symmetry. Isospin is explicitly treated for all hadrons. Cross sections in UrQMD depend on the particle types, their isospins, and their center-of-mass (c.m.) energy. Within the framework of the UrQMD model, a typical heavy-ion collision proceeds schematically in three stages: the prehadronic stage, described in terms of strings and constituent diquarks; the hadronic pre-equilibrium stage; and finally, the evolution toward hadronic kinetic equilibrium and freeze-out. Particle production in UrQMD takes place either via the decay of a meson or baryon resonance or via a string excitation and fragmentation. In this model, partonic interactions are not included except from quark coalescence during the string breakup.

On the other hand, AMPT is a hybrid transport model, which models an ultrarelativistic nuclear collision incorporating many tools of Monte Carlo simulation. As the initial condition it uses minijet partons from hard processes and strings from soft processes in the Heavy Ion Jet Interaction Generator (HIJING) model [\[13\]](#page-6-0). Time evolution of resulting minijet partons is then described by Zhang's parton cascade model [\[14\]](#page-6-0), which includes only parton-parton elastic scatterings with an in-medium cross section derived from pQCD, with an effective gluon screening mass taken as a parameter for fixing the magnitude and angular distribution of the parton scattering cross section. The subsequent transition from the partonic matter to the hadronic matter is based on

the Lund string fragmentation model as implemented in the PYTHIA [\[15\]](#page-6-0) generator, where the minijet partons, after they stop interacting, are combined with their parent strings, as in the HIJING model with jet quenching, to fragment into hadrons. The final-state hadronic scatterings are modeled by the ART model [\[16\]](#page-6-0). In the default AMPT model, minijets coexist with the remaining part of their parent nucleons, and together they form new excited strings, then the resulting strings fragment into hadrons following the Lund string fragmentation. However, in the string melting scenario, these strings are converted to soft partons. Interactions among these partons are again described by Zhang's parton cascade model. Because there are no inelastic scatterings, only quarks and antiquarks from the melted strings are present in the partonic matter. The transition from partonic matter to hadronic matter is then achieved using a simple coalescence model, which combines the two nearest quarks or antiquarks into mesons and the three nearest quarks or antiquarks into baryons or antibaryons that are close to the invariant mass of these partons.

In Fig. [1](#page-2-0) we present the results for the p_T dependence of *v*² of identified hadrons as predicted by UrQMD and AMPT at both top and intermediate FAIR energies. The upper panel shows results at 40*A* GeV, whereas the lower panel corresponds to a beam energy of 25*A* GeV. In all cases, we have selected midcentral Au $+$ Au collisions ($b = 5-9$ fm) and a rapidity window symmetric around the c.m. midrapidity.

In all three scenarios, we have found that v_2 is an increasing function of p_T at both energies. In addition, a mass ordering of v_2 for p_T up to 1.0 GeV/c is observed. At a given value of p_T , v_2 decreases with increase in hadron mass. In the default version of AMPT without partonic degrees of freedom, this ordering is maintained throughout the entire p_T window of investigation. But for UrQMD and AMPT with the string melting scenario, the ordering becomes inverse for $p_T > 1.2$ GeV/c. The heavier baryons are seen to attain larger flow compared to lighter mesons. In this context we would like to mention that these observations of hadron mass ordering at low p_T and its breaking at high p_T are consistent with the measurements at RHIC [\[7\]](#page-6-0). Measurement of v_2 as a function of transverse momenta p_T showed that in the low p_T region, the hadron v_2 values exhibit a mass-dependent ordering: heavier hadrons have smaller v_2 . Hydrodynamic model calculations predicted this effect [\[8\]](#page-6-0). This mass ordering has been attributed to the signification of a common velocity field. However, in the intermediate p_T region $(1.5 \text{ GeV}/c \leq p_T \leq 5 \text{ GeV}/c)$, hydrodynamic results were in gross disagreement with the $v_2(p_T)$ data. The mass ordering appears to switch over, and baryons are seen to have larger v_2 compared to mesons; the baryon and meson v_2 were found to get separated into two branches. In the FAIR energy range, however, the production of high p_T hadrons is small compared to that at RHIC. We cannot go beyond $p_T > 2$ GeV/c. Even for $p_T > 1.5$ GeV/c, simulated data suffer from large fluctuations.

Among the three cases investigated here, both the default AMPT and UrQMD, both of which are based on hadron/string degrees of freedom, give $v_2(p_T)$ values very similar in shape and comparable in magnitude. Inclusion of string melting in

FIG. 1. Variation of v_2 with p_T at 40A GeV (top) and 25A GeV (bottom) from different transport models. Left-hand panels show the results from UrQMD, whereas middle and right-hand panels show predictions from the string melting and the default version of AMPT, respectively.

AMPT, which assumes that the initially produced matter is fully partonic in nature, results in much higher values of v_2 owing to a longer phase of partonic interactions in the early reaction phase. For example, Fig. 2 shows the variation of $v_2(p_T)$ for protons at 25A GeV beam energy. It is clear from the figure that AMPT with the string melting scenario generates $v₂$ values larger compared than those for the default AMPT and UrQMD, neither of which possesses partonic degrees of freedom.

We would also like to mention that at top FAIR energy $(E_{Lab} = 40A \text{ GeV})$, the NA49 experiment at CERN-SPS has published results for the $v_2(p_T)$ of pions and protons for $Pb + Pb$ collisions [\[17\]](#page-6-0). For completeness we have compared the model predictions with the available data. The results are shown in Fig. [3.](#page-3-0)

FIG. 2. Variation of v_2 of protons with p_T at 25A GeV obtained from AMPT and UrQMD models. For AMPT both hadronic and partonic versions were used.

The v_2 values were calculated from the the data using the standard event plane method as well as the second- and fourth-order cumulant methods. The extracted v_2 values are found to suffer from a large amount of uncertainties, especially at high p_T . UrQMD is found to underestimate the experimental data at high p_T , whereas AMPT with partons is found to slightly overestimate the data. Owing to the large amount of systematic error in the data, it is difficult to draw any conclusive picture. It might be mentioned here that attempts have been made earlier at the SPS to describe the transverse momentum dependence of elliptic flow [\[18\]](#page-6-0). Analysis of the $v_2(p_T)$ data of pions for midcentral $(b = 5-9$ fm) Pb + Pb collisions at $E_{\text{Lab}} = 40A \text{ GeV}$ and $E_{\text{Lab}} = 160A \text{ GeV}$ showed that pure transport calculations (UrQMD) underpredict the data, especially at high p_T ; an integrated Boltzmann + hydrodynamics approach describes the data rather well.

But it is clearly visible from the figures that at both energies, AMPT with string melting yields larger v_2 values compared to UrQMD and default AMPT. This probably signals the enhancement of elliptic flow owing to partonic scattering. The buildup of v_2 is caused by different pressure gradients in different directions in the transverse plane. Microscopically the pressure is implemented by the rescattering among constituents. For hadronic-string transport models, the constituents are mainly hadrons, for which the ingredients (strings and diquarks) do not interact with others during their formation time [\[19\]](#page-6-0). For partonic models the partons interact with each other, and with the increase in partonic cross section, $v₂$ increases. In AMPT with the string melting scenario, the partonic cross section is a free parameter. It has been observed in STAR data analysis [\[4\]](#page-6-0) that the model describes the data well when the cross section is chosen in the range 3–10 mb. The best

FIG. 3. Variation of v_2 with p_T at 40*A* GeV. Results from different models are compared with the available data from NA49 [\[17\]](#page-6-0). Simulated data are for midcentral $Au + Au$ collisions in a rapidity window symmetric about midrapidity $(-1 \leq v_{\rm c.m.} \leq 1)$ in the center-of-mass frame. Experimental data, however, are for midcentral $Pb + Pb$ collisions. The rapidity coverage is slightly different for different methods. Left: results for protons. Right: results for charged pions.

and v_2 obeys a simple scaling law:

$$
v_2(p_T) \approx n_q v_2^q (p_T/n_q),\tag{3}
$$

where n_q is the number of valence quarks in the hadron and v_2^q is the elliptic flow parameter for them.

Such scaling behavior is thus considered as an indication that the early stage, when v_2 has been built up, is partonic in nature and the effective constituent quark degrees of freedom play an important role in the hadroniozation process. It has thus been speculated that this scaling is directly related to the formation of the color-deconfined phase, where all the hadrons are created at hadronization of the quark-gluon matter, by recombination or coalescence of partons. In this picture, the quantity $v_2^q(p_T/n_q)$ is interpreted as the elliptic flow of the constituent quarks. This means that the transverse expansion of the matter produced in energetic nuclear collisions develops in a phase dominated by partonic collectivity [\[25\]](#page-6-0). But before drawing a decisive picture, such as that NCQ scaling indicates color deconfinement in nuclear collisions, one needs to take into account all other possible nonpartonic scenarios that may lead to such scaling behavior. At top RHIC energy, an approximate NCQ scaling of the identified hadrons, which describes the data pretty well, has already been reproduced by hadronic transport models like RQMD and UrQMD [\[19,20\]](#page-6-0). This observed scaling at top RHIC energy is attributed to the hadronic cross sections, which depend only on the quark content of the colliding hadrons and roughly scale with the number of constituent quarks, as assumed by the additive quark model (AQM) [\[26\]](#page-6-0).

Here we have investigated the number of constituent quarks scaling behavior as a function of p_T for different hadrons at both beam energies. Figure [4](#page-4-0) shows the variation of v_2/n_a as a function of p_T / n_q at $E_{\text{Lab}} = 25A \text{ GeV}$ and $E_{\text{Lab}} = 40A \text{ GeV}$.

It is observed that over the entire p_T range under study, v_2/n_q as a function of p_T/n_q does not show any reasonable scaling behavior for either of the models. Results from these

fit, however, is obtained for a value around 5 mb. In our study we have set the partonic cross section as 6 mb. It may be noted in this context that UrQMD has been found to underpredict the strength of v_2 at the RHIC [\[19,20\]](#page-6-0). Analysis of v_2 values as a function of centrality showed that this hadron-string transport model fails by 40% to reach the absolute amplitude of v_2 measured at STAR [\[21\]](#page-6-0). The centrality dependencies are, however, very similar to the data. This underestimation is attributed to the lack of partonic interactions in the model, in the hot and dense early stages of evolution.

III. CONSTITUENT QUARK NUMBER SCALING OF ELLIPTIC FLOW

As mentioned earlier, one of the key observations of elliptic flow measurements for $Au + Au$ collisions at RHIC is the approximate quark-number scaling of v_2 of identified hadrons [\[7\]](#page-6-0). Measurement of v_2 for identified hadrons as a function of transverse momentum p_T exhibits mass ordering in the low p_T region and a switch-over resulting in higher v_2 values for baryons compared to mesons in the intermediate p_T region, as described in the last section. The baryon and meson v_2 's are separated into two branches. It is observed that v_2/n_q , where n_q is the number of constituent quarks, when plotted as a function of p_T / n_q , is approximately the same for all identified hadrons. Thus the differential elliptic flow is found to show a remarkable universal scaling with the number of valence quarks.

The observed scaling can be well reproduced with the calculations from the parton coalescence and recombination models for hadronization of quark-gluon plasma [\[22–24\]](#page-6-0). The basic assumption of these models is that the invariant spectrum of produced particles is proportional to the product of the invariant spectra of the constituents. This suggests that in the region of p_T where the recombination of partons dominates the process of hadronization, the effect of mass is minimal,

FIG. 4. Variation of v_2/n_q with p_T/n_q at top and intermediate FAIR energies with different models. Top: results for 40*A* GeV. Bottom: results for 25*A* GeV. In both cases left-hand panels represent results obtained from UrQMD, and middle and right-hand panels show predictions from the string melting scenario and default hadronic version of AMPT, respectively.

models, in the investigated p_T regime, do not show the behavior that would be expected if quark coalescence were dominant.

In addition to the transverse momentum p_T , the transverse kinetic energy KE_T is also used at RHIC, as a scaling variable to explore the NCQ scaling behavior of v_2 [\[27,28\]](#page-6-0). The transverse kinetic energy, KE_T (= $m_T - m_0$), is defined as the difference between the transverse mass and the rest of the mass of a particle, where m_T ($=\sqrt{m_0^2 + p_T^2}$) is the transverse mass of a particle of rest mass m_0 . It is considered a robust scaling variable because it takes care of relativistic effects, which are particularly important for lightest particles. The use of this variable stems from the idea that the anisotropic pressure gradients in the transverse plane, which give rise to azimuthal anisotropy, lead to collective transverse kinetic energy of the emitted particles. It has been observed at RHIC that, for different particle species, $v_2(KE_T)/n_q$ collapse onto a single curve over the entire range of $K E_T / n_q$. This is in contrast to p_T scaling, which is found to be poor for $p_T/n_q \leq 1$ GeV/c and becomes better for $p_T/n_q \geqslant 1.3$ GeV/c [\[28\]](#page-6-0). Even though an excellent scaling has ben observed over the entire range of KE_T/n_a values, this does not imply that quark coalescence describes the behavior from low p_T to intermediate p_T at RHIC. In our current understanding, the origin of KE_T scaling is different, and not necessarily related to quark coalescence.

Here we have investigated the NCQ scaling behavior as a function of KE_T for different hadrons at both beam energies. Figure [5](#page-5-0) shows the variation of v_2/n_q as a function of $K E_T/n_q$ at $E_{\text{Lab}} = 25A \text{ GeV}$ and $E_{\text{Lab}} = 40A \text{ GeV}$.

The default version of AMPT without partonic degrees of freedom does not show any remarkable scaling behavior with

respect to KE_T at either of the investigated beam energies. But reasonable scaling behavior can indeed be produced by both the string melting version of AMPT and UrQMD. Observation of the scaling behavior of v_2/n_a as a function of $K E_T / n_a$ over the entire p_T range under study is thus in line with the measurements of RHIC, where hydrodynamic mass scaling is believed to be applicable in the low *KET* region.

UrQMD is a hadronic-string transport model; it does not include quark and gluons as effective degrees of freedom. On the other hand, in the AMPT model with the string melting scenario, strings are melted into soft partons that can undergo only elastic scatterings among themselves, and hence the total number of partons in the system is exactly equal to the number of constituent quarks in the produced hadrons. Thus two different transport models with different degrees of freedom as input are found to exhibit a considerable amount of scaling for *v*2*/nq* of identified hadrons when plotted as a function of KE_T/n_q . In view of this observation it appears that the NCQ scaling of hadronic v_2 with respect to KE_T will probably not help us to draw any conclusive picture about the nature of the dense nuclear matter expected to be produced in relativistic nuclear collisions at FAIR. Relative values of v_2 might play a better role in distinguishing the two scenarios, that is, the partonic scenario and the hadronic scenario.

To extend our study further we have tried to investigate the degree of scaling exhibited by different models. We have plotted the deviation $\delta(v_2/n_q)$ of the scaled v_2 values of each hadron from their average value as a function of the scaled $K E_T$. The results for UrQMD and the string melting version of AMPT are shown in Fig. [6.](#page-5-0) Because the default version

FIG. 5. Variation of v_2/n_q with $K E_T/n_q$ at top and intermediate FAIR energies with different models. Top: results for 40*A* GeV. Bottom: results for 25*A* GeV. In both cases left-hand panels represent results obtained from UrQMD, and middle and right-hand panels show predictions from the string melting scenario and default hadronic version of AMPT, respectively.

of AMPT does not show any reasonable scaling behavior, we have not taken it into consideration.

For both models the baryons and mesons are clearly seen to be separated into two branches. Had the scaling been ideal, $\delta(v_2/n_a)$ would be zero for all particles and they would fall on one another. The nonzero value of $\delta(v_2/n_q)$ implies that the scaling is approximate and it is different for different particles. For mesons, $\delta(v_2/n_a)$ is positive, whereas for baryons it is negative. However, it appears that the deviation is greater in the case of AMPT compared to UrQMD, which possibly implies

that better scaling is exhibited by UrQMD than by AMPT. For a quantitative measure of the degree of scaling exhibited by these models, we define a variable α , the root mean square (RMS) value of the distribution of $\delta(v_2/n_a)$ for all particles over the given p_T range. A better scaling would then result in a lower value of *α*. Table [I](#page-6-0) reports the values of *α* for both models at the two investigated energies. We have found that at both energies, α_{UrOMD} is lower than α_{AMPT} , which signifies a better scaling exhibited by UrQMD than by AMPT in the FAIR energy regime.

FIG. 6. Variation of $\delta(v_2/n_q)$ with KE_T/n_q at top and intermediate FAIR energies with different models. Top: results for 40*A* GeV. Bottom: results for 25*A* GeV. In both cases left-hand panels represent results obtained from UrQMD, and right-hand panels show predictions from the string melting scenario of AMPT.

TABLE I. Comparison of the values of *α*.

E_{Lab} (GeV)	α_{UrOMD}	α AMPT
25	0.005633	0.01008
40	0.005381	0.01025

IV. SUMMARY

In summary, the differential elliptic flow $v_2(p_T)$ of identified hadrons and its scaling with the number of constituent quarks have been investigated in the FAIR energy regime employing both hadronic and partonic transport models. In both cases the observations are quite in line with the elliptic flow measurements at RHIC. It has been found that AMPT with string melting that includes partonic scatterings generates larger flow compared to the hadronic models.

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Neither of the models is found to produce the constituent quark number scaling behavior of hadronic v_2 with respect to p_T , over the p_T range studied in this paper, at either of the investigated energies at FAIR. Both models exhibit NCQ scaling behavior, to varying degrees, with respect to KE_T , which might be attributed to the hydrodynamicspredicted hadron mass ordering of $v_2(p_T)$, valid in the low p_T region [8]. The presence of constituent quark number scaling of hadronic v_2 in terms of KE_T , in the hadronic as well as the partonic model, makes this observable rather insensitive for distinguishing between hadronic and partonic phases at FAIR. A quantitative analysis of the scaling in terms of the scaling variable α further shows that UrQMD exhibits better scaling properties compared to AMPT. If at all, a universal scaling behavior of elliptic flow with respect to KE_T is observed at FAIR; whether or not it should be interpreted as a signature for the formation of a partonic medium remains a debated issue.

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