

$\pi \Xi$ correlations: An interweaving of resonance interaction, channel coupling, and Coulomb effects

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A method is presented for analysis of correlation function of two nonidentical particles with strong and Coulomb interactions, resonance formation, channel coupling, and spin structure. For resonance reactions we derive a formula giving the small distance contribution to the correlation function. The formalism is used to analyze the preliminary Relativistic Heavy Ion Collider data on $\pi^\pm \Xi^\mp$ correlation measurements. The $\Xi^*(1530)$ resonance is successfully described. The $\pi \Xi$ source size is obtained.

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I. INTRODUCTION

Measurements of momentum correlations of two low-relative-momentum particles produced in heavy-ion collisions provide a unique information on spatiotemporal picture of the emission source at the level of few fermis. The study of the collision region on the femtometer scale via two-particle correlations is called *femtoscopy* (see, for example, Refs. [1–7]). At the early stages the studies were focused on the identical pions correlations arising from the wave function symmetrization [8]. This type of analysis has a deep analogy with the “HBT interferometry” used in astronomy [9]. Since then, measurements have been performed for different systems of both identical and nonidentical hadrons. High-statistics data sets were accumulated in heavy-ion experiments at the BNL Alternating Gradient Synchrotron, CERN Super Proton Synchrotron, and BNL Relativistic Heavy Ion Collider (RHIC) accelerators [10–17]. Correlations are significantly affected by the Coulomb and/or strong final-state interaction (FSI) between outgoing particles. The nonidentical particle correlations due to the FSI provide information not only about space-time characteristics of the emitting source, but also about the average relative space-time separation between the emission points of the two particle species in the pair rest frame (PRF) [18].

Maybe the most exotic system studied recently by STAR Collaboration is $\pi - \Xi$ [19,20]: the particles composing the pair have one order of magnitude difference in mass plus $\Delta B = 1/\Delta S = 2$ gap in baryon/strangeness quantum numbers. It is challenging to study FSI of such exotic meson-baryon system and to extract information about the $\pi - \Xi$ scattering lengths. The other important reason to study $\pi - \Xi$ correlations is that multistrange baryons are expected to decouple earlier than other particle species because of their small hadronic cross sections [21], allowing one to extract the space-time interval between the different stages of the fireball evolution.

Preliminary results for the $\pi \Xi$ system are available from STAR Collaboration [19,20]. The following important observations were made:

- (i) Decomposition of the correlation function $C(\vec{k}) \equiv C(k, \cos \theta, \varphi)$ [\vec{k} is the center-of-mass (c.m.) momentum], from 10% of the most central Au + Au collisions into spherical harmonics, provided the first preliminary values of $R = (6.7 \pm 1.0)$ fm and $\Delta_{\text{out}} =$

(-5.6 ± 1.0) fm. The negative value of the shift parameter Δ_{out} indicates that the average emission point of Ξ is positioned more to the outside of the fireball than the average emission point of the pion.

- (ii) In addition to Coulomb interaction present in previous nonidentical particle analysis the $\pi^+ \Xi^-$ correlations at small relative momenta provide sufficiently clear signal of the strong FSI that reveals itself in a peak corresponding to the $\Xi^*(1530)$ resonance. The peak’s centrality dependence shows a high sensitivity to the source size.
- (iii) In Refs. [19,20] qualitative agreement was shown with the model calculations in the Coulomb region and overestimation of the peak in the $\Xi^*(1530)$ region.

It is clear that the problem of $\pi \Xi$ correlations deserves a thorough theoretical treatment which is undertaken in the present article. We will be concentrated on finding the expression for the $\pi \Xi$ wave function (WF) with Coulomb and strong interactions included.

The wave function enters as a building block into a correlation function (CF):

$$C(\vec{k}) = \int d\vec{r} S(\vec{r}) |\Psi_{\vec{k}}(\vec{r})|^2, \quad (1)$$

where $S(\vec{r})$ is a source function and $\Psi_{\vec{k}}(\vec{r})$ is the two final-state particles WF.

The influence of the Coulomb interaction can be taken into account following the standard procedure described in textbooks. A model-independent approach to strong FSI is absent since the low-energy hadron interactions cannot be described from the first principles of QCD. Phenomenological approach to the combined treatment of Coulomb and strong FSI is based on the effective range expansion of the strong amplitude. In case of $\pi \Xi$ system the problem is rather complicated due to the following factors: [22–25].

- (i) The superposition of strong and Coulomb interactions.
- (ii) The presence of $\Xi^*(1530)$ resonance.
- (iii) The spin structure of the WF including spin-flip.
- (iv) The fact that the $\pi^+ \Xi^-$ state is a superposition of $I = 1/2$ and $I = 3/2$ isospin states and that $\pi^+ \Xi^-$ state is coupled to the $\pi^0 \Xi^0$ and that the thresholds of the two channels are nondegenerate.

- (v) The contribution from inner potential region where the structure of the strong interaction is unknown.

The description of the $\pi \Xi$ correlation is a twofold problem. First, one has to construct the $\pi \Xi$ WF with all factors enumerated above included. Second, one has to use a reasonable model for the source function. In the present article we concentrate on the first task, while the source function is taken in the simplest Gaussian form. Limitations of the naive Gaussian model are well known and will be briefly discussed at the end of the article. In Ref. [19,20] the blast wave model for the source was used in combination with the FSI model from Ref. [22], the simultaneous description of the Coulomb and resonance regions was not obtained. Here we obtained successful description of the source size using simple Gaussian source model. It is clear that more elaborated source model is needed to describe simultaneously source sizes and emission asymmetries.

The article is organized as follows. Section II is devoted to a somewhat pedagogical introduction to the description of the FSI. Our purpose will be to expose the formalism in a way suited for the construction of the CF. In Sec. III we consider the spin-isospin structure of the WF and the coupling between charged and neutral $\pi \Xi$ channels. Here we also present the first fit to the experimental data. This fit encounters a problem in the resonance region, and the blame for this discrepancy lies in the disregard of the small distance region. The small distance contribution to the WF is derived in Sec. IV. Section V is devoted to the analysis of the experimental data. In Sec. VI the main results are presented and open problems are formulated.

II. THE BASIC FSI FORMALISM

As it was adumbrated above, the structure of the $\pi \Xi$ WF is rather complicated. To set the scene for the detailed treatment of the FSI in the $\pi \Xi$ system we start with a pedagogical introduction into the FSI problem in the context of femtoscopy. Almost all results presented in this paragraph may be found in the literature. What we have tried here is to gather in one place the properties of the out-state WF $\Psi_{\vec{k}}^{(-)}(\vec{r})$ needed to calculate the CF. Thus our treatment does not seriously overlap with any other in the literature.

There are two complete sets $\Psi_{\vec{k}}^{(\pm)}(\vec{r})$ of the continuous spectrum wave functions. In $\Psi_{\vec{k}}^{(+)}(\vec{r})$ the ingoing flux is nonzero only in the direction $\vec{n} = -\vec{m}$, $\vec{n} = \vec{k}/k$, $\vec{m} = \vec{r}/r$. This corresponds to particles moving along \vec{n} toward the center. The flux of outgoing particles in $\Psi_{\vec{k}}^{(+)}(\vec{r})$ consists of two parts: nonscattered ones moving along \vec{n} in the direction $\vec{n} = \vec{m}$ out of the center and scattered particles moving in all directions. On the contrary, in $\Psi_{\vec{k}}^{(-)}(\vec{r})$ ingoing particles are moving from all directions and outgoing particles are propagating only along the ray $\vec{n} = \vec{m}$. Asymptotically at $r \rightarrow \infty$ both functions $\Psi_{\vec{k}}^{(\pm)}(\vec{r})$ contain the same plane wave $e^{i\vec{k}\vec{r}}$:

$$\Psi_{\vec{k}}^{(+)}(\vec{r}) \sim e^{i\vec{k}\vec{r}} + f(\vec{n}\vec{m}) \frac{e^{ikr}}{r} \quad (2)$$

$$\Psi_{\vec{k}}^{(-)}(\vec{r}) \sim e^{i\vec{k}\vec{r}} + f^*(-\vec{n}\vec{m}) \frac{e^{-ikr}}{r}. \quad (3)$$

Coulomb interaction and channel coupling bring some distortions into these equations—see below. The following relation between $\Psi_{\vec{k}}^{(+)}(\vec{r})$ and $\Psi_{\vec{k}}^{(-)}(\vec{r})$ holds

$$\Psi_{\vec{k}}^{(-)}(\vec{r}) = \Psi_{-\vec{k}}^{(+)*}(\vec{r}). \quad (4)$$

The wave function $\Psi_{\vec{k}}^{(-)}(\vec{r})$ which is called the out-state is used to describe particles produced in some process. Suppose, for example, that some state $\phi(\vec{r})$ is created in the potential $U(\vec{r})$. Then the flow of the particles with the wave vector \vec{k} emitted from the center is given by

$$N(\vec{k}) = \int d\vec{r} \phi(\vec{r}) [\Psi_{\vec{k}}^{(-)}(\vec{r})]^*. \quad (5)$$

The out-state $\Psi_{\vec{k}}^{(-)}(\vec{r})$ has to be used in Eq. (1) for the correlation function.

Consider a pair of particles interacting via attractive Coulomb potential (like $\pi^+ - \Xi^-$). The WF $\Psi_{k_c}^{(-)}(\vec{r})$ reads

$$\Psi_{k_c}^{(-)}(\vec{r}) = e^{\pi\eta/2} \Gamma(1+i\eta) e^{i\vec{k}\vec{r}} F(-i\eta, 1, -i(kr + \vec{k}\vec{r})). \quad (6)$$

Here $\eta = 1/ka_B$, a_B is the Bohr radius, $a_B = 1/\alpha\mu$ ($=214$ fm for $\pi^+ - \Xi^-$), μ is the reduced mass ($=126$ MeV for $\pi^+ - \Xi^-$), $\alpha = 1/137$, and $F(\alpha, \gamma, z)$ is the confluent hyper-geometric function. The region $\eta \geq 1$ is called the atomic energy range. In this region Coulomb interaction dominates even in presence of strong interaction. The wave function is normalized so that

$$|\Psi_{k_c}^{(-)}(\vec{r} = 0)|^2 = \frac{2\pi\eta}{1 - e^{-2\pi\eta}} = c^2(k), \quad (7)$$

where $c^2(k)$ is Gamow's factor.

As an illustration let us take in (1) a Gaussian model for the source function

$$S(\vec{r}) = (8\pi^{3/2} R^3)^{-1} \exp(-r^2/4R^2). \quad (8)$$

Expanding $\Psi_{k_c}^{(-)}(\vec{r})$ in r/a_B and keeping only the first-order term we obtain the following result for the correlation function (1):

$$C(k) = c^2(k) \left\{ 1 - \frac{8}{\sqrt{\pi}} \frac{R}{a_B} + 8 \frac{R^2}{a_B^2} \right\}. \quad (9)$$

The partial wave expansion of the Coulomb WF is given by

$$\Psi_{k_c}^{(-)}(\vec{r}) = \frac{1}{kr} \sum_{l=0}^{\infty} i^l (2l+1) e^{-i\sigma_l} F_l(\eta, kr) P_l \left(\frac{\vec{k}\vec{r}}{kr} \right), \quad (10)$$

where σ_l is the Coulomb phase

$$\sigma_l = \arg \Gamma(l+1-i\eta), \quad (11)$$

$$e^{-i\sigma_l} = \frac{\Gamma(l+1+i\eta)}{|\Gamma(l+1+i\eta)|},$$

and $F_l(\eta, kr)$ is the regular solution of the Coulomb problem

$$F_l(\eta, kr) = N_l e^{-ikr} (kr)^{l+1} F(l+1+i\eta, 2l+2, 2ikr), \quad (12)$$

$$N_l = \frac{2^l}{(2l+1)!} \left\{ c^2(k) \prod_{m=1}^l (m^2 + \eta^2) \right\}^{1/2}. \quad (13)$$

Asymptotically $F(\eta, kr)$ behaves as

$$F_l(\eta, kr) \sim \sin \left[kr - \frac{l\pi}{2} + \eta \ln(2kr) + \sigma_l \right]. \quad (14)$$

Irregular solution is denoted by $G_l(\eta, kr)$ and it has the following asymptotes

$$G_l(\eta, kr) \sim \cos \left[kr - \frac{l\pi}{2} + \eta \ln(2kr) + \sigma_l \right]. \quad (15)$$

Asymptotically at $\xi = r(1 + \cos \theta) \gg 1/k$ one has

$$\Psi_{kc}^{(-)}(\vec{r}) \rightarrow \left(1 + i \frac{\eta^2}{k\xi} \right) \exp\{i\vec{k}\vec{r} + i\eta \ln(k\xi)\} + \frac{f_c^{(-)}(\theta)}{r} \exp\{-ikr - i\eta \ln(2kr)\}, \quad (16)$$

$$f_c^{(-)}(\theta) = \frac{\eta}{2k \cos^2 \frac{\theta}{2}} \frac{\Gamma(1+i\eta)}{\Gamma(1-i\eta)} \exp \left\{ -2i\eta \ln \left(\cos \frac{\theta}{2} \right) \right\}. \quad (17)$$

When the produced particles interact only via strong forces their wave function outside the interaction range (at $r > \epsilon \simeq 1$ fm) has the following form

$$\Psi_{ks}^{(-)}(\vec{r}) = \frac{1}{kr} \sum_{l=1}^{\infty} i^l (2l+1) e^{-i\delta_l} \chi_{kl}(r) P_l \left(\frac{\vec{k}\vec{r}}{kr} \right). \quad (18)$$

The radial wave function $\chi_{kl}(r)$ asymptotically behaves as

$$\chi_{kl}(r) \sim \sin \left(kr - \frac{l\pi}{2} + \delta_l \right). \quad (19)$$

Consider now a situation when FSI is caused by the combined action of the Coulomb and strong interactions. Then at $r > \epsilon$ we have

$$\Psi_{kcs}^{(-)}(\vec{r}) = \frac{1}{kr} \sum_{l=0}^{\infty} i^l (2l+1) e^{-i(\tilde{\delta}_l + \sigma_l)} u_{kl}(r) P_l \left(\frac{\vec{k}\vec{r}}{kr} \right). \quad (20)$$

The radial WF $u_{kl}(r)$ is a regular solution of the Schrodinger equation containing the sum of Coulomb and strong potentials. An important point is that $\tilde{\delta}_l$ entering into (20) is not identical with the pure hadronic phase shift δ_l in (18). The difference is called the Coulomb correction [26] and will be explicitly introduced later.

At $r > \epsilon$ the radial wave function in (20) has the form

$$u_{kl}(r) = F_l(\eta, kr) \cos \tilde{\delta}_l + G_l(\eta, kr) \sin \tilde{\delta}_l. \quad (21)$$

Making use of (21) and of the expression (10) for the Coulomb WF we obtain

$$\begin{aligned} \Psi_{kcs}^{(-)}(\vec{r}) &= \Psi_{kc}^{(-)}(\vec{r}) + \frac{1}{2kr} \sum_{l=0}^{\infty} i^l (2l+1) e^{-i\sigma_l} (e^{-2i\tilde{\delta}_l} - 1) \\ &\quad \times [F_l(\eta, kr) + iG_l(\eta, kr)] P_l \left(\frac{\vec{k}\vec{r}}{kr} \right) \\ &= \Psi_{kc}^{(-)}(\vec{r}) + \sum_{l=0}^{\infty} \frac{2l+1}{\sqrt{4\pi}} \varphi_l(kr) T_l(k) P_l \left(\frac{\vec{k}\vec{r}}{kr} \right), \quad (22) \end{aligned}$$

where

$$\varphi_l(kr) = \frac{\sqrt{4\pi}}{kr} i^l e^{-i\sigma_l} \left(\frac{F_l + iG_l}{i} \right), \quad (23)$$

$$T_l(k) = \frac{e^{-2i\tilde{\delta}_l} - 1}{-2i} \quad (24)$$

Recalling the asymptotic form of $\Psi_{kc}^{(-)}(\vec{r})$ given by Eqs. (16) and (17) and the asymptotic behavior of F_l and G_l defined by Eqs. (14) and (15) one arrives at the following asymptotic of $\Psi_{kcs}^{(-)}(\vec{r})$

$$\begin{aligned} \Psi_{kcs}^{(-)}(\vec{r}) &\sim \left(1 + i \frac{\eta^2}{k\xi} \right) e^{i\vec{k}\vec{r} + i\eta \ln(k\xi)} \\ &\quad + (f_c^{(-)}(\theta) + f_{cs}^{(-)}(\theta)) \frac{1}{r} e^{-i(kr + \eta \ln 2kr)}, \quad (25) \end{aligned}$$

where the Coulomb amplitude $f_c^{(-)}(\theta)$ is given by (17) and the Coulomb modified strong amplitude $f_{cs}^{(-)}(\theta)$ is equal to

$$f_{cs}^{(-)}(\theta) = \frac{i}{2k} \sum_{l=0}^{\infty} (-1)^l (2l+1) e^{-2i\sigma_l} (e^{-2i\tilde{\delta}_l} - 1) P_l \left(\frac{\vec{k}\vec{r}}{kr} \right). \quad (26)$$

Now let us turn to the low energy expansion of the WF (22). For the moment we are not interested in its spin-isospin structure. This problem will be addressed in the next section. Modification of the effective range expansion due to Coulomb interaction is a problem whose solution is well known [27–29]. Here we quote the results relevant for the construction of the out-state WF $\Psi_{kcs}^{(-)}(\vec{r})$. Coulomb corrections are most important for the S wave. The basic role in the low energy expansion is played by the effective range function $M(k)$. In the S wave we have

$$T_0 = \frac{e^{-2i\tilde{\delta}_0} - 1}{-2i} = \frac{kc^2(k)}{M(k) + \Delta + i\tilde{k}}. \quad (27)$$

Here Δ is the so-called Schwinger's correction [30]

$$\Delta = \frac{2}{a_B} \left(\ln \frac{2\epsilon}{a_B} + 2\gamma \right), \quad (28)$$

$\gamma \simeq 0.577$ is Eugler's constant and \tilde{k} is the Coulomb corrected momentum.

$$\tilde{k} = c^2(k)k - \frac{2i}{a_B} h(\eta), \quad (29)$$

$$h(x) = \ln(x) + \text{Re}\Psi \left(1 + \frac{i}{x} \right) \simeq \begin{cases} \frac{x^2}{12} & x \ll 1 \\ -\gamma + \ln(x) + \frac{1}{x^2} & x \gg 1 \end{cases} \quad (30)$$

with $\Psi(z) = \frac{d}{dz} \ln \Gamma(z)$. In what follows the Schwinger's correction will not be taken into account. The function $M(k)$ has the usual effective range expansion

$$M(k) = \frac{1}{a} + \frac{1}{2} R_0 k^2 + \dots \quad (31)$$

Similar procedure for the P -wave leads to the result

$$T_1 = \frac{e^{-2i\tilde{\delta}_1} - 1}{-2i} = \frac{k^3(1 + \eta^2)c^2(k)}{N(k) + i\tilde{k}k^2(1 + \eta^2)} \quad (32)$$

with

$$N(k) = \frac{1}{b} + \frac{1}{2}R_1k^2 + \dots, \quad (33)$$

where b is the scattering volume with the dimension fm^3 and $R_1 = -3/d$ with d having the interpretation of the range of forces. The Coulomb correction in the P wave similar to Δ [see (26)] can be neglected.

The important feature of the $\pi \Xi$ system is the existence of the P -wave resonance $\Xi(1530)P_{13}$ with a width $\Gamma = 9.1$ MeV. The corresponding value of the c.m. momentum is $k = 150$ MeV, $\eta = 1/ka_B \simeq 1/160$ and therefore Coulomb corrections in this region are small. Let us recast the P -wave term (32) into the Breit-Wigner form. We get

$$T_1 = \frac{k^3(1 + \eta^2)c^2(k)}{N(k) + i\tilde{k}k^2(1 + \eta^2)} \simeq -\frac{1}{2} \frac{\Gamma}{E - E_R - i\Gamma/2}, \quad (34)$$

where

$$k^2 = \frac{\lambda(s, M^2, m^2)}{4s} \simeq 2\mu E, \quad (35)$$

$$E = \sqrt{s} - (M + m), \quad (36)$$

$$E_R = M^* - (M + m), \quad (37)$$

$$\Gamma = \frac{2d}{3\mu}k^3 \quad (38)$$

with M and m being the masses of Ξ and π correspondingly, $M^* = 1530$ MeV is the mass of the Ξ^* resonance, $\mu = mM(M + m)^{-1}$. The scattering volume b is expressed in terms of the Breit-Wigner resonance as

$$b = \frac{d}{3\mu E_R}. \quad (39)$$

III. SPIN-ISOSPIN STRUCTURE OF THE WAVE FUNCTION

Until now the spin-isospin structure of the $\pi^\pm \Xi^\mp$ WF has been disregarded. In this section we shall wind up this lacuna.

It is sensible to begin by considering the angular momentum decomposition of the WF with a given isospin I . The isospin structure of the WF as well as the coupling to the $\pi^0 \Xi^0$ channel will be included at the next stage. The amplitude of meson-baryon scattering has the following form:

$$T = f + g(\vec{\sigma}\vec{n}) \quad (40)$$

$$\vec{n} = \frac{[\vec{k}, \vec{k}']}{||[\vec{k}, \vec{k}']||} \quad (41)$$

The second term in (40) corresponds to the spin-flip amplitude.¹ Now let us write down the expression for the WF with the spin-flip term included. This expression will replace the spin-less Eqs. (20),(22). For any type of the interaction the conserved quantum numbers are j^2 , l^2 and $j_z = m_j$. The WF having definite values of the above quantum numbers is

¹The importance of the elastic spin-flip and charge-exchange amplitudes in this problem was indicated to the authors by R. Lednicky.

constructed from the direct product of the WFs (20), (22) and the spin WFs

$$v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (42)$$

corresponding to $m_j = m_s = +1/2$ and $m_j = m_s = -1/2$, respectively. The resulting expressions read

$$\Psi_{m_j=1/2}^{(-)} = \Psi_{kc}^{(-)} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sum_{l=0}^{\infty} \phi_l(kr) \left[T_{l+} \sqrt{\frac{l+1}{2l+1}} \begin{pmatrix} \sqrt{l+1} Y_{l,0} \\ \sqrt{l} Y_{l,-1} \end{pmatrix} + T_{l-} \sqrt{\frac{l}{2l+1}} \begin{pmatrix} \sqrt{l} Y_{l,0} \\ -\sqrt{2l+1} Y_{l,-1} \end{pmatrix} \right] \quad (43)$$

$$\Psi_{m_j=-1/2}^{(-)} = \Psi_{kc}^{(-)} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \sum_{l=0}^{\infty} \varphi_l(kr) \left[T_{l+} \sqrt{\frac{l+1}{2l+1}} \begin{pmatrix} \sqrt{l+1} Y_{l,1} \\ \sqrt{l} Y_{l,0} \end{pmatrix} - T_{l-} \sqrt{\frac{l}{2l+1}} \begin{pmatrix} \sqrt{l+1} Y_{l,1} \\ -\sqrt{l} Y_{l,0} \end{pmatrix} \right]. \quad (44)$$

Here the symbols $(l\pm)$ correspond to $j = l \pm 1/2$. In deriving (43) and (44) use was made of the relation $Y_{l,m}^*(-\vec{k}/k) = (-1)^{l+m} Y_{l,-m}(\vec{k}/k)$.

The low-energy region of $\pi \Xi$ interaction up to the $\Xi^*(1530)$ resonance is dominated by S and P waves. Keeping only these two amplitudes we can rewrite (43) as follows

$$\Psi_{m_j=1/2}^{(-)} = \Psi_{kc}^{(-)} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \varphi_0 Y_{00} T_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{3}} \varphi_1 Y_{1,0} (2T_{1+} + T_{1-}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sqrt{\frac{2}{3}} \varphi_1 Y_{1,-1} (T_{1+} - T_{1-}) \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (45)$$

Similar expression holds for $m_j = -1/2$. The last term in (45) corresponds to spin-flip and this contribution vanishes if $\delta_{l+} = \delta_{l-}$. Recalling about isospin doubling we conclude that the WF (45) contains two S -wave amplitudes and four P -wave ones. To determine six amplitudes from femtoscopic experiments is hardly possible. To reduce the number of parameters we shall assume that the dominant interaction in the P wave occurs in a state with $j = l + 1/2 = 3/2$, $I = 1/2$ containing the $\Xi^*(1530)$ resonance [the amplitude T_{1+} in (45)]. Then the number of parameters becomes equal to three. The actual number is two since the parameters of the $\Xi^*(1530)$ resonance are known from the experiment. There might be one more parameter. The point is that expressions (20), (22), and (43)–(45) for the WF correspond to distances $r > \epsilon \sim 1$ fm where the strong interaction is assumed to vanish. We shall address this problem in the next section and show that the contribution of the inner region is important while ϵ is not a relevant parameter.

We now proceed to discuss the isospin structure of the WF and coupling between charged and neutral $\pi \Xi$ channels. There are two $\pi \Xi$ systems with opposite electric charges: $\pi^+ \Xi^-$ and $\pi^- \Xi^+$. For definitiveness we shall consider the charged channel $\pi^+ \Xi^-$ which is coupled to the neutral channel $\pi^0 \Xi^0$. The corresponding thresholds are not degenerate with the $\pi^+ \Xi^-$ threshold being by $\simeq 11$ MeV higher. The problem can be treated either in the channel basis $\{|\pi^+ \Xi^- \rangle, |\pi^0 \Xi^0 \rangle\}$ or

in the isospin basis $\{|I = \frac{1}{2}\rangle, |I = \frac{3}{2}\rangle\}$. The relation between the two frames is given by

$$\begin{pmatrix} |\pi^+ \Xi^- \rangle \\ |\pi^0 \Xi^0 \rangle \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} \end{pmatrix} \begin{pmatrix} |I = \frac{1}{2}\rangle \\ |I = \frac{3}{2}\rangle \end{pmatrix} \equiv \hat{U} \begin{pmatrix} |I = \frac{1}{2}\rangle \\ |I = \frac{3}{2}\rangle \end{pmatrix}. \quad (46)$$

Strong interaction is diagonal in the isospin basis while we are interested in the channel WF $|\pi^+ \Xi^- \rangle$ (we shall use the subscript α for the quantities corresponding to this channel and β —for those corresponding to $|\pi^0 \Xi^0 \rangle$). Similar problems have been treated previously by several authors [28,29,31]. We shall more or less follow the approach proposed by Shaw and Ross [28]. In Sec. II we have introduced the function $M(k) = k \cot \delta$ which allows us to perform the effective range expansion in presence of the Coulomb interaction [see Eqs. (27)–(31)]. The generalization to the two-channel system reads

$$\hat{T} = \hat{k}^{l+1/2} (\hat{M} - i \hat{k}^{2l+1})^{-1} \hat{k}^{l+1/2}, \quad (47)$$

where

$$\hat{k} = \begin{pmatrix} k_\alpha & 0 \\ 0 & k_\beta \end{pmatrix}, \quad \hat{M} = \begin{pmatrix} M_{\alpha\alpha} & M_{\alpha\beta} \\ M_{\beta\alpha} & M_{\beta\beta} \end{pmatrix}. \quad (48)$$

Consider first the S wave. In single channel case in the scattering length approximation one has $M = 1/a$ [see (31)]. In the two-channel case in the isospin basis one has

$$\hat{M}^I = \begin{pmatrix} \frac{1}{a_s} & 0 \\ 0 & \frac{1}{a_t} \end{pmatrix}, \quad (49)$$

where a_s corresponds to $I = 1/2$ and a_t to $I = 3/2$. Transformation to the channel basis reads

$$\hat{M} = \hat{U} \hat{M}^I \hat{U}^{-1} = \frac{1}{3} \begin{bmatrix} \frac{2}{a_s} + \frac{1}{a_t} & \sqrt{2} \left(-\frac{1}{a_s} + \frac{1}{a_t} \right) \\ \sqrt{2} \left(-\frac{1}{a_s} + \frac{1}{a_t} \right) & \frac{1}{a_s} + \frac{2}{a_t} \end{bmatrix}, \quad (50)$$

where the matrix \hat{U} was introduced in (46). According to (47) the two-channel generalization of (27) is

$$T_0 = k_\alpha c^2 (k_\alpha) (M_{\beta\beta} + i k_\beta) d^{-1}, \quad (51)$$

$$d = (M_{\alpha\alpha} + i \tilde{k}_\alpha) (M_{\beta\beta} + i k_\beta) - M_{\alpha\beta} M_{\beta\alpha}, \quad (52)$$

where as before the Schwinger's correction (28) has been neglected. We can now substitute (51) into (45) and obtain the closed expression for the S -wave component of the WF. However, in the two-channel case this would not be a complete answer. One should add the contribution from the coupling to the $|\pi^0 \Xi^0 \rangle$ channel, i.e., a term $\Psi^{(-)} \sim S_{\beta \leftarrow \alpha}^* \Psi_\beta^{(-)}$. The corresponding contribution to (45) is

$$\Psi_{m_j=1/2}^{(-)}(\beta \leftarrow \alpha)_S = \left(\frac{k_\beta}{k_\alpha} \right)^{1/2} \omega_0 Y_{00} R_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (53)$$

Here

$$\omega_l(k_\beta r_\beta) = \frac{\sqrt{4\pi}}{k_\beta r_\beta} i^l e^{-i\sigma_l(k_\alpha)} \hat{h}_l^{(-)}(k_\beta r_\beta), \quad (54)$$

$$R_0 = -c(k_\alpha) (k_\alpha k_\beta)^{1/2} M_{\alpha\beta} d^{-1}, \quad (55)$$

where $\hat{h}_l^{(\pm)}(z)$ is Hankel function

$$\hat{h}_0^{(\pm)}(z) = e^{\pm iz}, \quad \hat{h}_1^{(\pm)}(z) = (1 \pm i/z) e^{\pm i(z-\pi/2)}. \quad (56)$$

We note that $\omega_l(\rho) = \lim_{\eta \rightarrow 0} \varphi_l(\rho)$, where $\varphi_l(\rho)$ was introduced by (23).

As it was stated above the S -wave component of the $|\pi^+ \Xi^- \rangle$ WF depends on two parameters, namely the isospin scattering lengths a_s and a_t . With our sign convention moderate attraction corresponds to positive signs of the scattering lengths. Next we turn to the P wave. We assume that the $\Xi^*(1530)$ resonance with $I = 1/2$ plays the dominant role and therefore we neglect the P wave with $I = 3/2$. The resonance is coupled to both $\pi^+ \Xi^-$ and $\pi^0 \Xi^0$ channels. The corresponding amplitude T_{1+} and R_{1+} are evaluated using the matrix \hat{U} (46). Denoting the resonance width in the isospin basis by Γ we make the transition to the channel basis

$$\hat{U} \hat{\Gamma} \hat{U}^{-1} = \hat{U} \begin{pmatrix} \Gamma & 0 \\ 0 & 0 \end{pmatrix} \hat{U}^{-1} = \frac{1}{3} \begin{pmatrix} 2\Gamma & -\sqrt{2}\Gamma \\ -\sqrt{2}\Gamma & \Gamma \end{pmatrix}. \quad (57)$$

In line with (34) we shall omit the Coulomb corrections and write

$$T_{1+} = -\frac{1}{2} \frac{(\hat{U} \hat{\Gamma} \hat{U}^{-1})_{\alpha\alpha}}{E - E_R - i \frac{\Gamma}{2}} = -\frac{1}{3} \frac{\Gamma}{E - E_R - i \frac{\Gamma}{2}}, \quad (58)$$

$$R_{1+} = -\frac{1}{2} \frac{(\hat{U} \hat{\Gamma} \hat{U}^{-1})_{\alpha\beta}}{E - E_R - i \frac{\Gamma}{2}} = \frac{1}{3\sqrt{2}} \frac{\Gamma}{E - E_R - i \frac{\Gamma}{2}}. \quad (59)$$

In writing (59) we made the approximation

$$(\hat{U} \hat{\Gamma} \hat{U}^{-1})_{\alpha\beta} = -\Gamma \left[\frac{\Gamma_\alpha(k_\alpha)}{\Gamma} \right]^{1/2} \left[\frac{\Gamma_\beta(k_\beta)}{\Gamma} \right]^{1/2} \simeq -\frac{\sqrt{2}}{3} \Gamma(k_\alpha). \quad (60)$$

Two remarks are in order at this point. First, the amplitudes T_l and R_l are added non-coherently. This will be visualized in the expression for $|\Psi^{(-)}|^2$. Second, in order to calculate the CF according with (1), or a similar equation, one has to take $r_\beta = r_\alpha = r$. This is a reasonable approximation since the reduced masses in $\pi^+ \Xi^-$ and $\pi^0 \Xi^0$ channels are close to each other.

Now we can collect all pieces together and write the expressions for $\Psi_{m_j}^{(-)}$ and $|\Psi_{m_j}^{(-)}|^2$ with Coulomb interaction included in all partial waves and strong interaction in S and P waves. It is easy to see that

$$|\Psi_{m_j=1/2}^{(-)}|^2 = |\Psi_{m_j=-1/2}^{(-)}|^2 \quad (61)$$

and therefore

$$|\Psi^{(-)}|^2 = \frac{1}{2} \sum_{m_j=\pm 1/2} |\Psi_{m_j}^{(-)}|^2 = |\Psi_{m_j=1/2}^{(-)}|^2. \quad (62)$$

So, we write

$$\begin{aligned} \Psi_{m_j=1/2}^{(-)} &= A_\alpha \begin{pmatrix} Y_{00} \\ 0 \end{pmatrix} + A_\beta \begin{pmatrix} Y_{00} \\ 0 \end{pmatrix} + \sqrt{\frac{2}{3}} B_\alpha \begin{pmatrix} \sqrt{2} Y_{10} \\ Y_{1,-1} \end{pmatrix} \\ &+ \sqrt{\frac{2}{3}} B_\beta \begin{pmatrix} \sqrt{2} Y_{10} \\ Y_{1,-1} \end{pmatrix}, \end{aligned} \quad (63)$$

$$\begin{aligned}
 |\Psi^{(-)}|^2 &= Y_{00}(|A_\alpha|^2 + |A_\beta|^2) + \frac{4}{\sqrt{3}}Y_{00}Y_{10}(\text{Re}A_\alpha B_\alpha^* \\
 &\quad + \text{Re}A_\beta B_\beta^*) + \frac{4}{3}Y_{10}^2(|B_\alpha|^2 + |B_\beta|^2) \\
 &\quad + \frac{2}{3}|Y_{1,-1}|^2(|B_\alpha|^2 + |B_\beta|^2) \\
 &= \frac{1}{4\pi}(|A_\alpha|^2 + |A_\beta|^2 + |B_\alpha|^2 + |B_\beta|^2) \\
 &\quad + \frac{\cos\theta}{\pi}(\text{Re}A_\alpha B_\alpha^* + \text{Re}A_\beta B_\beta^*) \\
 &\quad + \frac{3\cos^2\theta}{4\pi}(|B_\alpha|^2 + |B_\beta|^2). \tag{64}
 \end{aligned}$$

Here

$$A_\alpha = \sqrt{4\pi}\Psi_{kc}^{(-)} + \varphi_0 T_0, \quad B_\alpha = \varphi_1 T_{1+}, \tag{65}$$

$$A_\beta = \left(\frac{k_\beta}{k_\alpha}\right)^{1/2} \omega_0 R_0, \quad B_\beta = \left(\frac{k_\beta}{k_\alpha}\right)^{1/2} \omega_1 R_{1+}. \tag{66}$$

Equation (64) is a sought for result which should allow to evaluate the CF according to (1). In the next paragraph we shall see that this is still not a complete answer.

IV. INCORPORATING THE INNER REGION

With the WF (64) at hand we can use Eq. (1) and try to fit the experimental data [19,20] on the CF. The results are shown in Fig. 1 by dashed line for the fireball radii $R = 7.0$ fm and zero scattering lengths. The low momentum region is properly described. However, in the region of the $\Xi^*(1530)$ resonance we see a dip-bump structure instead of the experimentally observed resonant behavior. The wiggly behavior of the calculated CF is explained by the interference at $\theta \sim \pi$ between $\Psi_{kc}^{(-)}$ and the resonant P wave, see (22), or (44).² Coulomb effects are not important here and for the sake of clarity they can be neglected, the phenomenon is present already in the general expression (3). Keeping in (22) the resonant P wave one has

$$\psi^{(-)}(\theta = \pi) \sim e^{-ikr} \left(1 + \frac{1}{kr} \frac{\Gamma}{E_R - E + i\Gamma/2} \right). \tag{67}$$

This leads to the interference term

$$|\psi_{\theta=\pi}^{(-)}|^2 \sim \frac{2}{kr} \frac{\Gamma(E_R - E)}{(E - E_R)^2 + \Gamma^2/4}. \tag{68}$$

The last expression clearly displays the dip-bump behavior seen in our calculations. The same result can be derived from the expression (64) for $|\psi^{(-)}|^2$.

Contrary to this result the experimental points shown in Fig. 1 exhibit a resonant structure. It means that something is missing in our calculations. Recall that the WFs (43), (44), and (63) correspond to the region $r > \epsilon$ where the strong potential is assumed to vanish. The contribution from the region $r < \epsilon$ is proportional to the time the impinging wave packet spends

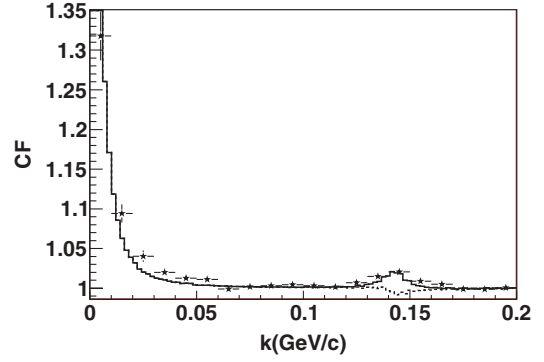


FIG. 1. The CF of $\pi^+ \Xi^-$ system as a function of PRF momentum k for $R = 7.7$ fm and zero scattering lengths without including the inner region correction (dashed line) and the full calculations (solid line), the STAR Collaboration experimental data points from Refs. [19,20] (black stars).

there. For the resonance this time is much larger than is needed to cross the sphere with the radius $r = \epsilon$.

We come now to a formal account of the small distance contribution. First, we split the CF (1) into two parts

$$C(\vec{k}) \simeq \int_{r>\epsilon} d\vec{r} S(\vec{r}) |\Psi_{\vec{k}}^{(-)}(\vec{r})|^2 + S(0) \int_{r<\epsilon} d\vec{r} |\Psi_{\vec{k}}^{(-)}(\vec{r})|^2. \tag{69}$$

The source function was taken out of the second integral because $S(\vec{r})$ has a characteristic scale $R \geq 4$ fm $\gg \epsilon$. The integral over the region $r < \epsilon$ entering into the last equation cannot be evaluated directly since the structure of the WF at small distances is unknown. However, this integral can be expressed in terms of the scattering phase shifts using the Luder-Wigner formula [26,31–33]. This approach is simple and effective when FSI is dominated by a narrow resonance. Otherwise one can apply the method proposed in Ref. [24] (see also Ref. [22]).

Consider the state with $j = 3/2$, $I = 1/2$ containing the $\Xi^*(1530)$ resonance. For the radial WF $\chi_{j,I}(r) = \chi_{3/2,1/2}(r)$ the following relation holds

$$\begin{aligned}
 J &= \int_0^\epsilon dr |\chi_{3/2,1/2}(r)|^2 \\
 &= \frac{1}{2k^2} \left[\frac{d\delta_{3/2,1/2}}{dk} + \epsilon - \frac{1}{2k} \sin 2(k\epsilon + \delta_{3/2,1/2} - \pi/2) \right]. \tag{70}
 \end{aligned}$$

The phase is given by

$$\delta_{3/2,1/2} = -\text{arctg} \frac{\Gamma}{2(E - E_R)}. \tag{71}$$

From (70) and (71) we obtain after a short calculation

$$\begin{aligned}
 2k^2 J &= \frac{k}{\mu} \frac{\Gamma/2}{(E - E_R)^2 + \Gamma^2/4} + \epsilon \\
 &\quad + \frac{[(E - E_R)^2 - \Gamma^2/4] \sin 2k\epsilon - \Gamma(E - E_R) \cos k\epsilon}{2k[(E - E_R)^2 + \Gamma^2/4]}, \tag{72}
 \end{aligned}$$

where μ is the $\pi \Xi$ reduced mass. This expression may be to a high accuracy approximated by the first term only. Indeed, at

²The interference at $\theta \sim \pi$ between the incident wave and the scattered one was observed and explained in Ref. [22].

$E = E_R$ the sum of the last two terms yields

$$\epsilon - \frac{\sin 2k\epsilon}{2k} \simeq \frac{2}{3}(k\epsilon)^2\epsilon \simeq \epsilon, \quad (73)$$

since $(k\epsilon) \sim 1$. The first term at $E = E_R$ is given by

$$\frac{2k}{\mu\Gamma} \simeq \frac{2}{\Gamma} \gg \epsilon. \quad (74)$$

Physically (74) means that particles spend inside the small distance region the time $\sim 1/\Gamma$ which is much larger than the time $\epsilon/v = \epsilon\mu/k$ needed to cross this region.

Therefore from (70) and (64) we obtain for the inner region contribution entering into (69) the following result

$$\begin{aligned} \int_{r<\epsilon} d\vec{r} |\Psi_{\alpha}^{(-)}(\vec{r})|^2 &= \frac{2}{3} \int_{r<\epsilon} d\vec{r} |\Psi_{3/2,1/2}^{(-)}(\vec{r})|^2 \\ &= \frac{2}{3} \int d\Omega \left(\frac{1+3\cos^2\theta}{4\pi} \right) \\ &\quad \times \int_0^{\epsilon} dr (4\pi |\chi_{3/2,1/2}(r)|^2) \\ &= \frac{16\pi}{3} \int_0^{\epsilon} dr |\chi_{3/2,1/2}(r)|^2 \\ &= \frac{8\pi}{3k\mu} \frac{\Gamma/2}{(E - E_R)^2 + \Gamma^2/4}. \end{aligned} \quad (75)$$

Here the factor $2/3$ is the isospin Clebsch-Gordan coefficient squared, the factor 4π in front of $|\chi_{3/2,1/2}(r)|^2$ results from our normalization condition, see (23) and (54). We can rewrite this relation in a relativistic form, as

$$\int_{r<\epsilon} d\vec{r} |\Psi_{\alpha}^{(-)}(\vec{r})|^2 = \frac{8\pi}{3k\mu_{\text{rel}}} \frac{2s\Gamma}{(s - M_R^2)^2 + s\Gamma^2}, \quad (76)$$

$$\mu_{\text{rel}}^{-1} = \frac{2\sqrt{s}(s + M_R^2)}{s^2 - (M^2 - m^2)^2}, \quad (77)$$

with M and m being the Ξ^- and π^+ masses, respectively, out of the resonance region $(s + M_R^2)$ in (77) is replaced by $2s$.

In Fig. 1 by solid line we show the CF calculated according to (69) with the inner region included.

V. ANALYSIS OF THE EXPERIMENTAL DATA

We now turn to the comparison of the calculated CF with the experimental data presented in Refs. [19,20]. From Fig. 1 we conclude that the data are fairly well described if we take the fireball radius $R = 7.7 \pm 0.7$ fm. The fitting was performed by the minimization of the functional $\chi^2 = \sum [\text{CF}(k)_{\text{exp}} - \text{CF}(k)_{\text{model}}]^2 / \sigma_{\text{exp}}^2$. The only fitted parameter was R , the scattering lengths a_s and a_t were put equal to zero (see below), calculations were performed for $\epsilon = 0.8$ fm, but as shown above for a narrow resonance the results are stable with respect to variations of ϵ at $\epsilon \ll R$.

Interpreting Fig. 1 one should keep in mind that in different momentum intervals the CF yields different physical information. Unless S -wave scattering lengths are large, which can hardly be expected for the $\pi \Xi$ system, and unless the fireball radius is anomalously small, the attractive Coulomb interaction becomes dominant already at $k \leq 0.05$ GeV/c. Strong interaction effects in this region are roughly speaking

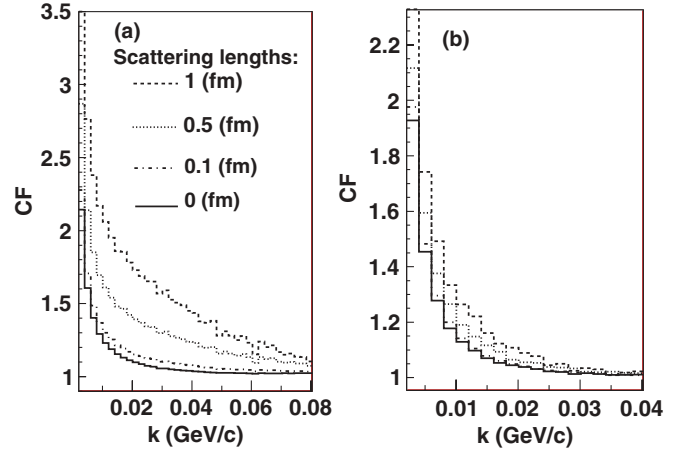


FIG. 2. The CF of $\pi^+ \Xi^-$ system as a function of PRF momentum k for $R = 2.0$ fm (a) and $R = 7.0$ fm (b) with the S -wave scattering lengths $a_{1/2} = a_{3/2}$: 0 fm (solid line), 0.1 fm (dashed-dotted line), 0.5 fm (dotted line), and 1.0 fm (dashed line).

proportional to a_i^2/R^2 ($i = s, t$) and therefore negligible for $a_i \ll R$. Therefore a good fit in Fig. 1 with $R = 7.7 \pm 0.7$ fm is obtained with zero scattering lengths. Only very precise experiments may allow us to determine the values of a_s and a_t . This conclusion is illustrated by Fig. 2 which shows the low momentum CF for different values of R and a_s, a_t .³ Given the present accuracy of the data we simply put the scattering lengths equal to zero and ignored correlations of their errors with errors of the source size.

Important information on the fireball radius and on the FSI can be inferred from the resonance region. An important point is that the resonance phenomena is very sensitive to the source size. This is illustrated by Fig. 3. The fit to the resonance effect allows us to obtain the source size value rather reliably. Based on the data points presented in Refs. [19,20] we conclude that the source size is $R = 7.7 \pm 0.7$ fm which agrees

³Recent lattice calculations yield the result $a(\pi^+ \Xi^0) = -0.098 \pm 0.017$ fm [34].

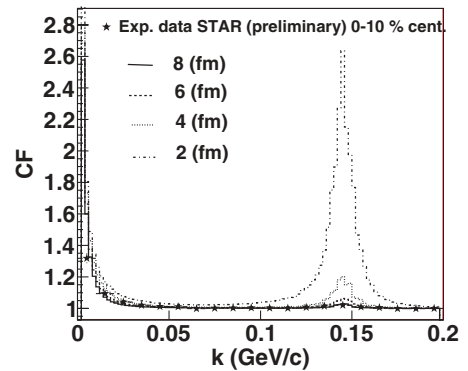


FIG. 3. The CF of $\pi^+ \Xi^-$ system as a function of PRF momentum k for $R = 2.0$ fm (dashed-dotted line), 4.0 fm (dotted line), 6.0 fm (dashed line), 8.0 fm (solid line), and the zero scattering lengths (solid line) the STAR Collaboration experimental data points from Refs. [19,20] (black stars).

with the value obtained in Refs. [19,20] where the low- k , Coulomb-dominated part of the CF was selected for fitting, excluding the region of the \mathbf{z} peak. Unlike in Refs. [19,20] we succeeded in describing the $\Xi^*(1530)$ resonance region.

It should be noted, however, that the source model which we used here (Gaussian in PRF) is oversimplified. The first step toward a more realible model of the source function is suggested by the expression (64) for the WF squared. This expression is tantamount to the expansion in terms of spherical harmonics with $l = 0, 1, 2$. Therefore the CF can be decomposed with the $l = 1$ term giving the emission point difference between π and Ξ . Such analysis was done in Ref. [19]. Other important points left beyond our present investigation are the influence of pions produced from resonance decays and non-Gaussian tail of the source function. Both factors are significant for pion and kaon production in heavy-ion collisions [35–37]. Experimental data on $\pi\Xi$ correlations were analyzed in Refs. [19,20] with the account of these effects. Roughly speaking, they lead to the increase of the source size but the problem has to be carefully investigated. The same is true for the distortion of the CF by the collective flows. As demonstrated in Ref. [38] for $d-\alpha$ correlations with resonance formation collective flow may reduce the source size and lower the resonance peak. One should, however, keep in mind that direct production of $\Xi^*(1530)$ may compensate for this effect.

VI. SUMMARY

We have developed a formalism to evaluate the CF for the system which comprises several features aggravating the

analysis. These features are: superposition of strong and Coulomb interaction, presence of a resonance, spin structure, and channel coupling. We have also proposed a new method to incorporate the contribution from the small distances where the structure of the interaction is unknown. This technique which is legitimate for large-enough source size has been successfully applied to the preliminary RHIC data on $\pi^+\Xi^-$ correlation measurements. The Gaussian source in the pair reference frame was considered. The resulting value of the source size radius is equal to $R = 7.7 \pm 0.7$ fm. There is no sensitivity of the data to the values of the $\pi^+\Xi^-$ - S -wave scattering lengths. There is a large sensitivity of the correlation function to the source size in the region of $\Xi^*(1530)$ resonance. A good description of the $\Xi^*(1530)$ resonance region was obtained. The developed formalism can be useful for femtoscopy studies of the systems with the narrow resonances born due to FSI, e.g., K^+K^- ($K^+K^- \rightarrow \phi$).

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