Gamow-Teller transitions from ^{9,11}Li to ^{9,11}Be

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Gamow-Teller (GT) transitions in β decays of ^{9}Li and ^{11}Li are investigated with theoretical calculations of antisymmetrized molecular dynamics. The calculated B(GT) values are small for transitions to low-lying states in Be isotopes, whereas relatively large B(GT) values are found for excited states at excitation energy $E_x \geqslant 10$ MeV. The sum of the B(GT) values is discussed for each spin-parity of final states. The calculated results seem to be inconsistent with an experimental report of the strongest GT transition from ^{9}Li to the $5/2^-$ state of ^{9}Be at 11.8 MeV.

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I. INTRODUCTION

Experimental studies of the β decays from unstable nuclei near drip lines are currently progressing owing to recent advances in experimental techniques. Gamow-Teller (GT) transition strength values B(GT) were extracted for transitions to not only low-lying states but also highly excited states. Moreover, the B(GT) values have also been determined in high-resolution experiments of charge-exchange reactions, as well as the β decays.

In recent years, the β decays of ^{9}Li and ^{11}Li have been measured in several experiments, for example, at the ISOLDE facility in CERN. They provide information on new states in ^{9}Be and ^{11}Be, such as their excitation energies, spins, and B(GT) values. In the β decay of ^{9}Li, the GT transitions to the low-lying ^{9}Be states at $E_x \leqslant 10$ MeV are weak, while strong transitions have been reported to the 11.81-MeV state [1,2]. The extracted B(GT) value for the ^{9}Be state at $E_x = 11.81$ MeV is surprisingly large, $B(\mathrm{GT}) = 8.9(1.9)$, which takes 62% of the Ikeda sum rule, $9(g_A/g_V)^2 \sim 14.3$. In a new measurement of the β decay [3], its spin and parity were assigned to the $5/2^-$ state. Because this value is larger by a factor of four than the B(GT) value for the mirror transition in ^{9}C β decays to the ^{9}B$(5/2^-)$ state at 12.19 MeV [4], it suggests the abnormally large mirror asymmetry [3]. However, the shell-model calculations in p-shell configurations indicate neither such large B(GT) values for the ^{9}Be$(5/2^-)$ state in ^{9}Li decays nor a large mirror asymmetry [5].

The β decay has also been measured experimentally for the drip-line nucleus, ^{11}Li [6–9]. Many excited states of ^{11}Be were observed in the low-energy region. The GT transitions to ^{11}Be states at $E_x \leqslant 10$ MeV are not strong and the extracted B(GT) values are less than $B(\mathrm{GT}) = 0.5$ for the low-lying states [8]. In a recent measurement, a new state was observed at 18 MeV in ^{11}Be, and $B(\mathrm{GT}) = 1.9$ for this state was extracted from the branching ratios [9].

As mentioned previously, in the β decays of neutron-rich Li isotopes, the GT transitions to the low-lying states of ^{9}Be and ^{11}Be are relatively weak compared with the rather strong GT transitions to highly excited states. This is easily understood because of the cluster features of Be isotopes. Theoretical studies of Be isotopes have suggested that $2\alpha + n$ and $2\alpha + 3n$ cluster structures develop in most of the low-lying states of ^{9}Be and ^{11}Be, respectively [10–16]. The GT transitions to such cluster states should be suppressed, because a GT transition to an ideal α cluster, written as the $(0s)^4$ configuration, is strictly forbidden owing to the Pauli principle. In other words, the high-lying states in $E_x \geqslant 10$ MeV with the strong GT transitions are expected to have significant cluster-breaking components.

In this article, I investigate the GT transition strengths of the β decays $^{9}\mathrm{Li} \to {}^{9}\mathrm{Be}$ and $^{11}\mathrm{Li} \to {}^{11}\mathrm{Be}$ with a theoretical method of antisymmetrized molecular dynamics (AMD) [17–19]. The distribution of the B(GT) values is analyzed for each spin of final states. Particular attention is paid to the strong GT transitions to the ^{9}Be$(5/2^-)$ state at 11.81 MeV suggested in the experimental report.

This article is organized as follows. In the next section, I describe the formulation of the present calculations. I explain the adopted effective interactions in Sec. III and show the calculated results and the experimental data in Sec. IV. Finally, in Sec. V, I give a summary and an outlook.

II. FORMULATION

I use an AMD method that has been proven to be a useful approach for studying the structures of light unstable nuclei. In the present work, I first perform the variation after projection (VAP) with respect to the total angular momentum and parity projection (spin-parity projection) in AMD [20] and extend the method to calculate the GT strength functions of final states in a wide energy region. The detailed formulation of the basic AMD method for nuclear structure study is described in Refs. [17–19].

In the AMD method, a wave function of an A-nucleon system is written as a Slater determinant of single-particle Gaussian wave packets,

$$\Phi_{\mathrm{AMD}}(\mathbf{Z}) = \frac{1}{\sqrt{A!}}\mathcal{A}\{\varphi_1, \varphi_2, \ldots, \varphi_A\}, \tag{1}$$

where the ith single-particle wave function is written as a product of spatial (ϕ), intrinsic spin (χ), and isospin (τ) wave functions,

$$\varphi_i = \phi_{\mathbf{X}_i}\chi_i\tau_i, \tag{2}$$

$$\phi_{\mathbf{X}_i}(\mathbf{r}_j) \propto \exp\left[-\nu\left(\mathbf{r}_j - \frac{\mathbf{X}_i}{\sqrt{\nu}}\right)^2\right], \tag{3}$$

$$\chi_i = \left(\tfrac{1}{2} + \xi_i\right)\chi_\uparrow + \left(\tfrac{1}{2} - \xi_i\right)\chi_\downarrow, \tag{4}$$

where ϕ and χ are represented by complex variational parameters X_{1i}, X_{2i}, X_{3i}, and ξ_i. The isospin function τ_i is fixed to be up (proton) or down (neutron). I take a fixed width parameter ν that is optimized for each nucleus. That is $\nu = 0.20$ fm^{-2} for ^{9}Be and ^{9}Li and $\nu = 0.18$ fm^{-2} for ^{11}Be and ^{11}Li in the present calculations. Accordingly, an AMD wave function is expressed by a set of variational parameters, $\mathbf{Z} \equiv \{\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_A, \xi_1, \xi_2, \ldots, \xi_A\}$.

For the lowest J^π state, I vary the parameters $\mathbf{X}_i$ and $\xi_i (i = 1 \sim A)$ to minimize the energy expectation value of the Hamiltonian, $\langle\Phi|H|\Phi\rangle/\langle\Phi|\Phi\rangle$, with respect to the spin-parity projected AMD wave function; $\Phi = P^{J\pi}_{MK}\Phi_{\rm AMD}(\mathbf{Z})$. Here, $P^{J\pi}_{MK}$ is the spin-parity projection operator. After the energy variation, the optimized parameter set $\mathbf{Z}_1^{J\pi}$ of the minimum-energy solution for the lowest J^π state is obtained. The solution $\mathbf{Z}_n^{J\pi}$ for the nth J^π state is determined by varying $\mathbf{Z}$ to minimize the energy of the wave function orthogonalized to the lower states,

$$|\Phi\rangle = \left|P^{J\pi}_{MK}\Phi_{\rm AMD}(\mathbf{Z})\right\rangle - \sum_{k=1}^{n-1}\left|\Phi^{(k)J\pi}_{MK}\right\rangle\left\langle\Phi^{(k)J\pi}_{MK}\left|P^{J\pi}_{MK}\Phi_{\rm AMD}(\mathbf{Z})\right.\right\rangle, \tag{5}$$

where $\Phi^{(k)J\pi}_{MK}$ is the normalized wave function determined for the lower states. This is the standard VAP calculation in AMD.

In the VAP calculations, the wave functions for the J^π_n states are obtained one by one from the lower energy states and it is not easy to calculate all final states of the GT transitions, which generally fragment into many high-lying states. To exhaust the GT strengths from an initial state, I extend the basis wave functions by applying one-body spin-isospin operators to the VAP wave functions for the parent nucleus. Let us consider the β^- decay of ^{9}Li. I perform the VAP calculation of the lowest $J^\pi = 3/2^-$ state of ^{9}Li and obtain the wave function $\Phi^{^9{\rm Li}}_{\rm AMD}(\mathbf{Z}_1^{3/2-})$ for the ^{9}Li ground state. Here I denote $\mathbf{Z}_1^{3/2-}$ for ^{9}Li as $\mathbf{Z}_{\rm init}$. Final states of the GT transitions from the ^{9}Li ground state are $J^\pi_n = 1/2^-, 3/2^-$, and $5/2^-$ in the daughter nucleus ^{9}Be. For the final states in ^{9}Be, I prepare the basis wave functions in two steps. I first perform the VAP calculations for the lowest two J^π states of ^{9}Be and obtain the wave functions $\Phi^{^9{\rm Be}}_{\rm AMD}(\mathbf{Z}_n^{J\pi})$ for $J^\pi_n = 1/2^-_1, 1/2^-_2, 3/2^-_1$, $3/2^-_2$, $5/2^-_1$, and $5/2^-_2$, which approximately describe the corresponding low-lying states. Next, I create other basis wave functions by applying the one-body spin-isospin operators to $\Phi^{^9{\rm Li}}_{\rm AMD}(\mathbf{Z}_{\rm init})$,

$$\Phi^{\sigma\tau}_{k,\alpha}(\mathbf{Z}_{\rm init}) = \frac{1}{\sqrt{A!}}\mathcal{A}\{\varphi_1, \varphi_2, \ldots, \sigma_\alpha\tau^-\varphi_k, \ldots, \varphi_A\}, \tag{6}$$

where φ_k $(k = 1, \ldots, 6)$ is a neutron wave function in ^{9}Li. $\sigma_\alpha\tau^-$ is the one-body GT transition operator, where $\sigma_\alpha (\alpha = x$, y, and $z)$ is the spin operator and τ^- is the charge changing operator. As a result, we get $6 \times 3 = 18$ basis wave functions, $\Phi^{\sigma\tau}_{k,\alpha}(\mathbf{Z}_{\rm init})$, which exhaust the GT transition strengths from $\Phi^{^9{\rm Li}}_{\rm AMD}(\mathbf{Z}_{\rm init})$.

Finally, I determine the wave functions of the final ^{9}Be states by spin-parity projection and the superposition of all the $6 + 18$ basis wave functions,

$$\Phi_{Jn} = \sum_{J'Km} c^{(Jn)}_{J',K,m} P^{J\pi}_{MK}\Phi^{^9{\rm Be}}_{\rm AMD}\left(\mathbf{Z}^{J'\pi}_m\right) + \sum_{Kk\alpha} c^{(Jn)}_{k,\alpha} P^{J\pi}_{MK}\Phi^{\sigma\tau}_{k,\alpha}(\mathbf{Z}_{\rm init}). \tag{7}$$

The coefficients $c^{(Jn)}_{J',K,m}$ and $c^{(Jn)}_{k,\alpha}$ are determined by diagonalizing the Hamiltonian and norm matrices.

The B(GT) value of the GT transition strength is written by the square of the reduced matrix element

$$B({\rm GT}) = (g_A/g_V)^2\left|\left\langle\mathcal{O}^{({\rm GT}-)}_\mu\right\rangle\right|^2 \tag{8}$$

of the GT transition operator,

$$\mathcal{O}^{({\rm GT}\pm)}_\mu = \sum_i \sigma_\mu(i)\tau^\pm(i), \tag{9}$$

where g_A and g_V are the axial-vector and vector coupling constants and are taken to be $(g_A/g_V)^2 = 1.26^2$ in the present calculations. It is worth mentioning again that the GT transition strengths for β^- decays from the ^{9}Li ground state written by the wave function $P^{J\pi=3/2-}_{MK}\Phi^{^9{\rm Li}}_{\rm AMD}(\mathbf{Z}_{\rm init})$ are exhausted by the final states, Φ_{Jn} $(n = 1, \ldots, 24)$, of ^{9}Be.

III. EFFECTIVE NUCLEAR FORCES

The effective nuclear interaction adopted in the present work consists of the central force $V_{\rm central}$, the spin-orbit force $V_{\rm ls}$, and the Coulomb force $V_{\rm coulomb}$. For the central force, the MV1 force (case 3) [21] containing finite-range two-body and zero-range three-body terms is used. For the spin-orbit force, the same two-range Gaussian form as in the G3RS force [22] is adopted.

The values of the interaction parameters in the MV1 force used in the present work are $b = h = 0$ and $m = 0.62$ of the Bartlett, Heisenberg, and Majorana parameters, and the strengths of the spin-orbit force are taken to be $u_{\rm I} = -u_{\rm II} = 3000$ MeV. These parameters are the same as those used in the AMD + VAP calculations of ^{12}C and ^{10}Be in Refs. [20,23]. Hereafter, I call this parameter set (A). To see the interaction dependence of the GT transition strengths, I also use two sets of modified parameters, (B) and (C). Set (B), $b = h = 0$, $m = 0.62$, and $u_{\rm I} = -u_{\rm II} = 2000$ MeV, has the same central force but a weaker spin-orbit force than (A), and set (C) $b = h = 0.15$, $m = 0.62$, and $u_{\rm I} = -u_{\rm II} = 3000$ MeV, contains the Bartlett and Heisenberg terms in the central force but has the same spin-orbit force strength as (A). Here, I comment that the $b = h = 0$ parametrization in (A) and (B) yields the same strength for the p-n interaction in S wave as that for the n-n channel, while the $b = h = 0.15$ values yields a stronger S-wave p-n interaction than the n-n one. The present values of the interaction parameters are also consistent with the parameter set ($b = h = 0.25$, $m = 0.62$, and $u_{\rm I} = -u_{\rm II} = 2800$ MeV) used for ^{11}B in Ref. [24] except for fine tuning of the $b = h$ and $u_{\rm I} = -u_{\rm II}$ values.

These effective interactions do not contain the tensor term as often-used effective two-body interactions such as the Gogny

and Skyrme forces. In such effective interactions without the tensor force, the major effect of the tensor force on binding energy is considered to be renormalized to the central part. However, it has been suggested that in the shell-model calculations [25], the GT strengths are affected by the tensor components. To see the effect of the tensor components on the GT strengths, I study the effect of the tensor matrix elements on the obtained basis wave functions as follows. The basis wave functions obtained by set (C) without the tensor force for ^{9}Be and ^{11}Be are adopted. By adding a tensor term $V_{\rm tensor}$ to the original interaction $V_{\rm central} + V_{\rm ls} + V_{\rm coulomb}$, the coefficients $c^{(Jn)}_{J',K,m}$ and $c^{(Jn)}_{k,\alpha}$ in Eq. (7) are recalculated by diagonalizing the Hamiltonian and norm matrices for the basis wave functions. Then the B(GT) values are calculated for the modified Φ_{Jn} states in ^{9}Be and ^{11}Be. For simplicity, the central and spin-orbit forces are not modified from the original forces. This is because the present model space is limited, and it is difficult to incorporate the effect of the tensor force directly. In other words, the major effect of the tensor force is still regarded as being renormalized to the central and spin-orbit parts, and the effect of the tensor matrix elements in the present configuration space is simply estimated in a perturbative way. As an additional tensor term, I use the tensor term of the Furutani potential [26], which is an effective two-body force with tensor force adjusted to the ^{4}Li properties within a ^{3}He + p cluster model. In the present calculations, I omit the odd part of the tensor force.

IV. RESULTS

A. GT transition from ^{9}Li to ^{9}Be

The results of the present VAP calculations for low-lying states of ^{9}Be show that $2\alpha + n$ cluster structure develops in the ground state ($3/2^-_1$) and excited states, $5/2^-_1$, $1/2^-_1$, $3/2^-_2$, and $5/2^-_2$. The calculated energy spectra of these low-lying states are consistent with those calculated with the $2\alpha + n$ cluster models [10,11,14] and the experimental energy levels (Table I).

As explained in Sec. II, by adding the basis wave functions $\Phi^{\sigma\tau}_{k,\alpha}(\mathbf{Z}_{\rm init})$ constructed from the ^{9}Li wave functions with spin-isospin applied to the VAP wave functions, the final wave functions of ^{9}Be are obtained and the B(GT) values for the transitions ^{9}Li $\rightarrow$ ^{9}Be are calculated. The calculated B(GT) values for transitions to the final ^{9}Be states up to $E_x \sim 13$ MeV are shown in Table I. The B(GT) values for transitions to the ground state ($3/2^-_1$) and the $5/2^-_1$, $1/2^-_1$, $3/2^-_2$, and $5/2^-_2$ states are small. This is because those states of ^{9}Be have the $2\alpha + n$ cluster structure and overlap little with the initial state of ^{9}Li, having no developed cluster structure. Figure 1 shows the density distribution in the intrinsic wave functions of the ^{9}Be ground state [$\Phi^{^9{\rm Be}}_{\rm AMD}(\mathbf{Z}^{3/2-}_1)$] and that of the ^{9}Li ground state [$\Phi^{^9{\rm Li}}_{\rm AMD}(\mathbf{Z}_{\rm init})$]. The intrinsic structure changes drastically from the initial to final state. Moreover, as mentioned previously, the GT transitions to such cluster states with the 2α core are suppressed because of the Pauli principle. In other words, the finite B(GT) values account for the slight dissociation of the ideal α clusters in ^{9}Be.

TABLE I. Experimental data on the GT strengths in the β^- decays of ^{9}C and ^{9}Li and theoretical values of the ^{9}Li decay. Experimental data on the B(GT) values are taken from Refs. [2,28].

J^π	Exp.		Exp.		Cal. (A)	
	^{9}Be E_x	^{9}C (β^+) B(GT)	^{9}Be E_x	^{9}Li (β^-) B(GT)	^{9}Be E_x	^{9}Li (β^-) B(GT)
$3/2^-_1$	0	0.0326(56)	0	0.0301(2)	0	0.069
$5/2^-_1$	2.361	0.0206(80)	2.43	0.0452(52)	2.0	0.091
$1/2^-_1$	2.80	0.0151(77)	2.78	0.011(6)	5.3	0.020
$3/2^-_2$			5.59		6.9	0.030
$5/2^-_2$			7.94	0.048(20)	8.2	0.068
$1/2^-_2$					12.0	2.1
$3/2^-_3$			11.28[b]	1.4(5)	12.1	0.99
$3/2^-_4$					12.4	0.68
$5/2^-_3$	12.19[a]	1.9(2)[a]	11.81[c]	8.9(1.9)	13.1	0.82

[a]The data is taken from Ref. [4].
[b]The spin and parity $3/2^-$ of the ^{9}Li (11.28 MeV) state is the assignment of Ref. [27] but it is not yet confirmed, and other spin-parity candidates, $7/2^-$ and $7/2^+$, are suggested [28].
[c]$5/2^-$ was assigned in Ref. [3].

In contrast to the small B(GT) values for transitions to the low-lying states, the excited states at $E_x = 12 \sim 13$ MeV show significant GT strengths, which come from the non-α-cluster states constructed from the spin-isospin operated ^{9}Li wave functions given in Eq. (6). The strongest GT transition is found for the $1/2^-_2$ state in the present results. In the experimental measurements of the β decay of ^{9}Li, large B(GT) values were reported for the states around $E_x = 12$ MeV [1–3]: B(GT) = 1.44 for the $E_x = 11.28$ MeV state and B(GT) = 8.9 for the $E_x = 11.81$ MeV state. The former is suggested to be a $3/2^-$ state [27]; it might correspond to the theoretical $3/2^-_3$ state in the present work. The calculated value of B(GT) = 0.99 agrees with the experimental value. The spin and parity of the latter $E_x = 11.81$ MeV state were assigned as $J^\pi = 5/2^-$ in a recent analysis of the β^- decay of ^{9}Li [3]. Although this state seems to correspond energetically to the calculated $5/2^-_3$ state, the theoretical B(GT) value is much smaller than the

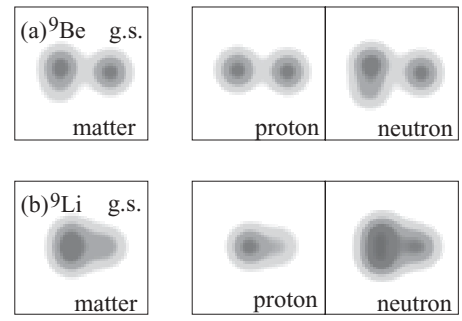

FIG. 1. Distributions of matter, proton, and neutron density in the intrinsic wave functions of (a) $\Phi^{^9{\rm Be}}_{\rm AMD}(\mathbf{Z}^{3/2-}_1)$ for the ^{9}Be ground state and (b) $\Phi^{^9{\rm Li}}_{\rm AMD}(\mathbf{Z}_{\rm init})$ for the ^{9}Li ground state calculated with interaction set (A).

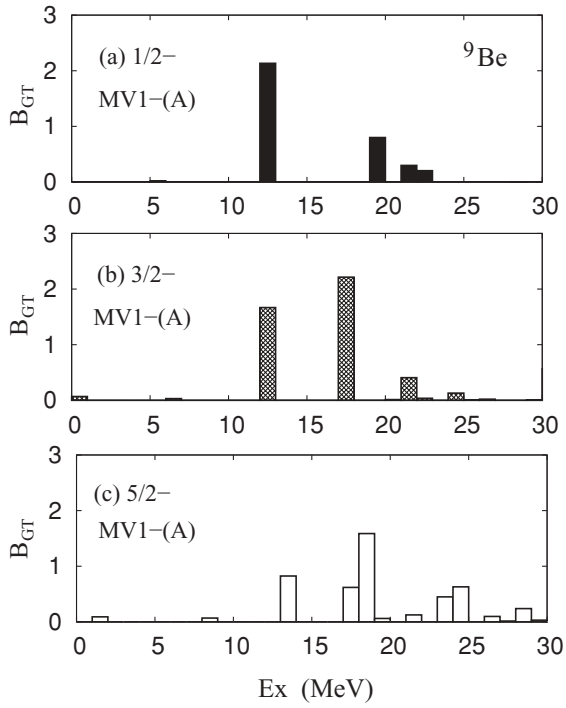

FIG. 2. B(GT) distribution to (a) $1/2^-$, (b) $3/2^-$, and (c) $5/2^-$ states of ^{9}Be in the β decay of ^{9}Li calculated with interaction set (A).

experimental value, $B(\mathrm{GT}) = 8.9$, which exhausts 62% of the Ikeda sum rule. Also in the shell-model calculation [5], it is difficult to reproduce the extraordinarily large B(GT) value of the experimental data for the $5/2^-$ state. As shown below, the present calculation yields no $5/2^-$ state having such a high GT strength compatible with $B(\mathrm{GT}) = 8.9$.

Figure 2 shows the B(GT) distribution to excited states of ^{9}Be up to $E_x = 30$ MeV calculated using interaction set (A). In the B(GT) values to $1/2^-$ states, the largest peak is found at $E_x \sim 12$ MeV, and some strengths are distributed around 20 MeV. The transition strengths to $3/2^-$ states are distributed mainly in the $E_x = 10$–20 MeV region. The B(GT) values to $5/2^-$ states in the $E_x < 15$ MeV region are relatively small compared with those to $1/2^-$ and $3/2^-$ states. To show the interaction dependence of the GT strength, I show the B(GT) distribution calculated with three interaction sets (A), (B), and (C) in Fig. 3. The calculated results for interaction set (C) with the tensor force are also shown. The B(GT) distribution is qualitatively similar for these three interaction sets, though the broadness of the distribution changes slightly. Comparing the results with and without the tensor force, we see that the the tensor force has a minor effect on the GT strength, at least in the present calculations. In all the results, the B(GT) values are very small for transitions to low-energy states ($E_x < 10$ MeV), while the strengths are distributed mostly in the range $E_x = 10$–20 MeV. The GT transition peaks in ^{9}Be are close to the energy positions of the isobaric analog state (IAS), shown by arrows. Compared with the shell-model calculations in Ref. [25], which show the B(GT) distribution concentrated at $E_x = 10$–12 MeV, the present calculations show significant fraction of the B(GT) values to highly excited states at $E_x > 15$ MeV.

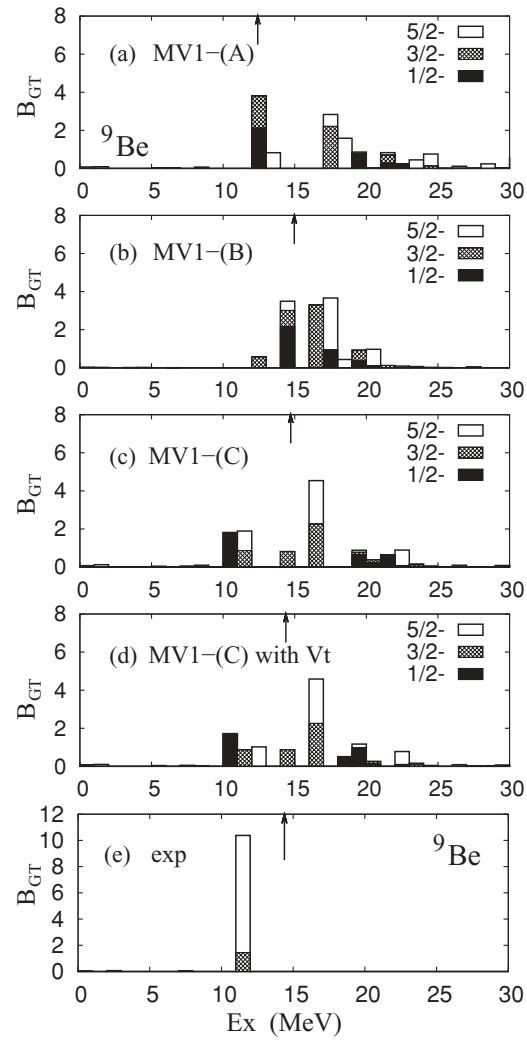

FIG. 3. B(GT) distribution for the decay ^{9}Li → ^{9}Be. The B(GT) values to $1/2^-$, $3/2^-$, and $5/2^-$ states of ^{9}Be. Panels (a), (b), and (c) show the results calculated with interaction sets (A), (B), and (C), respectively. Calculated results for set (C) with tensor force are shown in panel (d). Panel (e) shows the experimental data taken from the same references as those in Table I. An arrow in each panel indicates the energy of the IAS state.

B. GT transition from ^{11}Li to ^{11}Be

The B(GT) distribution in the ^{11}Li → ^{11}Be decay calculated with interaction set (A) is shown in Fig. 4. The B(GT) values to $1/2^-$ states are concentrated around $E_x =$ 16–18 MeV, and those to $3/2^-$ states are distributed widely in various excited states. The B(GT) values to the $5/2^-$ state exist in a higher energy region than those to $1/2^-$ and $3/2^-$ states. The results of the B(GT) distribution calculated with the three sets of interaction parameters, (A), (B), and (C), are displayed in Figs. 5(a), 5(b), and 5(c), respectively, along with the calculated results for interaction set (C) with the tensor force in Fig. 5(d). Experimentally observed B(GT) data are displayed in Fig. 5(e). The results are qualitatively similar for these three interaction sets. The effect of the tensor force on the GT strengths is minor in the present calculations. The B(GT) values are small at $E_x < 10$ MeV, whereas they are

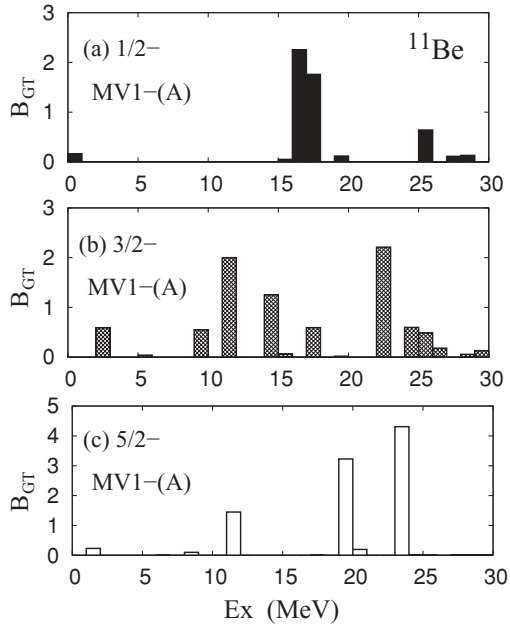

FIG. 4. B(GT) distribution to (a) $1/2^-$, (b) $3/2^-$, and (c) $5/2^-$ states in the ^{11}Li $\rightarrow$ ^{11}Be decay calculated with interaction set (A).

distributed widely in the $E_x = 10$–25 MeV region and show a broad peak structure centered around $E_x = 15$–20 MeV. The small B(GT) values for transitions to low-lying states are understood in terms of the 2α core structure, as was discussed in the previous study of ^{11}Be with the AMD method [15]. The highly excited ^{11}Be states with significant GT strengths are those with noncluster or less-cluster structures. This is a similar situation to that for ^{9}Be.

Let us discuss the relative positions of the GT peak and the IAS. In Fig. 5, the energy positions of the IAS are indicated by arrows. The IAS energy is sensitive to the interaction parameter, and that calculated with interaction set (C) is in good agreement with the experimental IAS energy, 21.16 ± 0.02 MeV [29], whereas those calculated with interaction sets (A) and (B) are smaller than the experimental value. In the results for (C), the GT peak appears at an energy around 4 MeV lower than the IAS energy in neutron-rich nuclei. This is consistent with the shell-model calculations in Ref. [30]. The energy difference between the GT peak and the IAS in ^{11}Li is larger than that in ^{9}Li, which has a GT peak close to the IAS energy. This result corresponds well to the theoretical suggestion that the GT peak occurs below the IAS in neutron-rich nuclei as a function of isospin asymmetry $(N - Z)/A$ [31].

Although ^{11}Li is known to be a neutron-rich nucleus with a neutron-halo structure, halo effects are not considered in the present calculations because single-particle wave functions in the present model are restricted to a Gaussian form and are not suitable for describing the long tail of the halo-neutron wave functions. The halo effects in the B(GT) values for the ^{11}Li $\rightarrow$ ^{11}Be decay were discussed in the shell-model study of Ref. [30], which showed that the halo structure results in a slight quenching of the B(GT) values.

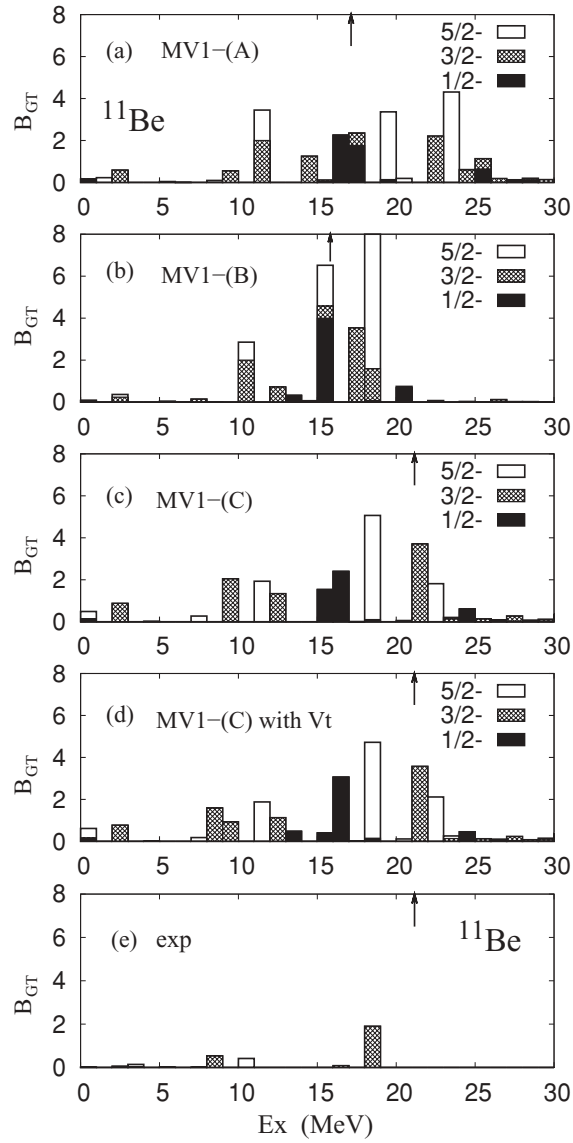

FIG. 5. B(GT) distribution for the decay ^{11}Li $\rightarrow$ ^{11}Be. B(GT) values to the $1/2^-$, $3/2^-$, and $5/2^-$ states of ^{11}Be. Panels (a), (b), and (c) show the values calculated with interaction sets (A), (B), and (C), respectively. Calculated results for set (C) with the tensor force are shown in panel (d). Panel (e) shows the experimental data taken from Refs. [8,9]. An arrow in each panel indicates the energy of the IAS state.

C. Sum rule

The Ikeda sum rule for the GT transition strengths is given as

$$S^{(\mathrm{GT}-)} - S^{(\mathrm{GT}+)} = \frac{g_A^2}{g_V^2}\langle 0|[\mathcal{O}^{(\mathrm{GT}+)}, \mathcal{O}^{(\mathrm{GT}-)}]|0\rangle = \frac{g_A^2}{g_V^2} 3(N - Z). \quad (10)$$

Here $S^{(\mathrm{GT}\pm)}$ is the sum of the B(GT) values for the $\beta^\pm$ decay and $|0\rangle$ is the initial state. Because $S^{(\mathrm{GT}+)}$ for the β^+ decays of neutron-rich nuclei is very small, the sum rule is often approximated as $S^{(\mathrm{GT}-)} = \frac{g_A^2}{g_V^2} 3(N - Z)$ by neglecting the $S^{(\mathrm{GT}+)}$ term.

TABLE II. Calculated results for the sum of the B(GT) values of ^{9}Li and ^{11}Li β decays to J_f^- states in ^{9}Be and ^{11}Be. The values of the Ikeda sum rule $3(N-Z)\times(g_A/g_V)^2$ are 14.3 and 23.8 for ^{9}Li (β^-) and ^{11}Li (β^-), respectively.

J_f^π	B (GT)		
^{9}Li$(3/2_1^-) \to {}^9$Be(J_f^π)	Cal. (A)	Cal. (B)	Cal. (C)
$1/2^-$	3.6	3.7	3.7
$3/2^-$	5.5	5.8	5.5
$5/2^-$	5.7	5.0	5.6
Total	14.8	14.5	14.9
^{11}Li$(3/2_1^-) \to {}^{11}$Be(J_f^π)			
$1/2^-$	5.3	5.3	5.3
$3/2^-$	8.9	9.0	8.9
$5/2^-$	9.8	9.6	9.7
Total	24.0	23.9	24.0

The values of the sum rule $3(N-Z)\times(g_A/g_V)^2$ are 14.3 and 23.8 for the ^{9}Li and ^{11}Li β^- decays, respectively. The theoretical values of the sum of the B(GT) values to $1/2^-$, $3/2^-$, and $5/2^-$ states in Be are listed in Table II. It is important that the sum values are determined only by the initial structure of the Li ground state but do not depend on the final states of Be. Compared with the results using (A), (B), and (C), the interaction dependence of the sum values is small. Thus, the wave functions of the ^{9}Li and ^{11}Li ground states are not sensitive to the adopted interactions in the present calculations. The sum of the B(GT) values to $5/2^-$ states is as large as that to $3/2^-$ states and it exhausts only 35%–40% of the calculated total sum $S^{(\mathrm{GT}-)}$ value in both the ^{9}Li and the ^{11}Li decays. This result contradicts the experimental report of $B(\mathrm{GT}) = 8.9$ to the $5/2^-$ state at $E_x = 11.81$ MeV in Ref. [3]. Theoretically, the sum of the B(GT) values for each spin is determined by the structure of the ^{9}Li ground state with spin and parity $3/2^-$, and it is unnatural that the highest-spin $5/2^-$ state exhausts such a large fraction of the Ikeda sum rule. In other words, the experimental value $B(\mathrm{GT}) = 8.9$ for the $5/2^-$ state seems too large to be described by theoretical calculations if the ^{9}Li ground state has an ordinary structure.

V. SUMMARY AND OUTLOOK

The GT transitions in the β^- decays ^{9}Li $\to$ ^{9}Be and ^{11}Li $\to$ ^{11}Be were investigated with a method of AMD. The calculated B(GT) values are small for transitions to low-lying states of Be isotopes because of the 2α-core structures in the final states. Significant strengths are found in the B(GT) distribution to noncluster states of ^{9}Be in the $E_x \geqslant 10$ MeV region. The sum of the B(GT) values for each spin-parity of the final states was also studied.

Particular attention was paid to the strong β-transition from ^{9}Li to ^{9}Be$(5/2^-)$ state at 11.8 MeV, which was reported experimentally [3]. The present results are inconsistent with the strong GT transition to the $5/2^-$ state, which shows a large fraction of the Ikeda sum rule value. Also in terms of the sum rule, the experimental value $B(\mathrm{GT}) = 8.9$ for the $5/2^-$ state seems too large to be described by theoretical calculations if the ^{9}Li ground state has an ordinary structure.

This work is the first in which the AMD method was applied to the study of GT transitions to highly excited states. One advantage of the present method is that one can describe various final states in the daughter nucleus (Be), such as low-lying cluster states and high-lying noncluster states. Although the present model space is not complete, it exhausts the GT transition strengths; that is, the sum of the calculated B(GT) for β^- and β^+ decays equals the Ikeda sum rule value.

Three sets of interaction parameters were used, and it was found that the interaction dependence of the GT strengths is not large, except for the energy of the IAS in ^{11}Be. Also, the effect of the tensor component was found to be minor in the present framework. The reason for such a small sensitivity to the tensor term is considered to be that the present model space is not large enough to incorporate the tensor effect. To investigate the tensor effect in more detail some extension of the AMD model space is required. There was a trial of extension of the AMD model space to treat the tensor force more explicitly [32]. In Ref. [32] it was shown that superposing different-width Gaussians and single-particle isospin mixing with the total charge projection are important, though neither of them were performed in the present calculations. Such extended AMD calculations were achieved only for very light systems smaller than ^{4}He, and its application to heavier systems is not easy because of numerical cost. Further improvement of the AMD framework is necessary to discuss the tensor effect on the GT strengths.

The present framework is a kind of bound-state approximation, and continuum states are not incorporated. Coupling with continuum states should be carefully considered in discussions of the widths of excited states above the thresholds.

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