BCS-BEC crossover in symmetric nuclear matter at finite temperature: Pairing fluctuation and pseudogap

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(Received 30 January 2010; published 26 March 2010)

By adopting a T-matrix-based method within the G_0G approximation for the pair susceptibility, we studied the effects of pairing fluctuation on the BCS-BEC crossover in symmetric nuclear matter. The pairing fluctuation induces a pseudogap in the excitation spectrum of a nucleon in both superfluid and normal phases. The critical temperature of the superfluid transition was calculated. It differs from the BCS result remarkably when density is low. We also computed the specific heat, which shows a nearly ideal BEC-type temperature dependence at low density, but a BCS-type behavior at high density. This qualitative change of the temperature dependence of specific heat may serve as a thermodynamic signal for the BCS-BEC crossover.

DOI: 10.1103/PhysRevC.81.034007 PACS number(s): 21.65.−f, 21.30.Fe, 26.60.−c, 74.20.−z

I. INTRODUCTION

One of the most common properties of the attractive fermion many-body system is the arising of a superfluid state at low temperature. Depending on the strength of the attractive interaction between two fermions, however, the physical contents of the superfluid state can be distinguishably altered. When the interaction is weak, the system can be well described by the BCS theory. In this case, the superfluidity is due to the condensate of loosely correlated Cooper pairs and the superfluid gap is much smaller than the Fermi energy. When the interaction becomes sufficiently strong, the two-fermion bound state can form, which may behave like a boson. In this situation, the superfluidity is due to the Bose-Einstein condensation (BEC) of the tightly bound two-fermion state and the superfluid gap can be much larger than the Fermi energy. Although the BCS and BEC limits have quite different physics, it was found that there is no true phase transition (traditional symmetry breaking) happening in between. The transition from a BCS state to a BEC state is smooth and is often called the BCS-BEC crossover [1–4]. Such a BCS-BEC crossover was recently realized in cold atomic experiments (see Refs. [5,6] and references therein).

It was well known that cold nuclear matter can be in a superfluid state, which plays a crucial role in a variety of nuclear many-body problems, from neutron stars, low-energy heavy-ion collisions, to finite nuclei. It was argued that the BCS-BEC crossover should also occur in nuclear matter where the BCS state of neutron-proton (np) Cooper pairs at high density undergoes a smooth transition into the BEC state of deuterons at low density [7–14]. At the same time, the chemical potential changes its sign at a certain density and finally approaches one-half of the deuteron binding energy at the low-density limit. Recently, a similar situation was also studied for the neutron-neutron (nn) pairs in the $^1S_0$ channel [14–25]. It was found that in certain (low) density regions the nn pairs can be strongly correlated. However, no assured BEC state was found for nn pairs.

So far, most of the investigations of nuclear BCS-BEC crossover in the literature focused on the ground-state crossover described by BCS theory. Although the BCS theory succeeds in describing the BCS-BEC crossover at zero temperature, it, as a mean-field theory, is not sufficient to describe low-density nuclear matter at finite temperature where the pairing fluctuation is substantial due to the strongly correlating nature of the system. Actually, as a consequence of the strong correlation, the low-density nuclear matter exhibits "pseudogap" phenomena above the critical temperature T_c of superfluid transition and has an exotic normal state that is different from the Fermi liquid normal state associated with BCS theory [26–28]. Similar situations were also found in other strongly correlated systems, such as high T_c superconductors [29–34] and cold atomic Fermi gases under Feshbach resonances [33,35–40]. To include the pairing fluctuation effects and investigate the pseudogap phenomena, we will adopt a T-matrix formalism based on a G_0G approximation for the pair susceptibility, which was first introduced by the Chicago group [34–40]. This formalism generalizes the early works of Kadanoff and Martin [41] and Patton [42] and can be considered as a natural extension of the BCS theory since they share the same ground state. Moreover, this formalism allows quasi-analytic calculations and gives a simple physical interpretation of the pseudogap phase. It clearly shows that the pseudogap is due to the incoherent pairing fluctuation.

Our focus will be put on the np pair in symmetric nuclear matter (mainly in the low-density region), since the interaction in this case is more attractive than in the nn or pp channels and it provides a very good playground for the BCS-BEC crossover. We will extend the early studies [7–14] to include pairing fluctuation effects and determine the magnitude of the pseudogap. Furthermore, the transition temperature for the onset of the superfluid and the thermodynamic properties will also be a concern. Such a study will be helpful to understand the strongly coupling nature of low-density nuclear matter and may give useful information on the physics of the surface of nuclei, expanding nuclear matter from heavy-ion collisions, collapsing stars, and so on.

The article is organized as follows. We give a brief summary of the effective nucleon-nucleon potential in Sec. II. In Sec. III, we give a detailed theoretical scheme of how the T-matrix-based formalism works at finite temperature. The numerical results are presented in Sec. IV. We summarize our results in Sec. V. Throughout this article, we use natural units $\hbar = k_B = c = 1$.

II. EFFECTIVE NUCLEON-NUCLEON POTENTIAL

The aim of this article is not to determine the precise values of the pairing gap, the critical temperature, and so on, but rather to perform a qualitative (or semiquantitative) study of the effects of pairing fluctuation on the BCS-BEC crossover. To highlight the essential physics, we will adopt a simple density-dependent contact interaction (DDCI) developed in Refs. [43,44]. The potential is of the form

$$V(\mathbf{x}-\mathbf{x}') = v_0 \left\{1 - \eta \left[\frac{\rho(\mathbf{x})}{\rho_0}\right]^\gamma \right\} \delta(\mathbf{x}-\mathbf{x}'), \tag{2.1}$$

where v_0, η, and γ are three adjustable parameters, $\rho(\mathbf{x}) = \rho_n(\mathbf{x}) + \rho_p(\mathbf{x})$ is the nuclear density, and $\rho_0 = 0.17\ \text{fm}^{-3}$ is the normal nuclear density. Taking suitable values of the parameters, one can reproduce the pairing gap $\Delta(k_F)$ as a function of Fermi momentum $k_F = (3\pi^2\rho/2)^{1/3}$ in the channels $L=0, I=1, I_z=\pm1, S=0$ and $L=0, I=0, S=1, S_z=0$ calculated from realistic nucleon-nucleon potentials [43,44], where L is orbital angular momentum, I denotes isospin, and S is spin. According to Garrido *et al.* [43,44], we will choose in the following numerical calculation $\eta = 0$, $v_0 = -530$ MeVfm^3 in the $I=0$, $^3S_1$ (np pairing) channel and an energy cutoff $\epsilon_c = 60$ MeV to regularize the integration. With these parameters one must use a density-dependent effective nucleon mass $m(\rho)$ corresponding to the Gogny interaction [43,44]

$$\left[\frac{m(\rho)}{m_0}\right]^{-1} = 1 + \frac{m_0}{2}\frac{k_F}{\sqrt{\pi}}\sum_{c=1}^{2}[W_c + 2(B_c - H_c) - 4M_c] \times \mu_c^3 e^{-x_c}\left[\frac{\cosh x_c}{x_c} - \frac{\sinh x_c}{x_c^2}\right], \tag{2.2}$$

where $x_c = k_F^2\mu_c^2/2$, $m_0 = 939$ MeV is the bare mass of the nucleon, and $\mu_c, W_c, B_c, H_c, M_c$ are the parameters corresponding to the Gogny force D1 [45,46], their values are listed in Table I.

III. T-MATRIX-BASED FORMALISM

We consider the nuclear matter as an infinite system of interacting fermions. In the low-density region, np pairing is realized mainly in the spin-triplet s-wave channel, so let us consider the following Lagrangian describing neutron and proton interaction via two-body attractive forces in the $^3S_1$, $S_z = 0$ channel,

TABLE I. Parameters in the effective mass of nucleon (2.2) corresponding to the Gogny interaction D1 [45,46].

c	μ_c [fm]	W_c [MeV]	B_c [MeV]	H_c [MeV]	M_c [MeV]
1	0.7	−402.4	−100.0	−496.2	−23.56
2	1.2	−21.30	−11.77	37.27	−68.81

$$\mathcal{L} = \sum_{i=n,p}\sum_{\sigma=\uparrow,\downarrow} \bar\psi_{i,\sigma}\left(-\partial_\tau + \frac{\nabla^2}{2m} + \mu\right)\Psi_{i,\sigma} + \frac{g}{2}(\bar\psi_{n\uparrow}\bar\psi_{p\downarrow} - \bar\psi_{p\uparrow}\bar\psi_{n\downarrow})(\Psi_{p\downarrow}\Psi_{n\uparrow} - \Psi_{n\downarrow}\Psi_{p\uparrow}), \tag{3.1}$$

where $g = -v_0 > 0$ is the coupling strength in the np channel and $\tau = it$ is the imaginary time. Introducing auxiliary fields $\bar\Delta \equiv (g/2)\,(\bar\psi_{n\uparrow}\bar\psi_{p\downarrow} - \bar\psi_{p\uparrow}\bar\psi_{n\downarrow})$ and $\Delta \equiv (g/2)\,(\Psi_{p\downarrow}\Psi_{n\uparrow} - \Psi_{n\downarrow}\Psi_{p\uparrow})$, we can recast Eq. (3.1) as

$$\begin{aligned}\mathcal{L} &= \sum_{i=n,p}\sum_{\sigma=\uparrow,\downarrow} \bar\psi_{i,\sigma}\left(-\partial_\tau + \frac{\nabla^2}{2m} + \mu\right)\Psi_{i,\sigma} - \frac{2}{g}\bar\Delta\Delta \\ &\quad + \bar\Delta(\Psi_{p\downarrow}\Psi_{n\uparrow} - \Psi_{n\downarrow}\Psi_{p\uparrow}) + (\bar\psi_{n\uparrow}\bar\psi_{p\downarrow} - \bar\psi_{p\uparrow}\bar\psi_{n\downarrow})\Delta \\ &= \Psi\mathcal{S}^{-1}\Psi - \frac{2}{g}\bar\Delta\Delta,\end{aligned} \tag{3.2}$$

where we introduced the Nambu-Gorkov spinor $\Psi = (\Psi_{n\uparrow}, \bar\psi_{p\downarrow}, \Psi_{p\uparrow}, \bar\psi_{n\downarrow})^{\mathrm{T}}$ and

$$\mathcal{S}^{-1} \equiv \begin{pmatrix} \mathcal{S}_1^{-1} & 0 \\ 0 & \mathcal{S}_2^{-1}\end{pmatrix}, \tag{3.3}$$

with

$$\mathcal{S}_1^{-1} \equiv \begin{pmatrix} -\partial_\tau + \nabla^2/(2m) + \mu & \Delta \\ \bar\Delta & -\partial_\tau - \nabla^2/(2m) - \mu\end{pmatrix}, \tag{3.4}$$

$$\mathcal{S}_2^{-1} \equiv \begin{pmatrix} -\partial_\tau + \nabla^2/(2m) + \mu & -\Delta \\ -\bar\Delta & -\partial_\tau - \nabla^2/(2m) - \mu\end{pmatrix}. \tag{3.5}$$

It is seen that $\mathcal{S}_2$ differs from $\mathcal{S}_1$ only by the minus signs in front of Δ and $\bar\Delta$. To make our formulas more compact, in the following discussions we will treat $\mathcal{S}_1$ only and neglect the subscript 1 without confusion.

In the rest of this section, following the works of the Chicago group, we will introduce the basic method of the T matrix. This T matrix is defined as an infinite series of ladder diagrams in a particle-particle channel (rather than a particle-hole channel) by constructing the ladder by one free nucleon propagator and one full nucleon propagator. Then, as usual, the T matrix enters the nucleon self-energy in place of the bare interaction vertex. The coupled T-matrix equation and the self-energy equation (as well as the number density equation) should be solved self-consistently. One can view this approach as the simplest generalization of the BCS scheme, which formally can also be cast in a T-matrix formalism. Let us discuss this point in the following section.

A. BCS theory

The BCS theory is based on the mean-field approximation to the (anomalous) self-energy, that is, Δ and $\bar\Delta$ are chosen as their mean-field values $\Delta = \Delta_{sf}$ and $\bar{\Delta} = \bar{\Delta}_{sf}$ (without loss of generality, we put Δ_{sf} and $\bar{\Delta}_{sf}$ to be constants and $\bar{\Delta}_{sf} = \Delta_{sf}$), which are regarded as order parameters for the superfluid phase transition. We start with the Nambu-Gorkov formalism in momentum space

$$\mathcal{S}_{mf}^{-1}(K) = \begin{pmatrix} \mathcal{G}_0^{-1}(K) & \Delta_{sf} \\ \Delta_{sf} & -\mathcal{G}_0^{-1}(-K) \end{pmatrix}, \tag{3.6}$$

where $K = (i\omega_n, \mathbf{k})$ and $i\omega_n = i(2n+1)\pi T$ is the fermion Matsubara frequency. $\mathcal{G}_0^{-1}(K) = i\omega_n - \xi_{\mathbf{k}}$ is the inverse of the free nucleon propagator, $\xi_{\mathbf{k}} = \mathbf{k}^2/(2m) - \mu$ is the dispersion relation of the free nucleon. From Eq. (3.6) one gets,

$$\mathcal{S}_{mf}(K) = \begin{pmatrix} \mathcal{G}_{mf}(K) & \mathcal{F}_{mf}(K) \\ \mathcal{F}_{mf}(K) & -\mathcal{G}_{mf}(-K) \end{pmatrix}, \tag{3.7}$$

where $\mathcal{F}_{mf}(K)$ is the anomalous propagator,

$$\mathcal{F}_{mf}(K) = \Delta_{sf}\mathcal{G}_{mf}(K)\mathcal{G}_0(-K) = \frac{-\Delta_{sf}}{(i\omega_n)^2 - E_{\mathbf{k}}^2}, \tag{3.8}$$

and $\mathcal{G}_{mf}(K)$ is the mean-field single nucleon propagator,

$$\mathcal{G}_{mf}(K) = \left[\mathcal{G}_0^{-1}(K) - \Sigma_{mf}(K)\right]^{-1} = \frac{i\omega_n + \xi_{\mathbf{k}}}{(i\omega_n)^2 - E_{\mathbf{k}}^2}, \tag{3.9}$$

with the mean-field dispersion relation of nucleon $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{sf}^2}$ and the mean-field self-energy

$$\Sigma_{mf}(K) = -\Delta_{sf}^2\mathcal{G}_0(-K). \tag{3.10}$$

The coupled gap and density equations read

$$\Delta_{sf} = \frac{g}{\beta V}\sum_K \mathcal{F}_{mf}(K) = \frac{g\Delta_{sf}}{V}\sum_{\mathbf{k}} \frac{1}{2E_{\mathbf{k}}}\left[1 - 2n_F(E_{\mathbf{k}})\right],$$
$$\rho = \frac{2}{\beta V}\sum_K e^{i\eta\omega_n}\mathcal{G}_{mf}(K) = \frac{2}{V}\sum_{\mathbf{k}}\left[1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}(1 - 2n_F(E_{\mathbf{k}}))\right], \tag{3.11}$$

where $n_F(x) = 1/[\exp(\beta x) + 1]$ is the Fermi-Dirac function and $e^{i\eta\omega_n}$ with $\eta \to 0$ is a convergence factor for the Matsubara summation. The prefactor 2 on the right-hand side of the density equation counts the degeneracy of $\mathcal{S}_1$ and $\mathcal{S}_2$.

In BCS theory, *np* pairs enter into the problem below T_c, but only through their condensates at zero momentum. By rewriting the mean-field self-energy $\Sigma_{mf}(K)$ in a manner of

$$\Sigma_{mf}(K) = \frac{1}{\beta V}\sum_Q t_{mf}(Q)\mathcal{G}_0(Q - K), \tag{3.12}$$

we are aware of the fact that these *condensed* pairs can be associated with a T matrix in the following form

$$t_{mf}(Q) = -\Delta_{sf}^2\delta(Q), \tag{3.13}$$

with $Q = (q_0, \mathbf{q})$, $q_0 = i\omega_\nu = i2\nu\pi T$, $\nu \in \mathbb{Z}$ being the boson Matsubara frequency and $\delta(Q) = \beta\delta_{\nu,0}\delta^{(3)}(\mathbf{q})$. Furthermore, if we define the mean-field pair susceptibility as

$$\chi_{mf}(Q) = \frac{1}{\beta V}\sum_K \mathcal{G}_{mf}(K)\mathcal{G}_0(Q - K), \tag{3.14}$$

we can re-write the gap equation in a superfluid phase as

$$1 - g\chi_{mf}(0) = 0, \quad T \leqslant T_c. \tag{3.15}$$

This suggests that one can consider the *uncondensed* pair propagator or T matrix to be of the form

$$t_{pair} = \frac{-g}{1 - g\chi_{mf}(Q)}, \tag{3.16}$$

and then the gap equation is given by $t_{pair}^{-1}(0) = 0$.

It is well known that the critical temperature T_c in the BCS theory is related to the appearance of a singularity in a T matrix in the form of Eq. (3.16), but with $\Delta_{sf} = 0$. This is the so-called Thouless criterion for T_c [47]. But the meaning of Eq. (3.15) is more general as stressed by Kadanoff and Martin [41]. It states that under an asymmetric choice of χ, the gap equation is equivalent to the requirement that the T matrix associated with the uncondensed pair remains singular at zero momentum and energy for all temperatures below T_c.

Although the construction of the uncondensed pair propagator (3.16) in the BCS scheme is quite natural, the uncondensed pair has no feedback to the nucleon self-energy (3.12). When the coupling is weak, such a feedback is not important, but if the system is strongly coupled, this feedback will be significant. The simplest way to include the feedback effects is to replace t_{mf} in Eq. (3.12) by $t_{mf} + t_{pair}$. But to make such an inclusion self-consistent, t_{pair} should be somewhat modified, which we discuss in the next section.

B. G_0G formalism at $T \leqslant T_c$

Physically, the BCS theory involves the contribution to nucleon self-energy below T_c only from those condensed pairs (i.e., the $\mathbf{q} = 0$ Cooper pairs). This is justified only at a weak-coupling region. Generally, in the superfluid phase, the self-energy consists of two distinctive contributions, one from the superfluid condensate and the other from thermal pair excitations. Correspondingly, it is natural to decompose the self-energy into two additive terms

$$\Sigma(K) = \frac{1}{\beta V}\sum_Q t(Q)\mathcal{G}_0(Q - K) = \Sigma_{mf}(K) + \Sigma_{pg}(K), \tag{3.17}$$

with the T matrix accordingly given by

$$t(Q) = t_{mf}(Q) + t_{pg}(Q),$$
$$t_{pg}(Q) = \frac{-g}{1 - g\chi(Q)}, \tag{3.18}$$

where the subscript pg indicates that this term will lead to the pseudogap in the nucleon dispersion relation as will become clear soon. See Fig. 1 for the Feynman diagrams for $t_{pg}(Q)$ and $\Sigma(K)$. Comparing with the BCS scheme, $t_{mf}(Q)$ in Eq. (3.12) is replaced by $t(Q)$ and $\Sigma(K)$ now contains the feedback of uncondensed pairs. The pair susceptibility $\chi(Q)$, as inspired by Eq. (3.14), is chosen to be the following asymmetric G_0G form

$$\chi(Q) = \frac{1}{\beta V}\sum_K \mathcal{G}(K)\mathcal{G}_0(Q - K). \tag{3.19}$$

In the spirit of Kadanoff and Martin, we now propose the superfluid instability condition or gap equation as [extension

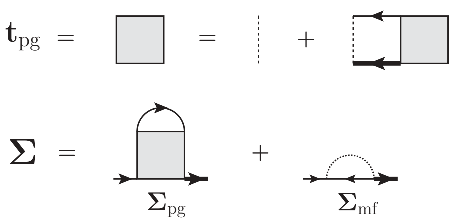

FIG. 1. Feynman diagrams for the T matrix of noncondensed pairs and the nucleon self-energy in the G_0G formalism.

of Eq. (3.15)]

$$1 - g\chi(0) = 0, \quad T \leqslant T_c. \tag{3.20}$$

We stress here that this condition has quite clear physical meaning in the BEC regime. The dispersion relation of the bound pair is given by $t^{-1}(Q) = 0$, hence $t^{-1}(0) \propto \mu_b$, with μ_b the effective chemical potential of the pairs. Then the BEC condition requires $\mu_b = 0$ for all $T \leqslant T_c$.

The gap equation (3.20) tells us that $t_{\rm pg}(Q)$ is highly peaked around $Q = 0$, so we can approximate $\Sigma_{\rm pg}$ as

$$\Sigma_{\rm pg}(K) \simeq -\Delta_{\rm pg}^2 \mathcal{G}_0(-K), \quad T \leqslant T_c, \tag{3.21}$$

where we defined the pseudogap parameter via

$$\Delta_{\rm pg}^2 = -\frac{1}{\beta V}\sum_Q t_{\rm pg}(Q). \tag{3.22}$$

The total self-energy now is

$$\Sigma(K) = -\Delta^2\mathcal{G}_0(-K), \tag{3.23}$$

with $\Delta^2 = \Delta_{\rm sf}^2 + \Delta_{\rm pg}^2$. It is clear that $\Delta_{\rm pg}$ also contributes to the energy gap in quasinucleon excitation. Physically, the pseudogap $\Delta_{\rm pg}$ below T_c can be interpreted as an extra contribution to the excitation gap of a nucleon quasiparticle: An additional energy is needed to overcome the residual attraction between nucleons in a thermal excited pair to produce fermion-like quasiparticles. One should note that the $\Delta_{\rm pg}$ is associated with the fluctuation of the pairs $\Delta_{\rm pg}^2 \sim \langle\Delta^2\rangle - \langle\Delta\rangle^2$ [34,37], hence it does not lead to superfluid (symmetry breaking).

With Eq. (3.21), the pair susceptibility reads

$$\begin{aligned}\chi(Q) &= \frac{1}{V}\sum_{\mathbf{k}}\Bigg[\frac{E_{\mathbf{k}}+\xi_{\mathbf{k}}}{2E_{\mathbf{k}}}\frac{n_F(-\xi_{\mathbf{q-k}}) - n_F(E_{\mathbf{k}})}{E_{\mathbf{k}}+\xi_{\mathbf{q-k}} - q_0 - i0^+} \\ &\quad - \frac{E_{\mathbf{k}}-\xi_{\mathbf{k}}}{2E_{\mathbf{k}}}\frac{n_F(E_{\mathbf{k}}) - n_F(\xi_{\mathbf{q-k}})}{E_{\mathbf{k}}-\xi_{\mathbf{q-k}} + q_0 + i0^+}\Bigg] \\ &= \frac{1}{V}\sum_{\mathbf{k},s=\pm}\frac{sE_{\mathbf{k}}+\xi_{\mathbf{k}}}{2sE_{\mathbf{k}}}\frac{n_F(-sE_{\mathbf{k}}) - n_F(\xi_{\mathbf{q-k}})}{sE_{\mathbf{k}}+\xi_{\mathbf{q-k}} - q_0},\end{aligned} \tag{3.24}$$

with $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta^2}$. The number equation remains unchanged except the replacement of $\Delta_{\rm mf} \to \Delta$.

Furthermore, the gap equation (3.20) suggests that we can make the following pole approximation to the pair propagator or T matrix $t_{\rm pg}(Q)$ as

$$t_{\rm pg}(Q) \simeq \frac{Z^{-1}}{q_0 - \mathbf{q}^2/(2m_b)}, \tag{3.25}$$

where the residue Z^{-1} and effective "boson" mass are given by

$$\begin{aligned} Z &= \frac{\partial\chi}{\partial q_0}\bigg|_{Q=0}, \\ \frac{Z}{m_b} &= -\frac{1}{3}\frac{\partial^2\chi}{\partial\mathbf{q}^2}\bigg|_{Q=0}. \end{aligned} \tag{3.26}$$

We stress here that, in general, the expansion of $t_{\rm pg}^{-1}(Q)$ should also contain a term $\propto q_0^2$, but such a term does not bring qualitative change to the crossover physics [38], hence we neglect it in Eq. (3.25).

A straightforward calculation gives

$$\begin{aligned} Z &= \frac{1}{V}\sum_{\mathbf{k}}\sum_{s=\pm}\frac{s}{2E_{\mathbf{k}}}\frac{n_F(E_{\mathbf{k}}) - n_F(s\xi_{\mathbf{k}})}{E_{\mathbf{k}} - s\xi_{\mathbf{k}}} \\ &= \frac{1}{\Delta^2}\left[\frac{\rho}{4} - \frac{1}{V}\sum_{\mathbf{k}} n_F(\xi_{\mathbf{k}})\right], \end{aligned} \tag{3.27}$$

and

$$\begin{aligned} \frac{Z}{m_b} &= \frac{1}{V}\sum_{\mathbf{k}}\sum_{s}\frac{1}{2sE_{\mathbf{k}}}\Bigg[\frac{1}{m}\frac{n_F(-sE_{\mathbf{k}}) - n_F(\xi_{\mathbf{k}})}{sE_{\mathbf{k}}+\xi_{\mathbf{k}}} \\ &\quad - \frac{2\mathbf{k}^2}{3m^2}\left(\frac{n_F(-sE_{\mathbf{k}}) - n_F(\xi_{\mathbf{k}})}{(sE_{\mathbf{k}}+\xi_{\mathbf{k}})^2} + \frac{n_F'(\xi_{\mathbf{k}})}{sE_{\mathbf{k}}+\xi_{\mathbf{k}}}\right)\Bigg] \\ &= \frac{Z}{m} - \frac{1}{V}\sum_{\mathbf{k}}\frac{2\mathbf{k}^2}{3m^2\Delta^2}n_F'(\xi_{\mathbf{k}}) - \frac{1}{V}\sum_{\mathbf{k}}\frac{\mathbf{k}^2}{3m^2E_{\mathbf{k}}\Delta^4} \\ &\quad \times \left\{\left(E_{\mathbf{k}}^2+\xi_{\mathbf{k}}^2\right)[1-2n_F(E_{\mathbf{k}})] - 2E_{\mathbf{k}}\xi_{\mathbf{k}}[1-2n_F(\xi_{\mathbf{k}})]\right\}. \end{aligned} \tag{3.28}$$

The expression in the square bracket of the right-hand-side of Eq. (3.27) is nothing but one-half the density of the pairs $\rho_b/2$, we then have $\rho_b = 2Z\Delta^2$.

Substituting Eq. (3.25) into Eq. (3.22) leads to

$$\Delta_{\rm pg}^2 = \frac{1}{ZV}\sum_{\mathbf{q}} n_B[\mathbf{q}^2/(2m_b)] = \frac{1}{Z}\left(\frac{Tm_b}{2\pi}\right)^{3/2}\zeta\left(\frac{3}{2}\right), \tag{3.29}$$

where $n_B(x) = 1/[\exp(\beta x) - 1]$ is the Bose-Einstein function and a vacuum term was regularized out. It should be stressed that at zero temperature $\Delta_{\rm pg}^2 = 0$, hence the G_0G scheme yields the BCS ground state. One should note that $\Delta_{\rm pg}^2 = \rho_b^{\rm uncondensed}/2Z$, and hence $\Delta_{\rm sf}^2 = \rho_b^{\rm condensed}/2Z$.

Now, Eqs. (3.20) and (3.29), as well as a number equation are coupled to determine the total excitation gap Δ, the pseudogap $\Delta_{\rm pg}$, and the nucleon chemical potential μ at a given density and temperature below T_c. In short, they are

$$\begin{aligned} 1 &= \frac{g}{V}\sum_{\mathbf{k}}\frac{1}{2E_{\mathbf{k}}}[1-2n_F(E_{\mathbf{k}})], \\ \rho &= \frac{2}{V}\sum_{\mathbf{k}}\left[1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}(1-2n_F(E_{\mathbf{k}}))\right], \\ \Delta_{\rm pg}^2 &= \frac{1}{Z}\left(\frac{Tm_b}{2\pi}\right)^{3/2}\zeta\left(\frac{3}{2}\right). \end{aligned} \tag{3.30}$$

C. G_0G formalism above T_c

Above T_c, Eq. (3.20) does not apply, hence Eq. (3.21) no longer holds. To proceed, we extend our more precise $T \leqslant T_c$ equations to $T > T_c$ in the simplest fashion. We will continue to use Eq. (3.23) to parametrize the self-energy, but with $\Delta = \Delta_{pg}$, and ignore the finite-lifetime effect associated with normal state pairs. It was shown that this is still a good approximation when temperature is not very high [36,38,40]. The T matrix $t_{pg}(Q)$ at small Q can be approximated now as

$$t_{pg}(Q) \simeq \frac{Z^{-1}}{q_0 - \Omega_{\mathbf{q}}}, \quad (3.31)$$

where $\Omega_{\mathbf{q}} = \mathbf{q}^2/(2m_b) - \mu_b$. Since there is no condensation in the normal state, the effective pair chemical potential μ_b is no longer zero, instead it should be calculated from

$$Z\mu_b \equiv t^{-1}(0) = -\frac{1}{g} + \chi(0) = -\frac{1}{g} + \frac{1}{V}\sum_{\mathbf{k}} \frac{1 - 2n_F(E_{\mathbf{k}})}{2E_{\mathbf{k}}}. \quad (3.32)$$

This is used as the modified gap equation. Similarly, above T_c the pseudogap Δ_{pg} is determined by

$$\Delta_{pg}^2 = \frac{1}{ZV}\sum_{\mathbf{q}} n_B(\Omega_{\mathbf{q}}) = \frac{1}{Z}\left(\frac{Tm_b}{2\pi}\right)^{3/2} \mathrm{Li}_{\frac{3}{2}}(e^{\mu_b/T}), \quad (3.33)$$

where $\mathrm{Li}_n(z)$ is the polylogarithm function. Then Eqs. (3.32), (3.33), and the number equation that remains unchanged determine Δ_{pg}, μ, and μ_b.

In summary, at $T > T_c$, the order parameter is zero and $\Delta = \Delta_{pg}$. The closed set of equations determining Δ, μ, and μ_b is

$$\begin{aligned} Z\mu_b &= -\frac{1}{g} + \frac{1}{V}\sum_{\mathbf{k}} \frac{1 - 2n_F(E_{\mathbf{k}})}{2E_{\mathbf{k}}}, \\ \rho &= \frac{2}{V}\sum_{\mathbf{k}}\left[1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}(1 - 2n_F(E_{\mathbf{k}}))\right], \\ \Delta_{pg}^2 &= \frac{1}{Z}\left(\frac{Tm_b}{2\pi}\right)^{3/2} \mathrm{Li}_{\frac{3}{2}}(e^{\mu_b/T}). \end{aligned} \quad (3.34)$$

D. Thermodynamics

The thermodynamics of the matter are governed by the thermodynamic potential, which reads

$$\Omega = \Omega_f + \Omega_b, \quad (3.35)$$

where Ω_f and Ω_b are the contributions from nucleons and thermal excited pairs

$$\Omega_f = 2\Delta^2\chi(0) - \frac{4T}{V}\sum_{\mathbf{k}}\left[\frac{E_{\mathbf{k}} - \xi_{\mathbf{k}}}{2} + \ln(1 + e^{-\beta E_{\mathbf{k}}})\right], \quad (3.36)$$

$$\Omega_b = \frac{2}{\beta V}\sum_{\mathbf{q}} \ln(1 - e^{-\beta\Omega_{\mathbf{q}}}). \quad (3.37)$$

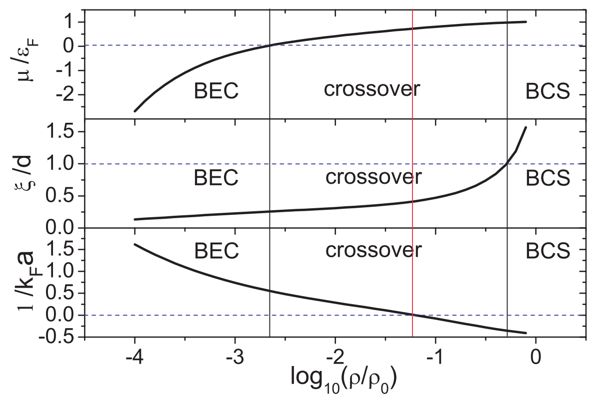

FIG. 2. (Color online) The nucleon chemical potential over Fermi energy ratio μ/ε_F, the scaled pair size ξ/d, and the scaled scattering length of np collision $1/(k_F a)$ as functions of density. The right and left vertical lines that separate the BCS, BEC, and crossover regions are, respectively, determined by the conditions $\xi/d = 1$ and $\mu/\varepsilon_F = 0$. The red vertical line in the crossover region denotes the unitary point where $1/(k_F a) = 0$.

Other thermodynamic quantities can be derived from Ω (e.g., the entropy density is given by $s = -\partial\Omega/\partial T$ and the specific heat c_V is given by $c_V = T\partial s/\partial T$).

IV. NUMERICAL RESULTS

We now discuss the results obtained by numerically solving Eq. (3.30) for $T \leqslant T_c$ and Eq. (3.34) for $T > T_c$. We will mainly focus on the intermediate (the crossover region, see Fig. 2) and low-density regions, since the high-density region is proven to be well understood in BCS theory. We begin with the results concerning the critical temperature for superfluid transition in the BCS-BEC crossover.

A. BCS-BEC crossover and critical temperature

At zero temperature, the G_0G formalism reproduces the usual BCS theory. To have a quantitative examination of the BCS-BEC crossover, it is convenient to define the condensed np Cooper pair wave function at zero temperature

$$\begin{aligned} \psi(\mathbf{r}) &\equiv C\langle \mathrm{BCS}|a_{n\uparrow}^\dagger(\mathbf{x})a_{p\downarrow}^\dagger(\mathbf{x}+\mathbf{r})|\mathrm{BCS}\rangle \\ &= C'\int \frac{d^3\mathbf{k}}{(2\pi)^3}\psi(\mathbf{k})e^{i\mathbf{kr}}, \end{aligned} \quad (4.1)$$

where $a_{n\sigma}^\dagger$ ($a_{p\sigma}^\dagger$) is the creation operator of a neutron (proton) with spin σ and $\psi(\mathbf{k})$ are the anomalous density distribution function

$$\psi(\mathbf{k}) = \langle \mathrm{BCS}|a_{n\uparrow}^\dagger(\mathbf{k})a_{p\downarrow}^\dagger(-\mathbf{k})|\mathrm{BCS}\rangle = \frac{\Delta}{2E_{\mathbf{k}}}. \quad (4.2)$$

After substituting Eq. (4.2) into the number and gap equations, we get the following Schrödinger-like equation

$$\frac{\mathbf{k}^2}{m}\psi(\mathbf{k}) - g(1 - 2n_{\mathbf{k}})\int \frac{d^3\mathbf{k}'}{(2\pi)^3}\psi(\mathbf{k}') = 2\mu\Psi(\mathbf{k}). \quad (4.3)$$

In the limit of vanishing density $n_{\mathbf{k}} \to 0$, this equation goes over into the Schrödinger equation for the np bound states (the deuterons) in the center-of-mass frame and the chemical potential 2μ then plays the role of the binding energy. Hence, one expects that at sufficiently low density and low temperature, the symmetric nuclear matter should be in the BEC phase.

To have a more quantitative description of the BCS-BEC crossover, we define other characteristic quantities: The mean-square-root size of the np pair

$$\xi^2 = \frac{\int d^3\mathbf{x}\mathbf{x}^2|\psi(\mathbf{x})|^2}{\int d^3\mathbf{x}|\psi(\mathbf{x})|^2}, \tag{4.4}$$

and the s-wave scattering length a that relates the coupling constant g to the low-energy limit of the two-body T matrix of np scattering in a vacuum

$$\frac{m}{4\pi a} = -\frac{1}{g} + \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{m}{\mathbf{k}^2}. \tag{4.5}$$

In the BCS region, ξ is expected to be larger than the average distance between the neutron and proton $d \equiv (\rho/2)^{-1/3}$ and at the same time the scattering length a should be negative to ensure that the interaction between the neutron and proton is attractive. In the BEC region, however, ξ/d should be very small reflecting the compactness of the pair and the scattering length will be positive to guarantee the appearance of the two-body bound state.

In Fig. 2, we show the nucleon chemical potential over Fermi energy ratio μ/ε_F, the scaled mean-square-root size ξ/d, and the inverse scattering length $1/(k_F a)$ as functions of nuclear density. Although ξ itself is not a monotonous function of ρ, the scaled one goes down monotonously with decreasing density and finally approaches zero at zero density. The right vertical line around $\rho/\rho_0 \sim 0.5$ indicates the position of $\xi = d$, which can be used to separate the BCS (weak coupling) region from the crossover (intermediately strong coupling) region. The chemical potential roughly equals the Fermi energy at the BCS region, but it drops down with decreasing density and becomes negative below $\rho/\rho_0 \sim 0.002$. The position where μ changes sign can be regarded as the boundary between the BEC (strong coupling) region in which μ is negative and the other region with positive μ. The third panel shows that $1/(k_F a)$ increases with decreasing density and becomes positive after $\rho/\rho_0 \sim 0.06$. This turning point is called the unitary limit, which we indicate by a red vertical line in the figure. We will discuss the unitary limit in next section.

The numerical result for the critical temperature is shown in Fig. 3. The dashed line shows the critical temperature over the Fermi energy ratio given by BCS theory, which blows up quickly when density goes down in the crossover and BEC regions ($T_c^{\rm BCS}$ is almost ten times larger than T_c at density $\rho = 0.0001\rho_0$). Physically, the ratio $T_c^{\rm BCS}/\varepsilon_F$ as well as $\Delta_{T=0}/\varepsilon_F$ measures the strength of the attraction between the neutron and proton. However, due to the lack of pairing fluctuation effect, in the crossover and BEC regions the BCS theory does not give the correct critical temperature for superfluid/normal transition, which is mainly determined by the bosonic degree of freedom in these regions. The solid line is for T_c obtained from Eq. (3.30). We can see that the evolution of T_c is smooth and the superfluid phase transition is second order in the whole density region. Also, it can be seen that T_c is not a monotonous function of density: There is a local maximum in the T_c curve, which is roughly located around the unitary point. One should notice that a similar local maximum also appears in the famous Nozieres-Schmitt-Rink approach for T_c [3]. At the low-density limit, all the nucleons participate into the deuterons, which are long lived at a temperature lower than T_c, the system is essentially a deuteron gas and the superfluid is totally due to the BEC of deuterons. In this case, we have, at T_c, $2Z\Delta_{\rm pg}(T = T_c) = \rho_b^{\rm uncondensed} = \rho/2$. Solving out T_c, we get

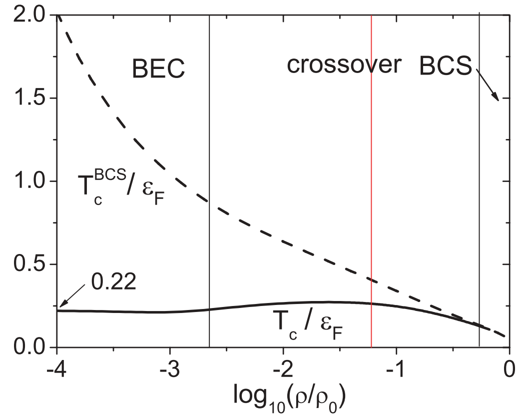

FIG. 3. (Color online) The critical temperature T_c scaled by the Fermi energy ε_F as a function of density. Also shown is the BCS prediction (dashed line).

$$T_c = \frac{2\pi}{m_b}\left[\frac{\rho}{4\zeta(3/2)}\right]^{2/3}. \tag{4.6}$$

This is just the BEC transition temperature for a boson of mass m_b. Adopting that $m_b \approx 2m$, we arrived at the well-known result,

$$T_c \approx 0.218\varepsilon_F, \tag{4.7}$$

which coincides well with our numerical result.

It should be stressed that the BCS critical temperature $T_c^{\rm BCS}$ was found to be a good approximation for the pair dissociation temperature T^* (above which the pairs are essentially dissociated by the thermal motion of the participators) [36]. So it is a pseudogap-dominated region in between T_c and $T_c^{\rm BCS}$.

Corresponding to the evolution of the critical temperature, it is indicative to see how the pseudogap evolves. In Fig. 4, we plot the zero temperature excitation gap $\Delta(T = 0)$ as well as the pseudogap $\Delta_{\rm pg}(T = T_c)$ at T_c. It can be seen that at low density $\rho \lesssim 0.01\rho_0$, $\Delta_{\rm pg}(T = T_c)$ is roughly equal to $\Delta(T = 0)$ (but they do not completely coincide) reflecting the strong coupled nature in this case; however, at high density, $\Delta_{\rm pg}(T = T_c)$ is much smaller than $\Delta(T = 0)$, indicating that the pairing fluctuation is not essential there and BCS theory can work well. In the following sections, we will focus on the crossover and BEC regions where BCS theory is not applicable at finite temperature.

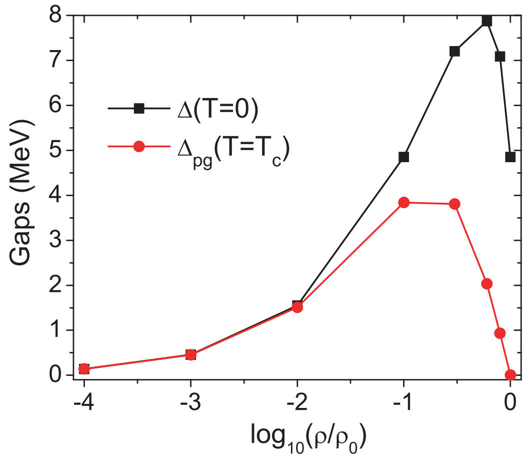

FIG. 4. (Color online) The excitation gap Δ at zero temperature and the pseudogap at T_c as functions of density.

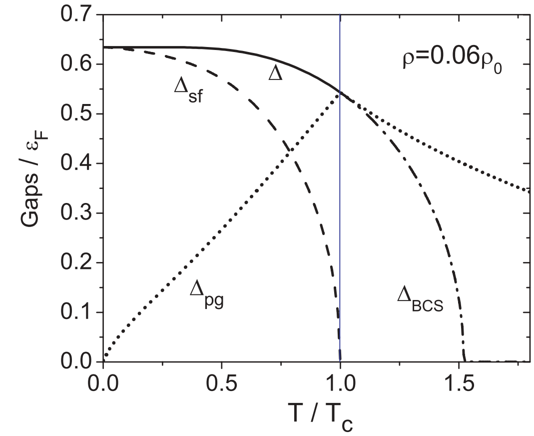

FIG. 5. (Color online) The superfluid gap $\Delta_{\rm sf}$ (dashed line), pseudogap $\Delta_{\rm pg}$ (dotted line), and the total gap Δ (solid line) as functions of the temperature at unitary point $\rho = 0.06\rho_0$. The BCS result (dot-dashed line) above T_c is also shown.

B. Unitary matter

As shown in the last section, the np scattering length $k_F a$ is infinite at the unitary point $\rho \approx 0.06\rho_0$. We call the nuclear matter at this point a unitary matter. It is interesting because it exhibits universal behaviors (i.e., the physical properties of unitary matter are independent of the details of the interactions [48]). Hence, the unitary nuclear matter behaves just like the unitary cold atomic Fermi gas that was realized in the laboratory through the Feshbach resonance. For unitary matter, the unique characteristic scale is given by the Fermi momentum k_F, so we have $\mu_{T=0} = \zeta\varepsilon_F$, $\Delta_{T=0} = \gamma\varepsilon_F$, $T_c = \alpha\varepsilon_F$, $\Delta_{\rm pg}(T = T_c) = \lambda\varepsilon_F$, and so on, with $\zeta, \gamma, \alpha, \lambda$, and so on, being universal constants. In our G_0G scheme, the universal coefficients are given by $\zeta \approx 0.59$, $\gamma \approx 0.64$, $\alpha \approx 0.26$, and $\lambda \approx 0.53$. For comparison, we would like to list the values obtained by Monte Carlo techniques: $\zeta \approx 0.42$ [49], $\gamma \approx 0.50$ [49], $\alpha \approx 0.157$ [50], or 0.25 [51]. Our results are larger than the Monte Carlo values. It is easy to show that the energy per particle in unitary matter is $E/N = \zeta(E/N)_{\rm free}$ where $(E/N)_{\rm free}$ is the energy per particle for free fermion gas. Moreover, the equation of state of unitary matter is the same as the free fermion gas $\varepsilon = 3P/2$, with P being the pressure.

In Fig. 5, we plot $\Delta_{\rm sf}$ (in units of ε_F, the same in the following), $\Delta_{\rm pg}$, and Δ as functions of temperature for unitary matter. As a comparison, we also plot the BCS result $\Delta_{\rm BCS}$ above T_c. As we can see from the figure, with decreasing temperature below T_c, $\Delta_{\rm pg}(T)$ is a monotonically decreasing function from its maximum value at T_c and it essentially vanishes at $T = 0$ roughly according to $\Delta_{\rm pg}(T) \propto T^{3/4}$ [see Eq. (3.29)], while $\Delta_{\rm sf}(T)$ and $\Delta(T)$ both increase monotonically and become coincident at $T = 0$. Such kinds of temperature dependence reflect the fact that the pseudogap is due to the thermally excited pairs with finite $\mathbf{q}$: When T grows higher and higher, more and more pairs are excited from the condensate and at T_c all the condensed pairs are thermally excited; after that the thermal motion of nucleons begins to dissociate the pairs and hence $\Delta_{\rm pg}$ (more exactly, $Z\Delta^2_{\rm pg}$) begins decreasing above T_c. In addition, the critical temperature T_c is smaller than the BCS prediction, which shows the fact that the pairing fluctuation tends to destroy the order of the system. Although the physical picture is clear, our formalism cannot be applied to very high temperature, where the effects of finite lifetime of the pairs become significant, which are not included in our formalism.

One should note that Δ and its derivative $d\Delta/\delta T$ are continuous at T_c. This is very different from the BCS case, where $d\Delta_{\rm BCS}/\delta T$ is discontinuous at $T_c^{\rm BCS}$. Such a difference can be reflected in thermodynamic quantities such as the specific heat. In Fig. 6, we illustrate the entropy density and the specific heat c_V for unitary matter. We compute c_V through $c_V = T\partial s/\partial T$, which involves the derivatives $\partial\mu/\partial T$, $\partial\Delta/\partial T$, $\partial m_b/\partial T$, and $\partial\mu_b/\partial T$, so it is a nontrivial calculation. As is well known, for the weak-coupling BCS case, the specific heat has a jump $\Delta c_V \propto d\Delta^2/\delta T$ at $T_c^{\rm BCS}$, which reflects the sudden opening of the excitation gap. For unitary matter, however, we found a continuous c_V at T_c. This continuity of c_V reflects the previous existence of the excitation gap above T_c due to the pairing fluctuation. It may serve as an experimentally

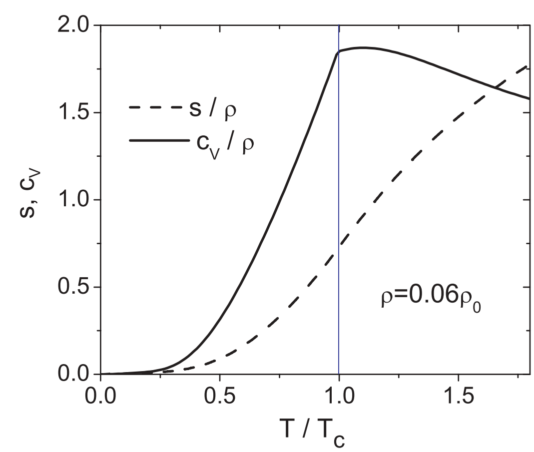

FIG. 6. (Color online) The entropy density and the specific heat of unitary matter as functions of temperature. Unlike the weak-coupling BCS case, there is no jump for the specific heat at T_c.

accessible signal for the existence of the pseudogap in the normal phase.

C. Deuteron gas

When density is very low, say $\rho < 0.002\rho_0$ from Fig. 2, the symmetric nuclear matter is effectively a Bose gas of deuteron. We, in this section, study the properties of deuteron gas based on our G_0G formalism. First, we observed from Figs. 2 and 3 that when $\rho \to 0$, $-\mu \gg T_c$. In this case, Δ, μ, and Z are almost temperature independent below T_c reflecting the strong *np* attraction. Then for $T < T_c$, the governing equations become (expanding in powers of $a^3\rho$ and Δ^2/μ^2) [38]

$$\frac{m}{4\pi a} = \frac{1}{V}\sum_{\mathbf{k}}\left(\frac{m}{\mathbf{k}^2} - \frac{1}{2E_{\mathbf{k}}}\right) \approx \frac{m\sqrt{2m|\mu|}}{4\pi}\left(1 + \frac{\Delta^2}{16\mu^2}\right),$$
$$\rho = \frac{2}{V}\sum_{\mathbf{k}}\left(1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right) \approx \frac{m^2\Delta^2}{2\pi\sqrt{2m|\mu|}}, \tag{4.8}$$
$$\Delta_{\text{pg}}^2 \approx \frac{4\Delta^2}{\rho}\left(\frac{Tm}{\pi}\right)^{3/2}\zeta\left(\frac{3}{2}\right).$$

Hence at low temperature, we have

$$\Delta^2 \approx \frac{2\pi\rho}{m^2 a}\left(1 - \frac{\pi a^3\rho}{2}\right),$$
$$\Delta_{\text{pg}}^2 \approx \frac{8\pi}{m^2 a}\zeta\left(\frac{3}{2}\right)\left(\frac{mT}{\pi}\right)^{3/2}\left(1 - \frac{\pi a^3\rho}{8}\right), \tag{4.9}$$
$$\mu \approx -\frac{1}{2ma^2}(1 - \pi a^3\rho).$$

These relations give how $\Delta(\rho)$, $\Delta_{\text{pg}}(\rho)$, $\mu(\rho)$, and $m_b(\rho)$ evolve with ρ at $T < T_c$ in the deep BEC region.

Next, let us study the temperature dependence of these characteristic quantities. To specify the problem, we fix the density as $\rho = 0.001\rho_0$. By solving the coupled equations (3.30), we get the transition temperature $T_c \simeq 0.22\varepsilon_F \approx 0.08$ MeV. In Fig. 7, we show the superfluid gap Δ_{sf}, the pseudogap Δ_{pg}, and the total excitation gap Δ as functions of temperature. Due to the stronger attraction, unlike for the unitary matter, now $\Delta(T)$ is almost a constant below T_c and at $T = 0$, Δ/ε_F is even larger than 1. But near zero temperature, $\Delta_{\text{pg}}(T)$ still behaves as $\Delta_{\text{pg}} \propto T^{3/4}$, as shown in Eq. (4.9).

In Fig. 8, we give the temperature dependence of the nucleon chemical potential $\mu(T)$ and the effective deuteron chemical potential $\mu_b(T)$. Below T_c, μ_b is zero meaning that the BEC superfluid is formed. Above T_c, both μ and μ_b decrease, corresponding to the thermal dissociating effect. Just above T_c, a simple calculation leads to that $\mu(T) - \mu(T_c) \propto \mu_b \propto (T - T_c)^2$. Since μ_b is related to μ and the deuteron binding energy E_b through $\mu_b = 2\mu + E_b$, we get that the binding energy E_b at zero temperature is roughly $0.6\varepsilon_F$ at $\rho = 0.001\rho_0$ from Fig. 8. The effective deuteron mass $m_b(T)$ is shown in Fig. 9. It is seen that at low temperature m_b is almost a constant, but it drops when temperature becomes higher. We note here that m_b can be regarded as the medium renormalized deuteron mass only in a deep BEC region [52,53]. It is renormalized because it contains indirectly the deuteron-deuteron interaction through the nucleon-deuteron coupling in the nucleon self-energy. Hence, this effective mass is not equal to $2m - E_b$, as one may intuitively expect. It

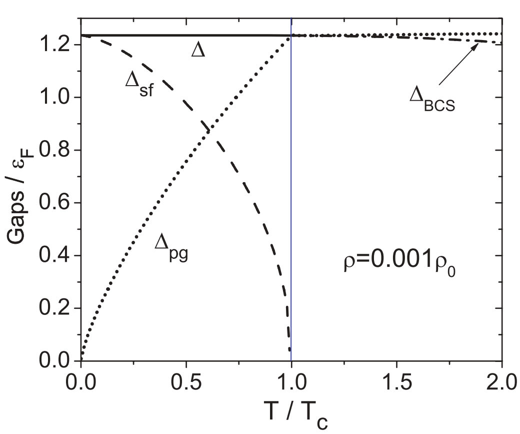

FIG. 7. (Color online) The superfluid gap Δ_{sf}, pseudogap Δ_{pg}, and the total gap Δ as functions of the temperature at density $\rho = 0.001\rho_0$. The BCS result is also shown above T_C.

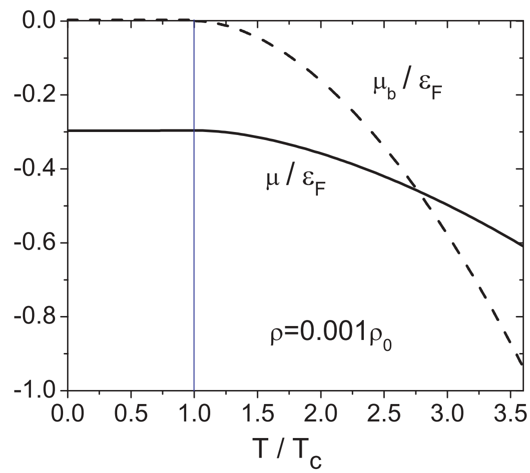

FIG. 8. (Color online) The nucleon chemical potential and the effective deuteron chemical potential as functions of T at density $\rho = 0.001\rho_0$.

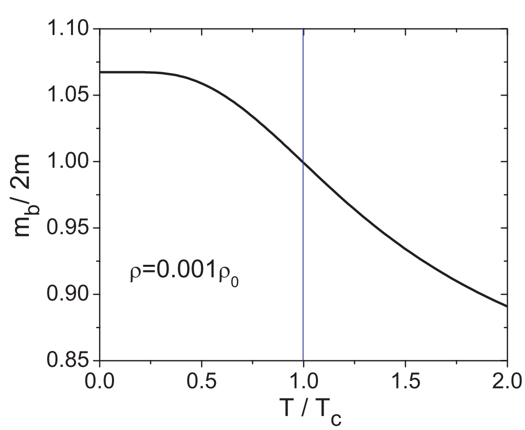

FIG. 9. (Color online) The effective deuteron mass parameter as a function of T at density $\rho = 0.001\rho_0$.

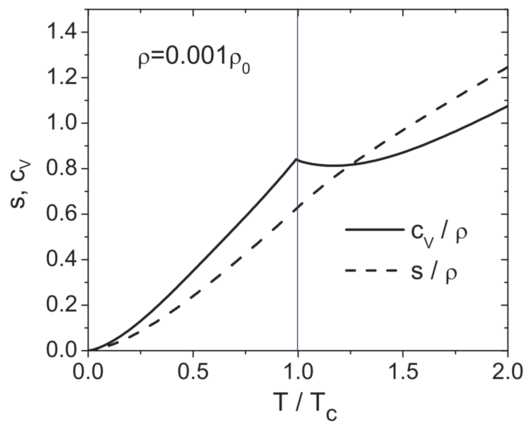

FIG. 10. (Color online) The entropy density and the specific heat of symmetric nuclear matter as functions of temperature at density $\rho = 0.001\rho_0$.

is actually a parameter measuring the effective size of the noncondensed pairs, hence it is also defined in intermediate coupling and even weak coupling regions. The drop of m_b at high temperature simply indicates that the effective size of the noncondensed pair is enlarged by the thermal motion of participate nucleons.

Finally, we depict the entropy density and the specific heat for deuteron gas in Fig. 10. One should note that at low temperature the behavior of c_V is very different from the prediction of BCS theory: It shows $T^{3/2}$ dependence at low T rather than an exponential suppression. Actually, since the condensate does not contribute to entropy, c_V at $T \ll T_c$ contains contributions from quasinucleons and from thermal excited pairs. The former contribution is just the BCS theory result

$$c_V^{\mathrm{BCS}} \propto \left(\frac{\Delta_0}{T}\right)^{1/2} e^{-\Delta_0/T}, \quad T \ll T_c, \tag{4.10}$$

with $\Delta_0 = \Delta_{T=0}$. The latter one is dominated by $T\partial\Delta_{\mathrm{pg}}^2/\partial T$ for other quantities are almost independent of T [see Eq. (4.9)], hence

$$c_V^{\mathrm{pg}} \propto T^{3/2}, \quad T \ll T_c, \tag{4.11}$$

which dominates c_V at low temperature. At the phase transition point, similarly with unitary matter, due to the continuity of the temperature derivative of the excitation gap, c_V does not get a discontinuity, but a λ-type behavior. Now, both the low temperature and the critical behaviors of c_V are quite similar with the situation found in an ideal BEC superfluid. It indicates that the symmetric nuclear matter at very low density is a nearly ideal deuteron gas.

Figures 6 and 10 inspire us that the specific heat jump at T_c may serve as a possible thermodynamic signal for the BCS-BEC crossover. We hence draw in Fig. 11 the specific heat jump $\Delta c_V \equiv c_V(T_c - 0^+) - c_V(T_c + 0^+)$ at T_c over the density ratio as a function of density. As density decreases, $\Delta c_V/\rho$ monotonously decreases and vanishes when density is smaller than $0.06\rho_0$ which is just the unitary point. The physical reason for such a kind of behavior is clear: as density decreases the system becomes more and more bosonic and the finite jump of the specific heat at T_c, which is a typical BCS feature gets suppressed.

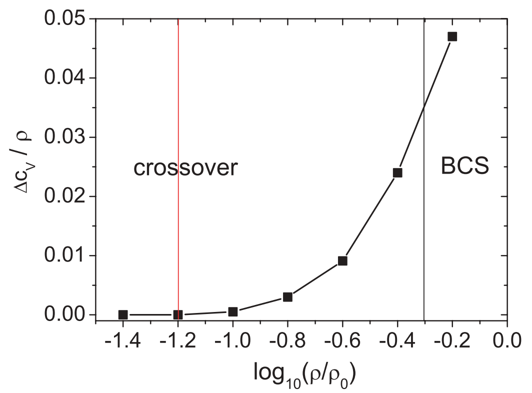

FIG. 11. (Color online) The specific heat jump at T_c over density ratio $\Delta c_V/\rho$ as a function of density.

V. SUMMARY AND DISCUSSION

As is well known, BCS theory is only applicable to a weak-coupling system or at zero temperature since it does not contain any pairing fluctuation effects. To study the BCS-BEC crossover at finite temperature, it is necessary to go beyond the BCS description. We, in this article, studied the effects of the pseudogap due to the pairing fluctuation on the BCS-BEC crossover problem in symmetric nuclear matter. For this purpose, we adopted a T-matrix method based on a G_0G approximation for the pair susceptibility. This method is a natural extension of BCS theory and has been widely used in theoretical studies of high T_c superconductor and BCS-BEC crossover problems in cold fermion atoms.

The pseudogap is determined by the density of thermally excited np pairs and vanishes at zero temperature. We found that its effects are substantial for intermediate and strong coupling regions (corresponding to intermediate and low-density regions) in the BCS-BEC crossover when temperature is not zero. At a high-density region, the pseudogap is essentially small and the G_0G theory recovers the BCS theory. Taking into account the pseudogap effects, we calculated the critical temperature for the superfluid phase transition shown in Fig. 3. At high density, T_c follows the BCS result, but at intermediate and low densities it deviates from the BCS prediction remarkably. At a dilute limit, T_c coincides with the BEC transition temperature of dilute deuterons.

The pseudogap persists in both the $T > T_c$ and $T < T_c$ regions. We investigated how the pseudogap affects the properties of unitary matter and dilute deuteron matter. At intermediate and low densities, the significant result is that due to the pseudogap, the specific heat is continuous at T_c. At low density, $c_V \propto T^{3/2}$ at low temperature and has a continuous λ-type behavior at T_c just like an ideal BEC superfluid. The qualitative change of the temperature dependence of specific heat from high density to low density also indicates the BCS-BEC crossover. Moreover, as density decreases, the jump of specific heat at T_c decreases and eventually disappears when $\rho \lesssim 0.06\rho_0$ as shown in Fig. 11. This may serve as a thermodynamic signal for the BCS-BEC crossover.

We stress that the G_0G approximation is, in principle, not applicable at a temperature much higher than T_c, since then the thermal dissociation effect will result in a finite width of the pairs, which we did not take into account. Besides, symmetric nuclear matter may favor the formation of α cluster [54], but we did not consider this situation in the present article. Finally, since most of the nuclear systems in nature do not contain equal numbers of neutrons and protons. It will be significant to extend the present formalism to asymmetric nuclear matter [14]. We leave all these challenges for future studies.

ACKNOWLEDGMENTS

We gratefully acknowledge H. Abuki, T. Brauner, and L. He for their careful reading of the manuscript and numerous suggestions. We also thank D. Rischke, A. Sedrakian, and P. Zhuang for helpful discussions. This work is supported, in part, by the Helmholtz Alliance Program of the Helmholtz Association, Contract No. HA216/EMMI "Extremes of Density and Temperature: Cosmic Matter in the Laboratory" and the Helmholtz International Center for FAIR within the framework of the LOEWE (Landesoffensive zur Entwicklung Wissenschaftlich-Ökonomischer Exzellenz) program launched by the state of Hesse.

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