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Role of shape dependence of dissipation on nuclear fission

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We examine the validity of extending Kramers' expression for fission width to systems with shape-dependent dissipations. For a system with a shape-dependent dissipation, Kramers' width obtained with the presaddle dissipation strength is found to be different from the stationary width obtained from the corresponding Langevin equations. It is demonstrated that the probability of a hot compound nucleus undergoing fission depends on both the presaddle and the postsaddle dynamics of collective nuclear motion. The predictions for prescission neutron multiplicity and evaporation residue cross section from statistical model calculations are also found to be different from those obtained from Langevin dynamical calculations when a shape-dependent dissipation is considered. For systems with shape-dependent dissipations, we conclude that the strength of "presaddle dissipation" determined by fitting experimental data in statistical model calculations does not represent the true strength of presaddle dissipation.

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Experimental and theoretical studies of heavy-ion-induced fusion-fission reactions at beam energies above Coulomb barriers have made significant contributions to the understanding of nuclear collective dynamics at high excitation energies in recent years. In particular, careful analyses of experimental data on multiplicities of prescission light particles (mainly neutrons and GDR γ's) [1–7], evaporation residue (ER) cross sections [8–10] and mass and kinetic energy distributions of fission fragments [1] have established that the fission dynamics of a hot compound nucleus is dissipative in nature. Consequently, fission has become a useful probe to study the dissipative properties of the nuclear bulk.

A dynamical model for fission of a hot nucleus was first proposed by Kramers [11] based on its analogy to the motion of a Brownian particle in a heat bath. In this model, the collective fission degrees of freedom represent the Brownian particle, while the rest of the intrinsic degrees of freedom of the compound nucleus correspond to the heat bath. The dynamics of such a system is governed by the appropriate Langevin equations or, equivalently, by the corresponding Fokker-Planck equation. Kramers solved the Fokker-Planck equation analytically with a few simplifying assumptions and obtained the stationary width of fission as

$$\Gamma_K = \frac{\hbar\omega_g}{2\pi} e^{-V_B/T} \left\{ \sqrt{1 + \left(\frac{\eta}{2\omega_s}\right)^2} - \frac{\eta}{2\omega_s} \right\}, \tag{1}$$

considering fission as diffusion of a Brownian particle across the fission barrier (V_B) placed in a hot and viscous fluid bath of temperature T and dissipation coefficient η. The frequencies of the harmonic oscillator potentials describing the nuclear potential at the ground state and the saddle configurations are ω_g and ω_s, respectively. Equation (1) was obtained assuming the dissipation coefficient η to be shape independent and constant for all deformations of the nucleus. Subsequently, the aforementioned stationary fission width predicted by Kramers was found to be in reasonable agreement with the asymptotic fission width obtained from numerical solutions of the Fokker-Planck [12–17] and Langevin [18–21] equations where shape-independent and constant values of dissipation were used. Kramers' fission width is extensively used in statistical model calculation for decay of compound nucleus. The coefficient η is often treated as a free parameter to fit experimental data. Efforts are also continuing to improve the modeling of the fission process to extract more reliable values of the dissipation coefficient [21–23].

It was first reported by Fröbrich *et al.* that the experimental data on prescission neutron multiplicity and fission cross section cannot be fitted by the same strength of the shape-independent dissipation in Langevin dynamical calculations [24]. While a smaller value of η can account for the fission excitation function, a larger value of η is required to describe the prescission multiplicity data. A shape-dependent nuclear dissipation was found necessary to simultaneously fit the prescission neutron multiplicity and fission cross-section data [24,25]. The required shape-dependent dissipation has a smaller value for small deformations of the compound nucleus and a larger value at large deformations. From considerations of chaos in single-particle motion within the nuclear volume, shape dependence of a similar nature is also predicted for one-body dissipation, considered to be mainly responsible for damping of nuclear motion [26,27]. A smaller dissipation strength in the presaddle region and a larger dissipation strength in the postsaddle region is found necessary in subsequent applications of Langevin equations for dynamics of fission [28,29].

Shape-dependent dissipation is also introduced in statistical model calculation for the decay of a compound nucleus, in the following manner [5–7,30]. One considers two dissipation strengths here: a smaller one (η_{in}) operating within the saddle-point region and a larger one (η_{out}) effective outside the saddle point. In a statistical model calculation of nuclear fission, it is assumed that the fission width is given by Γ_K^{in} [Γ_K in

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Eq. (1) with $\eta_{\rm in}$]. For a fission event, $\eta_{\rm out}$ is subsequently used to calculate the saddle-to-scission transition time during which further neutron evaporation can take place. However, the assumption that the fission width is given by $\Gamma_K^{\rm in}$ requires close scrutiny, as we are considering a shape-dependent dissipation here, while Kramers' width was originally obtained assuming a shape-independent dissipation. In the present communication, we examine the validity of determining the fission width from $\eta_{\rm in}$ alone when a shape-dependent dissipation is considered. To this end, we compare $\Gamma_K^{\rm in}$ with stationary widths from Langevin dynamical model calculations, considering the latter to represent the true fission width. We also compare the prescission neutron multiplicities and ER cross sections obtained from the statistical model with a shape-dependent dissipation with those obtained from the corresponding Langevin equations.

We choose the compound nucleus ^{224}Th for our calculation, mainly because its decay has been extensively investigated in the past [5,30,31]. We solve the Langevin equations using the "funny hills" shape parameters [32] to specify the collective coordinates for a dynamical description of nuclear fission. We presently study fission dynamics in one dimension where the elongation parameter c is the relevant coordinate and the Langevin equations are [27,33]

$$\begin{aligned} \frac{dp}{dt} &= -\frac{p^2}{2}\frac{\partial}{\partial c}\left(\frac{1}{m}\right) - \frac{\partial V}{\partial c} - \eta p + g\gamma(t), \\ \frac{dc}{dt} &= \frac{p}{m}, \end{aligned} \tag{2}$$

where γ represents the random force and its strength is given by the fluctuation-dissipation theorem as $g = \sqrt{mT\eta}$.

The Langevin equations are numerically integrated for a given compound nucleus at specified values of its spin and temperature. The initial collective coordinate is that of a spherical nucleus and its initial momentum distribution follows that of an equilibrated thermal system. A Langevin trajectory is considered to have undergone fission when it crosses the scission point. We choose the scission configuration to correspond to a neck radius of $0.3R$, where R is the radius of the initial shape of the compound nucleus [34]. The fission width is obtained from the time rate at which the Langevin trajectories cross the scission point using an ensemble of 10^6 trajectories for each calculation

Figure 1 shows the collective potential for ^{224}Th along with the shape-dependent dissipation coefficients used in the Langevin calculations. Denoting the elongation at which the dissipation changes its strength from $\eta_{\rm in}$ to $\eta_{\rm out}$ by c_η, the Langevin equations are solved for different values of c_η. Figure 2 shows the time-dependent fission widths from the Langevin equations for different values of c_η. The values of Kramers' fission widths $\Gamma_K^{\rm in}$ and $\Gamma_K^{\rm out}$ obtained with $\eta_{\rm in}$ and $\eta_{\rm out}$, respectively, in Eq. (1) are also shown in this figure. The values of stationary fission widths Γ_L from Langevin dynamics are subsequently plotted as a function of c_η in Fig. 3. It is immediately noted from Fig. 3 that for $c_\eta = 1.6$, which corresponds to the elongation at saddle, the stationary fission width (Γ_L) from Langevin equations is substantially smaller than the $\Gamma_K^{\rm in}$ obtained with a constant value of $\eta_{\rm in}$. This observation is contrary to the interpretation made in statistical model calculations employing shape-dependent dissipations, that $\Gamma_K^{\rm in}$ accounts for the fission rate. We further note in Fig. 3 that as c_η is shifted outward beyond the saddle point, Γ_L approaches $\Gamma_K^{\rm in}$. When c_η is moved inward, Γ_L approaches $\Gamma_K^{\rm out}$.

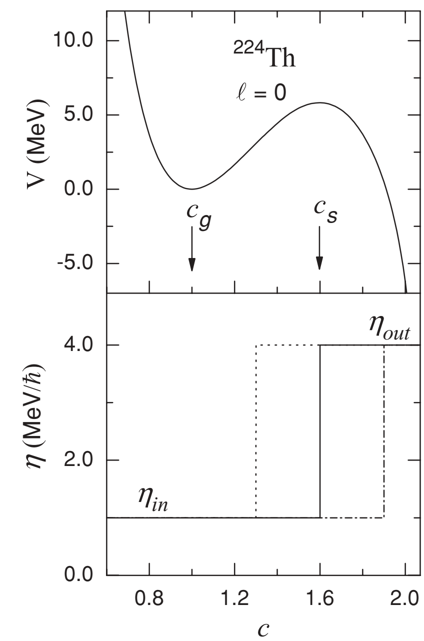

FIG. 1. Collective potential (top) and shape-dependent dissipation (bottom) for ^{224}Th. Different forms of the shape-dependent dissipation in the lower graph are represented by the solid line ($c_\eta = 1.6$), dotted line ($c_\eta = 1.3$), and dash-dotted line ($c_\eta = 1.9$).

The preceding observations are made when we choose $\eta_{\rm out} \gg \eta_{\rm in}$ in accordance with the applications of shape-dependent dissipation in statistical model calculations [5–7,30]. However, when the value of $\eta_{\rm out}$ is reduced toward $\eta_{\rm in}$, Fig. 4 shows that the stationary fission width from Langevin dynamics gets closer to Kramers' width for $\eta_{\rm in}$, as expected.

To understand the foregoing observations qualitatively, we proceed as follows. Kramers' width (Γ_K) [Eq. (1)] represents the steady-state diffusion rate of phase-space density ρ of Brownian particles across the fission barrier satisfying the appropriate Liouville equation, and the net flux or current

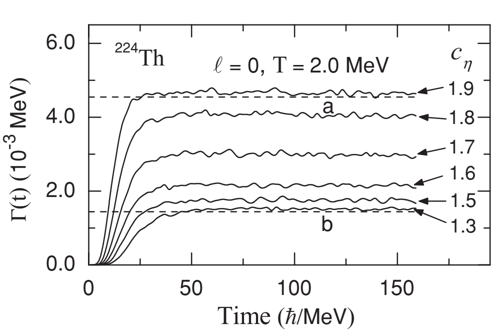

FIG. 2. Time-dependent fission rates from Langevin equations for different values of c_η. Values of Kramers' fission width (dashed lines) $\Gamma_K^{\rm in}$ and $\Gamma_K^{\rm out}$ are also labeled a and b, respectively.

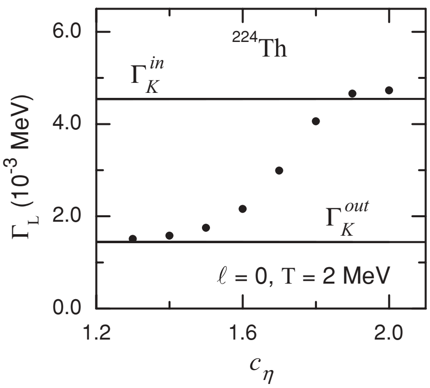

FIG. 3. Stationary values (Γ_L) of fission rate from Langevin equations as a function of c_η (filled circles). Kramers' widths $\Gamma_K^{\rm in}$ and $\Gamma_K^{\rm out}$ are shown by horizontal lines.

across the saddle is

$$j = \int_{-\infty}^{+\infty} \rho(c = c_s, p)\frac{p}{m_s}dp, \tag{3}$$

where both the outward (positive-p) and the inward (negative-p) fluxes are considered to obtain the net flux [11]. In terms of Langevin fission trajectories, while the outward flux is controlled by the dissipation within the saddle, the inward flux (from outside to inside the saddle) or the backstreaming trajectories experience the dissipation outside the saddle. Hence the net flux in Eq. (3) depends on both the "presaddle" and the "postsaddle" dissipation strengths, and the fission width is no longer determined by the presaddle dynamics alone. The stochastic nature of nuclear fission makes it dependent on the fission dynamics around the saddle, the extent of which is illustrated in Fig. 3.

We now compare the prescission neutron multiplicities ($n_{\rm pre}$) and ER cross sections obtained from statistical model calculation of compound nuclear decay with those from Langevin dynamical model calculation. Evaporation of neutrons, protons, α particles, and GDR γ's are considered along with the fission channel in both the calculations. While the particle and the γ emission widths used in both approaches are obtained from the Weisskopf formula [25], the fission

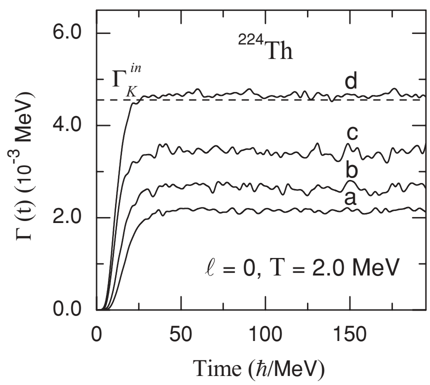

FIG. 4. Time-dependent Langevin fission widths, a–d, with $c_\eta = 1.6$, $\eta_{\rm in} = 1.0$ MeV/$\hbar$, and $\eta_{\rm out} = 4.0$, 3.0, 2.0, and 1.0 MeV/$\hbar$, respectively. $\Gamma_K^{\rm in}$ is shown by the horizontal dashed line.

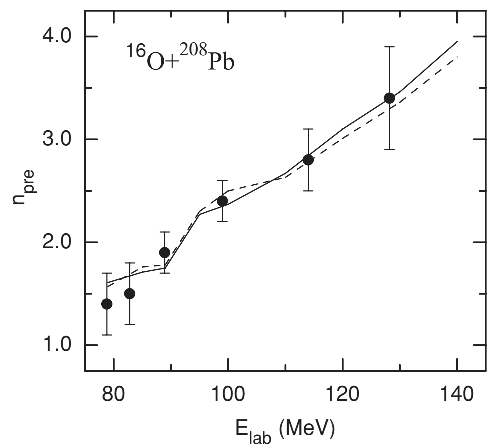

FIG. 5. Prescission neutron multiplicities from statistical (dashed line) and dynamical (solid line) model calculations with a shape-independent dissipation of value 3.5 MeV/$\hbar$. Experimental points are from Ref. [2].

width for the statistical model calculation is taken as $\Gamma_K^{\rm in}$. In the dynamical calculation, the particle and γ emissions are coupled with the Langevin equations for the fission degree of freedom [27]. We first consider the results obtained with a shape-independent strength of dissipation. Figure 5 shows the statistical and dynamical model predictions of $n_{\rm pre}$ excitation function calculated for the system $^{16}{\rm O} + {}^{208}{\rm Pb}$ along with the experimental data [2]. The dissipation strength is obtained here by fitting the data. A close agreement between the results from the two calculations is observed here, which reflects the validity of Kramers' width for shape-independent dissipation as demonstrated in Fig. 4.

We next perform statistical and Langevin dynamical model calculations where a shape-dependent dissipation is used. In the statistical model calculation, $\Gamma_K^{\rm in}$ is used as the fission width, while $\eta_{\rm out}$ is used to calculate the saddle-to-scission transition time. Additional neutrons are allowed to evaporate during this period [5]. The presaddle dissipation strength $\eta_{\rm in}$

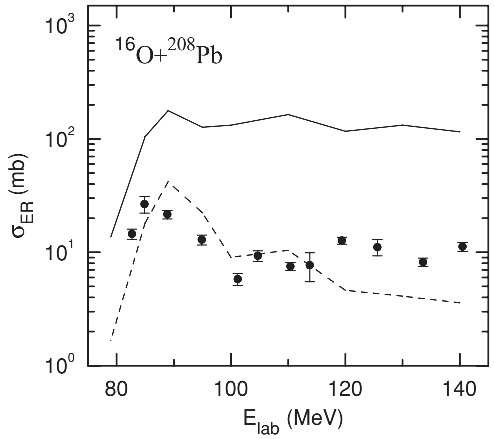

FIG. 6. Evaporation residue cross sections from statistical (dashed line) and dynamical (solid line) model calculations for a shape-dependent dissipation with $\eta_{\rm in} = 1.5$ MeV/$\hbar$ and $\eta_{\rm out} = 15$ MeV/$\hbar$. Experimental points are from Ref. [8].

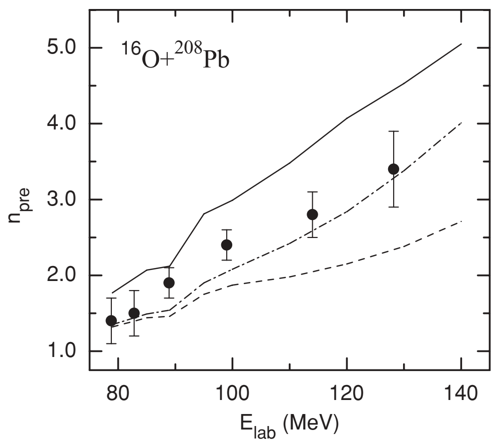

FIG. 7. Prescission neutron multiplicities from statistical and dynamical model calculations for a shape-dependent dissipation with $\eta_{\rm in} = 1.5\,{\rm MeV}/\hbar$ and $\eta_{\rm out} = 15\,{\rm MeV}/\hbar$. Dash-dotted and dashed lines represent statistical model calculation results with and without the saddle-to-scission neutrons, respectively. Langevin dynamical results are shown by the solid line. Experimental points are from Ref. [2].

and hence $\Gamma_K^{\rm in}$ are first obtained by fitting the experimental ER excitation function. The strength of $\eta_{\rm out}$ is subsequently adjusted to reproduce the experimental $n_{\rm pre}$ excitation function. Excitation functions for $n_{\rm pre}$ and ER are also obtained from the Langevin dynamical calculation using a shape-dependent dissipation given by the preceding values of $\eta_{\rm in}$ and $\eta_{\rm out}$. Figure 6 shows the calculated ER cross sections along with the experimental data. The dynamical model results are considerably larger than the statistical model predictions. This shows that the postsaddle dynamics controlled by $\eta_{\rm out}$ plays an important role in determining the fission probability of a compound nucleus, which in turn demonstrates the inadequacy of using only the $\eta_{\rm in}$ value in Eq. (1) to obtain the fission width. Figure 7 shows the calculated $n_{\rm pre}$ multiplicities and the experimental values. The statistical model results without including the additional saddle-to-scission neutrons are also given in this figure. The dynamical model predictions, however, turn out to be much higher than the statistical model results. Because the Langevin equations give the true description of dynamics of fission, the preceding differences between statistical and dynamical model results show that for shape-dependent dissipation, the assumptions of $\eta_{\rm in}$ accounting for the fission width and $\eta_{\rm out}$ controlling the saddle-to-scission neutrons are not consistent with the dynamical model results. Consequently, the fitted values of $\eta_{\rm in}$ and $\eta_{\rm out}$ from statistical model calculations when used in dynamical model calculations give rise to substantially different values of $n_{\rm pre}$ and ER cross sections.

We therefore conclude that due caution should be exercised when using Kramers' expression for fission width for systems with shape-dependent dissipation. In such cases, the Kramers' width obtained with a presaddle dissipation strength does not represent the true fission width, and consequently, the "presaddle dissipation strength" fitted to reproduce the experimental data in statistical model calculations does not represent the true strength of presaddle dissipation.

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