Axial charges of the nucleon and N^* resonances

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The axial charges of the nucleon and the well-established N^* resonances are studied within a consistent framework. For the first time the axial charges of the N^* resonances are produced for the relativistic constituent quark model. The axial charge of the nucleon is predicted close to experiment, and the ones of $N^*(1535)$ and $N^*(1650)$, the only cases where such a comparison is possible, agree well with results from quantum chromodynamics on the lattice that have recently become available. The relevance of the magnitudes of the N^* axial charges for the low-energy behavior of quantum chromodynamics is discussed.

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The axial charge g_A of the nucleon (N) is an essential quantity in understanding the electroweak and strong interactions within the standard model of elementary particles. In the first instance it is directly related to the neutron β decay, and its experimental value can be deduced from the ratio of the axial to the vector coupling constants $g_A/g_V = 1.2695 \pm 0.0029$ [1]; usually this is done under the assumption of conserved vector currents (CVC), which implies $g_V = 1$. The deviation of g_A from 1, the axial charge of a pointlike particle, can be attributed, according to the Adler-Weisberger sum rule [2,3], to the differences between the π^+N and π^-N cross sections in pion-nucleon scattering. Through the Goldberger-Treiman relation, $g_A = f_\pi g_{\pi NN}/M_N$, the axial charge is connected with the π decay constant f_π, the πNN coupling constant $g_{\pi NN}$, and the nucleon mass M_N [4]. Thus the axial charge of the N plays a key role for the spontaneous breaking of chiral symmetry (SBχS) of quantum chromodynamics (QCD) in low-energy hadron physics, a phenomenon that is manifested by the nonvanishing value of the light-flavor chiral condensate $\langle 0|q\bar{q}|0\rangle^{1/3} \approx -235$ MeV.

There have been a number of theoretical attempts to produce the axial charge of the N ground state with many different methods. We mention only the more novel approaches via the relativistic constituent quark model (RCQM) [5–7], by chiral perturbation theory [8], and within lattice QCD [9–15]. In general, the theoretical results come close to the experimental value of roughly 1.27, with the lattice-QCD predictions scattering over a range of approximately 1.10−1.40, depending on the various actions employed and a series of technical details entering the calculations by the different groups.

Recently, also the axial charges of the N^* resonances have come into the focus of interest, as it was suggested that their values should become small or even vanishing for excited states that could be parity partners in a scenario of chiral-symmetry restoration higher in the hadron spectra [16,17]. As the g_A values of N^* resonances can hardly be measured experimentally, this remains a highly theoretical question. However, the problem can be explored by *ab initio* calculations of QCD on the lattice. Corresponding first results have become available lately for just two of the N^* resonances, namely $N^*(1535)$ and $N^*(1650)$ [15]. Both of them have total angular momentum (intrinsic spin) $J = \frac{1}{2}$ and parity $P = -1$. Unfortunately, there is not yet any lattice-QCD result for positive-parity states, and the above issue relating to parity-doubling remains unresolved on this basis. In addition, a systematic study of g_A should give a clue to the magnitudes of meson dressing in N and N^* states.

It is thus most interesting to get insight into the N and N^* axial charges from different approaches. Especially by the RCQM we can investigate the problem comprehensively, as all the ground and resonant states are readily accessible. Here we report theoretical predictions of g_A for positive- as well as negative-parity N^* resonances up to $J = \frac{5}{2}$. The calculations are performed employing a RCQM with the quark-quark hyperfine interaction deduced from Goldstone-boson-exchange (GBE) dynamics. In particular, we use both existing versions of GBE RCQMs, the one with pseudoscalar (ps) spin-spin forces only [18] and the extended GBE (EGBE) RCQM with all the central, spin-spin, tensor, and spin-orbit force components included [19]. For the sake of comparison with another type of hyperfine interaction we employ also the RCQM with one-gluon-exchange (OGE) dynamics [20].

The calculations are performed in the framework of Poincarè-invariant quantum mechanics. In order to keep the numerical computations manageable, we have to restrict the axial current operator to the so-called spectator model (SM). It means that the weak-interaction gauge boson couples only to one of the constituent quarks in the baryon. This approximation has turned out to be very reasonable already in a previous study of the axial and induced pseudoscalar form factors of the nucleon [5], where the SM was employed specifically in the point form (PF) of relativistic quantum mechanics [21]. It has also been used in studies of the electromagnetic structure of the N, reproducing both the proton and neutron form factors in close agreement with the experimental data [6,22–24].

The axial charge is defined through the value of the axial form factor $G_A(Q^2)$ at $Q^2 = 0$, where $Q^2 = -q^2$ is the four-momentum transfer. The axial form factor $G_A(Q^2)$ can be derived from the relativistically invariant reduced matrix element of the axial current operator $\hat{A}^\mu_a(Q^2)$, with flavor index a, sandwiched between the eigenstates of N or N^*. We denote the latter generally by $|P, J, \Sigma\rangle$, i.e., as eigenstates of the four-momentum operator $\hat{P}^\mu$, the intrinsic-spin operator $\hat{J}$ and its z projection $\hat{\Sigma}$. Since $\hat{P}^\mu$ and the invariant mass operator $\hat{M}$ commute, these eigenstates can be obtained by solving the eigenvalue equation of $\hat{M}$

$$\hat{M}|P, J, \Sigma\rangle = M|P, J, \Sigma\rangle, \tag{1}$$

where M is the mass of N or N^*. For the various $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ states considered here, the axial charges g_A are thus computed from the matrix elements of the axial current operator $\hat{A}^\mu_a$ for zero-momentum transfer

$$\begin{aligned}
\left\langle P, \frac{1}{2}, \Sigma' \left| \hat{A}^\mu_a \right| P, \frac{1}{2}, \Sigma \right\rangle &= \bar{U}(P, \Sigma') g_A \gamma^\mu \gamma_5 \frac{\tau_a}{2} U(P, \Sigma), \\
\left\langle P, \frac{3}{2}, \Sigma' \left| \hat{A}^\mu_a \right| P, \frac{3}{2}, \Sigma \right\rangle &= \bar{U}^\nu(P, \Sigma') g_A \gamma^\mu \gamma_5 \frac{\tau_a}{2} U_\nu(P, \Sigma), \\
\left\langle P, \frac{5}{2}, \Sigma' \left| \hat{A}^\mu_a \right| P, \frac{5}{2}, \Sigma \right\rangle &= \bar{U}^{\nu\lambda}(P, \Sigma') g_A \gamma^\mu \gamma_5 \frac{\tau_a}{2} U_{\nu\lambda}(P, \Sigma).
\end{aligned} \tag{2}$$

Here $U(P, \Sigma)$ are the usual Dirac spinors for spin-$\frac{1}{2}$ and $U^\nu(P, \Sigma)$ and $U^{\nu\lambda}(P, \Sigma)$ are the Rarita-Schwinger spinors [25] for spin-$\frac{3}{2}$ and spin-$\frac{5}{2}$ particles, respectively, where we use the same notation and normalization as specified in the appendix of Ref. [26]. The γ^μ and γ_5 are the usual Dirac matrices and τ_a is the isospin matrix with Cartesian index a.

The matrix elements of $\hat{A}^\mu_a$ for any N or N^* read

$$\begin{aligned}
&\langle P, J, \Sigma' | \hat{A}^\mu_a (Q^2 = 0) | P, J, \Sigma \rangle \\
&= 2M \sum_{\sigma_i \sigma_i'} \int d^3\vec{k}_1 d^3\vec{k}_2 d^3\vec{k}_3 \frac{\delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)}{2\omega_1 2\omega_2 2\omega_3} \\
&\quad \times \Psi^\star_{PJ\Sigma'}(\vec{k}_1, \vec{k}_2, \vec{k}_3; \sigma_1', \sigma_2', \sigma_3') \\
&\quad \times \langle k_1, k_2, k_3; \sigma_1', \sigma_2', \sigma_3' | \hat{A}^\mu_a | k_1, k_2, k_3; \sigma_1, \sigma_2, \sigma_3 \rangle \\
&\quad \times \Psi_{PJ\Sigma}(\vec{k}_1, \vec{k}_2, \vec{k}_3; \sigma_1, \sigma_2, \sigma_3).
\end{aligned} \tag{3}$$

The Ψ's are the rest-frame wave functions of the N or N^* with corresponding mass M and total angular momentum J with z projections Σ and Σ'. Here they are represented as functions of the individual quark three-momenta $\vec{k}_i$, which sum up to $\vec{P} = \vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$; $\omega_i = \sqrt{m_i^2 + \vec{k}_i^2}$ is the energy of quark i with mass m_i, and the individual-quark spin orientations are denoted by σ_i.

The SM means that the matrix element of the axial current operator $\hat{A}^\mu_a$ between (free) three-particle states $|k_1, k_2, k_3; \sigma_1, \sigma_2, \sigma_3\rangle$ is assumed in the form

$$\begin{aligned}
&\langle k_1, k_2, k_3; \sigma_1', \sigma_2', \sigma_3' | \hat{A}^\mu_a | k_1, k_2, k_3; \sigma_1, \sigma_2, \sigma_3 \rangle \\
&= 3 \langle k_1, \sigma_1' | \hat{A}^\mu_{a,\mathrm{SM}} | k_1, \sigma_1 \rangle 2\omega_2 2\omega_3 \delta_{\sigma_2 \sigma_2'} \delta_{\sigma_3 \sigma_3'}.
\end{aligned} \tag{4}$$

For pointlike quarks this matrix element involves the axial current operator of the active quark 1 (with quarks 2 and 3 being the spectators) in the form

$$\langle k_1, \sigma_1' | \hat{A}^\mu_{a,\mathrm{SM}} | k_1, \sigma_1 \rangle = \bar{u}(k_1, \sigma_1') g_A^q \gamma^\mu \gamma_5 \frac{\tau_a}{2} u(k_1, \sigma_1), \tag{5}$$

where $u(k_1, \sigma_1)$ is the quark spinor and $g_A^q = 1$ the quark axial charge. A pseudovector-type current analogous to the one in Eq. (5) was recently also used in the calculation of $g_{\pi NN}$ and the strong πNN vertex form factor in Ref. [27].

If we are interested only in the axial charges g_A, the expression (5) specifies to $\mu = i = 1, 2, 3$ and can further be evaluated to give

$$\begin{aligned}
&\bar{u}(k_1, \sigma_1') \gamma^i \gamma_5 \frac{\tau_a}{2} u(k_1, \sigma_1) \\
&= 2\omega_1 \chi^*_{\frac{1}{2}, \sigma_1'} \left\{ \left[1 - \frac{2}{3}(1 - \kappa) \right] \sigma^i \right. \\
&\quad \left. + \sqrt{\frac{5}{3}} \frac{\kappa^2}{1 + \kappa} [[\vec{v}_1 \otimes \vec{v}_1]_2 \otimes \vec{\sigma}]^i_1 \right\} \frac{\tau_a}{2} \chi_{\frac{1}{2}, \sigma_1},
\end{aligned} \tag{6}$$

where $\kappa = 1/\sqrt{1 + v_1^2}$ and $\vec{v}_1 = \vec{k}_1 / m_1$. Herein σ^i is the i-th component of the usual Pauli matrix $\vec{\sigma}$ and v_1 the magnitude of the three-velocity $\vec{v}_1$. The symbol $[.\otimes .]^i_k$ denotes the i-th component of a tensor product $[.\otimes .]_k$ of rank k. We note that a similar formula was already published before by Dannbom *et al.* [28]; however, it was restricted to the case of total orbital angular momentum $L = 0$. Our expression holds for any L, thus allowing to calculate g_A for the most general wave function of a baryon specified by J^P.

TABLE I. Predictions for axial charges g_A of the EGBE RCQM in comparison to available lattice QCD results [9–15], the values calculated by Glozman and Nefediev (GN) within the SU(6) × O(3) nonrelativistic quark model [17], and the nonrelativistic (NR) limit from the EGBE RCQM.

State	J^P	EGBE	Lattice QCD	GN	NR
$N(939)$	$\frac{1}{2}^+$	1.15	$1.10 \sim 1.40$	1.66	1.65
$N^*(1440)$	$\frac{1}{2}^+$	1.16	–	1.66	1.61
$N^*(1535)$	$\frac{1}{2}^-$	0.02	$\sim$0.00	−0.11	−0.20
$N^*(1710)$	$\frac{1}{2}^+$	0.35	–	0.33	0.42
$N^*(1650)$	$\frac{1}{2}^-$	0.51	$\sim$0.55	0.55	0.64

In Table I we give the predictions of the EGBE RCQM for the N ground state and the first two N^* excitations of $J = \frac{1}{2}$ with positive as well as negative-parity P. It is seen that the result for g_A of the N comes close to the experimental value but falling slightly below it; the same result is obtained for the psGBE RCQM [5]. This might be linked, via the Goldberger-Treiman relation, to a similar behavior of the πNN coupling constant predicted by the GBE RCQM, where a value of $\frac{f^2_{\pi NN}}{4\pi} = 0.0691$ was found [27], which is to be compared to the phenomenological value of about 0.075 [29]. Even if these theoretical underestimates of both the g_A and the $f_{\pi NN}$ remain smaller than 10%, they could be interpreted as missing π-dressing effects in the RCQM. However, in this context one should also bear in mind that the phenomenological value of g_A ~1.27 is supposed under the conjecture of CVC, and there is a Goldberger-Treiman discrepancy of about ~10%. For RCQMs careful studies employing explicit meson-dressing mechanisms are still necessary to resolve these issues.

The EGBE RCQM prediction for g_A of the N is also quite congruent with available lattice-QCD results. The latter can be collected from various works that differ with regard to the actions used, the methods for (chiral) extrapolations employed and other input sources. A recent compilation of the lattice-QCD world data for g_A of the nucleon [30] exhibits a tendency

TABLE II. Mass eigenvalues and axial charges g_A of the N ground state and the positive-parity N^* resonances as predicted by the EGBE, the psGBE, and the OGE RCQMs.

State	J^P	EGBE		psGBE		OGE	
		Mass	g_A	Mass	g_A	Mass	g_A
$N(939)$	$\frac{1}{2}^+$	939	1.15	939	1.15	939	1.11
$N^*(1440)$	$\frac{1}{2}^+$	1464	1.16	1459	1.13	1578	1.10
$N^*(1710)$	$\frac{1}{2}^+$	1757	0.35	1776	0.37	1860	0.32
$N^*(1720)$	$\frac{3}{2}^+$	1746	0.35	1728	0.34	1858	0.25
$N^*(1680)$	$\frac{5}{2}^+$	1689	0.89	1728	0.83	1858	0.70

of undershooting the experimental value, where a possible reason is suspected in finite-volume effects. In any case, statistical as well as systematic errors are still of considerable sizes, and comparisons must be done with caution.

The congruency of the EGBE RCQM predictions with lattice-QCD results obviously also holds for the two other cases, where such data are already available, namely the $J^P = \frac{1}{2}^-$ resonances $N^*(1535)$ and $N^*(1650)$. Here, it should be emphasized that, due to the dynamics in its hyperfine interaction, the EGBE RCQM incorporates all possible couplings of orbital angular momentum and spin components in the three-quark states. It is certainly remarkable that a practically vanishing g_A of $N^*(1535)$ can obviously be achieved in a corresponding relativistic calculation without advocating a cancellation of $\{QQQ\}$ and $\{QQQQ\bar{Q}\}$ components as suggested in Ref. [31]. Similarly, a very small axial charge of $N^*(1535)$ was found in a chiral unitary approach in Ref. [32]. The simple SU(6) $\times$ O(3) nonrelativistic quark model used by Glozman and Nefediev cannot reproduce the g_A of the N and it yields exactly the same (too) big results for N and $N^*(1440)$. The corresponding axial charge of $N^*(1535)$ is nonzero but negative, only the results for $N^*(1710)$ and $N^*(1650)$ are similar to the ones of the EGBE RCQM. In the last column of Table I we also quote the results obtained in the nonrelativistic limit of the axial current operator of Eq. (5). By comparing with the figures in the first column, one can see that relativistic effects related to the current operator are considerable in all instances.

Next we are going to present and compare the relativistic predictions of g_A for the N ground state and all positive- as well as negative-parity N^* excitations with masses below ~1.9 GeV for three distinct types of RCQMs. The specific RCQMs considered here differ mainly with regard to their hyperfine interactions. The psGBE RCQM contains only the spin-spin part of the pseudoscalar meson exchange and carries an explicit flavor dependence. In the construction of this model the tensor and spin-orbit forces were omitted because they can only play a minor role in baryon spectroscopy, as is exhibited by small level splittings in the phenomenological data [33]. The same assumption was made also for the OGE RCQM; however, its spin-spin part has no flavor dependence, since it is derived from OGE [20]. The EGBE RCQM contains all force components generated by pseudoscalar meson exchange, including in particular the corresponding tensor force. The latter, however, is so strong that it destroys the remarkably small splittings of alike spin states in the baryon spectra. Its effect is compensated by the inclusion of vector-meson exchange, whose tensor force comes with opposite sign. The vector and the additional scalar meson exchanges, also included in the EGBE RCQM, can be interpreted as describing the effects of multiple Goldstone-boson exchanges [19]. It should be noted that for the baryon spectra the net effect of hyperfine forces other than the spin-spin ones is presumably small. This might not be the case with regard to other (dynamical) observables, as the full couplings of orbital angular momentum and spin components come into play when calculating transition matrix elements. Corresponding effects are naturally included by the EGBE RCQM, contrary to the other dynamical models considered here and also in previous works, such as Ref. [17].

Tables II and III show the relativistic results of g_A as obtained with the three different RCQMs. The EGBE and the psGBE RCQMs produce identical figures for g_A of the N ground state. In addition, the axial charges are very

TABLE III. Same as Table II but for the negative-parity N^* resonances.

State	J^P	EGBE		psGBE		OGE	
		Mass	g_A	Mass	g_A	Mass	g_A
$N^*(1535)$	$\frac{1}{2}^-$	1498	0.02	1519	0.09	1520	0.13
$N^*(1650)$	$\frac{1}{2}^-$	1581	0.51	1647	0.46	1690	0.44
$N^*(1520)$	$\frac{3}{2}^-$	1524	−0.64	1519	−0.21	1520	−0.15
$N^*(1700)$	$\frac{3}{2}^-$	1608	−0.10	1647	−0.50	1690	−0.47
$N^*(1675)$	$\frac{5}{2}^-$	1676	0.84	1647	0.83	1690	0.80

similar for most of the excited states, except for the $J^P = \frac{3}{2}^-$ states $N^*(1520)$ and $N^*(1700)$. Here, the remarkably large differences of the EGBE RCQM predictions relative to both the psGBE as well as OGE RCQMs have evidently to be attributed to tensor and/or spin-orbit forces prevailing in the former. In this context it is quite interesting to observe that at least the $N^*(1520)$, which has no parity partner, was also found exceptional regarding the large size of the π-coupling constant $f_{\pi N^* N}$ of the $N^*(1520) \rightarrow N\pi$ decay, and large effects from SBχS have been suspected to be responsible for this behavior [34].

Except for these two cases just discussed there are also no big differences between the results of both the EGBE and psGBE with the ones of the OGE RCQM, even though the theoretical resonance masses show sometimes considerable differences [20,35]. It will be very interesting to confront these predictions by the RCQMs with results by other approaches and in particular with further data from lattice QCD.

It is particularly satisfying to find the RCQM predictions for the axial charges not only of the nucleon N but also of the $N^*(1535)$ and $N^*(1650)$ resonances in agreement with the lattice-QCD results. We may thus be confident that at least in the limit of zero-momentum-transfer processes the mass eigenstates of the N and the above N^* excitations, especially as produced with the EGBE RCQM, are quite reasonable. It should be recalled that in this particular model the mutual interaction between constituent quarks is furnished by a linear confinement, whose strength is consistent with the string tension of QCD as well as the slopes of Regge trajectories [18], and by (all components of) a hyperfine interaction derived from GBE dynamics. The latter is supposed to model the SBχS property of low-energy QCD. This type of hyperfine interaction, which also introduces an explicit flavor dependence, has been remarkably successful in describing a number of phenomena in low-energy baryon physics. Most prominently, it produces the correct level orderings of the positive- and negative-parity N^* resonances and simultaneously the ones in the other hyperon spectra, notably the Λ spectrum [36]. The RCQM with GBE dynamics does not have any mechanism for chiral-symmetry restoration built in. As such it cannot be expected to produce parity doublets due to this reason. Nevertheless the GBE RCQM describes the pattern of N^* resonance masses in good agreement with the experimental data (mostly within the experimental error bars or at most exceeding them by 4%). This is due to the refined interplay of different force components in the effective interaction between constituent quarks. In view of these findings it will be most interesting if the present results for N^* axial charges derived within the RCQM will in the future be confirmed by lattice-QCD calculations.

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