Neutrino emission from triplet pairing of neutrons in neutron stars

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Neutrino emission resulting from the pair breaking and formation processes in the bulk triplet superfluid in neutron stars is investigated taking into account anomalous weak interactions. I consider the problem in the BCS approximation discarding Fermi-liquid effects. By this approach I derive self-consistent equations for anomalous vector and axial-vector vertices of weak interactions taking into account $^3P_2$-$^3F_2$ mixing. Further, I simplify the problem and consider pure $^3P_2$ pairing with $m_j = 0$, as is adopted in the minimal-cooling paradigm. As was expected because of current conservation, I have obtained a large suppression of neutrino emissivity in the vector channel. More exactly, the neutrino emission through the vector channel vanishes in the nonrelativistic limit $V_F = 0$. The axial channel is also found to be moderately suppressed. Total neutrino emissivity is suppressed by a factor of 1.9×10^{-1} relative to original estimates using bare weak vertices.

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I. INTRODUCTION

Thermal excitations in superfluid baryon matter of neutron stars, in the form of broken Cooper pairs, can recombine into the condensate by emitting neutrino pairs via neutral weak currents [1]. It is generally accepted that, for temperatures near the associated superfluid critical temperatures, emission from pair breaking and formation (PBF) processes dominates neutrino emissivities in many cases. Recently [2], it has been found, however, that the existing theory of PBF processes based on bare weak vertices violates conservation of vector weak current. Correct evaluations including anomalous interactions have shown the neutrino emission by a nonrelativistic singlet superfluid is substantially suppressed. Consistent estimates of the inhibition factor can be found in Refs. [2–4]. The suppression of neutrino emissivity from the $^1S_0$ PBF processes was studied also in Refs. [5–8], although these studies are controversial (see discussion in Refs. [3,4]).

Quenching of the neutrino emission found in the case of $^1S_0$ pairing leads to higher temperatures that can be reached in the crust of an accreting neutron star. This could explain the observed data of superburst triggering [9,10], which was in dramatic contrast with the previous theory of crust cooling. Numerical simulations of neutron star cooling in the minimal scenario [11] have shown that the suppression of the PBF processes in the crust of a neutron star has a significant effect at early times ($t < 1000$ years) and results in warmer crusts and increased crust relaxation times.

I now turn to PBF neutrino emission from bulk superfluid neutron matter, which is caused mostly by triplet neutron pairing. Neutrino energy losses due to triplet PBF processes have been initially derived in Ref. [12], ignoring anomalous weak interactions. From analogy with the singlet case, it is clear that conservation of the vector weak current is violated in this approach and thus neutrino emission in the vector channel, as obtained in Ref. [12], is a subject of inconsistency [13]. Moreover, in the triplet superfluid, the order parameter is sensitive also to the axial weak field. Therefore, the self-consistent axial response of the triplet superfluid must incorporate the anomalous contributions in the same degree of approximation as the vector response. This effect was not investigated until now.

In present article, I perform the corresponding self-consistent calculation. Formally, my approach is a development of the Larkin-Migdal-Leggett theory [14,15] to the triplet case. However, I discard residual particle-hole interactions because the Landau parameters are unknown for dense asymmetric baryon matter. Another reason is that the influence of the particle-hole interactions is not very significant in the PBF processes [4].

The article is organized as follows. Section II contains some preliminary notes. I discuss the order parameter and the quasiparticle propagators for the triplet pair-correlated system with strong interactions. I also recast the standard gap equation to the form convenient for considering the processes occurring in the vicinity of the Fermi surface. In Sec. III, I formulate a set of BCS equations for the calculation of the anomalous vertices and correlation functions of the triplet superfluid Fermi liquid at finite temperature involving a mixing of the $^3P_2$ and $^3F_2$ channels [16,17]. In Sec. IV, I present the general expression for the emissivity of the neutron PBF processes formulated in terms of the imaginary part of the current-current correlator. A widely used expression for the neutrino emissivity caused by the triplet pairing of neutrons was obtained in Ref. [12] with the aid of the Fermi "golden" rule. Therefore, before proceeding to the self-consistent calculation of the neutrino energy losses in Sec. V, I reproduce this formula using the calculation technique developed in present article so that an apposite comparison with Ref. [12] can be made. In Sec. VI, I consider the anomalous vertices and the self-consistent superfluid response in both the vector and the axial channels. Here I focus on the $^3P_2$ pairing with $m_j = 0$, as is adopted in the minimal-cooling paradigm [11]. Finally, in Sec. VI B, I evaluate the self-consistent neutrino energy losses from the PBF processes in the triplet neutron superfluid. Section VII contains a short summary of my findings and the conclusion.

In this work I use the standard model of weak interactions, the system of units $\hbar = c = 1$, and the Boltzmann constant $k_B = 1$.

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II. PRELIMINARY NOTES AND NOTATION

A. The order parameter and Green's functions

The order parameter, $\hat{D} \equiv D_{\alpha\beta}$, arising because of triplet pairing of quasiparticles, represents a 2×2 symmetric matrix in spin space, $(\alpha, \beta = \uparrow, \downarrow)$. The spin-orbit interaction among quasiparticles is known to dominate in nucleon matter of high density. Therefore, it is conventional to represent the triplet order parameter of the system $\hat{D} = \sum_{lm_j} \Delta_{lm_j} \Phi_{\alpha\beta}^{(jlm_j)}$ as a superposition of standard spin-angle functions of the total angular momentum (j, m_j),

$$\Phi_{\alpha\beta}^{(jlm_j)}(\mathbf{n}) \equiv \sum_{m_s+m_l=m_j} \left(\frac{1}{2}\frac{1}{2}\alpha\beta|sm_s\right)(slm_s m_l|jm_j)Y_{l,m_l}(\mathbf{n}). \tag{1}$$

For my calculations it will be more convenient to use vector notation that involves a set of mutually orthogonal complex vectors $\mathbf{b}_{lm_j}(\mathbf{n})$ defined as

$$\mathbf{b}_{lm_j}(\mathbf{n}) = -\tfrac{1}{2}\,\mathrm{Tr}\left(\hat{g}\hat{\boldsymbol{\sigma}}\hat{\Phi}_{jlm_j}\right), \tag{2}$$

where $\hat{\boldsymbol{\sigma}} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ are Pauli spin matrices and $\hat{g} = i\hat{\sigma}_2$. I will use the normalization condition

$$\int \frac{d\mathbf{n}}{4\pi} \mathbf{b}^*_{l'm'_j} \mathbf{b}_{lm_j} = \delta_{ll'}\delta_{m_j m'_j}. \tag{3}$$

If the most attractive channel of interactions is assumed in the states with $s = 1$, $j = 2$, and $l = j \pm 1$ (in the case of tensor forces), the order parameter can be written in the form

$$\hat{D}(\mathbf{n}) = \sum_{lm_j} \Delta_{lm_j}(\hat{\boldsymbol{\sigma}}\mathbf{b}_{lm_j})\hat{g}. \tag{4}$$

We are mostly interested in the values of quasiparticle momenta $\mathbf{p}$ near the Fermi surface $p \simeq p_F$, where the partial gap amplitudes, $\Delta_{lm_j}(p) \simeq \Delta_{lm_j}(p_F)$, are almost constants and the angular dependence of the order parameter is represented by the unit vector $\mathbf{n} = \mathbf{p}/p$, which defines the polar angles (θ, φ) on the Fermi surface.

The ground state (4) occurring in neutron matter has a relatively simple structure (unitary triplet) [16,17],

$$\sum_{lm_j} \Delta_{lm_j}\mathbf{b}_{lm_j}(\mathbf{n}) = \Delta\bar{\mathbf{b}}(\mathbf{n}), \tag{5}$$

where Δ is a complex constant (on the Fermi surface) and $\bar{\mathbf{b}}(\mathbf{n})$ is a real vector that we normalize by the condition

$$\int \frac{d\mathbf{n}}{4\pi}\bar{b}^2(\mathbf{n}) = 1. \tag{6}$$

Thus, the triplet order parameter can be written as

$$\hat{D}(\mathbf{n}) = \Delta\bar{\mathbf{b}}\hat{\boldsymbol{\sigma}}\hat{g}. \tag{7}$$

I will use the adopted graphical notation for the ordinary and anomalous propagators, as shown in Fig. 1.

The analytic form of the propagators can be found in the standard way [18,19], using the general form (7) of the gap matrix. Because the matter is assumed to be in thermal equilibrium at some temperature, I employ the Matsubara

FIG. 1. Diagrams depicting the ordinary and anomalous propagators of a quasiparticle.

calculation technique. Then

$$\begin{aligned}
\hat{G}\,(p_m, \mathbf{p}) &= aG(p_m, \mathbf{p})\delta_{\alpha\beta},\\
\hat{G}^-\,(p_m, \mathbf{p}) &= aG^-(p_m, \mathbf{p})\delta_{\alpha\beta},\\
\hat{F}^{(1)}\,(p_m, \mathbf{p}) &= aF(p_m, \mathbf{p})\bar{\mathbf{b}}\hat{\boldsymbol{\sigma}}\,\hat{g},\\
\hat{F}^{(2)}\,(p_m, \mathbf{p}) &= aF(p_m, \mathbf{p})\hat{g}\hat{\boldsymbol{\sigma}}\,\bar{\mathbf{b}},
\end{aligned} \tag{8}$$

where $a \simeq 1$ is the usual Green's-function renormalization constant, $p_m \equiv i\pi(2m+1)T$ with $m = 0, \pm1, \pm2, \ldots$ is the Matsubara's fermion frequency, and the scalar Green's functions are of the form

$$\begin{aligned}
G(p_m, \mathbf{p}) &= \frac{-ip_m - \varepsilon_\mathbf{p}}{p_m^2 + E_\mathbf{p}^2}, \quad G^-(p_m, \mathbf{p}) = \frac{ip_m - \varepsilon_\mathbf{p}}{p_m^2 + E_\mathbf{p}^2},\\
F(p_m, \mathbf{p}) &= \frac{-\Delta}{p_m^2 + E_\mathbf{p}^2}.
\end{aligned} \tag{9}$$

Here

$$\varepsilon_\mathbf{p} = \frac{p^2}{2M^*} - \frac{p_F^2}{2M^*} \simeq \frac{p_F}{M^*}(p - p_F), \tag{10}$$

with $M^* = p_F/V_F$ being the effective mass of a quasiparticle. The quasiparticle energy is given by

$$E_\mathbf{p} \equiv \sqrt{\varepsilon_\mathbf{p}^2 + \frac{1}{2}\mathrm{Tr}\hat{D}(\mathbf{n})\hat{D}^\dagger(\mathbf{n})} = \sqrt{\varepsilon_\mathbf{p}^2 + \Delta^2\bar{b}^2}, \tag{11}$$

where the (temperature-dependent) energy gap, $\Delta\bar{b}(\mathbf{n})$, is anisotropic. Here it is assumed that, in the absence of external fields, the gap amplitude Δ is real.

Green functions of a quasiparticle (8) involve the renormalization factor $a \simeq 1$ independent of ω, $\mathbf{q}$, and T (see, e.g., Ref. [19]). The final outcomes are independent of this factor; therefore, to shorten the equations, I will drop the renormalization factor by assuming that all the necessary physical values are properly renormalized.

The following notation will be used. I designate as $L_{X,X}(\omega, \mathbf{q}; \mathbf{p})$ the analytical continuation onto the upper-half plane of complex variable ω of the following Matsubara sums:

$$\begin{aligned}
L_{XX'}&\left(\omega_n, \mathbf{p}+\frac{\mathbf{q}}{2}; \mathbf{p}-\frac{\mathbf{q}}{2}\right)\\
&= T\sum_m X\left(p_m + \omega_n, \mathbf{p}+\frac{\mathbf{q}}{2}\right)X'\left(p_m, \mathbf{p}-\frac{\mathbf{q}}{2}\right),
\end{aligned} \tag{12}$$

where $X, X' \in G, F, G^-$ and $\omega_n = 2i\pi Tn$, with $n = 0, \pm1, \pm2, \ldots$.

It is convenient to divide the integration over the momentum space into an integration over the solid angle and an integration over the energy according to

$$\int \frac{d^3p}{(2\pi)^3}\ldots = \rho\int\frac{d\mathbf{n}}{4\pi}\frac{1}{2}\int_{-\infty}^{\infty} d\varepsilon_\mathbf{p}\ldots \tag{13}$$

and operate with integrals over the quasiparticle energy:

$$\mathcal{I}_{XX'}(\omega, \mathbf{n}, \mathbf{q}; T) \equiv \frac{1}{2}\int_{-\infty}^{\infty} d\varepsilon_\mathbf{p} L_{XX'}\left(\omega, \mathbf{p}+\frac{\mathbf{q}}{2}, \mathbf{p}-\frac{\mathbf{q}}{2}\right). \tag{14}$$

These are functions of ω, $\mathbf{q}$ and the direction of the quasiparticle momentum $\mathbf{p} = p\mathbf{n}$. Here and subsequently, $\rho = p_F M^*/\pi^2$ is the density of states near the Fermi surface.

The loop integrals (14) possess the following properties, which can be verified by a straightforward calculation:

$$\mathcal{I}_{G^-G} = \mathcal{I}_{GG^-}, \quad \mathcal{I}_{GF} = -\mathcal{I}_{FG}, \quad \mathcal{I}_{G^-F} = -\mathcal{I}_{FG^-}, \tag{15}$$

$$\mathcal{I}_{G^-F} + \mathcal{I}_{FG} = \frac{\omega}{\Delta}\mathcal{I}_{FF}, \tag{16}$$

$$\mathcal{I}_{G^-F} - \mathcal{I}_{FG} = -\frac{\mathbf{qv}}{\Delta}\mathcal{I}_{FF}. \tag{17}$$

For arbitrary ω, $\mathbf{q}$, and T, one can obtain also

$$\mathcal{I}_{GG^-} + \bar{b}^2\mathcal{I}_{FF} = A + \frac{\omega^2 - (\mathbf{qv})^2}{2\Delta^2}\mathcal{I}_{FF}, \tag{18}$$

where $\mathbf{v}$ is a vector with the magnitude of the Fermi velocity V_F and the direction of $\mathbf{n}$, and

$$A(\mathbf{n}) \equiv [\mathcal{I}_{G^-G}(\mathbf{n}) + \bar{b}^2(\mathbf{n})\mathcal{I}_{FF}(\mathbf{n})]_{\omega=0,\mathbf{q}=0}. \tag{19}$$

In the case of triplet superfluid, the key role in the response theory belongs to the loop integrals $\mathcal{I}_{FF}$ and $(\mathcal{I}_{GG} \pm \bar{b}^2\mathcal{I}_{FF})$. For further usage, I indicate the properties of these functions in the case of $\omega > 0$ and $\mathbf{q} \to 0$. A straightforward calculation yields

$$\mathcal{I}_{FF}(\omega, q = 0) = -2\Delta^2 \int_0^\infty \frac{d\varepsilon}{E}\frac{1}{(\omega + i0)^2 - 4E^2}\tanh\frac{E}{2T} \tag{20}$$

and

$$(\mathcal{I}_{GG} + \bar{b}^2\mathcal{I}_{FF})_{q\to 0} = 0, \tag{21}$$

$$(\mathcal{I}_{GG} - \bar{b}^2\mathcal{I}_{FF})_{q\to 0} = -2\bar{b}^2\mathcal{I}_{FF}(\omega, \mathbf{0}). \tag{22}$$

The imaginary part of $\mathcal{I}_{FF}$ arises from the poles of the integrand in Eq. (20) at $\omega = \pm 2E$:

$$\mathrm{Im}\mathcal{I}_{FF}(\omega > 0, q = 0) = \Theta(\omega^2 - 4\bar{b}^2\Delta^2)\frac{\pi\Delta^2}{\omega\sqrt{\omega^2 - 4\bar{b}^2\Delta^2}}\tanh\frac{\omega}{4T}, \tag{23}$$

where $\Theta(x)$ is Heaviside step function.

B. Gap equation

The block of the interaction diagrams irreducible in the channel of two quasiparticles, $\Gamma_{\alpha\beta,\gamma\delta}$, is usually generated by the expansion over spin-angle functions (1). Using the vector notation, the most attractive channel of pairing interactions with $j = 2$ can be written as

$$\rho\Gamma_{\alpha\beta,\gamma\delta}(\mathbf{p},\mathbf{p}') = -\sum_{l'lm_j} V_{ll'}(p,p')\big[\mathbf{b}_{lm_j}(\mathbf{n})\hat{\boldsymbol{\sigma}}\hat{g}\big]_{\alpha\beta}\big[\hat{g}\hat{\boldsymbol{\sigma}}\mathbf{b}^*_{l'm_j}(\mathbf{n}')\big]_{\gamma\delta}, \tag{24}$$

where $V_{ll'}(p, p')$ are the corresponding interaction amplitudes and $|l - l'| \leqslant 2$ in the case of tensor forces.

In vector notation, the set of equations for the triplet partial amplitudes Δ_{lm_j} is of the form

$$\Delta_{lm_j}(p) = -\sum_{l'}\frac{1}{2\rho}\int dp' p'^2 V_{ll'}(p,p')\Delta(p') \times \int\frac{d\mathbf{n}'}{4\pi}\mathbf{b}^*_{l'm_j}(\mathbf{n}')\bar{\mathbf{b}}(\mathbf{n}')T\sum_m\frac{1}{p_m^2 + E^2_{\mathbf{p}'}}, \tag{25}$$

where

$$\bar{\mathbf{b}}(\mathbf{n}) = \frac{1}{\Delta}\sum_{lm_j}\Delta_{lm_j}\mathbf{b}_{lm_j}(\mathbf{n}), \tag{26}$$

as defined in Eq. (5). These equations can be reduced to the standard form [17] with the aid of the identity

$$T\sum_m\frac{1}{p_m^2 + E^2_{\mathbf{p}'}} \equiv \frac{1}{2E(\mathbf{p}')}\tanh\frac{E(\mathbf{p}')}{2T} \tag{27}$$

and the relation

$$\tfrac{1}{2}\mathrm{Tr}\left(\hat{\Phi}_{jlm_j}\hat{\Phi}^*_{jl'm_j}\right) = \mathbf{b}_{lm_j}(\mathbf{n})\cdot\mathbf{b}^*_{l'm_j}(\mathbf{n}). \tag{28}$$

We are interested in the processes occurring in the vicinity of the Fermi surface. Therefore, one can recast the gap equation to a more convenient form. We notice that

$$\frac{1}{p_m^2 + E^2_{\mathbf{p}}} \equiv G(p_m,\mathbf{p})G^-(p_m,\mathbf{p}) + \bar{b}^2F(p_m,\mathbf{p})F(p_m,\mathbf{p}); \tag{29}$$

that is, Eq. (25) can be written as

$$\Delta_{lm_j}(p) = -\frac{1}{2\rho}\sum_{l'}\int dp' p'^2 V_{ll'}(p,p')\Delta(p') \times \int\frac{d\mathbf{n}'}{4\pi}\mathbf{b}^*_{l'm_j}(\mathbf{n}')\bar{\mathbf{b}}(\mathbf{n}')T\sum_m[G(p_m,\mathbf{p}') \times G^-(p_m,\mathbf{p}') + \bar{b}^2F(p_m,\mathbf{p}')F(p_m,\mathbf{p}')]. \tag{30}$$

To eliminate the integration over the regions far from the Fermi surface, one can renormalize the interaction as suggested in Ref. [15]:

$$V^{(r)}_{ll'}(p,p';T) = V_{ll'}(p,p') - V_{ll'}(p,p')(GG^-)_n V^{(r)}_{ll'}(p,p';T). \tag{31}$$

Here the loop $(GG^-)_n$ is evaluated in the normal (nonsuperfluid) state. In terms of $V^{(r)}_{ll'}$, the gap equation becomes

$$\Delta_{lm_j}(p) = -\frac{1}{2\rho}\sum_{l'}\int dp' p'^2 V^{(r)}_{ll'}(p,p')\Delta(p') \times \int\frac{d\mathbf{n}'}{4\pi}\mathbf{b}^*_{l'm_j}(\mathbf{n}')\bar{\mathbf{b}}(\mathbf{n}') \times T\sum_m[GG^- - (GG^-)_n + \bar{b}^2FF]_{p_m,\mathbf{p}'}, \tag{32}$$

and we may everywhere substitute $V^{(r)}_{ll'}$ for $V_{ll'}$ provided that at the same time we understand by the GG^- element the subtracted quantity $GG^- - (GG^-)_n$. [$(GG^-)_n$ is to be evaluated for $\omega = 0$, $\mathbf{q} = 0$ in all cases.] In the following, I will do this and drop the superscript r on $V^{(r)}_{ll'}$.

Because the function $GG^{-}+\bar{b}^{2}FF$ decreases rapidly along with a distance from the Fermi surface, we may replace Eq. (32) with

$$\Delta_{lm_j}=-\frac{1}{\rho}\sum_{l'}V_{ll'}\Delta\int\frac{d\mathbf{n}}{4\pi}\mathbf{b}^{*}_{l'm_j}(\mathbf{n})\bar{\mathbf{b}}(\mathbf{n})\times\frac{1}{2}\int dpp^{2}T\sum_{m}(GG^{-}+\bar{b}^{2}FF)_{p_m,\mathbf{p}},\qquad(33)$$

assuming that in the narrow vicinity of the Fermi surface the smooth functions $\Delta_{lm_j}(p)$, $V_{ll'}(p,p')$, and $\Delta(p')$ may be replaced with constants $\Delta(p)\simeq\Delta(p_F)\equiv\Delta$, etc.

The function (19) is now to be understood as

$$A(\mathbf{n})\rightarrow[\mathcal{I}_{G^{-}G}-\mathcal{I}_{(G^{-}G)_n}+\bar{b}^{2}\mathcal{I}_{FF}]_{\omega=0,\mathbf{q}=0},\qquad(34)$$

and the gap equations (33) become

$$\Delta_{lm_j}=-\Delta\sum_{l'}V_{ll'}\int\frac{d\mathbf{n}}{4\pi}\mathbf{b}^{*}_{l'm_j}(\mathbf{n})\bar{\mathbf{b}}(\mathbf{n})A\left(\mathbf{n}\right).\qquad(35)$$

The function (34) can be found explicitly after performing the Matsubara's summation:

$$A(\mathbf{n})=\frac{1}{4}\int_{-\infty}^{\infty}d\varepsilon\left(\frac{1}{\sqrt{\varepsilon^{2}+\Delta^{2}\bar{b}^{2}}}\tanh\frac{\sqrt{\varepsilon^{2}+\Delta^{2}\bar{b}^{2}}}{2T}-\frac{1}{\varepsilon}\tanh\frac{\varepsilon}{2T}\right).\qquad(36)$$

III. EFFECTIVE VERTICES AND THE CORRELATION FUNCTIONS

The field interaction with a superfluid should be described with the aid of four effective three-point vertices, as shown in Fig. 2.

There are two ordinary effective vertices corresponding to creation of a particle and a hole by the field that differ by direction of fermion lines. I denote these 2×2 matrices as $\hat{\tau}(\mathbf{n};\omega,\mathbf{q})\equiv\tau_{\alpha\beta}(\mathbf{n};\omega,\mathbf{q})$ and $\hat{\tau}^{-}(\mathbf{n};\omega,\mathbf{q})\equiv\tau_{\beta\alpha}(-\mathbf{n};\omega,\mathbf{q})$, respectively. The anomalous vertices correspond to the creation of two particles or two holes. I denote these matrices as $\hat{T}^{(1)}(\mathbf{n};\omega,\mathbf{q})$ and $\hat{T}^{(2)}(\mathbf{n};\omega,\mathbf{q})$, respectively.

Given by the sum of the ladder-type diagrams [14], the anomalous vertices are to satisfy Dyson's equations, depicted symbolically in Fig. 3.

Analytically, the equations reduce to the following (for brevity I omit the dependence of functions on ω

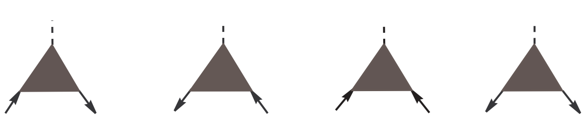

FIG. 2. Diagrams of the ordinary and anomalous vertices for the quasiparticle interacting with the external field shown by the dash line.

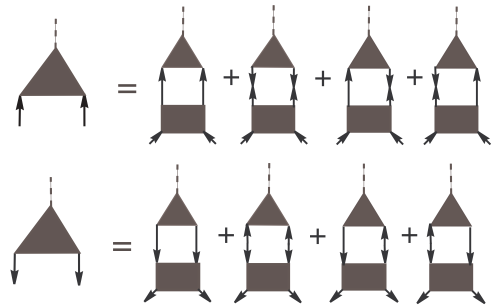

FIG. 3. Dyson's equations for the anomalous vertices. The shaded rectangles represent pairing interaction.

and $\mathbf{q}$):

$$T^{(1)}_{\alpha\beta}(\mathbf{n})=\sum_{lm_j}\left(\hat{\boldsymbol{\sigma}}\mathbf{b}_{lm_j}(\mathbf{n})\hat{g}\right)_{\alpha\beta}\sum_{l'}V_{ll'}\int\frac{d\mathbf{n}'}{8\pi}\mathrm{Tr}\left[\mathcal{I}_{GG^{-}}\hat{g}\times\left(\hat{\boldsymbol{\sigma}}\mathbf{b}^{*}_{l'm_j}\right)\hat{T}^{(1)}-\mathcal{I}_{FF}\left(\hat{\boldsymbol{\sigma}}\mathbf{b}^{*}_{l'm_j}\right)(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\hat{g}\hat{T}^{(2)}(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})-\mathcal{I}_{GF}(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\left(\hat{\boldsymbol{\sigma}}\mathbf{b}^{*}_{l'm_j}\right)\hat{\tau}+\mathcal{I}_{FG^{-}}\left(\hat{\boldsymbol{\sigma}}\mathbf{b}^{*}_{l'm_j}\right)(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\times(\hat{g}\hat{\tau}^{-}\hat{g})\right]_{\mathbf{n}'},\qquad(37)$$

$$T^{(2)}_{\alpha\beta}(\mathbf{n})=\sum_{lm_j}\left(\hat{g}\hat{\boldsymbol{\sigma}}\mathbf{b}^{*}_{lm_j}(\mathbf{n})\right)_{\alpha\beta}\sum_{l'}V_{ll'}\int\frac{d\mathbf{n}'}{8\pi}\mathrm{Tr}\left[\mathcal{I}_{G^{-}G}\times\left(\hat{\boldsymbol{\sigma}}\mathbf{b}_{l'm_j}\right)\hat{g}\hat{T}^{(2)}-\mathcal{I}_{FF}\left(\hat{\boldsymbol{\sigma}}\mathbf{b}_{l'm_j}\right)(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\hat{T}^{(1)}\hat{g}(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})+\mathcal{I}_{G^{-}F}\left(\hat{\boldsymbol{\sigma}}\mathbf{b}_{l'm_j}\right)\hat{g}\hat{\tau}^{-}\hat{g}(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})-\mathcal{I}_{FG}\left(\hat{\boldsymbol{\sigma}}\mathbf{b}_{l'm_j}\right)(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\hat{\tau}\right]_{\mathbf{n}'}.\qquad(38)$$

To obtain these equations, I used the identity $\hat{g}\hat{g}=-\hat{1}$ and a cyclic permutation of the matrices under the trace signs.

In general, the ordinary effective vertex is also to be found by ideal summation of the ladder diagrams incorporating residual particle-hole interactions. Unfortunately, the Landau parameters for these interactions in asymmetric nuclear matter are unknown; therefore, I simply neglect the particle-hole interactions and consider the pair correlation function in the BCS approximation. Thus, if the 2×2 matrix in spin space $\hat{\boldsymbol{\xi}}(\mathbf{n},k)$ is some vertex of a free particle, the ordinary vertices of a quasiparticle and a hole in the BCS approximation are to be taken as

$$\hat{\tau}(\mathbf{n},k)=\hat{\xi}(\mathbf{n},k),\quad\hat{\tau}^{-}(\mathbf{n},k)=\hat{\xi}^{T}(-\mathbf{n},k).\qquad(39)$$

Discarding the particle-hole interactions, I nevertheless assume that the "bare" vertices are properly renormalized [14] to get rid of the integration over regions far from the Fermi surface, $\varepsilon_p^{2}\gg\Delta^{2}$. As mentioned earlier in this article, I omit the renormalization factor everywhere.

Variation of the Green's function of a quasiparticle under the action of external field U,

$$\hat{G}'=\frac{\delta\tilde{G}}{\delta U},\qquad(40)$$

is given by the diagrams [19] shown in Fig. 4 and can be written analytically as

$$G'=GG\hat{\tau}+FF(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\hat{g}\hat{\tau}^{-}\hat{g}(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})+GF\hat{T}^{(1)}\hat{g}(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})+FG(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\hat{g}\hat{T}^{(2)},\qquad(41)$$

where $GG\equiv G(p_m+\omega_n,\mathbf{p}+\mathbf{q}/2)G(p_m,\mathbf{p}-\mathbf{q}/2)$, etc.

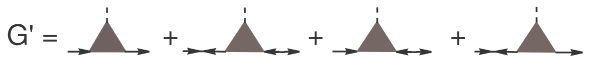

FIG. 4. Correction to the ordinary propagator of a quasiparticle in an external field.

The medium response onto an external field is given by the pair correlation function which can be found as the analytic continuation of the following Matsubara sum:

$$\Pi^{\tau}(\omega_n, q) = T \sum_m \int \frac{d^3\mathbf{p}}{8\pi^3} \operatorname{Tr}(\hat{\tau}\hat{G}'). \tag{42}$$

IV. GENERAL APPROACH TO NEUTRINO ENERGY LOSSES

The PBF processes are kinematically allowed, thanks to the existence of a superfluid energy gap, which admits the quasiparticle transitions with timelike momentum transfer $k = (\omega, \mathbf{q})$, as required by the final neutrino pair, $k = k_1 + k_2$. I consider the standard model of weak interactions. After integration over the phase space of escaping neutrinos and antineutrinos, the total energy that is emitted into neutrino pairs per unit volume and time is given by the following formula (see details, e.g., in Ref. [20]):

$$\epsilon = -\frac{G_F^2 \mathcal{N}_\nu}{192\pi^5} \int_0^\infty d\omega \int d^3q \frac{\omega\Theta(\omega - q)}{\exp\left(\frac{\omega}{T}\right) - 1} \operatorname{Im}\Pi^{\mu\nu}_{\text{weak}}(\omega, \mathbf{q}) \times (k_\mu k_\nu - k^2 g_{\mu\nu}), \tag{43}$$

where $\mathcal{N}_\nu = 3$ is the number of neutrino flavors, G_F is the Fermi coupling constant, and $\Theta(x)$ is the Heaviside step function. $\Pi^{\mu\nu}_{\text{weak}}$ is the retarded weak polarization tensor of the medium.

In general, the weak polarization tensor of the medium is a sum of the vector-vector, axial-axial, and mixed terms. The mixed axial-vector polarization has to be an antisymmetric tensor, and its contraction in Eq. (43) with the symmetric tensor $k_\mu k_\nu - k^2 g_{\mu\nu}$ vanishes. Thus, only the pure-vector and pure-axial polarizations should be taken into account. We then obtain $\operatorname{Im}\Pi^{\mu\nu}_{\text{weak}} \simeq C_{\text{V}}^2 \operatorname{Im}\Pi^{\mu\nu}_{\text{V}} + C_{\text{A}}^2 \operatorname{Im}\Pi^{\mu\nu}_{\text{A}}$, where C_{V} and C_{A} are vector and axial-vector weak coupling constants of a neutron, respectively.

V. PRESENT STATUS OF THE PROBLEM

The widely used expression for the neutrino emissivity caused by the triplet pairing of neutrons was obtained in Ref. [12] with the aid of the Fermi "golden" rule. Therefore, before proceeding to the self-consistent calculation of the neutrino energy losses, it is instructive to reproduce this formula using the calculation technique developed in present article. I will prove that the result of Ref. [12] can be obtained from my Eqs. (43) and (42) if to remove the field interactions through anomalous vertices [the two last terms in Eq. (41)]. I will label the corresponding results with tilde.

The authors of Ref. [12] state that the weak current of nonrelativistic neutrons is caused mostly by the temporal component of the vector current, $\hat{J}_0 = \Psi^+ \hat{1} \Psi$, and by the space components of the axial-vector current, $\hat{J}_i = \Psi^+ \hat{\sigma}_i \Psi$. Consequently, to reproduce their result, we need to evaluate the temporal component of the polarization tensor in the vector channel and the spatial part of the axial polarization. Omitting the anomalous contributions for the temporal component of the vector polarization, we have to substitute for

$$\hat{\tau} = \hat{\tau}^- \to \hat{1}, \quad \hat{\mathcal{T}}^{(1,2)} \to 0, \tag{44}$$

where $\hat{1}$ is a unit 2×2 matrix in spin space. Equation (42) is valid for each of the tensor components. Inserting the temporal component of the vector vertex into Eqs. (41) and (42), we find after a little algebra

$$\tilde{\Pi}^{00}_{\text{V}}(\omega, q) = 4\rho \int \frac{d\mathbf{n}}{4\pi} \frac{1}{2} (\mathcal{I}_{GG} - \bar{b}^2 \mathcal{I}_{FF}). \tag{45}$$

In obtaining this expression I used Eqs. (12) and (14) and the identity $(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}}) = \bar{b}^2$.

Only small transferred momenta, $q < \omega \sim T$, contribute into the neutrino energy losses. Because the transferred momentum comes in the polarization function in a combination $qV_F \ll \omega, \Delta$ (Fermi velocity V_F is small in a nonrelativistic system), to the lowest accuracy, we may evaluate the polarization tensor in the limit $\mathbf{q} = 0$. (In the same approximation the previously mentioned authors evaluate the matrix elements of a quasiparticle transition.) Then using Eqs. (22) and (23) we find

$$\operatorname{Im}\tilde{\Pi}^{00}_{\text{V}}(\omega > 0, q = 0) = -4\pi\rho \int \frac{d\mathbf{n}}{4\pi} \bar{b}^2 \Delta^2 \frac{\Theta(\omega - 2\bar{b}\Delta)}{\omega\sqrt{\omega^2 - 4\bar{b}^2\Delta^2}} \tanh\frac{\omega}{4T}. \tag{46}$$

The polarization tensor in the axial channel can be evaluated in the same way. In this case, omitting the anomalous contributions we have to take

$$\hat{\tau}(\mathbf{n},k) \to \hat{\sigma}_i, \quad \hat{\tau}^-(\mathbf{n},k) \to \hat{\sigma}_i^T, \quad \hat{\mathcal{T}}^{(1,2)} \to 0. \tag{47}$$

Then we find after some algebraic manipulations

$$\tilde{\Pi}^{ij}_{\text{A}}(\omega, q) = 4\rho \int \frac{d\mathbf{n}}{4\pi} \left[\frac{1}{2}(\mathcal{I}_{GG} - \bar{b}^2\mathcal{I}_{FF})\delta_{ij} + \mathcal{I}_{FF}\bar{b}_i\bar{b}_j\right]. \tag{48}$$

In obtaining this I used the identities $\hat{g}\hat{\boldsymbol{\sigma}}^T\hat{g} = \hat{\boldsymbol{\sigma}}$ and $\hat{\boldsymbol{\sigma}}(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}}) = 2\bar{\mathbf{b}} - (\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\hat{\boldsymbol{\sigma}}$.

With the aid of Eqs. (22) and (23), we find

$$\operatorname{Im}\tilde{\Pi}^{ij}_{\text{A}}(\omega > 0, q = 0) = -4\pi\rho \int \frac{d\mathbf{n}}{4\pi} \left(\delta_{ij} - \frac{\bar{b}_i\bar{b}_j}{\bar{b}^2}\right) \times \bar{b}^2\Delta^2 \frac{\Theta(\omega^2 - 4\bar{b}^2\Delta^2)}{\omega\sqrt{\omega^2 - 4\bar{b}^2\Delta^2}} \tanh\frac{\omega}{4T}. \tag{49}$$

Inserting the imaginary part of the polarization tensor into Eq. (43), we calculate the contraction of $\operatorname{Im}\tilde{\Pi}^{\mu\nu}_{\text{weak}}$ with the symmetric tensor $k_\mu k_\nu - k^2 g_{\mu\nu}$ to obtain

$$\operatorname{Im}\tilde{\Pi}^{\mu\nu}_{\text{weak}}(k_\mu k_\nu - k^2 g_{\mu\nu}) = -4\pi\rho \int \frac{d\mathbf{n}}{4\pi} \bar{b}^2\Delta^2 \frac{\Theta(\omega - 2\bar{b}\Delta)}{2\omega\sqrt{\omega^2 - 4\bar{b}^2\Delta^2}} \tanh\frac{\omega}{4T} \times \left\{C_{\text{V}}^2(q_\parallel^2 + q_\perp^2) + C_{\text{A}}^2[2(\omega^2 - q_\parallel^2) - q_\perp^2]\right\}, \tag{50}$$

where $q_\parallel$ and $q_\perp$ are defined as

$$q_\parallel^2 = \frac{1}{\bar{b}^2}(\mathbf{q}\bar{\mathbf{b}})^2, \quad q_\perp^2 = q^2 - q_\parallel^2. \tag{51}$$

After a little algebra, we obtain the neutrino emissivity in the form

$$\tilde{\epsilon} = \frac{G_F^2 \mathcal{N}_\nu}{120\pi^5} p_F M^* \int \frac{d\mathbf{n}}{4\pi} \Delta_\mathbf{n}^2 \int_{2\Delta_\mathbf{n}}^\infty d\omega \frac{\omega^5}{\left(1+\exp\frac{\omega}{2T}\right)^2} \times \frac{1}{\sqrt{\omega^2 - 4\Delta_\mathbf{n}^2}} \left(C_\mathrm{V}^2 + 2C_\mathrm{A}^2\right), \tag{52}$$

where $\Delta_\mathbf{n} \equiv \Delta \bar{b}(\mathbf{n})$.

With the aid of the change $\omega = 2T\sqrt{x^2 + \Delta_\mathbf{n}^2/T^2}$, one can recast this expression to the form obtained in Ref. [12],

$$\tilde{\epsilon} = \epsilon_{YKL} \equiv \frac{4G_F^2 \mathcal{N}_\nu}{15\pi^5} p_F M^* \left(C_\mathrm{V}^2 + 2C_\mathrm{A}^2\right) T^7 \int \frac{d\mathbf{n}}{4\pi} \frac{\Delta_\mathbf{n}^2}{T^2} \times \int_0^\infty dx \frac{z^4}{(1+\exp z)^2}, \tag{53}$$

where $z = \sqrt{x^2 + \Delta_\mathbf{n}^2/T^2}$.

Apparently, the contribution of the vector channel in this expression is a subject of inconsistency because conservation of the vector current in weak interactions requires $\omega \Pi_\mathrm{V}^{00}(\omega, q) = q_i \Pi_\mathrm{V}^{i0}(\omega, q)$, and thus one should expect $\Pi_\mathrm{V}^{00}(\omega > 0, q = 0) = 0$ for the correct result instead of Eq. (46). This, however, was not proved explicitly for the case of triplet pairing. I now focus on this calculation.

VI. ANOMALOUS CONTRIBUTIONS

A. Vector channel

The self-consistent longitudinal polarization function $\Pi_\mathrm{V}^{00}(\omega > 0, \mathbf{q})$ incorporates the anomalous contributions. At finite transferred space momentum, the problem of determining the vertex corrections is very complicated. Typically, massless Goldstone modes that arise due to symmetry breaking play a crucial role in conserving the vector current. In the anisotropic ${}^3P_2$ phase, rotational symmetry is broken and three Goldstone modes arise (termed angulons in Ref. [21]). However, because we are interested in the specific case of $\mathbf{q} = 0$, the temporal component of the anomalous vertex $\hat{\mathcal{T}}_\mu$ ($\mu = 0, 1, 2, 3$) can be retrieved from the Ward identity, which requires [3,19]

$$\omega \hat{\mathcal{T}}_0^{(1,2)}(\mathbf{n};\omega, \mathbf{q}) - \mathbf{q}\hat{\mathbf{\mathcal{T}}}^{(1,2)}(\mathbf{n};\omega, \mathbf{q}) = \pm 2\hat{D}(\mathbf{n}). \tag{54}$$

From this identity we immediately find

$$\hat{\mathcal{T}}_0^{(1)}(\mathbf{n};\omega, \mathbf{q} = \mathbf{0}) = \frac{2\Delta}{\omega} \bar{\mathbf{b}}\hat{\boldsymbol{\sigma}}\hat{g} \tag{55}$$

and

$$\hat{\mathcal{T}}_0^{(2)}(\mathbf{n};\omega, \mathbf{q} = \mathbf{0}) = -\frac{2\Delta}{\omega} \hat{g}\bar{\mathbf{b}}\hat{\boldsymbol{\sigma}}. \tag{56}$$

In the BCS approximation, the ordinary scalar vertices are to be taken, as given by Eq. (44). Inserting the previously mentioned vertices into Eqs. (41) and (42), we obtain after a little algebra

$$\Pi_\mathrm{V}^{00}(\omega, \mathbf{q} = 0) = 4\rho \int \frac{d\mathbf{n}}{4\pi} \left[\frac{1}{2}(\mathcal{I}_{GG} - \bar{b}^2 \mathcal{I}_{FF}) + \frac{2\Delta}{\omega} \bar{b}^2 \mathcal{I}_{FG}\right]_{\mathbf{q}=0}. \tag{57}$$

Using Eqs. (16) and (17) yielding

$$\mathcal{I}_{FG} = \frac{\omega + \mathbf{qv}}{2\Delta} \mathcal{I}_{FF}, \tag{58}$$

we finally find

$$\Pi_\mathrm{V}^{00}(\omega, \mathbf{q} = 0) = 2C_\mathrm{V}^2 \rho \int \frac{d\mathbf{n}}{4\pi} (\mathcal{I}_{GG} + \bar{b}^2 \mathcal{I}_{FF})_{\mathbf{q}=0}. \tag{59}$$

Comparing this with Eq. (21), we obtain $\Pi_\mathrm{V}^{00}(\omega, \mathbf{q} = 0) = 0$, as is required by the current conservation condition. Thus, the neutrino emissivity through the vector channel vanishes in the limit $\mathbf{q} = 0$. This proves explicitly that the neutrino emissivity via the vector channel, as obtained in Eq. (53), is a subject of inconsistency.

B. Axial channel

I now focus on the axial channel of the weak polarization. The order parameter in the triplet superfluid varies under the action of axial-vector external field. Therefore, the self-consistent axial polarization tensor also must incorporate anomalous contributions. Then from Eqs. (41) and (42) we obtain after simple algebraic manipulations

$$\Pi_\mathrm{A}^{ij}(\omega) = 4\rho \int \frac{d\mathbf{n}}{4\pi} \left\{\frac{1}{2}(\mathcal{I}_{GG} - \bar{b}^2 \mathcal{I}_{FF})\delta_{ij} + \bar{b}^2 \mathcal{I}_{FF} \frac{\bar{b}_i \bar{b}_j}{\bar{b}^2} - \frac{\omega}{2\Delta} \mathcal{I}_{FF} \frac{1}{4} \mathrm{Tr}\left[\hat{\sigma}_i \hat{\mathcal{T}}_j^{(1)} \hat{g}(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}}) - \hat{\sigma}_i (\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\hat{g}\hat{\mathcal{T}}_j^{(2)}\right]\right\}. \tag{60}$$

I again focus on the case $\mathbf{q} = 0$ and for brevity omit the dependence on $\mathbf{n}$ and ω. The anomalous axial-vector vertices $\hat{\mathcal{T}}_j^{(1,2)}$ ($j = 1, 2, 3$) are to be found from Eqs. (37) and (38), where the ordinary vertices are given by Eq. (47).

Up to this point I have not discussed the $\mathbf{n}$ dependence of $\mathbf{b}_{lm_j}(\mathbf{n})$. This makes Eq. (59) valid in the case of tensor forces resulting in the ${}^3P_2$-${}^3F_2$ mixing, because the general form of Eqs. (37) and (38) for the anomalous vertices takes into account not only spin-orbit interactions but the tensor interactions in the channel of two quasiparticles. Now I simplify the problem according to approximation adopted in simulations of neutron star cooling [11] and consider the case of paring in the ${}^3P_2$ channel, when $l = 1$ and $V_{ll'} = \delta_{ll'} V$ and the vectors $\mathbf{b}_{m_j}(\mathbf{n})$ are given by

$$\mathbf{b}_0 = \sqrt{\frac{1}{2}}(-n_1, -n_2, 2n_3),$$
$$\mathbf{b}_1 = -\mathbf{b}_{-1}^* = -\sqrt{\frac{3}{4}}(n_3, in_3, n_1 + in_2), \tag{61}$$
$$\mathbf{b}_2 = \mathbf{b}_{-2}^* = \sqrt{\frac{3}{4}}(n_1 + in_2, in_1 - n_2, 0),$$

where $n_1 = \sin\theta\cos\varphi, n_2 = \sin\theta\sin\varphi, n_3 = \cos\theta$. From here on I drop the subscript $l = 1$ by assuming $\mathbf{b}_{m_j} \equiv \mathbf{b}_{1,m_j}$, $\Delta_m \equiv \Delta_{1,m_j}$, etc.

I will focus on the p-wave condensation into the state $^3P_2$ with $m_j = 0$, which is conventionally considered as the preferable one in the bulk matter of neutron stars. In this case, Eq. (5) implies

$$\bar{\mathbf{b}}(\mathbf{n}) = \mathbf{b}_0(\mathbf{n}), \quad \Delta = \Delta_0 \tag{62}$$

and the gap equation (35) reads

$$1 = -V\int \frac{d\mathbf{n}}{4\pi}\bar{b}^2(\mathbf{n})A(\mathbf{n}). \tag{63}$$

From Eqs. (37) and (38) we obtain the vertex equations of the following form ($i = 1, 2, 3$):

$$\begin{aligned}\hat{T}_i^{(1)}(\mathbf{n}) = V\sum_{m_j}\hat{\boldsymbol{\sigma}}\mathbf{b}_{m_j}(\mathbf{n})\hat{g}\int\frac{d\mathbf{n}'}{8\pi}\{&\mathcal{I}_{GG^-}\mathrm{Tr}\left[\hat{g}\left(\hat{\boldsymbol{\sigma}}\mathbf{b}^*_{m_j}\right)\hat{T}_i^{(1)}\right]\\ &-\mathcal{I}_{FF}\mathrm{Tr}\left[\left(\hat{\boldsymbol{\sigma}}\mathbf{b}^*_{m_j}\right)(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\hat{g}\hat{T}_i^{(2)}(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\right]\\ &-\frac{\omega}{\Delta}\mathcal{I}_{FF}2i\left(\mathbf{b}^*_{m_j}\times\bar{\mathbf{b}}\right)_i\}_{\mathbf{n}'},\end{aligned}\tag{64}$$

$$\begin{aligned}\hat{T}_i^{(2)}(\mathbf{n}) = V\sum_{m_j}\hat{g}\hat{\boldsymbol{\sigma}}\mathbf{b}^*_{m_j}(\mathbf{n})\int\frac{d\mathbf{n}'}{8\pi}\{&\mathcal{I}_{G^-G}\mathrm{Tr}\left[\left(\hat{\boldsymbol{\sigma}}\mathbf{b}_{m_j}\right)\hat{g}\hat{T}_i^{(2)}\right]\\ &-\mathcal{I}_{FF}\mathrm{Tr}\left[\left(\hat{\boldsymbol{\sigma}}\mathbf{b}_{m_j}\right)(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\hat{T}_i^{(1)}\hat{g}(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\right]\\ &-\frac{\omega}{\Delta}\mathcal{I}_{FF}2i\left(\mathbf{b}_{m_j}\times\bar{\mathbf{b}}\right)_i\}_{\mathbf{n}'}.\end{aligned}\tag{65}$$

In obtaining the last line in these equations I used $\hat{\boldsymbol{\sigma}}(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}}) = 2\bar{\mathbf{b}} - (\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\hat{\boldsymbol{\sigma}}$ along with $\mathrm{Tr}\,[(\hat{\boldsymbol{\sigma}}\mathbf{b}^*_{m_j})(\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}})\hat{\boldsymbol{\sigma}}] = 2i(\mathbf{b}^*_{m_j}\times\bar{\mathbf{b}})$ and Eqs. (15) and (16).

Inspection of the equations reveals that the anomalous axial-vector vertices can be found in the following form:

$$\hat{\mathbf{T}}^{(1)}(\mathbf{n},\omega) = \sum_{m_j}\mathbf{B}^{(1)}_{m_j}(\omega)\left(\hat{\boldsymbol{\sigma}}\mathbf{b}_{m_j}\right)\hat{g}, \tag{66}$$

$$\hat{\mathbf{T}}^{(2)}(\mathbf{n},\omega) = \sum_{m_j}\mathbf{B}^{(2)}_{m_j}(\omega)\hat{g}\left(\hat{\boldsymbol{\sigma}}\mathbf{b}^*_{m_j}\right). \tag{67}$$

These general expressions can be simplified because the function $\mathcal{I}_{FF}(\mathbf{n};\omega)$ given by Eq. (20) is axial-symmetric and the last (free) term in Eqs. (64) and (65) can be averaged over the azimuth angle to give

$$\int\frac{d\varphi}{2\pi}(\mathbf{b}^*_0\times\bar{\mathbf{b}}) = \int\frac{d\varphi}{2\pi}(\mathbf{b}^*_2\times\bar{\mathbf{b}}) = \int\frac{d\varphi}{2\pi}(\mathbf{b}^*_{-2}\times\bar{\mathbf{b}}) = 0, \tag{68}$$

and

$$\begin{aligned}i\int\frac{d\varphi}{2\pi}(\mathbf{b}^*_1\times\bar{\mathbf{b}}) &= -\mathbf{e}\frac{\sqrt{6}}{4}\bar{b}^2,\\ i\int\frac{d\varphi}{2\pi}(\mathbf{b}^*_{-1}\times\bar{\mathbf{b}}) &= -\mathbf{e}^*\frac{\sqrt{6}}{4}\bar{b}^2,\end{aligned}\tag{69}$$

where $\mathbf{e} = (1, -i, 0)$ is a constant complex vector in spin space. The following relations can be also verified with a straightforward calculation,

$$\int\frac{d\varphi}{2\pi}\mathbf{b}^*_{m_j}\mathbf{b}_{m'_j} = \delta_{m_jm'_j}\mathbf{b}^*_{m_j}\mathbf{b}_{m_j}, \tag{70}$$

$$\int\frac{d\varphi}{2\pi}\left(\bar{\mathbf{b}}\mathbf{b}^*_{m_j}\right)\left(\bar{\mathbf{b}}\mathbf{b}_{m'_j}\right) = \delta_{m_jm'_j}\left(\bar{\mathbf{b}}\mathbf{b}^*_{m_j}\right)(\bar{\mathbf{b}}\mathbf{b}_{m_j}). \tag{71}$$

Relations (68) and (69) make it possible to conclude that $\mathbf{B}_0^{(1,2)} = \mathbf{B}_{\pm2}^{(1,2)} = 0$, and

$$\begin{aligned}\hat{\mathbf{T}}^{(1)}(\mathbf{n}) &= \left[\mathbf{B}^{(1)}_1(\hat{\boldsymbol{\sigma}}\mathbf{b}_1) + \mathbf{B}^{(1)}_{-1}(\hat{\boldsymbol{\sigma}}\mathbf{b}_{-1})\right]\hat{g},\\ \hat{\mathbf{T}}^{(2)}(\mathbf{n}) &= \hat{g}\left[\mathbf{B}^{(2)}_1(\hat{\boldsymbol{\sigma}}\mathbf{b}^*_1) + \mathbf{B}^{(2)}_{-1}(\hat{\boldsymbol{\sigma}}\mathbf{b}^*_{-1})\right].\end{aligned}$$

Inserting these expressions into Eqs. (64) and (65), taking the traces and using the orthogonality relations (3) along with relations (70) and (71) and

$$\bar{b}^2 \equiv \mathbf{b}_0^2, \quad \mathbf{b}^*_1\mathbf{b}_1 = \mathbf{b}^*_{-1}\mathbf{b}_{-1}, \tag{72}$$

$$(\bar{\mathbf{b}}\mathbf{b}^*_1)(\bar{\mathbf{b}}\mathbf{b}_1) = (\bar{\mathbf{b}}\mathbf{b}^*_{-1})(\bar{\mathbf{b}}\mathbf{b}_{-1}), \tag{73}$$

we obtain the equations

$$\begin{aligned}\mathbf{B}^{(1)}_{\pm1} = -V\int\frac{d\mathbf{n}}{4\pi}\Big\{&\mathcal{I}_{GG^-}\mathbf{B}^{(1)}_{\pm1}(\mathbf{b}_1\mathbf{b}^*_1) - \mathcal{I}_{FF}\mathbf{B}^{(2)}_{\mp1}[(\mathbf{b}^*_1\mathbf{b}_1)\bar{b}^2\\ &- 2(\mathbf{b}^*_1\bar{\mathbf{b}})(\bar{\mathbf{b}}\mathbf{b}_1)] - \frac{\omega}{\Delta}\mathcal{I}_{FF}\mathbf{e}\frac{\sqrt{6}}{4}\bar{b}^2\Big\}\end{aligned}\tag{74}$$

and

$$\begin{aligned}\mathbf{B}^{(2)}_{\pm1} = -V\int\frac{d\mathbf{n}'}{4\pi}\Big\{&\mathcal{I}_{G^-G}\mathbf{B}^{(2)}_{\pm1}(\mathbf{b}_1\mathbf{b}^*_1) - \mathcal{I}_{FF}\mathbf{B}^{(1)}_{\mp1}[(\mathbf{b}_1\mathbf{b}^*_1)\bar{b}^2\\ &- 2(\mathbf{b}_1\bar{\mathbf{b}})(\bar{\mathbf{b}}\mathbf{b}^*_1)] + \frac{\omega}{\Delta}\mathcal{I}_{FF}\mathbf{e}^*\frac{\sqrt{6}}{4}\bar{b}^2\Big\}.\end{aligned}\tag{75}$$

Solution to Eqs. (74) and (75) can be found in the form

$$\mathbf{B}^{(2)}_1 = -\mathbf{B}^{(1)}_{-1}, \quad \mathbf{B}^{(2)}_{-1} = -\mathbf{B}^{(1)}_1, \tag{76}$$

where

$$\mathbf{B}_1 = \mathbf{e}f(\omega), \quad \mathbf{B}_{-1} = \mathbf{e}^*f(\omega), \tag{77}$$

and the function $f(\omega)$ satisfies the equation

$$\begin{aligned}f = -V\int\frac{d\mathbf{n}}{4\pi}\Big[&(\mathcal{I}_{GG^-} + \bar{b}^2\mathcal{I}_{FF})(\mathbf{b}^*_1\mathbf{b}_1)f\\ &-2\mathcal{I}_{FF}(\mathbf{b}^*_1\bar{\mathbf{b}})(\bar{\mathbf{b}}\mathbf{b}_1)f - \frac{\omega}{\Delta}\mathcal{I}_{FF}\frac{\sqrt{6}}{4}\bar{b}^2\Big].\end{aligned}\tag{78}$$

Using Eq. (18) we can rewrite this as

$$\begin{aligned}f = -V\int\frac{d\mathbf{n}}{4\pi}\Big[&\left(A + \frac{\omega^2}{2\Delta^2}\mathcal{I}_{FF}\right)(\mathbf{b}^*_1\mathbf{b}_1)f\\ &-2\mathcal{I}_{FF}(\mathbf{b}^*_1\bar{\mathbf{b}})(\bar{\mathbf{b}}\mathbf{b}_1)f - \frac{\omega}{\Delta}\mathcal{I}_{FF}\frac{\sqrt{6}}{4}\bar{b}^2\Big].\end{aligned}\tag{79}$$

At this point it is convenient to recast the left-hand side of this equation according to Eq. (63):

$$f = -Vf\int\frac{d\mathbf{n}}{4\pi}\bar{b}^2(\mathbf{n})\,A(\mathbf{n}). \tag{80}$$

In this way we obtain the equation

$$f\int\frac{d\mathbf{n}}{4\pi}\left\{(\mathbf{b}_1^*\mathbf{b}_1-\bar{b}^2)A+2\left[\frac{\omega^2}{4\Delta^2}(\mathbf{b}_1^*\mathbf{b}_1)-(\mathbf{b}_1^*\bar{\mathbf{b}})(\bar{\mathbf{b}}\mathbf{b}_1)\right]\mathcal{I}_{FF}\right\}$$
$$=\sqrt{\frac{3}{2}}\frac{\omega}{2\Delta}\int\frac{d\mathbf{n}}{4\pi}\bar{b}^2\mathcal{I}_{FF}. \quad (81)$$

Because the function $\mathcal{I}_{FF}(\mathbf{n};\omega)$ is axial-symmetric and

$$\bar{b}^2=\tfrac{1}{2}\left(1+3n_3^2\right),\quad \mathbf{b}_1^*\mathbf{b}_1=\tfrac{3}{4}\left(1+n_3^2\right), \quad (82)$$

$$(\bar{\mathbf{b}}\mathbf{b}_1^*)(\bar{\mathbf{b}}\mathbf{b}_1)=\tfrac{3}{8}n_3^2\left(1-n_3^2\right), \quad (83)$$

Eq. (81) can be integrated over the azimuth angle, yielding the solution

$$f(\omega,q=0)=\frac{1}{\chi(\omega,q=0)}\sqrt{\frac{3}{2}}\frac{\omega}{2\Delta}\times\int_0^1 dn_3\frac{1}{2}\left(1+3n_3^2\right)\mathcal{I}_{FF}(n_3,\omega,T), \quad (84)$$

where

$$\chi(\omega,q=0)\equiv\int_0^1 dn_3\left\{\frac{1}{4}\left(1-3n_3^2\right)A(n_3,T)+\frac{3}{4}\left[\frac{\omega^2}{2\Delta^2}\times\left(1+n_3^2\right)-n_3^2\left(1-n_3^2\right)\right]\mathcal{I}_{FF}(n_3,\omega,T)\right\}, \quad (85)$$

and the functions $A(n_3,T)$ and $\mathcal{I}_{FF}(n_3,\omega,T)$ are given by Eqs. (36) and (20).

Explicit evaluation of Eq. (84) for arbitrary values of ω and T appears to require numerical computation. However, we can get a clear idea of the behavior of this function using the angle-averaged energy gap in the quasiparticle energy, $\langle\Delta^2\bar{b}^2\rangle\equiv\Delta^2$. (Replacing angle-dependent quantities in the gap equation with their angular average has been found to be a good approximation [22].) In this approximation the functions $\mathcal{I}_{FF}(\omega,T)$ and $A(T)$ in Eqs. (84) and (85) can be moved beyond the integrals. Using also the fact that

$$A\int_0^1 dn_3\left(1-3n_3^2\right)=0, \quad (86)$$

we find

$$f=\sqrt{\frac{3}{2}}\frac{\Delta\omega}{\omega^2-\Delta^2/5}. \quad (87)$$

Thus, in an approximation of the average gap, the function $f(\omega)$ is real valued and is independent of the temperature.

Poles of the vertex function correspond to collective eigenmodes of the system. Therefore, the pole at $\omega^2=\Delta^2/5$ signals the existence of collective spin oscillations. The decay of the collective oscillations into neutrino pairs gives the additive contribution into neutrino energy losses. However, examination of the collective modes deserves a separate study, which is beyond the scope of this article. Here I concentrate on the PBF processes discussed in the Introduction.

In this case, $\omega>2\Delta\bar{b}(\theta)\geqslant\sqrt{2}\Delta$ and, to obtain a simple analytic approximation, I omit a small term $\Delta^2/5$ in the denominator of Eq. (87), thus obtaining the axial-vector anomalous vertices in the following simple form:

$$\hat{\mathbf{T}}^{(1)}(\mathbf{n})=\sqrt{\frac{3}{2}}\frac{\Delta}{\omega}[\mathbf{e}(\hat{\sigma}\mathbf{b}_1)+\mathbf{e}^*(\hat{\sigma}\mathbf{b}_{-1})]\hat{g}, \quad (88)$$

$$\hat{\mathbf{T}}^{(2)}(\mathbf{n})=\sqrt{\frac{3}{2}}\frac{\Delta}{\omega}\hat{g}[\mathbf{e}(\hat{\sigma}\mathbf{b}_1)+\mathbf{e}^*(\hat{\sigma}\mathbf{b}_{-1})]. \quad (89)$$

Having obtained this simple result, we can evaluate the axial polarization function. Inserting Eqs. (88) and (89) into Eq. (60) gives

$$\Pi_{\rm A}^{ij}(\omega)=4\rho\int\frac{d\mathbf{n}}{4\pi}\left[\frac{1}{2}(\mathcal{I}_{GG}-\bar{b}^2\mathcal{I}_{FF})\delta_{ij}+\bar{b}^2\mathcal{I}_{FF}\frac{\bar{b}_i\bar{b}_j}{\bar{b}^2}+(\delta_{ij}-\delta_{i3}\delta_{j3})\frac{3}{4}\bar{b}^2\mathcal{I}_{FF}\right]_{\mathbf{q}=0}. \quad (90)$$

The first line in Eq. (90) can be evaluated with the aid of Eq. (22). We find

$$\Pi_{\rm A}^{ij}=-4\rho\int\frac{d\mathbf{n}}{4\pi}\left[\delta_{ij}-\frac{\bar{b}_i\bar{b}_j}{\bar{b}^2}-\frac{3}{4}(\delta_{ij}-\delta_{i3}\delta_{j3})\right]\times\bar{b}^2\mathcal{I}_{FF}(\omega,q=0). \quad (91)$$

Using Eq. (23) we obtain the imaginary part of axial polarization:

$$\mathrm{Im}\Pi_{\rm A}^{ij}(\omega>0,q=0)=-4\pi\rho\int\frac{d\mathbf{n}}{4\pi}\left[\delta_{ij}-\frac{\bar{b}_i\bar{b}_j}{\bar{b}^2}-\frac{3}{4}(\delta_{ij}-\delta_{i3}\delta_{j3})\right]\bar{b}^2\Delta^2\times\frac{\Theta(\omega^2-4\bar{b}^2\Delta^2)}{\omega\sqrt{\omega^2-4\bar{b}^2\Delta^2}}\tanh\frac{\omega}{4T}. \quad (92)$$

VII. SELF-CONSISTENT NEUTRINO ENERGY LOSSES

As we have obtained $\mathrm{Im}\Pi_{\rm V}^{\mu\nu}(\omega>0,q=0)=0$, using Eqs. (23) and (91), we find

$$\mathrm{Im}\Pi_{\rm weak}^{\mu\nu}=-\delta^{\mu i}\delta^{\nu j}C_{\rm A}^2 4\pi\rho\int\frac{d\mathbf{n}}{4\pi}\left[\delta_{ij}-\frac{\bar{b}_i\bar{b}_j}{\bar{b}^2}-\frac{3}{4}\times(\delta_{ij}-\delta_{i3}\delta_{j3})\right]\bar{b}^2\Delta^2\frac{\Theta(\omega^2-4\bar{b}^2\Delta^2)}{\omega\sqrt{\omega^2-4\bar{b}^2\Delta^2}}\tanh\frac{\omega}{4T}. \quad (93)$$

Contraction of this tensor with $(k_\mu k_\nu-k^2g_{\mu\nu})$ gives

$$\mathrm{Im}\Pi_{\rm weak}^{\mu\nu}(k_\mu k_\nu-k^2g_{\mu\nu})=-\frac{1}{4}C_{\rm A}^2[2(\omega^2-q_\parallel^2)-q_\perp^2]4\pi\rho\int\frac{d\mathbf{n}}{4\pi}\bar{b}^2\Delta^2\times\frac{\Theta(\omega-2\bar{b}\Delta)}{\omega\sqrt{\omega^2-4\bar{b}^2\Delta^2}}\tanh\frac{\omega}{4T}, \quad (94)$$

where

$$q_\parallel^2=\frac{1}{\bar{b}^2}(\mathbf{q}\bar{\mathbf{b}})^2,\quad q_\perp^2=q^2-q_\parallel^2. \quad (95)$$

The rest of the calculation is already performed in Sec. V. The neutrino energy losses can be written immediately after inspection of Eqs. (50) and (94). From this comparison it is clear that to obtain the correct neutrino energy losses, it is necessary to replace the factor $(C_V^2 + 2C_A^2)$ with $(1/2)C_A^2$ in Eq. (53). In this way, we obtain

$$\epsilon \simeq \frac{2}{15\pi^5} G_F^2 C_A^2 \mathcal{N}_\nu p_F M^* T^7 \int \frac{d\mathbf{n}}{4\pi} \frac{\Delta_{\mathbf{n}}^2}{T^2} \times \int_0^\infty dx \frac{z^4}{(1+\exp z)^2}, \tag{96}$$

where $\Delta_{\mathbf{n}}^2 \equiv \Delta^2 \bar{b}^2(\mathbf{n}) = \frac{1}{2} \Delta^2(1+3\cos^2\theta)$ and $z = \sqrt{x^2 + \Delta_{\mathbf{n}}^2/T^2}$. Comparison of this expression with Eq. (53) shows that the neutrino energy losses caused by the $^3P_2$ pairing in neutron matter are suppressed by the factor

$$\frac{1}{2} \frac{C_A^2}{(C_V^2 + 2C_A^2)} \simeq 0.19, \tag{97}$$

with respect to that predicted in Ref. [12].

For practical usage we reduce Eq. (96) to the traditional form,

$$\epsilon \simeq 5.85 \times 10^{20} \left(\frac{M^*}{M}\right) \left(\frac{p_F}{Mc}\right) T_9^7 \mathcal{N}_\nu C_A^2 F_t \frac{\text{erg}}{\text{cm}^3\text{s}}, \tag{98}$$

where M and M^* are the effective and bare nucleon masses, respectively, c is speed of light, and

$$F_t = \int \frac{d\mathbf{n}}{4\pi} \frac{\Delta_{\mathbf{n}}^2}{T^2} \int_0^\infty dx \frac{z^4}{(1+\exp z)^2}. \tag{99}$$

Notice the gap amplitude $\Delta(T)$ is $\sqrt{2}$ times larger than the gap amplitude $\Delta_0(T)$ used in Ref. [12], where the same anisotropic gap $\Delta_{\mathbf{n}}$ is written in the form $\Delta_{\mathbf{n}}^2 = \Delta_0^2(1+3\cos^2\theta)$. However, the function F_t, defined in Eq. (99), is independent of the particular choice of the gap amplitude; therefore, the analytic fit (B) suggested in Eq. (34) of Ref. [12], is valid and can be used in practical computations.

VIII. SUMMARY AND CONCLUSION

In this article I have performed a self-consistent calculation of the neutrino energy losses due to the PBF processes in the triplet-correlated neutron matter that is generally expected to exist in the interior of a neutron star. Because the existing theory of anomalous weak interactions in the fermion superfluid is well developed only for the case of $^1S_0$ pairing, I have generalized the corresponding equations for the triplet pairing, including the case when the attractive tensor coupling is operative.

Exact solution of the vertex equations is much complicated because of anisotropy of the triplet order parameter. Fortunately, only small values of the transferred space momenta are significant for the considered processes in the nonrelativistic approximation. Therefore, the weak vertices, as well as the polarization functions, can be evaluated in the limit $\mathbf{q} = 0$.

Before proceeding to the self-consistent calculation, I reproduced the neutrino energy losses as obtained in Ref. [12], using the calculation technique developed in present article. I have shown that the result of Ref. [12] can be obtained in the BCS approximation from my Eqs. (42) and (43) if one removes the field interactions through anomalous vertices.

The exact solution I found for the vector part of the weak polarization, $\Pi_V^{00}(\omega > 0, q = 0) = 0$, is consistent with the current conservation condition. This general result, which is obtained including the tensor couplings and the Fermi-liquid interactions, means that the neutrino emissivity in the vector channel, as obtained in Ref. [12], is a subject of inconsistency.

The self-consistent consideration of the axial weak polarization is more complicated. In this case, inclusion of the tensor forces and the Fermi-liquid effects requires numerical computations even in the limit of $\mathbf{q} = 0$. Therefore, to obtain a simple analytic result I have considered the $^3P_2$ pairing in the state with $m_j = 0$, which is conventionally considered as the preferable one in the minimal-cooling scenario of neutron stars. I have also neglected the residual particle-hole interactions because the Landau parameters are unknown for the neutron matter at high density.

Finally, I used the self-consistent polarization functions for evaluation of the neutrino energy losses due to PBF processes in the $^3P_2$ neutron superfluid with $m_j = 0$. The obtained self-consistent neutrino emissivity is given by Eq. (96). This expression needs to be compared to the emissivity (53) originally derived in Ref. [12], ignoring the anomalous weak interactions. One can see the neutrino emissivity is strongly suppressed because of the collective effects I have considered in this article. The suppression factor is $(1/2)C_A^2/(C_V^2 + 2C_A^2) \simeq 0.19$.

Because the neutron $^3P_2$ pairing occurs in the core, which contains more than 90% of the neutron star volume, the found quenching of the neutrino energy losses from the PBF processes can affect the minimal-cooling paradigm.

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