Effect of a logarithmic mesonic potential on nucleon properties in the coherent-pair approximation

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A logarithmic mesonic potential is proposed for determining nucleon properties that have been recently calculated in the mean-field approximation. The field equations have been solved in the coherent-pair approximation in which the variational method is used. The obtained nucleon properties have been compared with previous calculations and other models. The results indicate that the use of the logarithmic mesonic potential in the coherent-pair approximation provides good agreement with data for the axial-vector coupling constant $g_A(0)$, pion-nucleon coupling constant $g_{\pi NN}(0)$, and sigma commutator $\sigma(\pi N)$.

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I. INTRODUCTION

Quantum chromodynamics (QCD) is the fundamental theory underlying a strong interaction. This theory includes the properties confinement, asymptotic freedom, and spontaneous broken chiral symmetry [1]. The theory's non-Abelian color and flavor structures as well as its strong coupling constants [2], which lead to the processes involving strong interactions (mainly at low and intermediate energies) are difficult. Thus, it may be valuable to develop effective models based on hadronic degrees of freedom which incorporate relevant properties of QCD. The effective models, such as the Skyrme chiral soliton model, chiral perturbative quark model, and chiral quark sigma model, are formulated on the basis of the chiral symmetry and its spontaneous breakdown.

Decades ago, Skyrme formulated the topological soliton model, in which a conserved topological charge is identified with the baryonic number. The real existence of minimum energy configurations in this model is discussed in Ref. [3]. The model has chiral symmetry with its spontaneous breakdown and has further deeper motivation based on the large N_c QCD analysis [4,5]. Some modifications have been recently introduced as in Refs. [2,6]. In the recently introduced model, Braghin and Cavalcante [2] investigated the effect of coupling a classical scalar-isoscalar field to the usual Skyrmion model as a degree of freedom whose corresponding quantum would be the chiral partner of the pion. The dependence of the observables on the values of the scalar field mass m_σ and on the fourth-order coupling term e were also considered. Smaller values of the σ mass favor better agreement for most of the nucleon static properties.

The perturbative chiral quark model (PCQM) [7–11] is applied for the study of the low-energy nucleonic properties. This model contains several new features: (i) the generalization of the phenomenological confining potential; (ii) the $SU(3)$ extension of chiral symmetry to include the kaon and η-meson cloud contributions; (iii) the consistent formulation of perturbation theory both on the quark and baryon level by using renormalization techniques and allowing excited quark states to be depicted in the meson loop diagrams; (iv) the fulfillment of the constraints imposed by chiral symmetry (low-energy theorems), including the current quark mass expansion of the matrix elements; and (v) the possible consistency with chiral perturbation theory. The PCQM [7] is based on an effective chiral Lagrangian describing quarks as relativistic fermions moving in a self-consistent field (static potential). The latter is described by a scalar potential S providing confinement of quarks and the time component of a vector potential $\gamma^0 V$ responsible for short-range fluctuations of the gluon field configurations. The model potential defines unperturbed wave functions of quarks which are subsequently used in the calculation of baryon properties. The PCQM has been successfully applied to the charge and magnetic form factors of baryons, σ terms, ground state masses of baryons, the electromagnetic N-Δ transition, meson-nucleon σ terms, and other baryon properties (including exotic baryon states—pentaquarks); see Refs. [7–11] for reviews.

One of the effective theories in describing baryon properties is the linear σ model, which was previously suggested by Gell-Mann and Levy [12] to describe nucleons interacting via sigma (σ) and pion (π) exchanges. The linear σ model proposed a nucleonic structure that respected the constraints imposed by chiral symmetry. Spontaneous and explicit chiral symmetry breaking require the existence of the pion, whose mass vanishes in the limit as the current mass approaches zero. A few solutions for the Lagrangian of the chiral linear σ model as applied to the nucleon and δ have already been suggested. Birse and Banerjee [1] solved the linear chiral σ model in the mean-field approximation using the hedgehog ansatz for the pion field. After the variation, they performed an approximate projection on angular momentum and isospin, ignoring the contribution of the pions in this procedure. Birse [13] and Golli and Rosina [14] evaluated this model further by performing proper projections even before the variation in the hedgehog approximation. Fiolhais *et al.* [15] generalized the hedgehog and performed spin and isospin projections as well. Goeke *et al.* [16] obtained the static solitonic solution of the linear σ model using a coherent-pair trial Fock state with proper spin and isospin quantum numbers. The work of

Goeke *et al.* [16] has been re-examined by Aly *et al.* [17] and they have made some corrections.

In recent years, there has been growing interest in studying nucleon properties, resulting in modifications being suggested for the linear σ model within the framework of some aspects of QCD. Broniowski and Golli [18] analyzed a particular extension of the linear σ model coupled to valence quarks containing an additional term with two ingredients of the chiral fields and they investigated the dynamic consequences of this term and its relevance to the phenomenology of soliton models of the nucleons. In addition, this model is investigated in the mean-field approximation [19] and the coherent-pair approximation [20] for hadron properties. Dmitrasinovic and Myhrer [21] used an extended linear σ model [22] in which a pair of extra terms are added to the original linear σ model in order to improve pion-nucleon scattering and the nucleon σ term. Furthermore, Korchin [23] calculated the properties of the nucleon in a nonlocal σ model where conserved electromagnetic and vector currents and partially conserved axial vector currents are obtained. Along the same lines, Rashdan *et al.* [24–26] considered higher-order mesonic interactions in the linear σ model using mean-field approximation to get a better description of the nucleon properties.

The chiral logarithmic quark model, which is one of the most important ingredients in the chiral models, dynamically breaks the chiral symmetry, and the finite expectation value of σ generates the constituent quark and hadron masses. The logarithmic potential does not have any instability at any σ field value. Thus, the model is a good starting point to describe cold nuclear matter; see Ref. [27] for a review. Further, this model considers some aspects of QCD to get a better description of hadron properties in the mean-field approximation in Refs. [28,29].

The aim of this paper is to estimate the effect of the logarithmic mesonic potential on the hadron properties in the coherent-pair approximations using the logarithmic mesonic potential which was suggested for calculating nucleon properties in the mean-field approximation in Ref. [29].

This paper is organized as follows. In Sec. II, the logarithmic mesonic potential is explained briefly. The Fock state in the coherent-pair approximation and the variational principle are presented in Secs. III and IV, respectively. The derived nucleon properties are explained in Sec. V. The numerical calculations and discussion of the results are presented in Sec. VI.

II. LOGARITHMIC MESONIC POTENTIAL AND PHYSICAL SITUATION

We begin with the logarithmic σ model [28,29], in which the Lagrangian density of the logarithmic σ model that describes the interactions between quarks via the σ and π mesons is written as [29]

$$L(x) = i\overline{\Psi}\partial_\mu\gamma^\mu\hat{\Psi} + \frac{1}{2}(\partial_\mu\hat{\sigma}\partial^\mu\hat{\sigma} + \partial_\mu\hat{\pi}\cdot\partial^\mu\hat{\pi}) + g\hat{\overline{\Psi}}(\hat{\sigma} + i\gamma_5\hat{\tau}.\hat{\pi})\hat{\Psi} - U(\hat{\sigma},\hat{\pi}) \tag{1}$$

with

$$U(\hat{\sigma},\hat{\pi}) = \lambda_1^2(\hat{\sigma}^2+\hat{\pi}^2) - \lambda_2^2\ln(\hat{\sigma}^2+\hat{\pi}^2) - f_\pi m_\pi^2\hat{\sigma}, \tag{2}$$

$$\lambda_1^2 = \frac{1}{4}\left(m_\sigma^2 + m_\pi^2\right), \tag{3}$$

$$\lambda_2^2 = \frac{f_\pi^2}{4}\left(m_\sigma^2 - m_\pi^2\right), \tag{4}$$

where f_π is the pion decay constant, m_π is the pion mass, and m_σ and g are constants to be determined. The quark, σ, and $\boldsymbol{\pi}$ mesons are quantum fields denoted by (ˆ). Spontaneous symmetry breaking generates mass for the quark, which breaks the chiral symmetry and generates the small pion mass which would otherwise be zero, unlike the Goldstone boson from the theory in Refs. [27,29].

In the case of a massless pion ($m_\pi \simeq 0$), the minimum value for the logarithmic potential occurs at a finite value of σ mass ($\sigma \simeq f_\pi$), thus the logarithmic potential is shifted from the singularity which comes from the logarithmic term.

Now, we can rewrite the Hamiltonian density as in Ref. [17]:

$$\begin{aligned}\hat{H}(r) &= \tfrac{1}{2}\{\hat{P}_\sigma(r)^2 + [\nabla\hat{\sigma}(r)]^2 + \hat{P}_\pi(r)^2 + [\nabla\pi(r)]^2\} \\ &\quad + U(\hat{\sigma},\hat{\pi}) + \hat{\Psi}^\dagger(r)\times(-i\alpha\nabla)\hat{\Psi}(r) \\ &\quad - g(r)\hat{\Psi}^\dagger(r)[\beta\hat{\sigma}(r) + i\beta\gamma_5\hat{\tau}\cdot\hat{\pi}]\hat{\Psi}(r),\end{aligned} \tag{5}$$

where α and β are the usual Dirac matrices. In the above expression $\hat{\overline{\Psi}}$, $\hat{\sigma}$, and $\hat{\pi}$ are quantized field operators with the appropriate static angular momentum expansion [17],

$$\hat{\sigma}(r) = \int_0^\infty \frac{d^3k}{[2(2\pi)^3 W_\sigma(k)]^{\frac{1}{2}}}[\hat{c}^\dagger(k)e^{-ik.r} + \hat{c}(k)e^{+ik.x}], \tag{6}$$

$$\begin{aligned}\hat{\pi}(r) &= \left[\frac{2}{\pi}\right]^{\frac{1}{2}}\int_0^\infty dkk^2\left[\frac{1}{2W_\pi(k)}\right]^{\frac{1}{2}}\sum_{lmw} j_l(kr)Y_{lm}^*(\Omega_r) \\ &\quad\times\left[\hat{a}_{lm}^{1w\dagger}(k) + (-)^{m+w}\hat{a}_{l-m}^{1-w}(k)\right],\end{aligned} \tag{7}$$

$$\hat{\Psi}(r) = \sum_{njmw}\left(\langle r\upharpoonleft njmw\rangle\hat{d}_{njm}^{\frac{1}{2}w} + \langle r\upharpoonleft\overline{n}jmw\rangle\,\hat{d}_{njm}^{\frac{1}{2}w\dagger}\right), \tag{8}$$

where the $|njmw\rangle$ and $|\overline{n}jmw\rangle$ form a complete set of quark and antiquark spinors with angular momentum quantum numbers and spin-isospin quantum numbers j, m, and w, respectively. The corresponding conjugate momentum fields have the expansion [17]

$$\hat{P}_\sigma(r) = i\int_0^\infty d^3k\left[\frac{W_\sigma(k)}{2(2\pi)^3}\right]^{\frac{1}{2}}[\hat{c}^\dagger(k)e^{-\mathbf{k}.\mathbf{r}} - \hat{c}(k)e^{+\mathbf{k}.\mathbf{r}}], \tag{9}$$

$$\begin{aligned}\hat{P}_\pi(r) &= i\left[\frac{2}{\pi}\right]^{\frac{1}{2}}\int_0^\infty dkk^2\left[\frac{W_\pi(k)}{2}\right]^{\frac{1}{2}}\sum_{lmw} j_l(kr)\,Y_{lm}^*(\Omega_r) \\ &\quad\times\left[\hat{a}_{lm}^{1w\dagger}(k) - (-)^{m+w}\hat{a}_{l-m}^{1-w}(k)\right].\end{aligned} \tag{10}$$

Here $\hat{c}(k)$ destroys a σ quantum with momentum $\mathbf{k}$ and frequency $W_\sigma(k) = (k^2+m_\sigma^2)^{\frac{1}{2}}$ and $\hat{a}_{lm}^{1w}(k)$ destroys a pion with momentum $\mathbf{k}$ and corresponding $W_\pi(k) = (k^2+m_\pi^2)^{\frac{1}{2}}$ in the isospin-angular momentum state $\{lm; tw\}$.

III. THE FOCK STATE

For convenience, one constructs the configuration space pion field functions needed for the subsequent variational treatment by defining the alternative basis operators,

$$\hat{b}_{lm}^{1w} = \int dkk^2 \zeta_l(k) \hat{a}_{lm}^{1w}(k), \tag{11}$$

where $\hat{a}_{lm}^{1w}(k)$ are basis operators which create a free massive pion with isospin component w and orbital angular momentum (l, m), and $\zeta_l(k)$ is the variational function. Considering this in configuration space, the pion field function [17] is defined as

$$\Phi_l = \frac{1}{2\pi} \int_0^\infty dkk^2 \frac{\zeta_l(k)}{W_\pi(k)^{\frac{1}{2}}} j_l(r). \tag{12}$$

In the following, only the $l = 1$ value is used and the angular momentum label will be dropped. The Fock state for the nucleon is taken to be [17]

$$|NT_3J_z\rangle = [\alpha(|n\rangle \otimes |P^{00}\rangle)_{T_3J_z} + \beta(|n\rangle \otimes |P^{11}\rangle)_{T_3J_z} + \gamma(\delta > \otimes |P^{11}\rangle_{T_3J_z})|0\rangle] \Big|\sum\Big\rangle, \tag{13}$$

where $|\sum\rangle$ is the coherent sigma field state with the property $\langle\sum|\hat{\sigma}(r)|\sum\rangle = \hat{\sigma}(r)$, and $|P^{00}\rangle(|P_{1m}^{1w}\rangle)$ are pion coherent-pair states to be determined. The normalization of the nucleon state requires $\alpha^2 + \beta^2 + \gamma^2 = 1$. The permutation symmetric form of the $SU(2) \times SU(2) \times SU(2)$ quark wave functions imply that the source terms in the pion field equations will induce an angular momentum isospin correlation for the pion field (for details, see Ref. [17]).

IV. THE VARIATIONAL PRINCIPLE

The objective of this section is to seek the minimum of the total baryon energy, which is given by

$$E_B = \langle BT_3J_z| \int_0^\infty d^3r : H(r) : |BT_3J_z\rangle, \tag{14}$$

where $B = N$ or Δ. The field equations are obtained by minimizing the total energy of the baryon with respect to the variation of the fields, $\{u(r), v(r), \sigma(r), \Phi(r)\}$, as well as the Fock-space parameters, $\{\alpha, \beta, \gamma\}$, subjected to the normalization conditions. The total energy of the system is written as

$$E_B = 4\pi \int_0^\infty drr^2 \varepsilon_B(r). \tag{15}$$

Writing the quark Dirac spinor as

$$\Psi_{\frac{1}{2}m}^{\frac{1}{2}w}(\mathbf{r}) = \begin{pmatrix} u(r) \\ v(r)\sigma \cdot \hat{\mathbf{r}} \end{pmatrix} \chi_{\frac{1}{2}m} \zeta^{\frac{1}{2}w}, \tag{16}$$

the energy density is given by

$$\begin{aligned}\varepsilon_B(r) &= \frac{1}{2}\left(\frac{d\sigma}{dr}\right)^2 + \lambda_1^2\sigma^2(r) - m_\pi^2 f_\pi \sigma(r) + U_0 \\ &\quad + 3\left\{u(r)\left[\frac{dv}{dr} + \frac{2}{r}v(r)\right] - v(r)\frac{du}{dr}\right. \\ &\quad \left. + g\sigma(r)[u^2(r) - v^2(r)]\right\} + (N_\pi + x) \\ &\quad \times \left[\left(\frac{d\Phi}{dr}\right)^2 + \frac{2}{r^2}\Phi^2(r)\right] + (N_\pi - x)\Phi_p^2(r) \\ &\quad - \alpha\delta g(a+b)u(r)v(r)\Phi(r) \\ &\quad + \lambda_2^2 \ln[\sigma^2 + 2(N_\pi + x)\Phi^2(r)],\end{aligned} \tag{17}$$

where N_π is the average pion number

$$N_\pi = 9\left[\alpha^2 a^2 + \left(\beta^2 + \gamma^2\right) c^2\right], \tag{18}$$

and where δ takes the following values for nucleon or delta quantum numbers:

$$\delta_N = (5\beta + 4\sqrt{2}\gamma)/\sqrt{3}, \qquad \delta_\Delta = (2\sqrt{2}\beta + 5\gamma)/\sqrt{3}. \tag{19}$$

The function $\Phi_p(r)$ is obtained from $\Phi(r)$ by double folding:

$$\Phi_p(r) = \int_0^\infty w(r, \acute{r})\Phi(\mathbf{r})r^2 d\acute{r}, \tag{20}$$

$$w(r, \acute{r}) = \frac{2}{\pi}\int_0^\infty dkk^2 w(k) j_1(kr) j_1(kr'). \tag{21}$$

For fixed α, β, and γ, the stationary functional variations are expressed by

$$\delta\left[\int_0^\infty drr^2\{\varepsilon_B(r) - 3\epsilon[u^2(r) + v^2(r)] - 2k\Phi\Phi_p(r)\}\right] = 0, \tag{22}$$

where the parameter k enforces the pion normalization condition,

$$8\pi \int_0^\infty \Phi(r)\Phi_p(r)r^2 dr = 1, \tag{23}$$

and ϵ fixes the quark normalization,

$$4\pi \int_0^\infty [u^2(r) + v^2(r)]r^2 dr = 1. \tag{24}$$

Minimizing the Hamiltonian yields the four nonlinear coupled differential equations:

$$\frac{du}{dr} = -2(g\sigma + \epsilon)v(r) - \frac{1}{3}\alpha\delta\,(a+b)\,g\Phi(r)u(r), \tag{25}$$

$$\frac{dv}{dr} = -\frac{2}{r}v(r) - 2[g\sigma(r) - \epsilon]u(r) + \frac{1}{3}\alpha\delta\,(a+b) \times g\Phi\,(r)\,u(r), \tag{26}$$

$$\frac{d^2\sigma}{dr^2} = -\frac{2}{r}\frac{d\sigma}{dr} + 2\lambda_1^2\sigma(r) - m_\pi^2 f_\pi + 3g[u^2(r) - v^2(r)] + \frac{2\lambda_2^2\sigma(r)}{\sigma^2 + 2\,(N_\pi + x)\,\Phi^2(r)}, \tag{27}$$

$$\frac{d^2\Phi}{dr^2} = -\frac{2}{r}\frac{d\Phi}{dr} + \frac{2}{r^2}\Phi(r) + \frac{1}{2}\left(1 - \frac{x}{N_\pi}\right)m_\pi^2\Phi^2 - \frac{\alpha}{4N_\pi}\,(a+b)\,g\delta u(r)v(r) + \frac{\lambda_2^2(1 + \frac{x}{N_\pi})\Phi}{[\sigma^2 + 2\Phi^2(N_\pi + x)]}, \tag{28}$$

with eigenvalue ϵ and k. These consist of two quark equations for u and v where $\sigma(r)$ and $\Phi(r)$ appear as potentials and two

Klein-Gordon equations with $u(r)v(r)$ and $[u^2(r) - v^2(r)]$ as source terms. The boundary conditions are for $r \longrightarrow 0$,

$$v = \frac{d\sigma}{dr} = \Phi = \frac{du}{dr} = 0, \tag{29}$$

and for $r \longrightarrow \infty$,

$$[r(gf_\pi - \epsilon)^{\frac{1}{2}} + (gf_\pi + \varepsilon)^{-\frac{1}{2}}]u(r) - r(gf_\pi + \epsilon)^{\frac{1}{2}} v(r) = 0, \tag{30}$$

$$\left(2 + 2m_\pi r + m_\pi^2 r^2\right)\Phi(r) + r(1 + m_\pi r)\Phi(r) = 0, \tag{31}$$

$$r\sigma(r) + [\sigma(r) - f_\pi](1 + m_\sigma r) = 0. \tag{32}$$

The field equations are solved for the fixed coherence parameter (x) and the fixed Fock-space parameters (α, β, γ) as in Ref. [17].

V. THE NUCLEON PROPERTIES

The expectation value of the energy is minimized with respect to (α, β, γ) by diagonalizing the energy matrix

$$\begin{bmatrix} H_{\alpha\alpha} & H_{\alpha\beta} & H_{\alpha\gamma} \\ H_{\alpha\beta} & H_{\beta\beta} & H_{\beta\gamma} \\ H_{\alpha\beta} & H_{\beta\gamma} & H_{\gamma\gamma} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = E \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}, \tag{33}$$

each H entry of the matrix is related to a corresponding density as follows:

$$H_{\alpha\beta} = 4\pi \int r^2 E_{\alpha\beta}(r) dr, \tag{34}$$

and analogously for the other entries. The functions for a nucleon are

$$E_{\alpha\alpha} = E_0(r) + 9a^2 \left[\left(\frac{d\Phi}{dr}\right)^2 + \frac{2}{r^2}\Phi^2\right] + 9a^2\Phi_p^2(r), \tag{35}$$

$$E_{\beta\beta} = E_0(r) + 9c^2 \left[\left(\frac{d\Phi}{dr}\right)^2 + \frac{2}{r^2}\Phi^2\right] + 9c^2\Phi_p^2(r), \tag{36}$$

$$E_{\gamma\gamma} = E_0(r) + 9c^2 \left[\left(\frac{d\Phi}{dr}\right)^2 + \frac{2}{r^2}\Phi^2\right] + 9c^2\Phi_p^2(r), \tag{37}$$

$$E_{\alpha\beta} = -2g(a + b)\Phi(r)u(r)v(r)\frac{2\sqrt{2}}{\sqrt{3}}, \tag{38}$$

$$E_{\alpha\gamma} = -2g(a + b)\Phi(r)u(r)v(r)\frac{5}{\sqrt{3}}, \tag{39}$$

where

$$\begin{aligned} E_0(r) = {} & \frac{1}{2}\left(\frac{d\sigma}{dr}\right)^2 + \lambda_1^2\sigma^2(r) - m_\pi^2 f_\pi \sigma(r) + U_0 \\ & + 3\left\{u(r)\left[\frac{dv}{dr} + \frac{2}{r}v(r)\right] - v(r)\frac{du}{dr}\right. \\ & \left. + g\sigma(r)[u^2(r) - v^2(r)]\right\} \\ & + x\left[\left(\frac{d\Phi}{dr}\right)^2 + \frac{2}{r^2}\Phi^2(r)\right] + x\Phi_p^2(r) \\ & - +\lambda_2^2 \ln[\sigma^2 + 2(N_\pi + x)\Phi^2(r)]. \end{aligned} \tag{40}$$

A. Mass of the nucleon

In this subsection, we calculate the total energy of the nucleon, which consists of quark, σ, pion, quark-σ interaction, quark-pion interaction, and meson static energy contributions. We derive the nucleon mass as in Ref. [17]. We obtain

$$(\text{K.E.})_{\text{quark}} = \int_0^\infty [g\sigma\rho_s(r) + \epsilon\rho_w(r) + g\pi\rho_p(r)]r^2 dr, \tag{41}$$

where ρ_s, ρ_p, and ρ_w are the quark scalar density, pseudoscalar density, and vector density, respectively. Similarly, we find the meson kinetic contribution:

$$\begin{aligned} (\text{K.E.})_\sigma = {} & \frac{1}{2}\int_0^\infty \sigma(r)\left\{2\lambda_1^2\sigma(r) - m_\pi^2 f_\pi + 3g[u^2(r)\right. \\ & \left. - v^2(r)] + \frac{2\lambda_2^2\sigma(r)}{\sigma^2 + 2(N_\pi + x)\Phi^2(r)}\right\}r^2 dr, \end{aligned} \tag{42}$$

$$\begin{aligned} (\text{K.E.})_{\text{pion}} = {} & \frac{1}{2}\int_0^\infty \Phi(r)\left\{\frac{2}{r^2}\Phi(r) + \frac{1}{2}\left(1 - \frac{x}{N_\pi}\right)m_\pi^2\Phi^2\right. \\ & - \frac{\alpha}{4N_\pi}(a + b)g\delta u(r)v(r) \\ & \left. + \frac{\lambda_2^2\left(1 + \frac{x}{N_\pi}\right)\Phi}{[\sigma^2 + 2\Phi^2(N_\pi + x)]}\right\}r^2 dr. \end{aligned} \tag{43}$$

The quark-meson interaction energy is

$$E_{q\text{-}\sigma} = -\int_0^\infty g\sigma\rho_s(r)r^2 dr, \tag{44}$$

$$E_{q\text{-}pion} = -\int_0^\infty g\sigma\rho_p(r)r^2 dr. \tag{45}$$

The meson-meson interaction energy is

$$\begin{aligned} E_{\text{meson-meson}} = {} & \int_0^\infty \left[\lambda_1^2(\hat{\sigma}^2 + \hat{\pi}^2) - \lambda_2^2 \ln(\hat{\sigma}^2 + \hat{\pi}^2)\right. \\ & \left. - f_\pi m_\pi^2 \hat{\sigma} - U_0\right]r^2 dr, \end{aligned} \tag{46}$$

U_0 is the minimum of potential U at $(\sigma = f_\pi, \pi = 0.0)$.

B. Magnetic moment, axial-vector coupling constant $(\frac{g_A}{g_v})$, pion-nucleon coupling constant $g_{\pi NN}(0)$, and σ commutator $\sigma(\pi N)$

The magnetic moments of the proton and neutron are given by as in Ref. [17]:

$$\begin{aligned} \frac{\mu_p(r)}{4\pi e} = {} & \frac{ruv}{81}(54\alpha^2 + 2\beta^2 + \gamma^2 + 32\sqrt{2}\beta\gamma) \\ & + \frac{x}{729a^2}(9a^2 + x)(4\beta^2 + \gamma^2)\Phi^2, \end{aligned} \tag{47}$$

$$\begin{aligned} \frac{\mu_n(r)}{4\pi e} = {} & \frac{ruv}{81}(-36\alpha^2 - 8\beta^2 + \gamma^2 - 32\sqrt{2}\beta\gamma) \\ & - \frac{x}{729a^2}(9a^2 + x)(4\beta^2 + \gamma^2)\Phi^2. \end{aligned} \tag{48}$$

The axial-vector coupling constant, measured in neutron β decay, is a matrix element of the space part of the isovector-axial vector current. Explicitly, one is interested in $\frac{g_A}{g_v}$, where g_v is the corresponding matrix element of the isovector-vector current

$$\mathbf{J}_{\mu}^{V} = \tfrac{1}{2}\overline{\Psi}\gamma^{\mu}\hat{\Psi} + \hat{\pi} \times \partial^{\mu}\hat{\pi}. \tag{49}$$

The values of g_A and g_v are taken from the z component in space and the third component in isospace. Since the vector part yields $\frac{1}{2}$, one obtains [17]

$$\frac{g_A}{g_v} = 4\pi \int_0^{\infty} dr r^2 \left[\left(\frac{5}{3}\alpha^2 + \frac{5}{27}\beta^2 + \frac{25}{27}\gamma^2 + \frac{32\sqrt{2}}{27}\beta\gamma \right) \times \left(u^2(r) - \frac{v^2(r)}{3} \right) + \frac{8}{3\sqrt{3}}\alpha\beta(a+b)\frac{d\sigma}{dr}\Phi \right], \tag{50}$$

$$\sigma(\pi N) = 4\pi f_{\pi} m_{\pi}^2 \int_0^{\infty} dr r^2 [\sigma(r) - f_{\pi}]. \tag{51}$$

The change in the magnetic moments, coupling constant ($\frac{g_A}{g_v}$), and σ commutator $\sigma(\pi N)$ is induced by the dynamics of the fields in Eqs. (25)–(28).

VI. DISCUSSION OF RESULTS

The set of nonlinear differential equations have been solved in the same manner as Aly *et al.* [17]. The iteration procedure is implemented in the following calculations. As to the fixed values α, β, and γ, the set of differential equations with the corresponding boundary conditions are solved by using the modified numerical package (COLSYS) as used in Ref. [17]. The solutions of the system are mixed and repeated until self-consistency is achieved. The present work contains two free parameters: the coupling constant g and the σ mass as in Refs. [12,16,17].

The corresponding fields for a typical solution are shown in Fig. 1. They agree with those obtained by the original work of Aly *et al.* [17] and the results in the mean-field approximation model in Ref. [29], where the pion field takes the P-wave and so it rises linearly from zero at the center of the soliton and takes its maximum value at the radius of soliton ($r = 0.57$). In addition, the sigma wave function its takes maximum value at $r = 0.0$. From Fig. 2, we note that the kinetic energy of a quark is greater than the kinetic energy reported in the original work [17]. The kinetic energy of the mesons is slightly less than that of the original work [17]. The effect of meson-quark interactions is investigated and is shown in Fig. 3. The behavior is the same in the two models, but the logarithmic mesonic potential has greater values for quark-meson interactions in comparison with the original work of Aly *et al.* [17]. The largest values for the delta and nucleon masses are obtained, wherever the masses are increasing with increasing coherence parameter x; thus, the best results for the nucleon and delta masses ($M_N = 1175$, $M_{\Delta} = 1231$) are obtained for $x = 2.64$ and the δ mass is in good agreement with the data (see Fig. 4).

Next consider the nucleon properties. Table I shows the results for several nucleon properties compared with the previous calculation [17] and data. The values of the observables to the nucleon are improved. Comparing the predictions of the logarithmic quark model to those of the work Aly *et al.* [17], we find that the quark contributions to each observable are different; however, the coherence parameter x is increased up to 4, which is not possible in the previous calculation of [17]

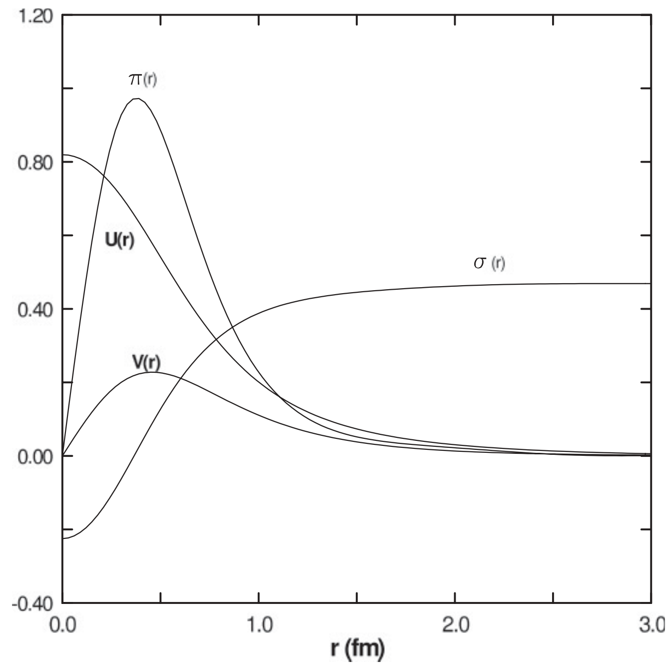

FIG. 1. Components of the quark fields, σ and π fields as functions of r. The calculations are performed for the pion-coupling constant $g = 5.0$, $m_{\sigma} = 550$ MeV, and $x = 4$.

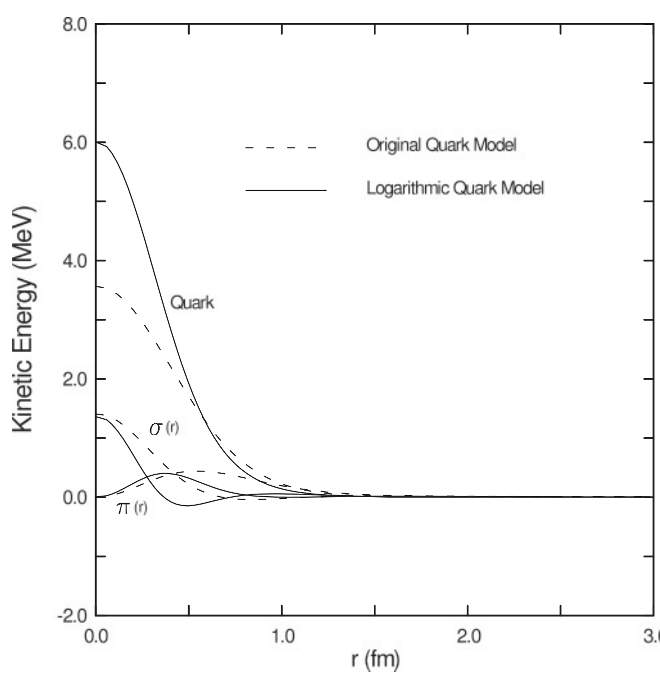

FIG. 2. Comparison of the kinetic energy for the logarithmic quark model and the original quark model using a coherence parameter of $x = 4$ with $g = 5$ and $m_{\sigma} = 550$ MeV.

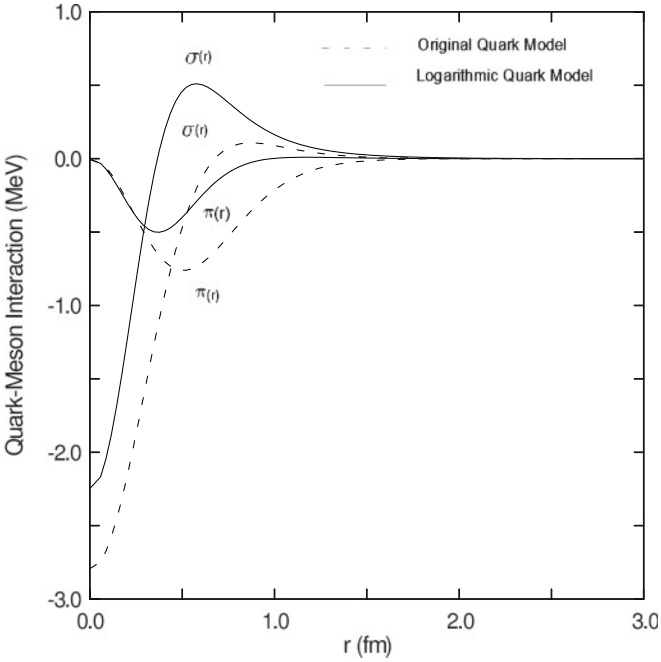

FIG. 3. Comparison of the quark-meson interaction energies for the logarithmic quark model and the original quark model using a coherence parameter of $x = 4$ with $g = 5$ and $m_\sigma = 550$ MeV.

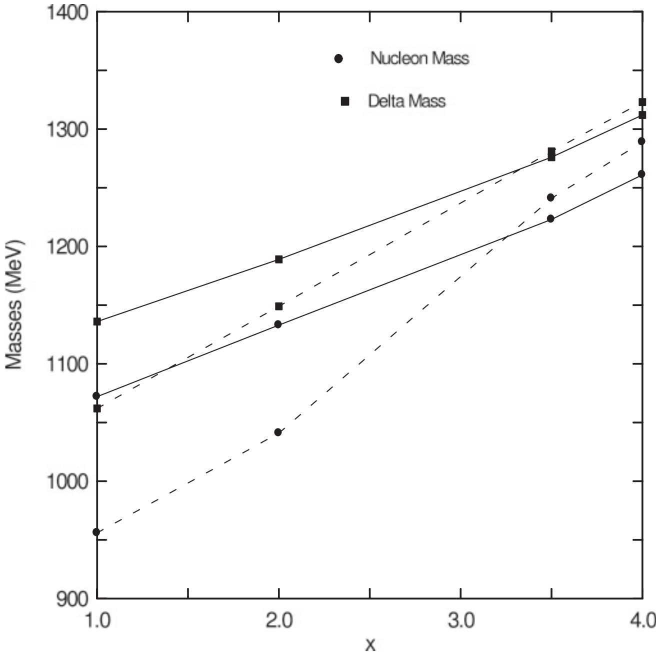

FIG. 4. The dependence of the δ and nucleon masses on the coherence parameter x where solid curves represent the logarithmic quark model and dashed curves represent the original quark model.

wherever the largest values in $(\frac{g_A}{g_v})$, $g_{\pi NN}(0)$, and $\sigma(\pi N)$ are obtained and conflicts with the data.

Considering the square nucleon-charge radius, we find that the quark and meson contributions are greater than the contributions in Ref. [17]. However, the proton radius is improved so that the relative error is about 14% compared to the data. Also, the square of the neutron radius is corrected so that the relative error is about 97% compared to the data in Ref. [17]; therefore this error has been reduced by 25% in our approach. The situation is similar for the magnetic moments. The quark contribution and mesonic contribution are increased due to the increase pionic degrees by increasing the coherence parameter x leading to an increase in the nucleon magnetic moments of about 9%.

The problems in the linear σ model are the greater values of the axial-vector coupling constant $(\frac{g_A}{g_v})$, pion-nucleon coupling constant $g_{\pi NN}(0)$, and σ commutator $\sigma(\pi N)$ in comparison with the data. With our approach, we were able to successfully predict these quantities since the logarithmic potential did not include the higher-order mesonic contributions of σ^2 and π^2, which leads to a reduction in the mesonic contribution in these quantities.

The axial-vector coupling constant is in good agreement with the data. This is an important property, in particular, for a chiral model such as the present one. The result obtained is $\frac{g_A}{g_v} = 1.23$, which is the ratio of the axial-vector coupling constant to the vector coupling constant. If one uses the Goldberger-Treiman relation and evaluates $g_{\pi NN}(0)$ equal to $M_n(\frac{g_A}{g_V})f_\pi^{-1}$ as obtained in Table I, the value of $g_{\pi NN}(0)$ is found to be equal to 0.92, which is in good agreement with the data.

TABLE I. Observables for the coherent-pair nucleon with a coherence parameter of $x = 4$ with $m_\sigma = 441$ MeV and $g = 5$. The magnetic moments are in nuclear magnetons. The charge radius is in fm. For comparison, the results from the coherent-pair approximation model of Aly *et al.* [17] are also presented.

Quantity	Present approach			Aly *et al.* [17]			Expt.
	Quark	Meson	Total	Quark	Meson	Total	
$\langle r^2\rangle_p$	0.71	0.17	0.88	0.533	0.023	0.556	0.70
$\langle r^2\rangle_n$	0.02	−0.17	−0.15	0.019	−0.023	−0.004	−0.12
$\langle\mu_p\rangle$	1.710	0.1396	1.850	1.53	0.18	1.71	2.79
$\langle\mu_n\rangle$	−1.263	−0.139	−1.402	−1.13	−0.18	−1.31	−1.91
$\frac{g_A(0)}{g_V(0)}$	1.13	0.0896	1.227	1.07	0.39	1.46	1.25
$g_{\pi NN}(0)\frac{m_\pi}{2M_N}$	0.848	0.067	0.92	1.11	0.24	1.35	1.0

The σ commutator $\sigma(\pi N)$ is the fundamental parameter of low-energy hadron physics since it provides a direct measure of the scalar quark condensates in baryons and constitutes a test for the mechanism of chiral symmetry breaking [30]. We obtain a good value for the σ commutator, 45 MeV when $x = 4$, $m_\sigma = 680$, and $g = 5$. Our result for the $\sigma(\pi N)$ is in perfect agreement with the value obtained by Gasser, Leutwyler, and Sainio [31] and with the results of other theoretical approaches such as the cloudy bag model [32], chiral quark soliton model [33], lattice QCD [34], and the perturbative chiral quark model [30] where the pion cloud contribution was properly taken into account.

Finally, from Table II, the σ commutator $\sigma(\pi N)$ has been improved by about 49% with respect to the original model [17] and by, 35% compared to the result in the mean-field approximation [29].

VII. COMPARISON WITH OTHER MODELS

It is interesting to compare the nucleon properties in the present approach with other models. Here we consider three models: the perturbative chiral quark model [7–11], the mean-field model [29], and the extended Skyrme model [2]. The perturbative chiral quark model is an effective model of baryons based on chiral symmetry. The baryon is described as a state of three localized relativistic quarks supplemented by a pseudoscalar meson cloud as dictated by chiral symmetry requirements. In this model, the effect of the meson cloud is evaluated perturbatively in a systematic fashion. The model has been successfully applied to the nucleon properties (see Table II). We obtained reasonable results in comparison with the perturbative chiral quark model which is based on a nonlinear σ model Lagrangian and provides a good description of the nucleon properties. In particular, the axial-vector coupling constant $(\frac{g_A}{g_v})$, pion-nucleon coupling constant $g_{\pi NN}(0)$, and σ commutator $\sigma(\pi N)$ have been improved. The original Skyrme model Lagrangian [35] consists of the nonlinear σ model term and the fourth-order derivative term, which guarantees the stabilization of the soliton so that the degree of freedom of the σ field may be replaced by a variable chiral radius, which becomes the new dynamical degree of freedom and plays an important role in the modified Skyrmion Lagrangian density [2], leading to a better description of nucleon properties. In comparison with the extended Skyrme model [2], the results obtained for the nucleon and δ masses and the axial-vector coupling constant $(\frac{g_A}{g_v})$, pion-nucleon coupling constant $g_{\pi NN}(0)$, and σ commutator $\sigma(\pi N)$ have been improved (see Table II).

The mean-field approximation model has been introduced in Ref. [29]. In this model, the meson fields are treated as time-independent classical fields. This means that we replace the powers and products of the (MFT) by the corresponding powers and products of their expectation values [1]. In [29], it is argued that spontaneous symmetry breaking of the quantum chromodynamics (QCD) Lagrangian gives rise to an effective chiral Lagrangian of Gell-Mann–Levy σ model by involving explicit quark, scalar-isoscalar σ-meson, and pseudoscalar-isovector pion-meson degrees of freedom. There is no confinement in this model, and nucleons appear as bound states of a three-quark system. The bound states of the model have been solved in the mean field using the hedgehog ansatz [1] which assumes a configuration-space-isospin correlation for the pion field, $\pi = \pi\hat{\mathbf{r}}$ and for the quarks. One drawback of this ansatz is that it breaks both rotational J and isospin invariance I (although the grand spin $G = I + J$ remains conserved) requiring some projection onto physical states at the end. In spite of this drawback, the model is successful at predicting baryon properties (see Table II). Another weakness of their approach (MFT) was the use of semiclassical mean-field approximation to describe the very light pion. Logarithmic mesonic potential is investigated in the mean-field approximation, which gives a good description for hadron properties as in Refs. [28,29], but has the greatest values for the axial-vector coupling constant $(\frac{g_A}{g_v})$ and pion-nucleon coupling constant $g_{\pi NN}(0)$ compared with the data (see Table II). By considering the quantum effects in our approach, we obtain a good description of these values which closely agrees with the data. In addition, the results of Ref. [29] were successfully applied to the study of σ-term physics, which results in a $\sigma(\pi N)$ term equal to 70 MeV, but it still has a large value in comparison with the value (45 ± 5) in Ref. [34]. We successfully predicted the value of $\sigma(\pi N) = 45$ MeV for $(x = 4,\ g = 5, m_\sigma = 680)$, which is one of the advantages of this approach. Also, we have calculated the radius of the proton and neutron in this approach, which is not calculated in the mean-field approximation model [29].

TABLE II. Observables of the nucleon with the logarithmic σ model [29], modified Skyrme model [2], and perturbative chiral quark model [7–11].

Quantity	Present work	[17]	[29]	[2]	[7–11]	Expt.
M_N	1175	1073	966	1436	828.5	938
M_Δ	1231	1224	1348	1722	1124.9	1232
$\langle r^2\rangle_p$	0.88	0.556	–	0.71	–	0.7
$\langle r^2\rangle_n$	−0.15	−0.004	–	−0.18	–	−0.12
μ_p	1.85	1.71	2.77	2.9	2.62 ± 0.02	2.79
μ_n	−1.40	−1.31	−2.11	−2.1	-2.0 ± 0.02	−1.91
$\sigma(\pi N)$	45	88.9	70	54	54.7	45 ± 5
$g_A(0)$	1.23	1.35	1.76	0.78	1.19	1.25
$g_{\pi NN}(0)\frac{m_\pi}{2M_N}$	0.921	1.46	1.44	0.61	–	1.0

VIII. SUMMARY AND CONCLUSION

In the present work, we examine the effect of coherent-pair approximation on hadron properties in the logarithmic quark model [29]. The coherent-pair approximation (CPA) has some advantages in comparison with the mean-field approximation and provides a systematic expansion method for the description of a boson field. In addition, it avoids assumptions like the hedgehog structure of the quark and pion fields [16]. The CPA is too restrictive for the pionic degree of freedom [16]. The average number of pions in the nucleon is calculated to be about one-half, whereas the mean-field approximation yields twice as much [17].

From the results, we have obtained a good description for nucleon properties. In particular, the axial-vector coupling constant ($\frac{g_A}{g_v}$), pion-nucleon coupling constant $g_{\pi NN}(0)$, σ commutator $\sigma(\pi N)$, and the squared neutron radius $\langle r^2\rangle_n$ are in good agreement with the data.

The best results for nucleon and δ masses ($M_N = 1175$, $M_\Delta = 1231$) are obtained for $x = 2.64$, with a δ mass that is in good agreement with data and a relative error in the nucleon mass of about 9% compared to original model [17] and a relative error of 25% with respect to the data due to the increase in the kinetic energies of the σ and pion mesons, so we need to extend the chiral symmetry to include the kaon and η-meson cloud contributions to improve this quantity as in the perturbative chiral quark model [7]. The second solution is to increase the higher-order mesonic contributions in the framework of the chiral symmetry without introducing more parameters in our approach.

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