PHYSICAL REVIEW C **81**, 015202 (2010)

# Magnetic moments of the proton and of octet baryons in a quasiparticle diquark model

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(Received 29 June 2009; revised manuscript received 18 November 2009; published 11 January 2010)

The strange contribution to the proton magnetic moment is investigated assuming a uuds$\bar{s}$ configuration for the proton in the context of a quasiparticle diquark model. The magnetic moment and mass of octet pentaquark baryons are also investigated considering the diquark-diquark-antiquark scheme. The results are found to be in agreement with the existing theoretical and experimental predictions.

DOI: 10.1103/PhysRevC.81.015202 PACS number(s): 12.39.Mk, 14.20.Dh, 14.20.Gk, 24.85.+p

## I. INTRODUCTION

Recent experiments [1] have suggested a strange quark contribution to the proton magnetic moment and have predicted that the contribution is positive, whereas most theoretical calculations suggest that it is negative [2]. The magnetic moment is an important quantum number for studying the internal structure of a particle. It gives us an idea about the gyromagnetic ratio of the particle and its electroproduction. It is well understood, as new experimental results are obtained, that our knowledge of proton and neutron structures is still limited and a complete description must include the sea of gluons and the virtual $q\bar{q}$ pair arising from the interaction of quarks. It has been suggested that at least 5%–10% of the magnetic moment of the proton comes from the contribution of the sea of strange quarks [1]. These experimental results suggest a pentaquark configuration of the proton as uuds$\bar{s}$ [3]. The contribution to the proton magnetic moment owing to strange quarks has been examined by a number of authors. Zou and Riska [3] investigated the strange quark contribution to the proton in the context of uuds$\bar{s}$ considering possible configurations that yield both a positive and a negative value of $\mu_s$. Bijker [4] investigated the strange form factors and $\mu_s$ of protons in the context of a two-component model consisting of a three-quark intrinsic structure surrounded by a meson cloud. He obtained a positive value of $\mu_s$. Riska [5] studied the $\mu_s$ of protons in a chiral quark model and obtained a much lower value of $\mu_s$.

Pentaquarks are of special interest since the discovery of the $\theta^+$ particle by several groups [6]. Wang [6] investigated the magnetic moment of the $\theta^+$ particle in the light cone quantum chromodynamics (QCD) sum rule considering a diquark-diquark-antiquark configuration and suggested that $\mu_{\theta^+} = -(0.49 \pm 0.06)\mu_n$. The chiral soliton model [7] predicts a low-lying antidecuplet state with three-flavor generalization. Jaffe and Wilczek (JW) [8] suggested that pentaquarks may belong to flavor octet $8_f$ and antidecuplet $10_f$ states. They stated that a pentaquark may be described as a diquark-diquark-antiquark configuration. Liu *et al.* [9] estimated the magnetic moments of octet pentaquarks in addition to the antidecuplet in the context of different models. The mass of octet pentaquarks was investigated by Majee and Raychaudhuri [10] in the context of QCD, and they suggested that the splitting between the mass of the octet state and that of the corresponding antidecuplet state is typically at 500 to 600 MeV.

In the present work we study the magnetic moment and mass of octet pentaquarks in the framework of a quasiparticle diquark model with a diquark-diquark-antiquark configuration. We suggest a quasiparticle model for the diquark in analogy with an electron in a crystal lattice. The strangeness contribution to the proton magnetic moment is estimated considering the pentaquark uuds$\bar{s}$ configuration for the proton. Our quasiparticle scheme of the diquark is used to investigate the properties of octet pentaquarks and the proton in the diquark-diquark-antiquark scheme. Some interesting observations are made.

## II. QUASIPARTICLE PICTURE OF THE DIQUARK

The role of the diquark in baryon spectroscopy has been discussed by a number of authors [11]. Diquarks are described as strongly correlated quark pairs, and in QCD both the one-gluon exchange interaction and the instanton-induced interaction favor a spin singlet and color anti-antisymmetric diquark combination. The exact nature of the diquark is under extensive study at present. We have suggested a model for the diquark in which it is described as a quasiparticle, in analogy to an electron in a crystal lattice [12]. It is well known that quasiparticles are particlelike entities arising in some systems of interacting particles. They are low-lying, excited, hypothetical states possessing an energy very close to the ground state, and to a considerable extent, the properties of a system can be determined by investigating the properties of the quasiparticles [12]. Such states are often observed in the usual condensed matter physics. An electron in a crystal is subjected to two types of forces, namely, the effect of the crystal field ($\nabla V$) and an external force ($F$) that accelerates the electron [12]. Under the influence of the two forces, an electron in a crystal behaves like a quasiparticle whose effective mass $m^*$ reflects the inertia of an electron that is already in a crystal field. The bare electron (with a normal mass) is affected by the lattice force $-\nabla V$ and the external force $F$ so that the ratio of the normal mass $m$ to the effective mass $m^*$ can be expressed

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as [12]

$$m/m^* = 1 - \frac{1}{F}\left[\frac{\delta \overline{V}}{\delta x}\right]. \tag{1}$$

We have proposed a similar scheme for the diquark $[ud]_0$ as a quasiparticle inside a baryon. It is now well understood experimentally that hadrons are complicated systems and a complete description includes the virtual sea of gluons, virtual $q\bar{q}$ pairs in addition to valance quarks. So the situation may resemble the situation of an electron in a crystal lattice. We have assumed that the diquark is a fundamental constituent and an independent body behaving like a quasiparticle inside the hadron. It is under the influence of two types of forces inside the hadron, simulating the many-body interaction of the complicated structure of hadrons. One is due to the background meson cloud, which is represented by potential $\overline{V} = (-\frac{2}{3})\alpha_s/r$ and resembles the crystal field on a crystal electron. For the external force we consider an average harmonic oscillator type of force of the confinement type. Hence the potential may be expressed as

$$V_{ij} = -\frac{\alpha}{r} + (F_i F_j)\left(-\frac{1}{2}Kr^2\right), \tag{2}$$

where the coupling constant $\alpha = \frac{2}{3}\alpha_s$, $F_iF_j = -\frac{2}{3}$ for a $qq$ interaction [13], and $K$ is the strength parameter, so that $V_{ij}$ becomes $V_{ij} = -(\alpha/r) + ar^2$, where $a = K/3$. Here $V = -(\alpha/r)$ corresponds to $V$ in Eq. (1) and $F = -\nabla ar^2 = -2ar$ replaces $F$ in Eq. (1). Now $dV/dr = \alpha/r^2$, which yields $(1/F)(dV/dr) = -\alpha/2ar^3$. Hence in analogy with Eq. (1), the ratio of the constituent mass to the effective mass of the diquark can be expressed as

$$\frac{m}{m^*} = 1 + \frac{\alpha}{2ar^3}, \tag{3}$$

where $a = K/3$, $m$ is the constituent mass, $m^*$ is the effective mass of the diquark, and $r$ is the radius parameter of the diquark. To calculate the effective mass of the diquark we need its radius parameter $r$, which is not exactly known. We have considered the radius of the $[ud]_0$ diquark as the radius of the $\pi$ meson, $r_{ud} = 5.38\,\text{GeV}^{-1}$ [14], whereas the radius of the $[us]_0$ diquark is computed as $r_{us} = 6.06\,\text{GeV}^{-1}$, fitting the experimental mass of the $\Xi$ with a configuration of two $[us]_0$ diquarks and one $\bar{u}$. With $m_u = m_d = 360\,\text{MeV}$, $m_s = 540\,\text{MeV}$, $\alpha = \frac{2}{3}\alpha_s = 0.393$ as $\alpha_s = 0.58$, and $K = 241.5\,\text{MeV fm}^{-2}$ [15], the diquark masses are estimated as $[m_{ud}] = 506\,\text{MeV}$ and $[m_{us}] = 700\,\text{MeV}$. It is noteworthy that in QCD both the gluon exchange interaction and the instanton-induced interaction favor a spin singlet and color antisymmetric diquark combination, and the diquark mass is predicted to be 420 MeV. The diquark mass has been computed by a number of authors in the context of different models. The JW [8] model suggests the values $m_{ud} = 420\,\text{MeV}$ and $M_{us} = 600\,\text{MeV}$, whereas the Karliner and Lipkin model suggests that these values are 720 and 900 MeV, respectively [16].

## III. STRANGE CONTRIBUTION TO THE PROTON MAGNETIC MOMENT

Recent experiments [1] suggest a strange quark contribution to proton spin and have predicted that the strangeness magnetic moment of the proton is positive. To incorporate the strangeness contribution to proton spin, a pentaquark configuration like uuds$\bar{s}$ for the proton has been suggested. Note that the spin contribution is positive in the uuds$\bar{s}$ system when $\bar{s}$ is in the ground state and the uuds system is in the P state. Considering a proton consisting of two diquarks, $[ud]_0$ and $[us]_0$, which are supposed to be in the P state and have the configuration $[ud] \times [us]$—that is, two scalar diquarks—and the antiquark $\bar{s}$ in the ground state, the strangeness magnetic moment can be expressed as in [3]:

$$\mu_s = \frac{m_p}{3m_s}\left(1 + \frac{2m_s}{m_{ud} + m_{us}}\right)P_{s\bar{s}}, \tag{4}$$

where $P_{s\bar{s}}$ is the probability of the $s\bar{s}$ component. With the spin flavor wave function of the proton as in [3] and $P_{s\bar{s}} \sim \mu_s^{\text{exp}}$ [3], we estimated $\mu_p^s$ with the input of the diquark mass as computed in the framework of the quasiparticle model and obtained a strange quark contribution to the proton spin of $0.467\mu_n$. Note that various calculations predict a value of $\mu_p^s$ ranging from $0.8\mu_n$ to $0.003\mu_n$ [17], whereas the lattice QCD calculation [18] predicts a value of $(-0.046 \pm 0.019)\mu_n$. Pate *et al.* [19] have investigated the strange quark contribution to proton spin by analyzing data from the G0 and HAPPEX Collaborations [1] and argued that the contribution is negative based on their investigation of the $Q^2$ dependence of the axial strange form factor. Note that the G0 Collaboration predicted that 5% of the proton magnetic moment is from strange quarks, whereas Jlab predicted the strange quark contribution to be $\sim$10%. Recently Acha *et al.* [1] (HAPPEX Collaboration) reported the strange contribution to be $G_E^s + 0.09G_M^s = 0.007 \pm 0.011 \pm +0.005$ at $Q^2 = 0.109\,\text{GeV}^2$, whereas Baunack *et al.* (A4 Collaboration) [1] predicted $G_M^s = 0.14 \pm 0.11 \pm 0.11$ at $Q^2 \sim 0.1\,\text{GeV}^2$. These values are low compared to the previous results of Aniol *et al.* (SAMPLE) [1]. Our result seems to be in agrement with the SAMPLE experiment, which predicts the value $\mu_s = 0.37 \pm 0.2 \pm 0.26 \pm 0.07$. The strange contribution was found to be $0.315\mu_n$ by Bijker [4] in the context of a two-component model composed of a three-quark intrinsic structure surrounded by a meson cloud.

## IV. MAGNETIC MOMENT OF MULTIQUARK STATES

The magnetic moment of a compound system is the sum of the magnetic moments from the spin and orbital contributions of each constituent,

$$\vec{\mu} = \sum \vec{\mu_i} = \sum(\vec{g_i}\,\vec{s_i} + \vec{l_i})\mu_i, \tag{5}$$

where $g$ is the $g$ factor of the $i$th constituent and $\mu_i$ is the magneton of the $i$th constituent. Now considering the pentaquark to consist of two scalar diquarks and a spin-$\frac{1}{2}$ quark, a three-body system, the magnetic moment can be

TABLE I. Magnetic moments of pentaquark states. Set I [16]: $m_{ud} = 720$ MeV, $m_{us} = m_{ds} = 900$ MeV. Set II [8]: $m_{ud} = 420$ MeV, $m_{us} = m_{ds} = 600$ MeV. Set III (present work): $m_{ud} = 506$ MeV, $m_{us} = m_{ds} = 700$ MeV.

| $Y, I, I_3$ | Set I | Set II | Set III |
|---|---|---|---|
| $1, \frac{1}{2}, \frac{1}{2}$ | 0.018 | 0.21 | 0.139 |
| $1, \frac{1}{2}, -\frac{1}{2}$ | 0.50 | 0.65 | 0.594 |
| $0, 1, 1$ | 0.007 | 0.14 | 0.095 |
| $0, 1, 0$ | −0.13 | −0.13 | −0.121 |
| $0, 1, -1$ | −0.27 | −0.41 | −0.337 |
| $-1, \frac{1}{2}, \frac{1}{2}$ | 0.41 | 0.46 | 0.439 |
| $-1, \frac{1}{2}, \frac{1}{2}$ | −0.35 | −0.52 | −0.447 |
| $0, 0, 0$ | 0.25 | 0.37 | 0.332 |

written as

$$\vec{\mu} = (g_i \vec{0} + \vec{l_1})\mu_1 + (g_2 \vec{0} + \vec{l_2})\mu_2 + \left(g_3 \vec{\tfrac{1}{2}} + \vec{0}\right)\mu_3 = \vec{l_1}\mu_1 + \vec{l_2}\mu_2 + g_3 \vec{\tfrac{1}{2}}\mu_3. \quad (6)$$

Considering the diquark to be a bosonlike particle with spin 0 as in the JW model [8], the magnetic moment for a pentaquark system with two scalar diquarks and one antiquark, with relative angular momentum $l = 1$ state, can be written as [13]

$$\vec{\mu} = \vec{l_1}\mu_1 + \vec{l_2}\mu_2 + g_3 \vec{\tfrac{1}{2}}\mu_3. \quad (7)$$

As diquarks are considered to be spin-0 particles, the magnetic moment arises because of the relative momentum of the diquarks and the spin of the antiquark. Considering the wave function for diquarks of Liu *et al.* [9], we estimated the magnetic moments of octet pentaquark baryons; they are reported in Table I (set III) along with results from the JW model [9] with two sets of diquark masses (sets I and II) [8], [16]. Note that the magnetic moment is very sensitive to the diquark mass. Our results show agreement with set II [8].

## V. ESTIMATION OF THE OCTET PENTAQUARK MASS

To estimate the mass of octet pentaquarks we considered the diquark-diquark-antiquark configuration for the octet pentaquark as in [9]. The diquark in the present model is considered to be an independent body behaving as a quasiparticle analogous to the quasiparticle in a crystal lattice simulating a many-body interaction and resembling low-lying excitation. They are noninteracting entities. These elementary excitations, simulating the effect of a many-body interaction, are such that they behave like scalar bosons within a hadron. Hence these low excitations may be regarded as separate entities, behaving as quasiparticles within the system. They create an environment around themselves and act as free objects but simulate the interaction through the effective mass. The vacuum then may behave like a superfluid and quasiparticles may resemble phonons. Therefore, as in an ideal gas of quasiparticles, their energies are simply additive [20] and the effective mass characterizes the dynamic properties of the system. Hence the mass of an octet pentaquark can be represented as

TABLE II. Mass of pentaquark states.

| Particle | $(Y, I)$ | $I_3$ | Our results | JW model |
|---|---|---|---|---|
| $p_8$ | $(1, \frac{1}{2})$ | $\frac{1}{2}$ | 1746 | 1460 |
| $n_8$ | | $-\frac{1}{2}$ | 1746 | 1460 |
| $\Sigma_8^+$ | (0, 1) | 1 | 1566 | 1360 |
| $\Sigma_8^0$ | | 0 | 1566 | 1360 |
| $\Sigma_8^-$ | | −1 | 1566 | 1360 |
| $\Lambda_8$ | (0,0) | 0 | 1566 | 1533 |
| $\Xi_8^0$ | $(-1, \frac{1}{2})$ | $\frac{1}{2}$ | 1760 | 1520 |
| $\Xi_8^-$ | | $-\frac{1}{2}$ | 1760 | 1520 |

$$M_x = 2M_D + M_{\bar{q}}, \quad (8)$$

where $M_D$ is the mass of the diquark. The results are reported in Table II along with the results of Zhu [21]. In a recent work Majee and Raychaudhuri [10] investigated the mass of octet pentaquarks in the context of a chiral SU(6) model considering the triquark-diquark configuration suggested by the Karliner and Lipkin model and obtained the mass of $N_8$, $\Sigma_8$, and $\Xi_8$ as 2057, 2217, and 2326 MeV, respectively.

## VI. CONCLUSION

In the present paper the contribution of strange quarks to proton spin has been investigated considering the pentaquark configuration of a proton with two scalar diquarks and an antiquark structure. The quasiparticle model for the diquark has been suggested for study of the strange quark contribution to proton spin. The strange quark content of the nucleon is a very important issue that could shed light on the understanding of the internal structure of the proton and the dynamics of interactions inside the hadron, which are yet to be understood clearly. The existing theoretical and experimental results for $\mu_p^s$ are somewhat inconclusive. The group theoretical calculation yields the results $(+0.155 \pm 0.0022)\mu_n$ and $(+0.161 \pm 0.028)\mu_n$ for two different fits. Forkel [22] has studied the nucleon strange form factor using a dispersive approach and made a detailed investigation of the JW three-pole analysis. He observed that a 40% increase in strangeness radius yields a 20% reduction in $\mu_s$ for the nucleon, yielding $\mu_s = 0.26\mu_n$. Wang *et al.* [23] have investigated the $\theta^+$ magnetic moment in the QCD sum rule using an external weak electromagnetic field and light cone model. They suggested the magnetic moment $\mu_{\theta^+} = -(0.11 \pm 0.02)\mu_n$. In the present work the magnetic moments of octet pentaquarks have been estimated in the context of a quasiparticle model of the diquark. The masses of octet pentaquarks have also been computed and compared with other theoretical estimates. A model of the diquark has been suggested and used to investigate the properties of protons and exotics. In the present work the diquark has been described as a quasiparticle that embodies the interaction of the background medium and is suggested to be the fundamental building block of the hadron. Diquarks are low-lying excited states behaving independently, simulating a many-body interaction. The results obtained here are in reasonably good agreement with the existing experimental and theoretical estimates. Many experimental efforts are now focusing on octet and decuplet pentaquarks, so at this juncture theoretical estimates are of immense importance. Recently Thomas [24] made a detailed analysis of the development in the vector and scalar matrix elements of strange quarks in the proton along with recent experimental predictions. He mentioned that at present the theoretical calculations hold the precision lead and there is need for new ideas so that the experimental determinations may also reach such a level [24]. However, further experiments in the near-future could reveal the exact dynamics underlying the formation of pentaquarks.

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