# <span id="page-0-0"></span>Nuclear suppression factor for  $P<sub>T</sub>$  increasing towards the kinematic limit

Thorsten Renk\*

*Department of Physics, P.O. Box 35 FI-40014, University of Jyväskylä, Finland and Helsinki Institute of Physics, P.O. Box 64 FI-00014, University of Helsinki, Finland* (Received 26 October 2009; published 25 January 2010)

The suppression of high-transverse-momentum  $(P_T)$  hadron production in ultrarelativistic heavy-ion (A-A) collisions as compared to the scaled expectation from proton-proton (p-p) collisions expressed as the nuclear modification factor *R*<sub>AA</sub> is experimentally well established and can be traced back to interactions between the hard parton shower and the soft bulk matter. Intuition suggests that the medium modification should cease to be important when the hard scale of parton production is much larger than the typical momentum scale in the medium (e.g., the temperature *T*) and that consequently  $R_{AA}(P_T) \to 1$  for  $P_T \gg T$ . However,  $R_{AA}$  is not a "simple" observable but rather results from many different mechanisms that influence what happens when  $P_T$ increases. In particular,  $R_{AA}$  does not necessarily approach unity even if the hadron momentum is probed at the kinematic limit of the reaction. The aim of this work is to identify and discuss such mechanisms, to present different model expectations of what one would find if one could measure suppression to the kinematic limit for hard hadron production, and to predict the *PT* dependence at both Relativistic Heavy-Ion Collider (RHIC) and Large Hadron Collider (LHC).

DOI: [10.1103/PhysRevC.81.014906](http://dx.doi.org/10.1103/PhysRevC.81.014906) PACS number(s): 25*.*75*.*Gz

## **I. INTRODUCTION**

Jet quenching [i.e., the energy loss of hard partons created in the first moments of an ultrarelativistic heavy-ion (A-A) collision resulting from interactions with the surrounding soft medium] has long been regarded a promising tool to study properties of the soft bulk medium created along with the hard process  $[1-6]$ . The effect of the medium is apparent from a comparison of high-transverse-momentum *PT* hadron observables measured in A-A collisions with the same observables in proton-proton (p-p) collisions. The current range of such observables includes the suppression in single inclusive hard hadron spectra  $R_{AA}$  [\[7\]](#page-7-0), the suppression of back-to-back correlations [\[8,9\]](#page-7-0) and single hadron suppression as a function of the emission angle with the reaction plane [\[10\]](#page-7-0). Recently, preliminary measurements of fully reconstructed jets also have become available [\[11\]](#page-7-0).

Single hadron observables and back-to-back correlations above 6 GeV (where hadron production is dominated by hard processes) are well described in detailed model calculations using the concept of energy loss  $[12-14]$ , that is, under the assumption that the process can be described by a mediuminduced shift of the leading parton energy by an amount  $\Delta E$ where the probability of energy loss is governed by a distribution  $P(\Delta E)$ , followed by a fragmentation process using vacuum fragmentation of a parton with the reduced energy. This can be cast into the form of a modified fragmentation function (MMFF). If the vacuum fragmentation function, that is, the distribution of hadrons produced from a parton at fractional momentum *z* given a hadronization scale  $\mu^2$ , is  $D(z, \mu^2)$ , then the MMFF given the medium-induced energy

loss probability  $P(\Delta E)$  can be written as

$$
\tilde{D}(z,\mu^2) = \int_0^E d\Delta E P(\Delta E) \frac{D\{[z/(1-\Delta E/E)], \mu^2\}}{1-\Delta E/E}.
$$
 (1)

Beyond the leading parton approximation in which energy loss and fragmentation factorize, one has to solve the full partonic shower evolution equations in the medium while assuming that the nonperturbative hadronization takes place outside the medium. At least for light subleading hadrons in a shower, factorizing hadronization from the medium-modified parton shower is a reasonable assumption at both Relativistic Heavy-Ion Collider (RHIC) and Large Hadron Collider (LHC) kinematics. There are several calculations that utilize such medium-modified showers analytically [\[15–17\]](#page-7-0). Recently, Monte Carlo (MC) codes for in-medium shower evolution, which are based on MC shower simulations developed for hadronic collisions such as PYTHIA  $[18]$  and HERWIG  $[19]$ , also have become available  $[20-25]$ . Unlike current analytical computations, these have full energy-momentum conservation enforced at each branching vertex. In these calculations, the MMFF is obtained directly rather than from an expression such as Eq. (1).

So far, the different pictures for the parton-medium interaction have been explored and compared with data over a finite kinematical window where  $P_T < 20$  GeV. There is a widespread expectation that if the  $P_T$  range of the measurement could be extended, at either RHIC or LHC, the medium effect would eventually disappear. This expectation is based on the idea that the medium is able to modify the hard-parton kinematics at a typical scale set by its temperature *T* , whereas the parton dynamics takes place at a partonic hard scale  $p_T$ , and if  $p_T \gg T$ , then the hard kinematics should be essentially unchanged, which can be realized for large hadronic *PT* . For example, in the case of the nuclear suppression factor,

<sup>\*</sup>thorsten.i.renk@jyu.fi

<span id="page-1-0"></span>this expectation would imply that  $R_{AA}(P_T)$  approaches unity for  $P_T \gg T$ . It is our aim in this article to discuss the physics contained in the shape of  $R_{AA}(P_T)$  and to present which current models, based on both the energy-loss concept and the in-medium parton shower concept, predict for the shape of *R*AA at very large momenta at both RHIC and LHC.

# **II. NUCLEAR SUPPRESSION IN THE ENERGY-LOSS PICTURE**

## **A. Qualitative estimates**

A qualitative argument for why  $R_{AA}$  should increase with *PT* can be made as follows: parton spectra can be approximated by a power law as  $dN/dp_T = \text{const.}/p_T^n$  where  $n \approx 7$ . Assume that one can approximate the effect of the medium by the mean value energy loss  $\langle \Delta E \rangle$  (for realistic energy-loss models, this is not a good approximation because fluctuations around the mean are large). In this case, the energy loss shifts the spectrum. This can be described by the replacement  $p_T \rightarrow p_T + \langle \Delta E \rangle$  in the expression for the parton spectrum.  $R_{AA}(p_T)$  can then be approximated by the ratio of the parton spectra before and after energy loss as

$$
R_{\rm AA}(p_T) \approx \left(\frac{p_T}{p_T + \langle \Delta E \rangle}\right)^n = \left(1 - \frac{\langle \Delta E \rangle}{p_T + \langle \Delta E \rangle}\right)^n, \qquad (2)
$$

and it is easily seen that this expression approaches unity for  $p_T \gg \langle \Delta E \rangle$ . However, it is not readily obvious under which conditions the limit is reached even if the medium properties are known. Parametrically, the medium temperature *T* governs both the medium density and the typical medium momentum scale, but the total energy loss represents the cumulative effect of the medium (i.e., a line integral of medium properties along the path of the partons), and furthermore the physics of medium-induced radiation is rather complicated because interference between different radiation graphs plays a significant role and therefore the mean energy loss is not simply  $\sim$ *T*. Thus, in realistic calculations, the mean energy loss at RHIC conditions is  $\langle \Delta E \rangle \approx O(10)$  GeV even for *T <* 0*.*35 GeV [\[14\]](#page-7-0), and hence it current data are relatively far from the limit.

There are five main points that may be raised against the approximation in Eq. (2).

(i) The estimate holds for partons and does not take into account fragmentation.

However, this is not a crucial issue for the question at hand. The fragmentation function  $D(z, \mu^2)$  is steeply falling with *z*, and as a result, fragmentation processes at low *z* are preferred. However, for a given momentum scale of the hadron spectrum, low *z* implies high parton momentum, and this is suppressed because the parton spectrum is also falling steeply with  $p<sub>T</sub>$ . As a result, there is some typical intermediate  $\langle z \rangle$  (dependent on hadron and parton type) that relates hadron and parton momentum for quarks fragmenting into light hadrons at RHIC kinematics  $\langle z \rangle \approx 0.5{\text -}0.7$ . This means that hadronic  $R_{AA}$  is to first approximation simply scaled by this factor as compared to partonic  $R_{AA}$ . Fluctuations around the average tend to smear out structures in the

partonic  $R_{AA}$  through the hadronization process but do not alter the shape of  $R_{AA}(P_T)$  beyond that. Thus, qualitatively Eq. (2) holds also on the hadronic level.

(ii) The estimate does not distinguish between quarks and gluons.

This is moderately important because energy loss is expected to be stronger for gluons by a factor of 9*/*4 (the ratio of the Casimir color factors). At low  $P_T$ , hadron production is driven by gluonic processes because gluons are copiously available in the low-*x* region in parton distribution functions (PDFs) [\[26–30\]](#page-7-0). However, hadron production at higher  $P_T$  probes more in the high-*x* region in the parton distributions and eventually valence quark scattering dominates. The hadronic  $R_{AA}$  should therefore show a rise from gluonic  $R_{AA}$  to the larger value of quark  $R_{AA}$  that corresponds to the transition from gluon- to quark-dominated hadron production. As shown in Ref. [\[31\]](#page-7-0), this is likely to be the mechanism underlying the rising trend observed in  $R_{AA}$ at RHIC. However, for asymptotically high energies, the mechanism is not relevant, because this is always a quark-dominated regime.

(iii) The estimate neglects fluctuations around the average energy loss.

In the presence of fluctuations,  $P(\Delta E)$  can be written as the sum of three terms, corresponding to transmission without energy loss, shift of the parton energy by finite energy loss, or parton absorption, as

$$
P(\Delta E) = \tilde{T}\delta(\Delta E) + \tilde{S} \cdot \tilde{P}(\Delta E) + \tilde{A} \cdot \delta(\Delta E - E),
$$
\n(3)

where  $\tilde{P}(\Delta E)$  is a normalized probability distribution such that  $\tilde{T} + \tilde{S} + \tilde{A} = 1$ . By inserting this form to average over Eq. (2) with the proper weights, one finds

$$
R_{\rm AA} \approx \tilde{T} + \int d\Delta E \cdot \tilde{S} \cdot \tilde{P}(\Delta E) \left(1 - \frac{\Delta E}{p_T + \Delta E}\right)^n.
$$
\n(4)

It follows that  $R_{AA}$  obtained with this expression is always bounded by  $\tilde{T}$  from below (if a fraction of partons escapes unmodified, no amount of modification to the rest will alter this) and by  $(1 - A)$  from above (if partons are absorbed *independent of their energy*, *R*<sub>AA</sub> will never approach unity). In many calculations, *A* is determined by the condition that a parton is absorbed whenever its calculated energy loss exceeds its energy, that is, *A* and *S* are dependent on the initial parton energy. In particular, in the Armesto-Salgado-Wiedemann  $(ASW)$  formalism  $[32]$ , the energy loss can be formally larger than the initial energy, because the formalism is derived for asymptotically high energies  $E \rightarrow \infty$  and small energy of radiated gluons  $\Delta E \ll E$ but is commonly applied to kinematic situations in which these conditions are not fulfilled.  $R_{AA}$  at given  $p_T$  is then equal to the transmission term  $\tilde{T}$  plus a contribution that is proportional to the integral of  $P(\Delta E)$  from zero up to the energy scale  $E_{\text{max}}$  of the parton, *seen through the filter* of the steeply falling

<span id="page-2-0"></span>

FIG. 1. (Color online) The leading-order (LO) pQCD parton spectrum for  $\sqrt{s} = 200$  GeV compared with a power law fit to the region from 20 to 50 GeV.

parton spectrum. Thus, in the presence of fluctuations,  $R_{AA}$  is dominated by fluctuations toward the low  $\Delta E$ , and *R*<sub>AA</sub> grows with *p<sub>T</sub>* because  $E_{\text{max}}$  grows linearly with  $p_T$ . If  $P(\Delta E)$  includes fluctuations up to a maximal energy loss  $\Delta E_{\text{max}}$ , then for  $p_T \gg \Delta E_{\text{max}}$ the original argument made for constant energy loss applies and  $R_{AA}$  approaches unity. In practice, this may not be observable; the energy loss probability for RHIC kinematics may be substantial up to scales of  $O(100)$  GeV  $[14]$ , that is, of the order of the kinematic limit.

(iv) The perturbative quantum chromodynamics (pQCD) parton spectrum is not a power law.

Although the pQCD parton spectrum is approximated well by a power law in a limited kinematic range, at about  $\sqrt{s}/4$  the power law fit becomes a bad description of the spectrum. This is shown in Fig. 1. Close to the kinematic limit at  $\sqrt{s}/2$ , the parton spectrum falls very steeply. If one would attempt a local power law fit in this region, the region of validity for the fit would be small and *n* would be very large. One can readily see from Eq. [\(2\)](#page-1-0) that even for  $p_T \gg$  $\langle \Delta E \rangle$   $R_{AA}$  does not approach unity when at the same time  $n \to \infty$ . In other words, close to the kinematic limit, even a small  $\Delta E$  causes a massive suppression simply because there are no partons available higher up in the spectrum that could be shifted down. For this reason, close to the kinematic limit,  $R_{AA} \rightarrow 1$ cannot be expected, and instead (dependent on the details of modeling) something like  $R_{AA} \rightarrow \tilde{T}$  should be expected. However, note also that the validity of factorization into a hard process and a fragmentation function has been assumed for hadron production up to the kinematic limit. This may not be true; higher twist mechanisms such as direct hadron production in the hard subprocess (in the context of heavy-ion collisions; see, e.g., [\[33\]](#page-7-0)) may represent a different contribution that, due to color transparency, remains unaffected by the medium at all  $P_T$  and that may be



FIG. 2. (Color online)  $R_{AA}(P_T)$  calculated with nuclear *initialstate* effects only, obtained from the nuclear PDF set [\[28\]](#page-7-0) and the EKS98 set [\[29\]](#page-7-0) of nuclear parton distributions calculated for the whole kinematic range at RHIC.

significantly stronger than fragmentation close to the kinematic limit. This could be effectively absorbed into a modified coefficient  $\tilde{T}$ , which, however, ceases to have a probabilistic interpretation.

(v) The nuclear initial state effects have not been taken into account.

The initial-state nuclear effects, that is, the difference in nucleon  $[26,27]$  and nuclear  $[28-30]$  PDFs, are often thought to be a small correction to the final-state medium effects. Over a large kinematic range, that is quite true. However, as one approaches the kinematic limit and forces the distributions into the  $x \to 1$  valence quark distributions, one probes the Fermi motion region in the nuclear parton distributions where the difference to nucleon PDFs is sizable.

In Fig. 2,  $R_{AA}(P_T)$  is shown for RHIC kinematics, taking into account only the nuclear initial-state effects with two different sets of nuclear PDFs but no final-state medium-induced energy loss. It is readily apparent that over most of the kinematic range  $R_{AA}(P_T) \approx 1$ , but that there is a strong enhancement visible beyond 80 GeV.

### **B. Detailed calculation**

The detailed calculation of  $R_{AA}$  in the energy-loss models presented here follows the Baier-Dokshitzer-Mueller-Peigne-Schiff (BDMPS) formalism for radiative energy loss [\[2\]](#page-7-0) using quenching weights as introduced by Salgado and Wiedemann [\[32\]](#page-7-0), commonly referred to as the ASW formalism.

The probability density  $P(x_0, y_0)$  for finding a hard vertex at the transverse position  $\mathbf{r}_0 = (x_0, y_0)$  and impact parameter **b** is given by the product of the nuclear profile functions as

$$
P(x_0, y_0) = \frac{T_A(\mathbf{r_0} + \mathbf{b}/2)T_A(\mathbf{r_0} - \mathbf{b}/2)}{T_{AA}(\mathbf{b})},
$$
 (5)

where the thickness function is given in terms of Woods-Saxon, the nuclear density  $\rho_A(\mathbf{r}, z)$  is  $T_A(\mathbf{r}) = \int dz \rho_A(\mathbf{r}, z)$ , and  $T_{AA}(\mathbf{b})$  is the standard nuclear overlap function  $T_{AA}(\mathbf{b}) =$  $d^2s T_A(s)T_A(s - b).$ 

<span id="page-3-0"></span>If the angle between outgoing parton and the reaction plane is  $\phi$ , the path of a given parton through the medium  $\zeta(\tau)$  (i.e., its trajectory *ζ* as a function of proper medium evolution time *τ* ) is determined in an eikonal approximation by its initial position **r**<sub>0</sub> and the angle  $\phi$  as  $\zeta(\tau) = [x_0 + \tau \cos(\phi), y_0 + \tau \sin(\phi)]$ where the parton is assumed to move with the speed of light  $c = 1$  and the *x* direction is chosen to be in the reaction plane. The energy-loss probability  $P(\Delta E)_{\text{path}}$  for this path can be obtained by evaluating the line integrals along the eikonal parton path

$$
\omega_c(\mathbf{r_0}, \phi) = \int_0^\infty d\zeta \zeta \hat{q}(\zeta) \quad \text{and} \quad \langle \hat{q} L \rangle(\mathbf{r_0}, \phi) = \int_0^\infty d\zeta \hat{q}(\zeta),\tag{6}
$$

with the relation

$$
\hat{q}(\zeta) = K \cdot 2 \cdot \epsilon^{3/4}(\zeta)(\cosh \rho - \sinh \rho \cos \alpha) \tag{7}
$$

assumed between the local transport coefficient  $\hat{q}(\zeta)$  (specifying the quenching power of the medium), the energy density  $\epsilon$ , and the local flow rapidity  $\rho$  with angle  $\alpha$  between flow and parton trajectory [\[34,35\]](#page-7-0). The medium parameters  $\epsilon$  and  $\rho$  are obtained from a 2 + 1 – *d* hydrodynamical simulation of bulk-matter evolution [\[36\]](#page-7-0), chosen to have the RHIC and the LHC media described within the same framework. The  $\omega_c$  is the characteristic gluon frequency, setting the scale of the energy-loss probability distribution, and  $\langle \hat{q} L \rangle$  is a measure of the path length weighted by the local quenching power. The parameter  $K$  is a tool to account for the uncertainty in the selection of the strong coupling  $\alpha_s$  and possible nonperturbative effects that increase the quenching power of the medium (see the discussion in Ref. [\[9\]](#page-7-0)) and adjusted such that pionic  $R_{AA}$  for central Au-Au collisions is described at one value of *PT* .

By using the numerical results of  $[32]$  and the previous definitions, we can now obtain the energy-loss probability distribution given a parton trajectory as a function of the initial vertex and direction  $(\mathbf{r_0}, \phi)$  as  $P[\Delta E; \omega_c(\mathbf{r}, \phi), R(\mathbf{r}, \phi)]_{\text{path}} \equiv$  $P(\Delta E)_{\text{path}}$  for  $\omega_c$  and  $R = 2\omega_c^2/\langle \hat{q}L \rangle$ . From the energy-loss distribution given a single path, one can define the averaged energy-loss probability distribution  $P(\Delta E)$ <sub>*T*AA</sub> as

$$
\langle P(\Delta E) \rangle_{T_{AA}} = \frac{1}{2\pi} \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} dx_0 \int_{-\infty}^{\infty} dy_0 P(x_0, y_0) P(\Delta E)_{\text{path}}.
$$
\n(8)

The energy-loss probability  $P(\Delta E)_{\text{path}}$  is derived in the limit of infinite parton energy [\[32\]](#page-7-0); however, in the following the formalism is applied to finite kinematics. To account for the finite energy *E* of the partons  $\langle P(\Delta E) \rangle_{T_{AA}}$  is truncated at  $\Delta E = E$ , and  $\delta(\Delta E - E) \int_E^{\infty} d\Delta E P(\Delta E)$  is added to the truncated distribution to ensure proper normalization. The meaning of this correction is that all partons are considered absorbed if their energy loss is formally larger than their initial energy. The momentum spectrum of hard partons is calculated in LO pQCD (explicit expressions are given in Ref. [\[9\]](#page-7-0) and references therein). The medium-modified perturbative production of hadrons can then be computed from

the expression

$$
d\sigma_{\text{med}}^{\text{AA}\to h+X} = \sum_{f} d\sigma_{\text{vac}}^{\text{AA}\to f+X} \otimes \langle P(\Delta E) \rangle_{T_{\text{AA}}} \otimes D^{f \to h}(z, \mu^2),\tag{9}
$$

where  $d\sigma_{\text{vac}}^{\text{AA}\rightarrow f+X}$  is the partonic cross section for the inclusive production of a parton *f* and  $D^{f \to h}(z, \mu^2)$  is the vacuum fragmentation function for the hadronization of a parton *f* into a hadron *h* with momentum fraction *z* and hadronization scale  $\mu$ . From this, the nuclear modification factor  $R_{AA}$  follows as

$$
R_{AA}(p_T, y) = \frac{dN^h_{AA}/dp_T dy}{T_{AA}(b)d\sigma^{pp}/dp_T dy}.
$$
 (10)

# **III. NUCLEAR SUPPRESSION IN THE MEDIUM-MODIFIED SHOWER PICTURE**

#### **A. Qualitative arguments**

In a medium-modified (MM) shower picture, the whole partonic in-medium evolution of a parton shower following a hard process is studied, leading to a modification of the fragmentation function (FF) that is more general than Eq. [\(1\)](#page-0-0). In this framework,  $R_{AA} \rightarrow 1$  is realized if the MMFF becomes sufficiently similar to the vacuum FF. A qualitative argument for why the MMFF should approach the vacuum FF for  $P_T \gg$ *T* can be made by considering, for example, the RAD (radiative energy loss) scenario of the MC code YaJEM (yet another jet energy-loss model). This model is described in detail in Refs. [\[21,22\]](#page-7-0).

The parton shower developing from a highly virtual initial hard parton in this model is described as a series of  $1 \rightarrow$ 2 splittings,  $a \rightarrow bc$ , where the virtuality scale decreases in each splitting (i.e.,  $Q_a > Q_b$ ,  $Q_c$ ) and the energy is shared among the daughter partons *b*, *c* as  $E_b = zE_a$  and  $E_c =$  $(1 - z)E_a$ . The splitting probabilities for a parton *a* in terms of  $Q_a$ ,  $E_a$  are calculable in pQCD, and the resulting shower is computed event by event in an MC framework. In the absence of a medium, the evolution of the shower is obtained using the PYSHOW routine  $[37]$ , which is part of the PYTHIA package  $[18]$ .

In the presence of a medium, the main assumption of YaJEM is that the parton kinematics or the splitting probability is modified. In the RAD scenario, the relevant modification is a virtuality gain,

$$
\Delta Q_a^2 = \int_{\tau_a^0}^{\tau_a^0 + \tau_a} d\zeta \,\hat{q}(\zeta),\tag{11}
$$

of a parton during its lifetime through the interaction with the medium. To evaluate Eq.  $(11)$  during the shower evolution, the momentum space variables of the shower evolution equations need to be linked with a space-time position in the medium. This is done via the uncertainty relation for the average formation time as

$$
\langle \tau_b \rangle = \frac{E_b}{Q_b^2} - \frac{E_b}{Q_a^2} \tag{12}
$$

<span id="page-4-0"></span>and randomized for every splitting by sampling from the distribution

$$
P(\tau_b) = \exp\left[-\frac{\tau_b}{\langle \tau_b \rangle}\right].
$$
 (13)

The limit in which the medium modification is unimportant is then given by  $Q^2 \gg \Delta Q^2$ , that is, if the influence of the medium on the parton virtuality is small compared with the virtuality itself, the evolution of the shower takes place as in vacuum. Note that there is always a kinematical region in which the condition can never be fulfilled: The region  $z \rightarrow 1$  in the fragmentation function represents showers in which there has been essentially no splitting. Because the initial virtuality determines the amount of branchings in the shower, this means one probes events in which the initial virtuality  $Q_0$  is not (as in typical events) of the order of the initial parton energy  $E_0$ , but rather  $Q_0 \sim m_h$ , where  $m_h$  is a hadron mass. Because  $m_h$ is, at least for light hadrons, of the order of the medium temperature,  $Q^2 \gg \Delta Q^2$  cannot be fulfilled in the region  $z \approx 1$ of the MMFF; here the medium effect is always visible.

This is illustrated in Fig. 3. Here the ratio of mediummodified over-vacuum fragmentation function  $D^{q \to h^{-}}(z, \mu_p^2)$ as obtained in YaJEM is shown for a constant medium for different initial partonic scales  $\mu_p \equiv E$ . For an initial scale of  $E = 20$  GeV, one observes that the whole range between  $z = 0.2$  and  $z = 1$  is suppressed in the medium, whereas the region less than  $z = 0.1$  shows enhancement because of the hadronization of the additional medium-induced radiation. For larger initial scales, the region of enhancement becomes confined to smaller and smaller *z*, and the fragmentation function ratio approaches unity across a large range. However, in the region  $z \approx 1$ , suppression due to the medium always persists, as expected. As a consequence, one can expect  $R_{AA} \rightarrow 1$  for  $P_T \gg T$  [where  $\Delta Q^2$  is assumed to be parametrically  $O(T^2)$ ] except near the kinematic limit  $P_T \approx \sqrt{s}/2$ , where the region



FIG. 3. (Color online) Ratio of medium-modified over-vacuum quark fragmentation function into charged hadrons obtained in YaJEM for a constant medium 5 fm long and  $\hat{q} = 2 \text{ GeV}^2/\text{fm}$ . Shown are the results for different initial quark energies *E*.

 $z \approx 1$  of the MMFF is probed and suppression is expected to persist.

## **B. Detailed calculation**

The detailed computation of  $R_{AA}$  within YaJEM is outlined in Ref. [\[22\]](#page-7-0). It shares many steps with the computation within the energy-loss picture as described previously, in particular the medium-averaging procedure.

The basic quantity to compute is the MMFF, given a path through the medium. Because of the approximate scaling law identified in Ref. [\[21\]](#page-7-0), it is sufficient to compute the line integral

$$
\Delta \mathcal{Q}_{\text{tot}}^2 = \int d\zeta \,\hat{q}(\zeta) \tag{14}
$$

in the medium to obtain the full MMFF  $D_{MM}(z, \mu_p^2, \zeta)$  from a YaJEM simulation for a given eikonal path of the showerinitiating parton, where  $\mu_p^2$  is the *partonic* scale. The link between  $\hat{q}$  and medium parameters is given as previously by Eq. [\(7\)](#page-3-0), albeit with a different numerical value for *K*. The medium-averaged MMFF is then computed as

$$
\langle D_{\text{MM}}(z, \mu_{p}^{2}) \rangle_{T_{\text{AA}}} = \frac{1}{2\pi} \int_{0}^{2\pi} d\phi \int_{-\infty}^{\infty} dx_{0} \int_{-\infty}^{\infty} dy_{0} P(x_{0}, y_{0}) D_{\text{MM}}(z, \mu_{p}^{2}, \zeta).
$$
\n(15)

From this, the medium-modified production of hadrons is obtained from

$$
d\sigma_{\text{med}}^{\text{AA}\to h+X} = \sum_{f} d\sigma_{\text{vac}}^{\text{AA}\to f+X} \otimes \left\langle D_{\text{MM}}(z, \mu_{p}^{2}) \right\rangle_{T_{\text{AA}}} \quad (16)
$$

and  $R_{AA}$  via Eq. [\(10\)](#page-3-0). A crucial issue when computing  $R_{AA}$ for a large momentum range is that YaJEM provides the MMFF for a given *partonic* scale, whereas a factorized QCD expression such as Eq. (16) utilizes a fragmentation function at givem *hadronic scale*. In previous publications [\[21,22\]](#page-7-0), the problem has been commented on but not addressed because the variation in momentum scale for current observables is not substantial. In this article, the matching between partonic and hadronic scales is done as follows:

For several partonic scales,  $\langle D_{\text{MM}}(z, \mu_p^2) \rangle_{T_{AA}}$  is computed, and the exponent *n* of a power law fit to the parton spectrum at scale  $\mu_p$  is determined. The maximum of  $z^n \langle D_{MM}(z, \mu_p^2) \rangle_{T_{AA}}$ corresponds to the most likely value  $\tilde{z}$  in the fragmentation process, and thus the partonic scale choice is best for a hadronic scale  $P_T = \tilde{z}\mu_p$ . The hadronic  $R_{AA}$  is then computed by interpolation between different optimal scale choices from runs with different partonic scales. Finally, in the region  $P_T \rightarrow$  $\sqrt{s}/2$ ,  $\langle D_{MM}(z, s/4) \rangle_{T_{AA}}$  is always the dominant contribution to hadron production.

The matching procedure between hadronic and partonic scales choice also leads to a significant improvement in the description of  $R_{AA}$  in the measured momentum range at RHIC as compared to previous results [\[21,22\]](#page-7-0).

### **IV. RESULTS FOR RHIC**

The nuclear suppression factor for 200-AGeV central Au-Au collisions at RHIC, calculated both in the energy-loss



FIG. 4. (Color online) The nuclear suppression factor  $R_{AA}$  at RHIC across the full kinematic range in 10% central 200-AGeV Au-Au collisions. Shown are PHENIX data [\[7\]](#page-7-0), a calculation in the energy-loss picture (ASW) with quark and gluon results shown separately, and a calculation in the medium-modified parton shower picture (YaJEM).

picture (represented by the ASW model) and the mediummodified shower picture (represented by the MC code YaJEM), is shown over the full kinematic range in Fig. 4 and compared with PHENIX data [\[7\]](#page-7-0). For the ASW calculation, the partonic *R*AA is also indicated separately for quarks and gluons.

Before discussing details of the plot, let us recapitulate the main differences between the energy-loss picture as exemplified by the ASW model and the medium-modified shower picture as represented by YaJEM:

- (i) ASW is derived for infinite parton energy; hence,  $P(\Delta E)$  is independent of the initial parton energy, and the only energy dependence arises from the prescription to assign contributions where  $\Delta E > E$  into an absorption term *A*. In contrast, YaJEM is a finite-energy framework where the MMFF explicitly depends on the initial parton energy. In particular, within ASW there is an energy-independent transmission probability  $\tilde{T}$  that bounds  $R_{AA}$  from below.
- (ii) In the energy-loss picture, it is not specified what happens to the lost energy. In contrast, within YaJEM the energy lost from the leading shower partons is recovered explicitly in a low-*z* enhancement of the MMFF.

In Fig 4, these differences are apparent as follows: in the lowest  $P_T$  region from 6 GeV and above, there is small rise of  $R_{AA}$  with  $P_T$  observed in ASW, which is not seen in YaJEM. As apparent from the comparison of the ASW result for pions to the results for quarks and gluons, the rise in this region in the ASW model is driven by the transition from a gluon-dominated to a quark-dominated regime; the ASW hadronic result subsequently approaches the quark result for larger momenta. This transition is also present in YaJEM; however, it is masked by the onset of the low- $P_T$  enhancement, which just starts to become significant below 6 GeV and corresponds to a decreasing trend of  $R_{AA}$  with increasing  $P_T$ .

As a result, the two opposing effects roughly cancel, and the YaJEM result appears flatter than the ASW result between 6 and 25 GeV.

For higher  $P_T$ , there is a region up to 50 GeV in which both the ASW and the YaJEM results decrease slightly. This can be traced back to the facts that the pQCD spectrum is not a power law and that local power law fits result in increasing *n* for higher  $p_T$ . The two curves run in parallel until ≈75 GeV; then the predictions of the two models are strikingly different.

The ASW curve turns upward beyond  $P_T = 75$  GeV. A comparison with Fig. [2](#page-2-0) shows that this has nothing to do with the final-state energy loss but rather reflects the Fermi motion region in the nuclear PDFs. At the kinematic boundary, the curve finally turns over to reach the transmission probability  $\tilde{T}$ , as all shifts in the spectrum at the kinematic boundary result in substantial suppression and the only remaining contributions are unmodified partons. In contrast, the YaJEM result shows a strong suppression from 75 GeV to the kinematic limit. This corresponds to the region  $z \rightarrow 1$  in the MMFF in which suppression was always observed in Fig. [3,](#page-4-0) regardless of the initial energy. This suppression is strong enough to mask the enhancement from the nuclear PDF. In contrast to the ASW model, YaJEM does not include an *E*-independent transmission term; thus  $R_{AA}$  becomes very small toward the kinematic limit. In this, the finite-energy nature of the suppression in YaJEM is apparent. Note that the YaJEM result cannot be computed all the way to the kinematic limit because of a lack of statistics in the MC results at  $z \to 1$ .

It is also clear that there is no region throughout the whole kinematic range at RHIC in either model for which  $R_{AA} \rightarrow 1$ could be observed.

### **V. RESULTS FOR LHC**

It is then natural to expect that  $R_{AA} \rightarrow 1$  could be realized by probing even higher momenta beyond the RHIC kinematic limit, for example, by studying  $R_{AA}$  at the LHC. However, in going to collisions at larger  $\sqrt{s}$ , not only is the kinematic limit is changed but also the production of bulk matter is increased; that is, higher  $\sqrt{s}$  corresponds to a modification of both hard probe *and* medium. However, there is reason to expect that eventually one will find a region in which  $P_T \gg T$ ,  $\Delta E_{\text{max}}$ ,  $\sqrt{\Delta Q^2}$ , and  $R_{AA} \rightarrow 1$  can be realized: the kinematic limit  $\sqrt{s}/2$  increases linearly with  $\sqrt{s}$ . However, the medium density does not. There are different models that try to extrapolate how the rapidity density of produced matter increases with <sup>√</sup>*s*. The Eskola-Kajantie-Ruuskanen-Tuominen (EKRT) [\[38\]](#page-7-0) model is among the models with the strongest predicted increase, and it has the scaling  $dN/dy \approx \sqrt{s}^{0.574}$ . Thus, the rapidity density of bulk matter increases significantly more slowly than the kinematic limit for increasing  $\sqrt{s}$ .

Although the medium lifetime may increase substantially with  $\sqrt{s}$  as well, the more relevant scale is the transverse size of the medium, as high- $p_T$  partons move with the speed of light and will exit the medium once they reach its edge. However, the transverse size of the medium is approximately given by the overlap of the two nuclei and hence to zeroth approximation independent of  $\sqrt{s}$  (beyond, there is the weak



FIG. 5. (Color online) The nuclear suppression factor  $R_{AA}$  at LHC across the full kinematic range in 10% of central 5.5-ATeV Pb-Pb collisions. Shown are results in the energy-loss picture (ASW), with the quark and gluon results shown separately, and a calculation in the medium-modified parton shower picture (YaJEM).

logarithmic growth of all total cross sections with  $\sqrt{s}$ ). Thus, for asymptotically high energies, the integration limits for a line integral along the parton path through the medium will not grow arbitrary large, and the integrand (i.e., the density distribution) will be the main change. All these arguments indicate that  $R_{AA} \rightarrow 1$  can thus be realized for large  $\sqrt{s}$ despite the increased production of bulk matter. The remaining question is if the LHC energy  $\sqrt{s} = 5.5$  ATeV is large enough.

The result of the detailed calculation shown in Fig. 5 indicates that this is not the case. For this calculation, a hydrodynamical evolution based on an extrapolation of RHIC results using the EKRT saturation model [\[36\]](#page-7-0) has been used to account for the increased medium density and lifetime. All other differences to the RHIC result are either plain kinematics or can be traced back to the scale evolution of the MMFF.

As far as the shape of  $R_{AA}(P_T)$  is concerned, the LHC predictions for both ASW and YaJEM agree; however, quantitatively they differ substantially. At the heart of this difference is that ASW is an infinite energy formalism in which the larger  $\sqrt{s}$  of LHC as compared to RHIC is chiefly reflected in the harder slope of the parton spectra but not directly in  $P(\Delta E)$ . In contrast, within YaJEM, in addition to the harder slope of the parton spectrum, there is an explicit scale evolution of the medium effect in the MMFF (see Fig. [3\)](#page-4-0). Because both mechanisms tend to increase  $R_{AA}$ , the combined effect of scale evolution and parton spectrum slope leads, all things considered, to less final-state suppression in the YaJEM result.

The shape of  $R_{AA}(P_T)$  can be understood by the mechanisms also observed in the RHIC case. The initial steep rise and subsequent flattening reflects the changing slope of the parton spectra. Note that the transition from gluon-dominated to quark-dominated hadron production is not an issue over most of the LHC kinematical range. The final enhancement at more than 2 TeV is again driven by the Fermi motion region in

the nuclear PDFs. Unlike in the RHIC case, at LHC kinematics the suppression obtained from YaJEM for this region is not strong enough to mask the enhancement. Finally, close to the kinematic limit, a small  $R_{AA}$  is obtained.

These results indicate that there is no reason to expect that the limit  $R_{AA} \rightarrow 1$  can be observed even with LHC kinematics. However, the general trend for larger  $R_{AA}$  observed in the transition from RHIC to LHC indicates that the limit could be reached for asymptotically high energies over a large kinematic range but not close to the kinematic boundary.

### **VI. DISCUSSION**

So far, the nuclear suppression factor  $R_{AA}$  has been observed experimentally in only a very limited kinematical region. In this region, no strong  $P_T$  dependence has been observed. The main expectation of how  $R_{AA}$  changes if observed over a larger kinematical range is that the suppression should eventually vanish and *R*<sub>AA</sub>should approach unity. The results presented here show that this expression is too simplistic.

In particular, it is wrong to think that the shape of  $R_{AA}(P_T)$ is the result of any single cause. Instead, many effects, among them the slope change of the pQCD parton spectrum, the scale evolution of the medium modification effect, the transition from gluon-dominated to quark-dominated hadron production, and also the initial-state nuclear effects, influence  $R_{AA}(P_T)$  in a characteristic way. Moreover, it is not sufficient to think of going to higher  $P_T$  to see the lessening of the suppression; it matters how one approaches higher  $P_T$ , in particular if one can push a measurement further up in  $P<sub>T</sub>$  with higher statistics or if one measures a different system with higher <sup>√</sup>*s*. Based on the results presented here, it appears unlikely that the simple limit  $R_{AA} \rightarrow 1$  for sufficiently high  $P_T$  can be reached even at LHC kinematics.

These findings may largely be of little practical value because of the impossibility of reaching out to a substantial fraction of the kinematic limit experimentally. However, theoretically they serve well to illustrate that even a hard probe observable such as  $R_{AA}$  is never simple, in the sense that it reflects directly tomographic properties of the medium, but rather that it is a convolution of many different effects that all need to be understood and discussed carefully. In particular,  $R_{AA}$  cannot be interpreted as an observable reflecting properties of the medium causing a final-state effect. The shape of the underlying parton spectrum or initial-state effects are equally important to understanding  $R_{AA}$ .

#### **ACKNOWLEDGMENTS**

Discussions with Will Horowitz, Kari Eskola, and Hannah Petersen are gratefully acknowledged. This work was supported by an Academy Research Fellowship from the Finnish Academy (Project 130472) and from Academy Project 115262. The numerical computations were carried out with generous support by Helen Caines on the **bulldogk** cluster at Yale University.

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