Spin-polarized states in neutron matter in a strong magnetic field

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Spin-polarized states in neutron matter in strong magnetic fields up to 10^{18} G are considered in the model with the Skyrme effective interaction. By analyzing the self-consistent equations at zero temperature, it is shown that a thermodynamically stable branch of solutions for the spin-polarization parameter as a function of density corresponds to the negative spin polarization when the majority of neutron spins are oriented opposite to the direction of the magnetic field. Besides, beginning from some threshold density dependent on magnetic field strength, the self-consistent equations also have two other branches of solutions for the spin-polarization parameter with the positive spin polarization. The free energy corresponding to one of these branches turns out to be very close to that of the thermodynamically preferable branch. As a consequence, in a strong magnetic field, the state with the positive spin polarization can be realized as a metastable state in the high-density region in neutron matter, which, under decreasing density, at some threshold density changes to a thermodynamically stable state with the negative spin polarization.

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I. INTRODUCTION

Neutron stars observed in nature are magnetized objects with magnetic field strength at the surface in the range of 10^9–10^{13} G [1]. For special classes of neutron stars such as soft γ-ray repeaters and anomalous X-ray pulsars, the field strength can be much larger and is estimated to be about 10^{14}–10^{15} G [2]. These strongly magnetized objects are called magnetars [3] and comprise about 10% of the whole population of neutron stars [4]. However, in the interior of a magnetar the magnetic field strength may be even larger, reaching values of about 10^{18} G [5,6]. The possibility of the existence of such ultra-strong magnetic fields is not yet excluded because what we can learn from observations of magnetar periods and spin-down rates or from hydrogen spectral lines is limited to a magnetar's surface fields. There is still no general consensus regarding the mechanism by which magnetars generate such strong magnetic fields, although different scenarios have been suggested, such as a turbulent dynamo amplification mechanism in a neutron star with the rapidly rotating core at first moments after it goes supernova [2] and the possibility of spontaneous spin ordering in the dense quark core of a neutron star [7].

Under such circumstances, the issue of interest is the behavior of neutron star matter in a strong magnetic field [5,6,8,9]. In a recent study [9], neutron star matter was approximated by pure neutron matter in a model with effective Skyrme and Gogny forces. It has been shown that the behavior of the spin polarization of neutron matter in the high-density region in a strong magnetic field depends crucially on whether neutron matter develops a spontaneous spin polarization (in the absence of a magnetic field) at several times the nuclear matter saturation density typical for the Skyrme forces or the appearance of spontaneous polarization is not allowed at the relevant densities (or delayed to much higher densities), as in the case with the Gogny D1P force. In the former case, a ferromagnetic transition to a totally spin polarized state occurs; in the latter case, a ferromagnetic transition is excluded at all relevant densities and the spin polarization remains quite low even in the high-density region. Note that the issue of the spontaneous appearance of spin-polarized states in neutron and nuclear matter is a controversial one. On the one hand, models with the Skyrme effective nucleon-nucleon (NN) interaction predict the occurrence of spontaneous spin instability in nuclear matter at densities in the range of ϱ_0–$4\varrho_0$ for different parametrizations of the NN potential [10–22] (where $\varrho_0 = 0.16\,\mathrm{fm}^{-3}$ is the nuclear saturation density). For the Gogny effective interaction, a ferromagnetic transition in neutron matter occurs at densities larger than $7\varrho_0$ for the D1P parametrization and is not allowed for D1 or D1S parametrizations [23]. However, for the D1S Gogny force, an antiferromagnetic phase transition happens in symmetric nuclear matter at a density of $3.8\varrho_0$ [24]. On the other hand, for the models with the realistic NN interaction, no sign of spontaneous spin instability has been found so far at any isospin asymmetry up to densities well above ϱ_0 [25–31].

Here we study thermodynamic properties of spin-polarized neutron matter in a strong magnetic field in a model with the Skyrme effective forces. As a framework for consideration, we choose a Fermi liquid approach for the description of nuclear matter [32–34]. Proceeding from the minimum principle for the thermodynamic potential, we get self-consistent equations for the spin-order parameter and the chemical potential of neutrons. In the absence of a magnetic field, the self-consistent equations have two degenerate branches of solutions for

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the spin-polarization parameter, corresponding to the case in which the majority of neutron spins are oriented along the spin quantization axis and that in which they are oriented opposite to it (positive and negative spin polarization, respectively). In the presence of a magnetic field, these branches are modified differently. A thermodynamically stable branch corresponds to the state with the majority of neutron spins aligned opposite to the magnetic field. In a strong magnetic filed, the branch corresponding to the positive spin polarization splits into two branches with positive spin polarization as well. The last solutions were missed in Ref. [9]. We perform a thermodynamic analysis based on a comparison of the respective free energies and conclude that the formation of metastable states in neutron matter with the majority of neutron spins directed along the strong magnetic field is possible. Such a possibility exists because of the strong spin-dependent medium correlations in neutron matter with the Skyrme forces at high densities.

Note that we consider thermodynamic properties of spin-polarized states in neutron matter in a strong magnetic field up to the high-density region relevant for astrophysics. Nevertheless, we take into account the nucleon degrees of freedom only, although other degrees of freedom, such as pions, hyperons, kaons, or quarks, could be important at such high densities.

II. BASIC EQUATIONS

The normal (nonsuperfluid) states of neutron matter are described by the normal distribution function of neutrons $f_{\kappa_1\kappa_2} = \mathrm{Tr}\,\varrho a^+_{\kappa_2} a_{\kappa_1}$, where $\kappa \equiv (\mathbf{p}, \sigma)$, $\mathbf{p}$ is momentum, σ is the projection of spin on the third axis, and ϱ is the density matrix of the system [20,21]. Further, it will be assumed that the third axis is directed along the external magnetic field $\mathbf{H}$. The energy of the system is specified as a functional of the distribution function f, $E = E(f)$, and determines the single-particle energy

$$\varepsilon_{\kappa_1\kappa_2}(f) = \frac{\partial E(f)}{\partial f_{\kappa_2\kappa_1}}. \tag{1}$$

The self-consistent matrix equation for determining the distribution function f follows from the minimum condition of the thermodynamic potential [32,33] and is

$$f = [\exp(Y_0\varepsilon + Y_4) + 1]^{-1} \equiv [\exp(Y_0\xi) + 1]^{-1}. \tag{2}$$

Here the quantities ε and Y_4 are matrices in the space of κ variables, with $Y_{4\kappa_1\kappa_2} = Y_4\delta_{\kappa_1\kappa_2}$, $Y_0 = 1/T$, and $Y_4 = -\mu_0/T$ being the Lagrange multipliers, μ_0 the chemical potential of neutrons, and T the temperature.

Given the possibility for alignment of neutron spins along or opposite to the magnetic field $\mathbf{H}$, the normal distribution function of neutrons and single-particle energy can be expanded in the Pauli matrices σ_i in spin space:

$$\begin{aligned} f(\mathbf{p}) &= f_0(\mathbf{p})\sigma_0 + f_3(\mathbf{p})\sigma_3, \\ \varepsilon(\mathbf{p}) &= \varepsilon_0(\mathbf{p})\sigma_0 + \varepsilon_3(\mathbf{p})\sigma_3. \end{aligned} \tag{3}$$

Using Eqs. (2) and (3), one can express the distribution functions f_0 and f_3 in terms of the quantities ε:

$$\begin{aligned} f_0 &= \tfrac{1}{2}[n(\omega_+) + n(\omega_-)], \\ f_3 &= \tfrac{1}{2}[n(\omega_+) - n(\omega_-)]. \end{aligned} \tag{4}$$

Here $n(\omega) = [\exp(Y_0\omega) + 1]^{-1}$ and

$$\begin{aligned} \omega_\pm &= \xi_0 \pm \xi_3, \\ \xi_0 &= \varepsilon_0 - \mu_0, \quad \xi_3 = \varepsilon_3. \end{aligned} \tag{5}$$

As follows from the structure of the distribution functions f, the quantity $\omega_\pm$, being the exponent in the Fermi distribution function n, plays the role of the quasiparticle spectrum. The spectrum is twofold split because of the spin dependence of the single-particle energy $\varepsilon(\mathbf{p})$ in Eq. (3). The branches $\omega_\pm$ correspond to neutrons with spin up and spin down.

The distribution functions f should satisfy the normalization conditions

$$\frac{2}{\mathcal{V}}\sum_{\mathbf{p}} f_0(\mathbf{p}) = \varrho, \tag{6}$$

$$\frac{2}{\mathcal{V}}\sum_{\mathbf{p}} f_3(\mathbf{p}) = \varrho_\uparrow - \varrho_\downarrow \equiv \Delta\varrho. \tag{7}$$

Here $\varrho = \varrho_\uparrow + \varrho_\downarrow$ is the total density of neutron matter and $\varrho_\uparrow$ and $\varrho_\downarrow$ are the neutron number densities with spin up and spin down, respectively. The quantity $\Delta\varrho$ may be regarded as the neutron spin-order parameter. It determines the magnetization of the system $M = \mu_n\Delta\varrho$, with μ_n being the neutron magnetic moment. The magnetization may contribute to the internal magnetic field $B = H + 4\pi M$. However, we will assume, analogously to Refs. [6,9], that the contribution of the magnetization to the magnetic field B remains small for all relevant densities and magnetic field strengths; hence,

$$B \approx H. \tag{8}$$

This assumption holds true because of the tiny value of the neutron magnetic moment $\mu_n = -1.9130427(5)\mu_N \approx -6.031 \times 10^{-18}\mathrm{MeV/G}$ [35] (μ_N being the nuclear magneton) and is confirmed numerically by finding solutions of the self-consistent equations in two approximations, corresponding to preserving and neglecting the contribution of the magnetization.

To get the self-consistent equations for the components of the single-particle energy, one must set the energy functional of the system. In view of the approximation (8), this reads [21,33]

$$\begin{aligned} E(f) &= E_0(f, H) + E_{\mathrm{int}}(f) + E_{\mathrm{field}}, \\ E_0(f, H) &= 2\sum_{\mathbf{p}} \varepsilon_0(\mathbf{p}) f_0(\mathbf{p}) - 2\mu_n H \sum_{\mathbf{p}} f_3(\mathbf{p}), \\ E_{\mathrm{int}}(f) &= \sum_{\mathbf{p}} [\tilde{\varepsilon}_0(\mathbf{p}) f_0(\mathbf{p}) + \tilde{\varepsilon}_3(\mathbf{p}) f_3(\mathbf{p})], \\ E_{\mathrm{field}} &= \frac{H^2}{8\pi}\mathcal{V}, \end{aligned} \tag{9}$$

where

$$\begin{aligned} \tilde{\varepsilon}_0(\mathbf{p}) &= \frac{1}{2\mathcal{V}}\sum_{\mathbf{q}} U_0^n(\mathbf{k}) f_0(\mathbf{q}), \; \mathbf{k} = \frac{\mathbf{p} - \mathbf{q}}{2}, \\ \tilde{\varepsilon}_3(\mathbf{p}) &= \frac{1}{2\mathcal{V}}\sum_{\mathbf{q}} U_1^n(\mathbf{k}) f_3(\mathbf{q}). \end{aligned} \tag{10}$$

Here, $\varepsilon_0(\mathbf{p}) = \frac{\mathbf{p}^2}{2m_0}$ is the free single-particle spectrum, m_0 is the bare mass of a neutron, $U_0^n(\mathbf{k})$ and $U_1^n(\mathbf{k})$ are the normal Fermi liquid (FL) amplitudes, and $\tilde{\varepsilon}_0$ and $\tilde{\varepsilon}_3$ are the FL corrections to the free single-particle spectrum. Note that in this study we are not interested in the total energy density or pressure in the interior of a neutron star. Therefore, the field contribution E_{field}, being the energy of the magnetic field in the absence of matter, can be omitted. Using Eqs. (1) and (9), we get self-consistent equations in the form

$$\xi_0(\mathbf{p}) = \varepsilon_0(\mathbf{p}) + \tilde{\varepsilon}_0(\mathbf{p}) - \mu_0, \quad \xi_3(\mathbf{p}) = -\mu_n H + \tilde{\varepsilon}_3(\mathbf{p}). \quad (11)$$

To obtain numerical results, we utilize the effective Skyrme interaction. The amplitude of the NN interaction for the Skyrme effective forces reads [36]

$$\hat{v}(\mathbf{p},\mathbf{q}) = t_0(1 + x_0 P_\sigma) + \frac{1}{6} t_3(1 + x_3 P_\sigma)\varrho^\beta + \frac{1}{2\hbar^2} t_1(1 + x_1 P_\sigma)(\mathbf{p}^2 + \mathbf{q}^2) + \frac{t_2}{\hbar^2}(1 + x_2 P_\sigma)\mathbf{pq}, \quad (12)$$

where $P_\sigma = (1 + \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)/2$ is the spin exchange operator and t_i, x_i, and β are phenomenological parameters specifying a given parametrization of the Skyrme interaction. In Eq. (12), the spin-orbit term, which is irrelevant for uniform matter, is omitted. The normal FL amplitudes can be expressed in terms of the Skyrme force parameters [33,34]:

$$U_0^n(\mathbf{k}) = 2t_0(1 - x_0) + \frac{t_3}{3}\varrho^\beta(1 - x_3) + \frac{2}{\hbar^2}[t_1(1 - x_1) + 3t_2(1 + x_2)]\mathbf{k}^2, \quad (13)$$

$$U_1^n(\mathbf{k}) = -2t_0(1 - x_0) - \frac{t_3}{3}\varrho^\beta(1 - x_3) + \frac{2}{\hbar^2}[t_2(1 + x_2) - t_1(1 - x_1)]\mathbf{k}^2 \equiv a_n + b_n\mathbf{k}^2. \quad (14)$$

Further, we do not take into account the effective tensor forces, which lead to the coupling of the momentum and spin degrees of freedom [37–39] and, correspondingly, to anisotropy in the momentum dependence of FL amplitudes with respect to the spin quantization axis. Then,

$$\xi_0 = \frac{p^2}{2m_n} - \mu, \quad (15)$$

$$\xi_3 = -\mu_n H + \left(a_n + b_n\frac{\mathbf{p}^2}{4}\right)\frac{\Delta\varrho}{4} + \frac{b_n}{16}\langle\mathbf{q}^2\rangle_3, \quad (16)$$

where the effective neutron mass m_n is defined by the formula

$$\frac{\hbar^2}{2m_n} = \frac{\hbar^2}{2m_0} + \frac{\varrho}{8}[t_1(1 - x_1) + 3t_2(1 + x_2)], \quad (17)$$

and the renormalized chemical potential μ should be determined from Eq. (6). The quantity $\langle\mathbf{q}^2\rangle_3$ in Eq. (16) is the second-order moment of the distribution function f_3:

$$\langle\mathbf{q}^2\rangle_3 = \frac{2}{V}\sum_{\mathbf{q}}\mathbf{q}^2 f_3(\mathbf{q}). \quad (18)$$

In view of Eqs. (15) and (16), the branches $\omega_\pm \equiv \omega_\sigma$ of the quasiparticle spectrum in Eq. (5) read

$$\omega_\sigma = \frac{p^2}{2m_\sigma} - \mu + \sigma\left(-\mu_n H + \frac{a_n\Delta\varrho}{4} + \frac{b_n}{16}\langle\mathbf{q}^2\rangle_3\right), \quad (19)$$

where m_σ is the effective mass of a neutron with spin up ($\sigma = +1$) and spin down ($\sigma = -1$):

$$\frac{\hbar^2}{2m_\sigma} = \frac{\hbar^2}{2m_0} + \frac{\varrho_\sigma}{2}t_2(1 + x_2) + \frac{\varrho_{-\sigma}}{4}[t_1(1 - x_1) + t_2(1 + x_2)],\ \varrho_{+(-)} \equiv \varrho_{\uparrow(\downarrow)}. \quad (20)$$

Note that for totally spin-polarized neutron matter

$$\frac{m_0}{m^*} = 1 + \frac{\varrho m_0}{\hbar^2}t_2(1 + x_2), \quad (21)$$

where m^* is the effective neutron mass in the fully polarized state. Because usually for Skyrme parametrizations $t_2 < 0$, we have the constraint $x_2 \leqslant -1$, which guarantees the stability of totally polarized neutron matter at high densities.

It follows from Eq. (19) that the effective chemical potential μ_σ for neutrons with spin up ($\sigma = 1$) and spin down ($\sigma = -1$) can be determined as

$$\mu_\sigma = \mu + \sigma\left(\mu_n H - \frac{a_n\Delta\varrho}{4} - \frac{b_n}{16}\langle\mathbf{q}^2\rangle_3\right). \quad (22)$$

Thus, taking into account expressions (4) for the distribution functions f, we obtain the self-consistent Eqs. (6), (7), and (18) for the effective chemical potential μ, spin-order parameter $\Delta\varrho$, and second-order moment $\langle\mathbf{q}^2\rangle_3$.

III. SOLUTIONS OF SELF-CONSISTENT EQUATIONS AT $T = 0$. THERMODYNAMIC STABILITY

Here we solve directly the self-consistent equations at zero temperature and present the neutron spin-order parameter as a function of density and magnetic field strength. In solving numerically the self-consistent equations, we utilize the SLy4 and SLy7 Skyrme forces [40], which were constrained originally to reproduce the results of microscopic neutron matter calculations (pressure-versus-density curve). Note that the density dependence of the nuclear symmetry energy, calculated with these Skyrme interactions, gives neutron star models in broad agreement with observable data such as the minimum rotation period, gravitational mass-radius relation, and the binding energy released in supernova collapse [41]. Also, these Skyrme parametrizations satisfy the constraint $x_2 \leqslant -1$ obtained from Eq. (21).

We consider magnetic fields up to the values allowed by the scalar virial theorem. For a neutron star with mass M and radius R, equating the magnetic field energy $E_H \sim (4\pi R^3/3)(H^2/8\pi)$ with the gravitational binding energy $E_G \sim GM^2/R$, one gets the estimate $H_{\max} \sim \frac{M}{R^2}(6G)^{1/2}$. For a typical neutron star with $M = 1.5\,M_\odot$ and $R = 10^{-5}\,R_\odot$, this yields $H_{\max} \sim 10^{18}$ G for the maximum value of the magnetic field strength. This magnitude can be expected in the interior of a magnetar, while recent observations report surface values up to $H \sim 10^{15}$ G, as inferred from the hydrogen spectral lines [42].

To characterize spin ordering in neutron matter, it is convenient to introduce a neutron spin-polarization parameter:

$$\Pi = \frac{\varrho_\uparrow - \varrho_\downarrow}{\varrho} \equiv \frac{\Delta\varrho}{\varrho}. \quad (23)$$

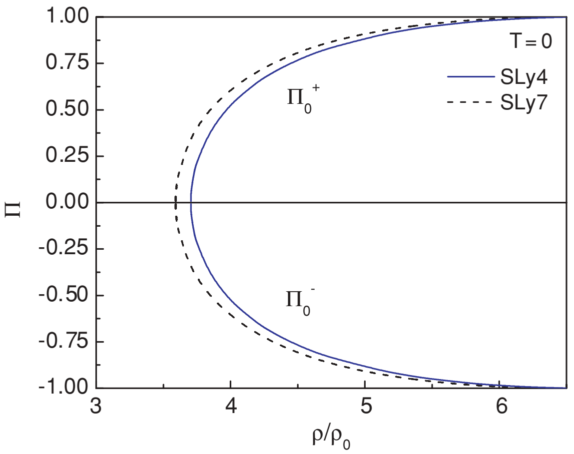

FIG. 1. (Color online) Neutron spin-polarization parameter as a function of density at vanishing temperature and magnetic field.

Figure 1 shows the dependence of the neutron spin-polarization parameter on density, normalized to the nuclear saturation density ϱ_0, at zero temperature in the absence of the magnetic field. Spontaneous polarization develops at $\varrho = 3.70\varrho_0$ for the SLy4 interaction ($\varrho_0 = 0.16\,\mathrm{fm}^{-3}$) and at $\varrho = 3.59\varrho_0$ for the SLy7 interaction ($\varrho_0 = 0.158\,\mathrm{fm}^{-3}$), which reflects the instability of neutron matter with the Skyrme interaction at such densities against spin fluctuations. Because the self-consistent equations at $H = 0$ are invariant with respect to the global flip of neutron spins, we have two branches of solutions for the spin-polarization parameter, $\Pi_0^+(\varrho)$ (upper) and $\Pi_0^-(\varrho)$ (lower), which differ only by sign, $\Pi_0^+(\varrho) = -\Pi_0^-(\varrho)$.

Figure 2 shows the neutron spin-polarization parameter as a function of density for a set of fixed values of the magnetic field. The branches of spontaneous polarization are modified differently by the magnetic field because the self-consistent equations at $H \neq 0$ lose the invariance with respect to the global flip of the spins. At nonvanishing H, the lower branch $\Pi_1(\varrho)$, corresponding to the negative spin polarization, extends down to the very low densities. There are three characteristic density domains for this branch. At low densities $\varrho \lesssim 0.5\varrho_0$, the absolute value of the spin-polarization parameter increases with decreasing density. At intermediate densities $0.5\varrho_0 \lesssim \varrho \lesssim 3\varrho_0$, there is a plateau in the $\Pi_1(\varrho)$ dependence whose characteristic value depends on H (e.g., $\Pi_1 \approx -0.08$ at $H = 10^{18}$ G). At densities $\varrho \gtrsim 3\varrho_0$, the magnitude of the spin-polarization parameter increases with density, and neutrons become totally polarized at $\varrho \approx 6\varrho_0$.

Note that the results in the low-density domain should be considered as a first approximation to the real complex picture, because, as discussed in detail in Ref. [9], the low-density neutron-rich matter in β equilibrium possesses a frustrated state, "nuclear pasta," arising as a result of competition between Coulomb long-range interactions and nuclear short-range forces. In our case, where pure neutron matter is considered, there is no mechanical instability due to the absence of the Coulomb interaction. However, the possibility of the appearance of low-density nuclear magnetic pasta and its impact on the neutrino opacities in the protoneutron star early cooling stage should be explored in a more detailed analysis.

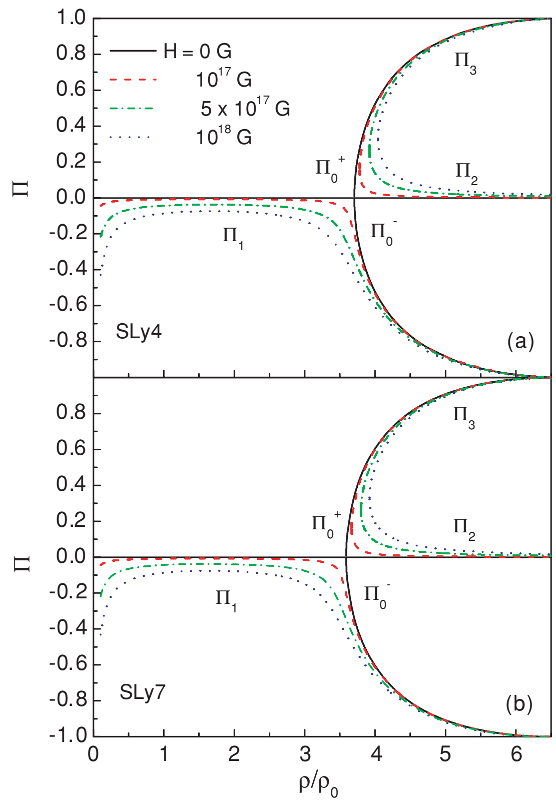

FIG. 2. (Color online) Neutron spin-polarization parameter as a function of density at $T = 0$ and different magnetic field strengths for (a) the SLy4 interaction and (b) the SLy7 interaction. The branches of spontaneous polarization Π_0^- and Π_0^+ are shown by solid curves.

Let us consider the modification of the upper branch of spontaneous polarization $\Pi_0^+(\varrho)$ in a nonvanishing magnetic field. Figure 2 shows that, beginning from some threshold density, the self-consistent equations at a given density have two positive solutions for the spin-polarization parameter (apart from one negative solution). These solutions belong to two branches, $\Pi_2(\varrho)$ and $\Pi_3(\varrho)$, characterized by different dependencies on density. For the branch $\Pi_2(\varrho)$ the spin-polarization parameter decreases with density and tends to zero, whereas for the branch $\Pi_3(\varrho)$ it increases with density and is saturated. These branches appear stepwise at the same threshold density $\varrho_{\rm th}$ dependent on the magnetic field and which is larger than the critical density of spontaneous spin instability in neutron matter. For example, for the SLy7 interaction, $\varrho_{\rm th} \approx 3.80\,\varrho_0$ at $H = 5 \times 10^{17}$ G and $\varrho_{\rm th} \approx 3.92\,\varrho_0$ at $H = 10^{18}$ G. Because of the negative value of the neutron magnetic moment, the magnetic field tends to orient the neutron spins opposite to the magnetic field direction. As a result, the spin-polarization parameter for the branches $\Pi_2(\varrho)$ and $\Pi_3(\varrho)$ with positive spin polarization is smaller than that for the branch of spontaneous polarization Π_0^+; vice versa, the magnitude of the spin-polarization parameter for the branch $\Pi_1(\varrho)$ with negative spin polarization is larger than the corresponding value for the branch of spontaneous polarization Π_0^-. Note that the impact of even a strong magnetic field such as $H = 10^{17}$ G

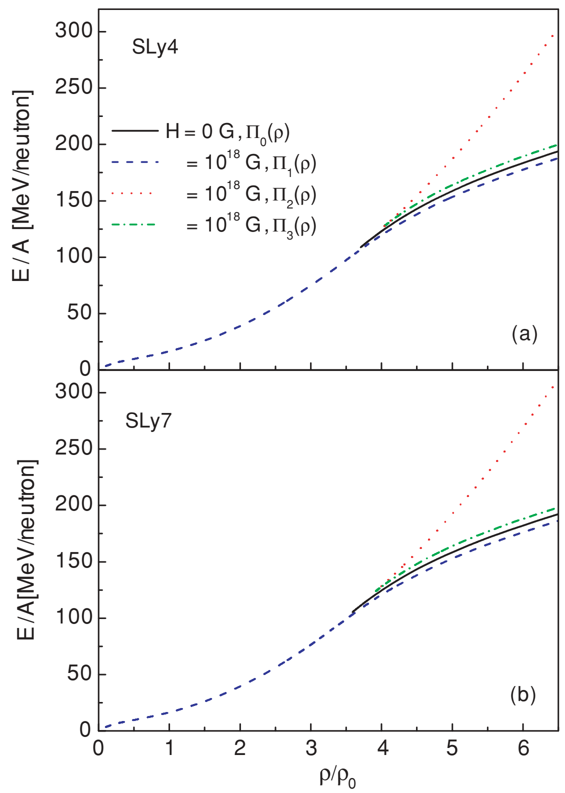

FIG. 3. (Color online) Energy per neutron as a function of density at $T = 0$ for different branches $\Pi_1(\varrho)$–$\Pi_3(\varrho)$ of solutions of the self-consistent equations at $H = 10^{18}$ G for the (a) SLy4 and (b) SLy7 interactions, including a spontaneously polarized state.

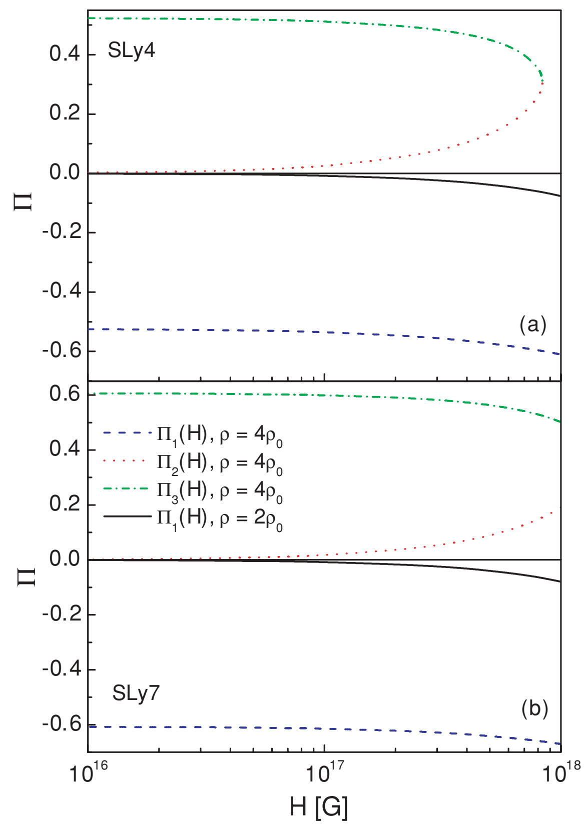

FIG. 4. (Color online) Spin-polarization parameter as a function of magnetic field strength at $T = 0$ for different branches $\Pi_1(H)$–$\Pi_3(H)$ of solutions of the self-consistent equations at $\varrho = 4\varrho_0$ and for the branch $\Pi_1(H)$ at $\varrho = 2\varrho_0$ for (a) the SLy4 interaction and (b) the SLy7 interaction.

is small: The spin-polarization parameter for all three branches $\Pi_1(\varrho)$–$\Pi_3(\varrho)$ is either close to zero or close to its value in the state with spontaneous polarization, which is governed by the spin-dependent medium correlations.

Thus, at densities larger than ϱ_{th}, we have three branches of solutions: one of them, $\Pi_1(\varrho)$, with negative spin polarization, and two others, $\Pi_2(\varrho)$ and $\Pi_3(\varrho)$, with positive polarization. To clarify which branch is thermodynamically preferable, we should compare the corresponding free energies. Figure 3 shows the energy per neutron as a function of density at $T = 0$ and $H = 10^{18}$ G for these three branches compared with the energy per neutron for a spontaneously polarized state [the branches $\Pi_0^{\pm}(\varrho)$]. It is seen that the state with the majority of neutron spins oriented opposite to the direction of the magnetic field [the branch $\Pi_1(\varrho)$] has the lowest energy. This result is intuitively clear because the magnetic field tends to direct the neutron spins opposite to $\mathbf{H}$, as mentioned earlier. However, the state described by the branch $\Pi_3(\varrho)$ with positive spin polarization has energy very close to that of the thermodynamically stable state. This means that despite the presence of a strong magnetic field $H \sim 10^{18}$ G, the state with the majority of neutron spins directed along the magnetic field can be realized as a metastable state in the dense core of a neutron star in the model consideration with the Skyrme effective interaction. In this scenario, because such states exist only at densities $\varrho \geqslant \varrho_{th}$, under decreasing density (going from the interior to the outer regions of a magnetar) a metastable state with positive spin polarization at the threshold density ϱ_{th} changes to a thermodynamically stable state with negative spin polarization.

At this point, note some important differences between the results in our study and those obtained in Ref. [9]. First, in the study [9] of neutron matter in a strong magnetic field, only one branch of solutions for the spin-polarization parameter was found in the model with the Skyrme interaction (for the same SLy4 and SLy7 parametrizations). However, we have seen that the degenerate branches of spontaneous polarization (for zero magnetic field) with positive and negative spin polarization are modified differently by the magnetic field and, as a result, in the Skyrme model, in general, there are three different branches of solutions of the self-consistent equations for a nonvanishing magnetic field. Besides, the only branch considered in Ref. [9] and corresponding to our thermodynamically stable branch Π_1 is characterized by positive spin polarization, contrary to our result with $\Pi_1 < 0$. This disagreement is explained by the incorrect sign before the term with the magnetic field in the equation for the quasiparticle spectrum in Ref. [9] [analogous to Eq. (19) in our case]. Clearly, in the equilibrium configuration the majority of neutron spins are aligned opposite to the magnetic field.

Figure 4 shows the spin-polarization parameter as a function of magnetic field strength at zero temperature for different

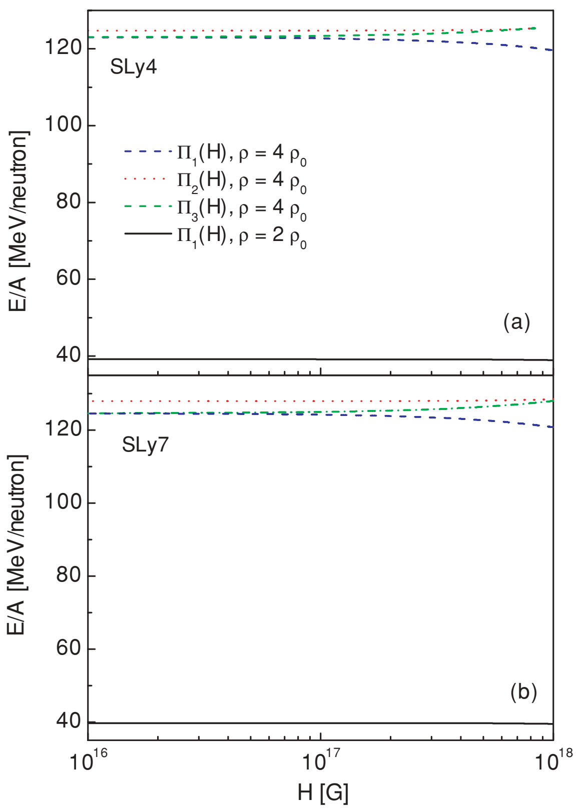

FIG. 5. (Color online) Same as in Fig. 4, but for energy per neutron.

branches $\Pi_1(H)$–$\Pi_3(H)$ of solutions of the self-consistent equations at $\varrho = 4\varrho_0$ compared with that for the branch $\Pi_1(H)$ at $\varrho = 2\varrho_0$. It is seen that up to the field strengths $H = 10^{17}$ G, the influence of the magnetic field is rather marginal. For the branches $\Pi_1(H)$ and $\Pi_2(H)$, the magnitude of the spin-polarization parameter increases with the field strength, whereas for the $\Pi_3(H)$ branch it decreases. Interestingly, as is clearly seen from the top panel for the SLy4 interaction, at the given density, there is some maximum magnetic field strength H_m at which the branches Π_2 and Π_3 converge and do not continue at $H > H_m$.

Figure 5 shows the energy of neutron matter per particle as a function of the magnetic field strength at $T = 0$ under the same assumptions as in Fig. 4. It is seen that the state with the negative spin polarization [branch $\Pi_1(H)$] becomes more preferable with increasing magnetic field, although the total effect of changing the magnetic field strength by two orders of magnitude on the energy corresponding to all three branches $\Pi_1(H)$–$\Pi_3(H)$ remains small. It is also seen that the increase of the density by a factor of 2 leads to the increase in the energy per neutron roughly by a factor of 3, reflecting the fact that the medium correlations play a more important role in building the energetics of the system than the impact of a strong magnetic field.

IV. CONCLUSIONS

We have considered spin-polarized states in neutron matter in a strong magnetic field in the model with the Skyrme effective NN interaction (SLy4 and SLy7 parametrizations). The self-consistent equations for the spin-polarization parameter and chemical potential of neutrons have been obtained and analyzed at zero temperature. It has been shown that the thermodynamically stable branch of solutions for the spin-polarization parameter as a function of density corresponds to the case when the majority of neutron spins are oriented opposite to the direction of the magnetic field (negative spin polarization). This branch extends from the very low densities to the high-density region where the spin-polarization parameter is saturated, and, respectively, neutrons become totally spin polarized. Besides, beginning from some threshold density, $\varrho_{\rm th}$ being dependent on magnetic field strength, the self-consistent equations also have two other branches (upper and lower) of solutions for the spin-polarization parameter, corresponding to the case when the majority of neutron spins are oriented along the magnetic field (positive spin polarization). For example, for the SLy7 interaction, $\varrho_{\rm th} \approx 3.80\,\varrho_0$ at $H = 5 \times 10^{17}$ G and $\varrho_{\rm th} \approx 3.92\,\varrho_0$ at $H = 10^{18}$ G. The spin-polarization parameter along the upper branch increases with density and is saturated, whereas along the lower branch it decreases and vanishes. The free energy corresponding to the upper branch turns out to be very close to the free energy corresponding to the thermodynamically preferable branch with negative spin polarization. As a consequence, in a strong magnetic field, the state with positive spin polarization can be realized as a metastable state in the high-density region in neutron matter, which, under decreasing density (going from the interior to the outer regions of a magnetar), at the threshold density $\varrho_{\rm th}$ changes to a thermodynamically stable state with negative spin polarization.

In this study, we have considered the zero-temperature case, but as was shown in Ref. [9], the influence of finite temperatures on spin polarization remains moderate in the Skyrme model, at least up to the temperatures relevant for protoneutron stars; hence, one can expect that the scenario considered will be preserved at finite temperatures as well. The possible existence of a metastable state with positive spin polarization will affect the neutrino opacities of a neutron star matter in a strong magnetic field, and, hence, will lead to the change of cooling rates of a neutron star compared to cooling rates in the scenario with the majority of neutron spins oriented opposite to the magnetic field [43].

The calculations of the neutron spin-polarization parameter and energy per neutron show that the influence of the magnetic field remains small at field strengths up to 10^{17} G. Note that in Ref. [9] the consideration has also been done for the Gogny effective NN interaction (D1S and D1P parametrizations) up to densities of $4\varrho_0$. Because for the D1S parametrization there is no spontaneous ferromagnetic transition in neutron matter for all relevant densities, and for the D1P parametrization this transition occurs at densities greater than $7\varrho_0$ [23], no sign of a ferromagnetic transition in a strong magnetic field up to densities of $4\varrho_0$ for these Gogny forces was found in Ref. [9]. According to our results, one can expect that the metastable states with positive spin polarization in neutron matter in a strong magnetic field could appear at densities larger than $7\varrho_0$ for the D1P parametrization, whereas the scenario with the only branch of solutions corresponding to negative spin polarization would be realized for the D1S force.

It is also worth noting that in the given research neutron star matter was approximated by pure neutron matter. This approximation allows one to get the qualitative description of the spin-polarization phenomena and should be considered as a first step towards a more realistic description of neutron stars taking into account a finite fraction of protons with charge neutrality and β equilibrium conditions. In particular, some admixture of protons can affect the onset densities of enhanced polarization in a neutron star matter with the Skyrme interaction.

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