

Longitudinal boost invariance of the charge balance function in hadron-hadron and nucleus-nucleus collisions

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(Received 9 October 2009; published 22 December 2009)

Using Monte Carlo generators of the PYTHIA model for hadron-hadron collisions and a multiphase transport (AMPT) model for nucleus-nucleus collisions, the longitudinal boost invariance of charge balance function and its transverse-momentum dependence are carefully studied. It shows that the charge balance function is boost invariant in both $p + p$ and Au + Au collisions in these two models, consistent with experimental data. The balance function properly scaled by the width of the pseudorapidity window is independent of the position or the size of the window and is corresponding to the balance function of the whole pseudorapidity range. This longitudinal property of balance function also holds for particles in small transverse-momentum ranges in the PYTHIA and the AMPT default models, but is violated in the AMPT with string melting. The physical origin of the results is discussed.

DOI: [10.1103/PhysRevC.80.064910](https://doi.org/10.1103/PhysRevC.80.064910)

PACS number(s): 25.75.Gz, 25.75.Ld

I. INTRODUCTION

Charge balance function (BF) has been widely used as an effective means of exploring the hadronization scheme in hadron-hadron collisions at the ISR energies [1] and $e^+ + e^-$ annihilations at the PETRA energies [2]. Recently, the charge BF gained special attention in clocking hadronization at relativistic heavy-ion collisions. A narrowing of the BF is suggested as a signature for delayed hadronization [3,4].

The dependence of the BF on centrality and system size was reported by several relativistic heavy-ion experiments [5,6]. However, most of the current heavy-ion experiments are limited by the pseudorapidity range [5–7]; it is impossible to quantitatively compare the results from the experiments with the coverage at different pseudorapidity ranges. The dependence of the BF on the pseudorapidity window is essential for understanding the physics of the BF [5,6,8,9], and was carefully studied by the NA22 [9] and STAR experiments for hadron-hadron and relativistic heavy-ion collisions, respectively.

The NA22 experiment has full 4π acceptance and excellent momentum resolution [9]. It is found in the experiment that the BF in π^+p and K^+p collisions at 22 GeV is invariant under the longitudinal boost over the whole rapidity range of produced particles, despite the non-boost-invariance of the single-particle density. Moreover, the BF of the whole rapidity range can be deduced from the BF properly scaled by the width of rapidity windows [9].

The STAR experiment covers a finite but relatively wide pseudorapidity range. The scaling property of the BF in Au + Au collisions at 200 GeV is further observed in the experiment [10]. This scaling property of the balance function is also found in different p_T ranges of final-state particles.

These results from both hadron-hadron and nuclear collisions indicate that the charge balance of produced particles in strong interactions is boost invariant in the longitudinal phase space, in contrast with the single-particle density.

Therefore, it is interesting to see if those properties are taken into account in the models that are successfully described as hadron-hadron and nuclear collisions, and how they associate with the mechanisms of particle production in the models.

II. CHARGE BALANCE FUNCTION AND IMPLEMENT MODELS

Charge balance function measures how the conserved electric charge compensates in the phase space, that is, how the surrounding net charges are rearranged if the charge of a selected point changes [1]. In high-energy collisions, the production of charged particles is constrained by charge balance in the phase space. The BF therefore provides a direct access to collision dynamics.

The BF was originally defined in terms of a combination of four kinds of charge-related conditional densities in pseudorapidity [1]:

$$B(\eta_1|\eta_2) = \frac{1}{2}[\rho(+, \eta_1|-, \eta_2) - \rho(+, \eta_1|+, \eta_2) + \rho(-, \eta_1|+, \eta_2) - \rho(-, \eta_1|-, \eta_2)], \quad (1)$$

where the notation $\rho(a, \eta_a|b, \eta_b)$ represents the ratio $\rho_{ab}(\eta_a, \eta_b)/\rho_b(\eta_b) = \langle n_{ab}(\eta_a, \eta_b) \rangle / \langle n_b(\eta_b) \rangle$ with a, b standing for + or - charged particles. Projecting to the pseudorapidity difference $\delta\eta = \eta_1 - \eta_2$ in a pseudorapidity window η_w , it becomes [3,5]

$$B(\delta\eta|\eta_w) = \frac{1}{2} \left[\frac{\langle n_{+-}(\delta\eta) \rangle - \langle n_{++}(\delta\eta) \rangle}{\langle n_+ \rangle} + \frac{\langle n_{-+}(\delta\eta) \rangle - \langle n_{--}(\delta\eta) \rangle}{\langle n_- \rangle} \right], \quad (2)$$

where $n_{ab}(\delta\eta)$ is the total number of pairs of opposite charged particles with the pseudorapidity difference $\delta\eta$ in the pseudorapidity window η_w ; n_+ and n_- are the number of positively and negatively charged particles in the window

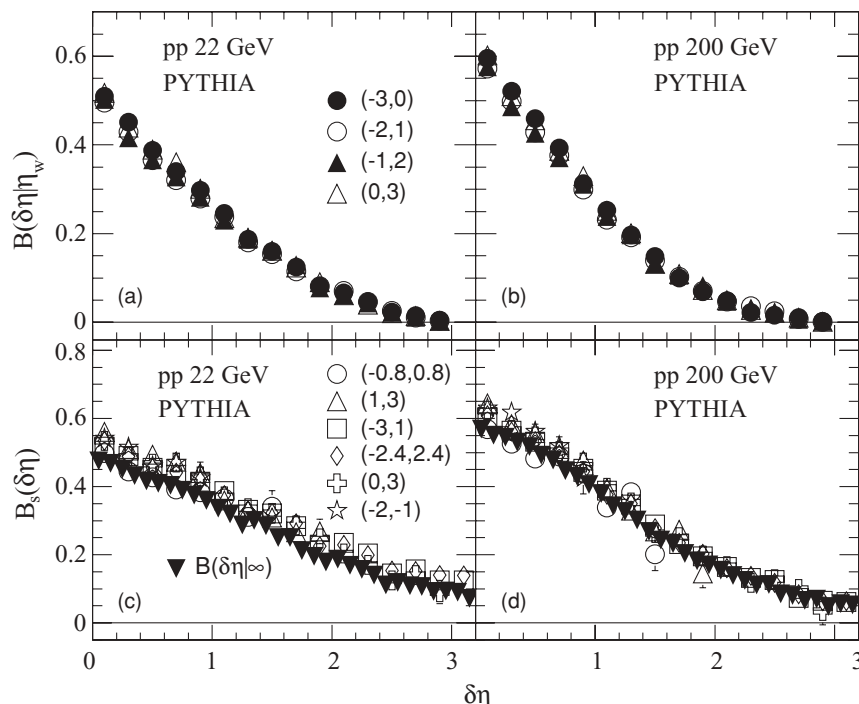


FIG. 1. (Top) The $B(\delta|\eta_w)$ in four pseudorapidity windows with equal size $|\eta_w| = 3$ at the different positions for $p + p$ collisions at (a) $\sqrt{s} = 22$ GeV and (b) $\sqrt{s} = 200$ GeV by the PYTHIA model. (Bottom) The scaled balance function, $B_s(\delta|\eta)$, deduced from the directly measured BF at six different sizes and positions of pseudorapidity windows for $p + p$ collisions at (c) $\sqrt{s} = 22$ GeV and (d) $\sqrt{s} = 200$ GeV by the PYTHIA model. The solid down triangle is the BF of the whole η range.

η_w , respectively; and $\langle \dots \rangle$ is the average over the whole event sample.

From the findings of the BF at the NA22 [9] and STAR experiments [10], the BF is boost invariant in the whole rapidity range in hadron-hadron collisions and may be in nuclear collisions as well. In this case, the properly scaled BF is corresponding to the BF of the whole pseudorapidity range and is deduced by

$$B_s(\delta|\eta) = \frac{B(\delta|\eta_w)}{1 - \frac{\delta|\eta}{|\eta_w|}}, \quad (3)$$

where $|\eta_w|$ is the width of the pseudorapidity window.

The PYTHIA 5.720 [11] is well set up for $p + p$ collisions. It is a standard Monte Carlo generator with string fragmentation as the hadronization scheme. Two versions of a multiphase transport (AMPT) model [12] are used to study Au + Au collisions. One is the AMPT default and the other one is the AMPT with string melting. In both versions, the initial conditions are obtained from the heavy-ion jet interaction generator (HIJING) model, and then the scattering among partons is given by the Zhangs parton cascade (ZPC) model. In the AMPT default model, the partons recombine with their parent strings when they stop interacting, and the resulting strings are converted to hadrons using the Lund string fragmentation model, whereas in the AMPT model with string melting, quark coalescence is used in combining partons into hadrons. The dynamics of the hadronic matter is described by the ART model.

It is commonly believed that in relativistic heavy-ion collisions, the charge ordering during the string fragmentation in elementary collisions is no longer valid, and it should be replaced by the quark-coalescence mechanism in hadronization [13]. Thus, it is interesting to see whether the boost invariance of the BF is sensitive to the mechanisms of hadronization.

In this article, we first study the boost invariance of the BF for $p + p$ collisions at $\sqrt{s} = 22$ and $\sqrt{s} = 200$ GeV using the PYTHIA model, and for Au + Au collisions at $\sqrt{s} = 200$ GeV using two versions of the AMPT model. The transverse-momentum dependence of the longitudinal scaling property of the BF is then examined in the models. The obtained results are compared with the corresponding experimental data and discussed.

III. BOOST INVARIANCE AND LONGITUDINAL SCALING OF THE BF

To demonstrate directly whether the BF is invariant under a longitudinal Lorentz transformation over the whole rapidity in hadron-hadron collisions, we choose four equal-size ($|\eta_w| = 3$) pseudorapidity windows located at different positions: $(-3, 0)$, $(-2, 1)$, $(-1, 2)$, and $(0, 3)$. The results for $p + p$ collisions at $\sqrt{s} = 22$ and $\sqrt{s} = 200$ GeV are shown in Figs. 1(a) and 1(b), respectively. The statistic errors are smaller than the markers. It is clear that the BF measured in four windows is approximately identical to each other at two incident energies. This indicates that the charge compensation is essentially the same in any longitudinally Lorentz-transformed frame for $p + p$ collisions in the PYTHIA model, consistent with the data from the NA22 experiment. These results show that the string fragmentation mechanism implemented in PYTHIA well describes the production mechanisms of charge particles and their charge balance in longitudinal phase space.

Figures 1(c) and 1(d) are the scaled balance function $B_s(\delta|\eta)$ at two incident energies. They are deduced from directly measured $B(\delta|\eta_w)$ at six different pseudorapidity windows, $(-0.8, 0.8)$; open circles), $(1, 3)$; open triangles), $(-3, 1)$; open squares), $(-2.4, 2.4)$; open diamonds), $(0, 3)$; open crosses), and $(-2, -1)$; open stars). From the figures we can see that all the $B_s(\delta|\eta)$ deduced from different windows coincide with each

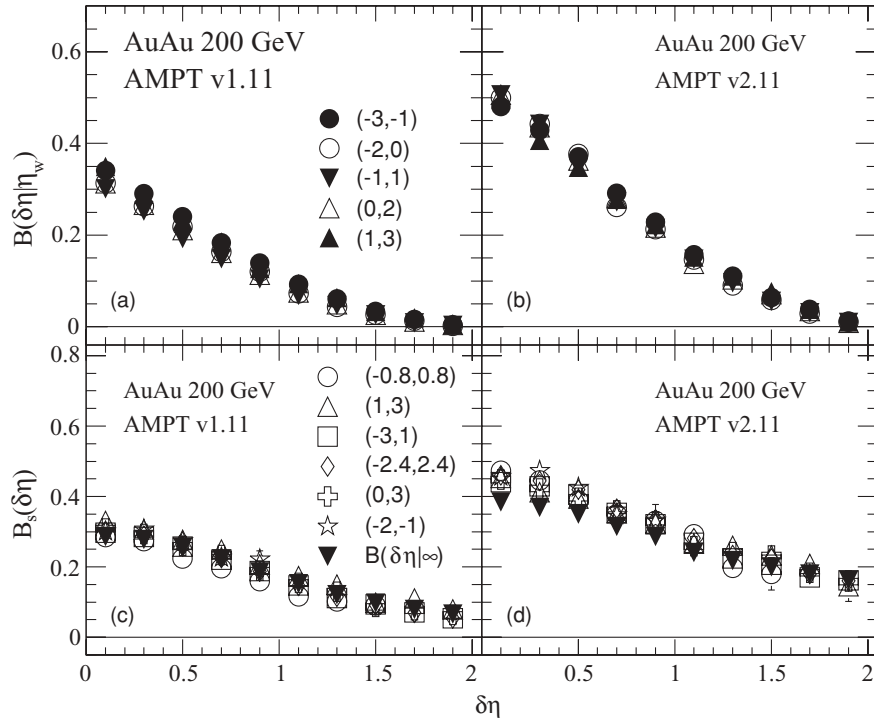


FIG. 2. (Top) The $B(\delta\eta|\eta_w)$ in five pseudorapidity windows with equal size $|\eta_w| = 2$ at the different positions for Au + Au collisions at $\sqrt{s} = 200$ GeV by (a) the AMPT default and (b) the AMPT with string melting. (Bottom) The scaled balance function, $B_s(\delta\eta)$, deduced from the directly measured BF at various pseudorapidity windows with different sizes and positions for Au + Au collisions at $\sqrt{s} = 200$ GeV by (c) the AMPT default and (d) the AMPT with string melting.

other within errors, as expected from the boost invariance of the BF [4]. The solid down triangles in the same figures are the BF of the whole pseudorapidity range $B(\delta\eta|\eta_\infty)$. It is close to the scaled balance function $B_s(\delta\eta)$. These results indicate that the scaled BF is, in fact, corresponding to the BF of the whole pseudorapidity range $B(\delta\eta|\infty)$ [4].

It is then interesting to see whether the boost invariance of the BF is held in nucleus-nucleus collisions. The STAR experiment only observes the boost invariance of BF in the central pseudorapidity range $-1 < \eta < 1$ [10], where the single-particle distribution is almost flat, or boost invariant. Now, in the model investigation, we can carefully examine the property in the whole pseudorapidity range.

The top panel of Fig. 2 are the BF in five pseudorapidity windows with equal size $\eta_w = 2$ at different positions: $(-3, -1)$, $(-2, 0)$, $(-1, 1)$, $(0, 2)$, and $(1, 3)$. Figures 2(a) and 2(b) are the results from the AMPT default (v1.11) and the AMPT with string melting (v2.11), respectively. Both figures show that the BF is boost invariant in the pseudorapidity range $(-3, 3)$ in two versions of the AMPT model.

The bottom panel of Fig. 2 are the scaled balance functions, which are obtained from the directly measured BF at six different windows as indicated in the legend of the figure, where the solid down triangles are the BF in the pseudorapidity range $(-4, 4)$. It shows that the scaled BF does not depend on the size and position of the windows, and corresponds to the BF of the whole pseudorapidity in two versions of the AMPT, consistent with the results of $p + p$ collisions in the PYTHIA model.

IV. THE TRANSVERSE-MOMENTUM DEPENDENCE OF THE BOOST INVARIANCE OF THE BF

The longitudinal property of the boost invariance of the BF comes from the special longitudinal interaction of

charged particles under the constraint of global electric charge balance. Global electric charge conservation not only applies to all final-state charged particles, but also constrains particles that are produced at the same proper time of evolution. It is argued that the transverse momentum of final-state particles may be roughly used as a scale of the proper time of their production in the expansion of nuclear collisions [14–17]. Examining the p_T dependence of the longitudinal property of the BF will provide direct access on whether particles in the specified p_T range are consistent to be produced simultaneously with well-balanced electric charge.

Thus, we turn to check whether the boost invariance of BF holds for particles in different p_T ranges. Figure 3 shows the BF for $p + p$ collisions at $\sqrt{s} = 22$ and $\sqrt{s} = 200$ GeV from PYTHIA in three transverse-momentum bins: $(0 < p_T < 0.2)$, $(0.2 < p_T < 0.4)$, and $(p_T > 0.2)$ GeV/c, respectively. These p_T bins are selected to make the multiplicity in each bin comparable. The result shows that the points at a given $\delta\eta$ in a restricted p_T interval approximately coincide with each other (i.e., the boost invariance of the BF holds in small p_T ranges). It indicates that particles produced at different p_T ranges are also boost invariant for hadron-hadron collisions in the PYTHIA model.

The same study for Au + Au 200-GeV collisions from the two versions of the AMPT model are presented in the top and bottom panels of Fig. 4, respectively, where the four p_T bins are $(0.15, 0.4)$, $(0.4, 0.7)$, $(0.7, 1)$, and $(1, 2)$ GeV/c. From the top panel of the figure, we can see that the BF of the different pseudorapidity windows in each p_T bin is close to each other, consistent with the data from the STAR experiment [10]. However, in the AMPT model with string melting, as shown in the bottom panel of the figure, the BF of the different pseudorapidity

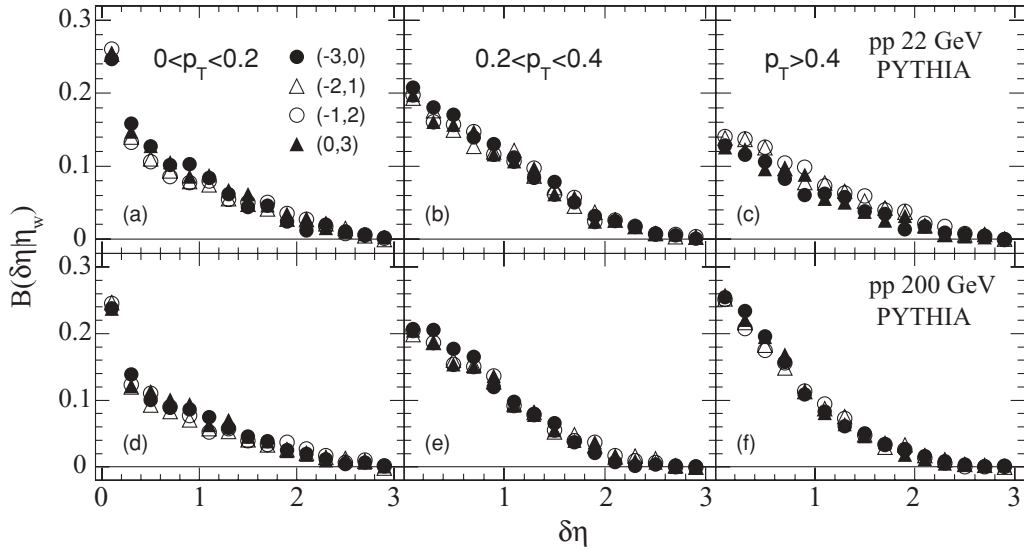


FIG. 3. For each of three p_T ranges, the $B(\delta n|\eta_w)$ in four pseudorapidity windows with equal size $|\eta_w| = 3$ at the different positions for $p + p$ collisions at $\sqrt{s} = 22$ and $\sqrt{s} = 200$ GeV in the top and bottom panels, respectively.

windows is not as close to each other as those in the top panel.

This is because in the AMPT with string melting, each parton in the evolution of nuclear collision has its own freeze-out time, which lasts a very long period after the interaction of two nucleus [18]. The particles in the same transverse-momentum range are not freeze-out simultaneously with the well-balanced charge, and therefore the longitudinal boost invariance of the BF in small p_T ranges is violated. In the AMPT default, the partons recombined with their parent strings immediately after they stopped interacting, and converted to hadrons. Thus, the charge balance of the produced particles in the same p_T ranges is preserved and the boost invariance of the BF holds.

V. SUMMARY

In this article, we systematically study the longitudinal boost invariance of the charge balance function and its p_T dependence for $p + p$ and Au + Au collisions using the PYTHIA and the AMPT models. This shows that charge balance function is boost invariant in both hadron-hadron and nuclear interactions, in contrast to the single-particle density. As expected, this boost invariance of the BF results that the BF properly scaled by window size is independent of the window and corresponds to the BF of the whole pseudorapidity range. Therefore, the BF is a good measure, free from the restriction of finite longitudinal acceptance.

This work further shows that the boost invariance of the BF in the specified p_T range is valid in the PYTHIA model for

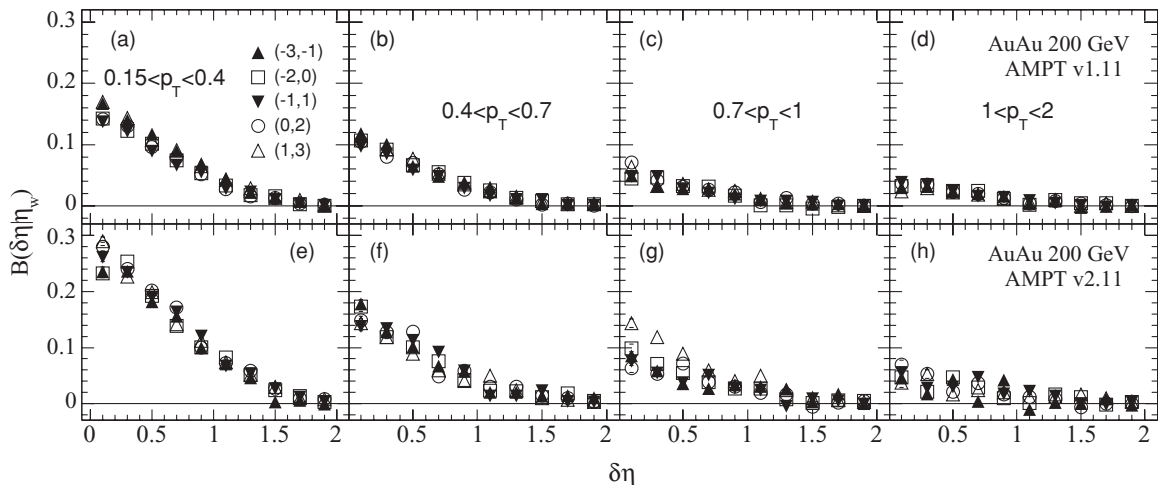


FIG. 4. For each of four p_T ranges, $B(\delta n|\eta_w)$ in five pseudorapidity windows with equal size $|\eta_w| = 2$ at the different positions for Au + Au collisions at $\sqrt{s} = 200$ GeV from the AMPT default (top panel) and the AMPT with string melting (bottom panel).

hadron-hadron collisions and the AMPT default for Au + Au collisions. But the AMPT with string melting fails to reproduce this property due to the different schemes at hadronization. So the p_T dependence of the longitudinal property of the BF can be served as a sensitive probe for charge balance in hadronization mechanism.

ACKNOWLEDGMENTS

We thank Professor Liu Lianshou and Dr. Yu Meiling for valuable discussions and remarks. This work is supported in part by the National Natural Science Foundation of China (Project Nos. 10835005 and 10647124) and the Ministry of Education of China (Project Nos. IRT0624 and B08033).

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