Deformed shell model results for two-neutrino positron double-β decay of ^{74}Se

A. Shukla,[1,2] R. Sahu,[3] and V. K. B. Kota[2,4]
[1]*Rajiv Gandhi Institute of Petroleum Technology, Raebareli 229 316, Utter Pradesh, India*
[2]*Physical Research Laboratory, Ahmedabad 380 009, Gujarat, India*
[3]*Physics Department, Berhampur University, Berhampur 760 007, Orissa, India*
[4]*Department of Physics, Laurentian University, Sudbury, Ontario P3E 2C6, Canada*
(Received 13 April 2009; revised manuscript received 15 September 2009; published 30 November 2009)

Half-lives $T_{1/2}^{2\nu}$ for two-neutrino positron double-β decay modes β^+ EC/ECEC are calculated for ^{74}Se, a nucleus of current experimental interest, using the deformed shell model based on Hartree-Fock states and employing a modified Kuo interaction in ($^2p_{3/2}$, $^1f_{5/2}$, $^2p_{1/2}$, $^1g_{9/2}$) space. The calculated half-life for the ECEC mode is $\sim 10^{26}$yr, and it may be possible to observe this in future experiments.

DOI: 10.1103/PhysRevC.80.057305 PACS number(s): 23.40.Hc, 21.10.Tg, 21.60.Jz, 27.50.+e

Double-β decay (DBD) is a rare weak-interaction process in which two identical nucleons inside the nucleus undergo decay with or without emission of neutrinos. The two-neutrino DBD ($2\nu\,\beta^-\beta^-$), which was first predicted by Meyer [1], is fully consistent with the standard model and has been observed experimentally in more than 10 nuclei. The neutrinoless DBD ($0\nu\,\beta^-\beta^-$), which involves emission of two electrons and no neutrinos, has not been observed experimentally, and it violates lepton-number conservation. The Heidelberg-Moscow group's [2] claim to have observed $0\nu\,\beta^-\beta^-$ decay of ^{76}Ge is controversial and has yet to be confirmed by other experiments. DBD is one of the best probes for studying physics beyond the standard model. To extract the mass of a neutrino via $0\nu\,\beta^-\beta^-$ decay, it is necessary to have good nuclear-structure models for reliably calculating the nuclear transition-matrix elements (NTME). A large number of theoretical studies for various muclei candidates for $2\nu\,\beta^-\beta^-$ and $0\nu\,\beta^-\beta^-$ decay, using various nucleus models, have been carried out to establish the NTME [3,4].

The shell model is the best choice for calculating the NTME because it attempts to solve the nuclear many-body problem as exactly as possible. Recently, there have been large-scale shell model predictions for neutrinoless DBD half-lives of seven important nuclei [5]. However, quasiparticle random-phase approximation (QRPA) with various extensions has emerged as successful model for calculating half-lives for $2\nu\,\beta^-\beta^-$ and $0\nu\,\beta^-\beta^-$ modes (and the e^+ DBD modes discussed later in this article), and this model has been applied to a greater number of nuclei [6,7]. The advantage of QRPA calculations compared to the conventional shell model calculations is in QRPA's ability to treat large model spaces, typically consisting of two major oscillator shells. However, in the QRPA approach, more uncertainty may occur in deformed nuclei because deformation is usually ignored. Although extensive attempts are being made to improve shell model computational techniques and also to improve the QRPA model for more reliable calculations, it is important to complement them with other nuclear models to check the role of the valence space and deformation in the study of DBD. Results have been reported using the pseudo-$SU(3)$ model [8], interacting boson model [9], and projected Hartree-Fock-Bogoliubov (PHFB) approach [10,11]. We attempt to employ the so-called deformed shell model (DSM), which is proved to be successful for spectroscopy of nuclei with mass $A = 44$–80.

In contrast to the $2\nu\,\beta^-\beta^-$ decay, the positron decay modes [$2\nu\,\beta^+\beta^+/\beta^+$ EC/ECEC decay modes (hereafter, these three are all called $2\nu\,e^+$ DBD)] have not yet been observed experimentally (the exception being the evidence for ^{130}Ba decay derived from geochemical methods [12], although these have not yet been confirmed by results from direct counting methods), and hence there are not many theoretical studies of the NTME involved in $2\nu\,e^+$ DBD. However, in the past few years, serious attempts have been made using direct counting methods to measure half-lives for $2\nu\,e^+$ DBD modes in the upper ($pfg_{9/2}$) shell nuclei ^{78}Kr [13], ^{64}Zn [14], and ^{74}Se [15]. In the past, attempts have also been made for ^{106,108}Cd [16] and ^{130,132}Ba [17] nuclei. Prompted by this experimental interest, recently we [18] have carried out calculations for ^{78}Kr using the DSM by employing a modified Kuo interaction in ($^2p_{3/2}$, $^1f_{5/2}$, $^2p_{1/2}$, $^1g_{9/2}$) space. The predictions of DSM for $2\nu\,e^+$ DBD half-lives are close to those of QRPA and PHFB models. To extend the study in Ref. [18] further, we have carried out DSM calculations for $2\nu\,e^+$ DBD half-lives for the ^{74}Se nucleus, and the results are reported in this brief report. We did not consider ^{64}Zn as spherical shell model is well suited [19] for the three nuclei ^{64}Zn, ^{64}Cu, and ^{64}Ni. They are not that deformed and have proton numbers close to the N = 28 closed core.

Over the years, we have has success using DSM based on Hartree-Fock (HF) states to study spectroscopic properties, such as band structures, shapes, nature of band crossings, and electromagnetic-transition probabilities, for medium-heavy nuclei with $A = 64$–80 [20–22]. More recently, this model was applied to $N = Z$ and $N = Z + 1$ nuclei by including isospin projection [23,24]. The spectroscopic properties, especially electromagnetic transitions such as $B(E2)$ and $B(M1)$ values, provide a stringent test for the goodness of the nuclear wave functions generated using the model. It is also important to add that DSM results are being used by many groups in the discussion of experimental data for $A \sim 64$–80 nuclei [25]. In addition, DSM was used to calculate transition-matrix elements for μ–e conversion in ^{72}Ge [26] and to analyze data for inelastic scattering of electrons from fp-shell nuclei [27]. This model has also been used to study 2ν DBD transition-matrix elements for ^{76}Ge $\rightarrow ^{76}$Se [28] with considerable success. More recently, in [18] we have applied DSM to study β-decay half-lives, Gamow-Teller (GT) distributions, electron capture rates, and $2\nu\, e^+$ DBD in ^{78}Kr. All these confirm that DSM generates good nuclear-wave functions for nuclei in the mass region $A \sim 64$–80. Here, we briefly discuss first the DSM formalism and then the results for ^{74}Se.

The half-life for the $2\nu\ e^+$ DBD decay modes for the $0_I^+ \rightarrow J_F^+$ transitions, with J_F^+ for the daughter nucleus being $J_F^+ = 0_1^+$ or 2_1^+, is given by [7,29],

$$\left[T_{1/2}^{2\nu}(k, J_F)\right]^{-1} = G_{2\nu}(k, J_F)\ |M_{2\nu}(J_F)|^2, \qquad (1)$$

where k denotes the modes $\beta^+\beta^+$, β^+ EC, and ECEC. Besides the $0_1^+ \rightarrow 0_1^+$ transition, the ECEC mode for $0_1^+ \rightarrow 2_1^+$ is also of experimental interest, and we have considered both $J_F^+ = 0_1^+$ and 2_1^+ in Eq. (1). The integrated kinematical factors $G_{2\nu}(k, J_F)$ are independent of nuclear structure (except for the dependence on the excitation energy E_F of the J_F state of the daughter nucleus), and they can be calculated with good accuracy [29–32]. Further, the NTME $M_{2\nu}$ are nuclear-model dependent and are given by [7,11,29]

$$M_{2\nu}(J_F) = \frac{1}{\sqrt{J_F+1}} \sum_N \frac{\langle J_F^+||\sigma\tau^-||1_N^+\rangle\langle 1_N^+||\sigma\tau^-||0_I^+\rangle}{[E_0 + E_N - E_I]^{J_F+1}}, \qquad (2)$$

where $|0_I^+\rangle$, $|J_F^+\rangle$, and $|1_N^+\rangle$ are the initial, final, and virtual intermediate states respectively and $E_N(E_I)$ is the energy of intermediate (initial) nucleus. Note that $E_0 = \frac{1}{2}W_0$, where W_0 is the total energy released for different $2\nu\ e^+$ DBD modes. For $0_1^+ \rightarrow 0_1^+$ transitions, as given in Refs. [7,11], $W_0(\beta^+\beta^+) = Q_{\beta^+\beta^+} + 2m_e$, $W_0(\beta^+\text{EC}) = Q_{\beta^+\text{EC}} + e_b$, and $W_0(\text{ECEC}) = Q_{\text{ECEC}} - 2m_e + e_{b1} + e_{b2}$. The Q values for different $2\nu\ e^+$ DBD modes can be defined as in Ref. [7]: $Q_{\beta^+\beta^+} = M(A,Z) - M(A,Z-2) - 4m_e$, $Q_{\beta^+\text{EC}} = M(A,Z) - M(A,Z-2) - 2m_e$, and $Q_{\text{ECEC}} = M(A,Z) - M(A,Z-2)$. Here M denotes the neutral atomic mass (atomic masses are taken from the tabulations in Ref. [33]), and e_b is the binding energy of the captured atomic electron. For the $0_1^+ \rightarrow 2_1^+$ ECEC transition, denoted by ECEC*, we have $W_0(\text{ECEC}^*) = Q_{\text{ECEC}} - \Delta E - 2m_e + e_{b1} + e_{b2}$, where ΔE is the excitation energy of the 2_1^+ state. Energies in the denominator in Eq. (2) are taken in the units of electron mass. It should be noted that the Q value should be positive for the corresponding $2\nu\ e^+$ DBD decay possible. With the atomic mass difference being 1209.7 ± 0.6 keV for ^{74}Se decay, it should be clear that the $\beta^+\beta^+$ DBD mode is forbidden for ^{74}Se.

In DSM, for a given nucleus, starting with a model space consisting of a given set of single-particle orbitals and effective two-body Hamiltonian, the lowest prolate and oblate intrinsic states are obtained by solving the HF single-particle equation consistently. Excited intrinsic configurations are obtained by making particle-hole excitations over the lowest intrinsic state. These intrinsic states will not have good angular momentum, and good angular-momentum states are obtained by angular-momentum projection from these intrinsic states. In general, the projected states with same J but coming from different intrinsic states will not be orthogonal to each other. Hence, they are orthonormalized, and then band-mixing calculations are performed. DSM is well established as a successful model for transitional nuclei (with $A = 64$–80) when sufficient intrinsic states are included in the band-mixing calculations; see Ref. [18] and references therein. By performing DSM calculations for the parent, daughter, and the intermediate odd-odd nucleus (here we need only the 1^+ states) and then using the DSM wave functions, we calculate the $\sigma\tau^-$ matrix elements in Eq. (2). For further details, see Ref. [18]. Now, we discuss the results for ^{74}Se.

In our calculations of ^{74}Se $2\nu\ e^+$ DBD half-lives, for the structure of the nuclei ^{74}Se, ^{74}As, and ^{74}Ge, we have used a modified Kuo effective interaction [34] in the ($^2p_{3/2}$, $^1f_{5/2}$, $^2p_{1/2}$, $^1g_{9/2}$) space with ^{56}Ni as the inert core. The single-particle energies of these orbitals are taken as 0.0, 0.78, 1.08, and 4.5 MeV respectively. DSM with modified Kuo effective interaction has been quite successfully used by us to describe many important features of nuclei in the $A \sim 60$–80 region. In particular, shape coexistence in spectra, observed $B(E2)$ values, and band crossings in ^{70,72,74}Se isotopes are well described by DSM [35]. We have also verified that ^{74}Ge spectroscopic properties are well described by DSM. For $2\nu\ e^+$ DBD half-life calculations, we first performed axially symmetric HF calculations and obtained the lowest prolate HF intrinsic states. In DSM, the HF calculation in a limited configuration space is used to generate a DSM basis. A few low-lying deformed configurations (usually 20 to 30) are sufficient to give most of the important features and systematics of different properties of transitional/deformed nuclei in the $A = 64$–80 region. The lowest HF single-particle spectra for ^{74}Se, ^{74}Ge, and ^{74}As nuclei are shown in Fig. 1. Only prolate intrinsic states are considered in these calculations, and the oblate intrinsic states are ignored just as in the previous ^{78}Kr analysis [18] using DSM. The reason for neglecting the oblate states has been discussed in an earlier publication [36]. For these three nuclei, we found that the spectroscopic results obtained with only oblate states compare poorly with the experiment, and hence we did not include oblate states in the final calculation. We have also seen in the band-mixing calculations that oblate states do not mix significantly with prolate states, and hence they are not expected to affect our final results. By particle-hole excitations from the lowest intrinsic states, shown in Fig. 1, excited intrinsic states are generated. For ^{74}Se ground state 0^+, 10 intrinsic states with $K = 0^+$ are used for band mixing, and similarly for ^{74}Ge, 24 with $K = 0^+$ are employed. For the intermediate ^{74}As nucleus, we considered 65 intrinsic states in band-mixing calculations, and the lowest 14 1^+ eigenstates (up to 2 MeV excitation) are used in the sum in Eq. (2). Each of the single-particle levels (occupied as well as unoccupied) in the HF single-particle spectrum shown in Fig. 1 have certain single-particle quadrupole moments. The occupancy of the single-particle levels generating the various intrinsic states is different. Since the quadrupole moment for a given intrinsic state is the sum of the single-particle quadrupole moments of the occupied orbits, the 1^+ levels generated by projection and band mixing from these intrinsic states will have different deformations. We have verified that the lowest 14 1^+ eigenstates included in the calculations provide adequate description of $2\nu\ e^+$ DBD. Further increase in the

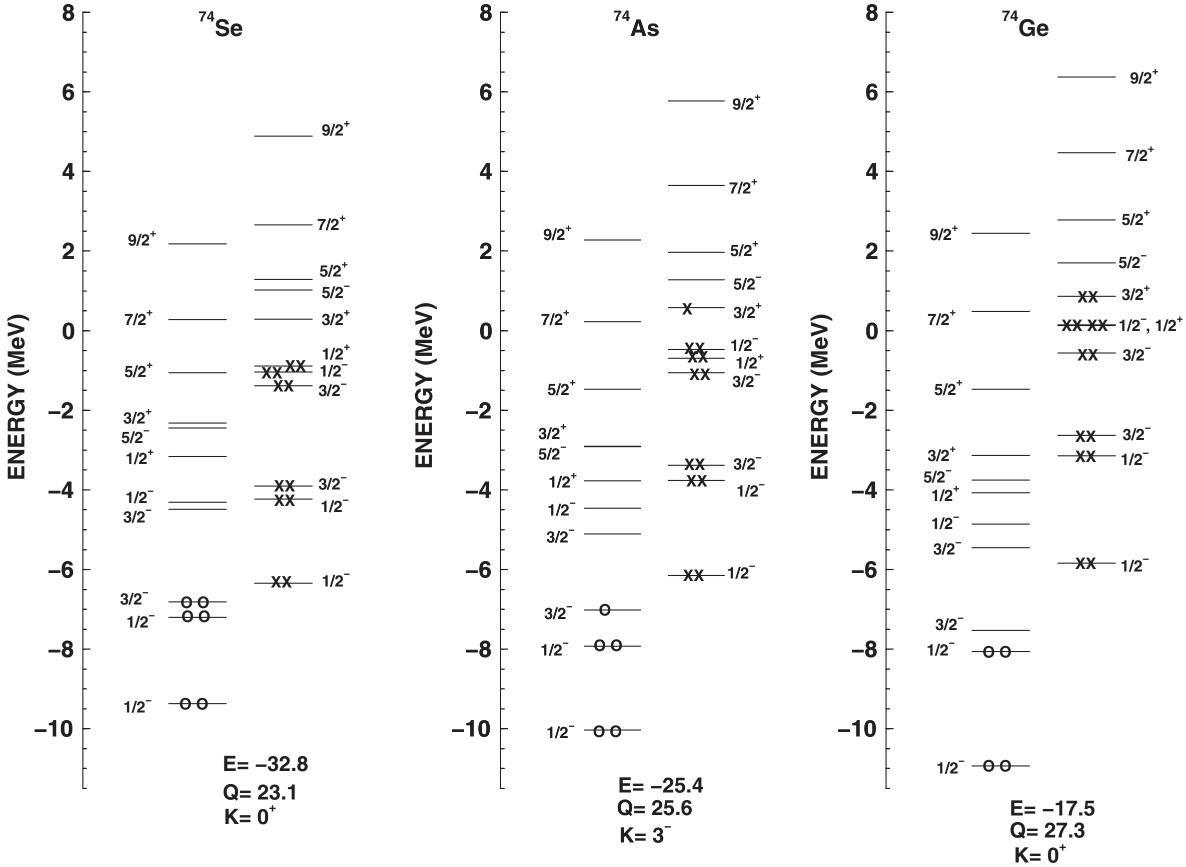

FIG. 1. HF single-particle spectra for ^{74}Se, ^{74}As, and ^{74}Ge. Circles represent protons, and crosses represent neutrons. The HF energy (E) in MeV, mass quadrupole moment (Q) in units of the square of the oscillator length parameter, and the total K quantum number of the lowest intrinsic states are given.

number of states does not change the results significantly. However, following the recent experimental results [37], which corroborated with shell-model calculations [38] for ^{48}Ca DBD, it can be argued that the present DSM calculation with 14 intermediate 1^+ states saturating the NTME may not prove that the results converged to the corresponding shell-model values. In the future, DSM versus shell model for DBD will be investigated.

Using the wave functions generated by DSM, $2\nu\beta^+$ EC and ECEC half-lives for $^{74}\text{Se} \rightarrow {}^{74}\text{Ge}$ transitions are calculated, and the results are shown in Table I. The integrated kinematical factors $G_{2\nu}(k, J_F)$ have been calculated following the prescription given by Doi and Kotani [29]. Let us add that we did not include any quenching factor for the GT operator (if we use a typical value of 0.74, the half-life will increase by a factor 4). The limits for β^+ EC processes in ^{74}Se were determined only recently, in the SuperNEMO project. Measurements of an Se sample consisting of natural Se powder using a 400-cm^3 high performance Germanium (HPGe) detector resulted in the first $T_{1/2}$ limits to be calculated as $>10^{18}$–10^{19} yr [15] for $2\nu\beta^+$ EC and ECEC. The DSM results in Table I are the first theoretical estimates for the half-lives for positron double-decay modes of ^{74}Se; no other model calculations exist. Let us recall here the statement in Ref. [15]: "It is necessary to stress that ^{74}Se has never been investigated before—neither has this isotope been investigated theoretically; thus there are no predictions with which to compare." Therefore, the results presented in Table I will be of immediate interest to experimentalists.

In this brief report, by extending our recent results for ^{78}Kr [18], we have presented results for positron DBD half-lives for ^{74}Se. They were obtained using the DSM model with a modified Kuo interaction in $(^2p_{3/2}, {}^1f_{5/2}, {}^2p_{1/2}, {}^1g_{9/2})$ space. Because spectroscopic properties of Se isotopes (and also many other nuclei with $A = 64$–80) are well described by

TABLE I. Experimental limit on half-lives $T_{2\nu}^{1/2}$ along with theoretical estimates in DSM and corresponding phase-space factor $G_{2\nu}$ for possible decay modes for $^{74}\text{Se} \rightarrow {}^{74}\text{Ge}$. Note that the natural abundance of ^{74}Se is 0.89% [39]. $G_{2\nu}$ are calculated using $g_A/g_V = 1$. The ranges (a–b) given in parentheses for the theoretical estimate of the half-life are given for $g_A/g_V = 1.261$ and 1, respectively.

Decay mode	$G_{2\nu}$ (yr^{-1})	$T_{2\nu}^{1/2}$ (yr)	
		Expt. [15]	Theory
β^+EC	2.05×10^{-29}	$>1.9 \times 10^{18}$	$(14.99\text{–}37.9) \times 10^{30}$
ECEC	2.63×10^{-24}		$(7.56\text{–}19.12) \times 10^{25}$
ECEC[a]	3.06×10^{-27}	$>7.7 \times 10^{18}$	$(15.55\text{–}39.32) \times 10^{30}$

[a]Represents ground state to 2_1^+ state transition ($\Delta E = 595.8$ keV).

DSM, the half-lives calculated for $2\nu\, e^+$ DBD modes of ^{74}Se, given in Table I, can be taken as useful predictions. The calculated half-life for the ECEC mode is $\sim 10^{26}$ yr, and it may be possible to observe this in future experiments. Finally, to apply DSM for DBD NTME calculations for $A > 80$ nuclei (most of the DBD nuclei have $A > 80$), first it is necessary to carry out extensive tests of the DSM truncation for $A > 80$ nuclei. However, this is for future studies.

R.S. is thankful to the Department of Science and Technology (India) for financial support.

[1] M. Meyer, Phys. Rev. **48**, 512 (1935).
[2] H. V. Klapdor-Kleingrothaus *et al.*, Mod. Phys. Lett. A **16**, 2409 (2001); Eur. Phys. J. A **12**, 147 (2001).
[3] S. R. Elliott and P. Vogel, Annu. Rev. Nucl. Part. Sci. **52**, 115 (2002).
[4] F. T. Avignone III, S. R. Elliott, and J. Engel, Rev. Mod. Phys. **80**, 481 (2008).
[5] E. Caurier, J. Menéndez, F. Nowacki, and A. Poves, Phys. Rev. Lett. **100**, 052503 (2008); J. Menéndez, A. Poves, E. Caurier, and F. Nowacki, Nucl. Phys. **A818**, 139 (2009).
[6] J. Suhonen and O. Civitarese, Phys. Rep. **300**, 123 (1998); F. Šimkovic, A. Faessler, V. Rodin, P. Vogel, and J. Engel, Phys. Rev. C **77**, 045503 (2008); M. S. Yousef, V. Rodin, A. Faessler, and F. Simkovic, *ibid.* **79**, 014314 (2009).
[7] M. Hirsch, K. Muto, T. Oda, and H. V. Klapdor-Kleingrothaus, Z. Phys. A **347**, 151 (1994).
[8] J. G. Hirsch, Rev. Mex. Fis. **48**, 87 (2002).
[9] J. Barea and F. Iachello, Phys. Rev. C **79**, 044301 (2009).
[10] K. Chaturvedi, R. Chandra, P. K. Rath, P. K. Raina, and J. G. Hirsch, Phys. Rev. C **78**, 054302 (2008).
[11] A. Shukla, P. K. Raina, R. Chandra, P. K. Rath, and J. G. Hirsch, Eur. Phys. J. A **23**, 235 (2005); A. Shukla, P. K. Raina, and P. K. Rath, J. Phys. G: Nucl. Part. Phys. **34**, 549 (2007).
[12] A. P. Meshik, C. M. Hohenberg, O. V. Pravdivtseva, and Y. S. Kapusta, Phys. Rev. C **64**, 035205 (2001). In this article, half-life for ^{130}Ba for all $(0\nu + 2\nu)e^+$ DBD decay modes has been determined using geochemical methods, and the result quoted is $2.2 \pm 0.5 \times 10^{21}$ yr (68% confidence limit).
[13] J. M. Gavriljuk *et al.*, Phys. At. Nucl. **69**, 2124 (2006); C. Sáenz *et al.*, Phys. Rev. C **50**, 1170 (1994).
[14] P. Belli *et al.*, Phys. Lett. **B658**, 193 (2008).
[15] A. S. Barabash, P. Hubert, A. Nachab, and V. Umatov, Nucl. Phys. **A785**, 371 (2007).
[16] P. Belli *et al.*, Astropart. Phys. **10**, 115 (1999).
[17] R. Cerulli *et al.*, Nucl. Instrum. Methods Phys. Res. A **525**, 535 (2004).
[18] S. Mishra, A. Shukla, R. Sahu, and V. K. B. Kota, Phys. Rev. C **78**, 024307 (2008).
[19] E.-W. Grewe *et al.*, Phys. Rev. C **77**, 064303 (2008).
[20] R. Sahu and S. P. Pandya, J. Phys. G **14**, L165 (1988); Nucl. Phys. **A548**, 64 (1992); K. C. Tripathy and R. Sahu, J. Phys. G **20**, 911 (1994); Int. J. Mod. Phys. E **11**, 531 (2002).
[21] K. C. Tripathy and R. Sahu, Nucl. Phys. **A597**, 177 (1996).
[22] R. Sahu and S. P. Pandya, Nucl. Phys. **A571**, 253 (1994).
[23] R. Sahu and V. K. B. Kota, Phys. Rev. C **66**, 024301 (2002); **67**, 054323 (2003); Eur. Phys. J. A **24**, 5 (2005).
[24] S. Mishra, R. Sahu, and V. K. B. Kota, Prog. Theor. Phys. **118**, 59 (2007).
[25] D. C. Zheng and L. Zamick, Phys. Lett. **B266**, 5 (1991); S. L. Tabor and J. Doring, Phys. Scr. **T56**, 175 (1995); H. Sun *et al.*, Phys. Rev. C **59**, 655 (1999); B. S. Narasingh *et al.*, Phys. Rev. C **75**, 061301(R) (2007).
[26] T. S. Kosmas, A. Faessler, and R. Sahu, Phys. Rev. C **68**, 054315 (2003).
[27] R. Sahu, K. H. Bhatt, and D. P. Ahalpara, J. Phys. G **16**, 733 (1990).
[28] R. Sahu, F. Šimkovic, and A. Faessler, J. Phys. G **25**, 1159 (1999).
[29] M. Doi and T. Kotani, Prog. Theor. Phys. **87**, 1207 (1992).
[30] M. Doi, T. Kotani, and E. Takasugi, Prog. Theor. Phys. **83**, 1 (1985).
[31] T. Tomoda, Rep. Prog. Phys. **54**, 53 (1991).
[32] F. Boehm and P. Vogel, *Physics of Massive Neutrinos*, 2nd edition (Cambridge University Press, Cambridge, 1992).
[33] G. Audi, A. H. Wapstra, and C. Thibault, Nucl. Phys. **A729**, 337 (2003).
[34] D. P. Ahalpara, K. H. Bhatt, and R. Sahu, J. Phys. G **11**, 735 (1985).
[35] R. Sahu, D. P. Ahalpara, and S. P. Pandya, J. Phys. G **13**, 603 (1987).
[36] K. C. Tripathy and R. Sahu, Nucl. Phys. **A597**, 177 (1996).
[37] K. Yako *et al.*, Phys. Rev. Lett. **103**, 012503 (2009).
[38] M. Horoi, S. Stoica, and B. A. Brown, Phys. Rev. C **75**, 034303 (2007).
[39] J. K. Boheke *et al.*, J. Phys. Chem. Ref. Data **34**, 57 (2005).