Nonperturbative relativistic approach to pion form factors: Predictions for future JLab experiments

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Some predictions concerning possible results of the future experiments at the Thomas Jefferson National Accelerator Facility (JLab) on the pion form factor $F_\pi(Q^2)$ are made. The calculations exploit the method proposed previously by the authors and based on the instant-form Poincaré invariant approach to pions, considered as quark-antiquark systems. This model has predicted with surprising accuracy the values of $F_\pi(Q^2)$, which were measured later in JLab experiments. The results are almost independent from the form of wave function. The pion mean square radius $\langle r_\pi^2\rangle$ and the decay constant f_π also agree with experimental values. The model gives powerlike asymptotic behavior of $F_\pi(Q^2)$ at high momentum transfer in agreement with QCD predictions.

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I. INTRODUCTION

The recent highly accurate experiments on the measurement of the pion form factor in the range of Q^2 up to 2.45 GeV^2 [1,2] ($Q^2 = -q^2$, where q is momentum transfer) and future Thomas Jefferson National Accelerator Facility (JLab) experiments up to $Q^2 \approx 6$ GeV^2 [3,4] enhanced the interest in theoretical descriptions of pions at high Q^2.

It is usually believed that these future experiments will provide a meaningful test of the transition between perturbative and nonperturbative regions, which is expected at much lower Q^2 in the case of pions than of other hadrons, in particular, nucleons. At the present time, different theoretical approaches to the pion form factor $F_\pi(Q^2)$ exist. They are partly listed and described in Ref. [2] (Sec. IV) (see also Ref. [5]). In the frameworks of some of these models, a certain agreement with existing experimental data is obtained for soft $F_\pi(Q^2)$. For the region of high momentum transfer, the theoretical results differ from one another to a great extent. It seems us that one has almost no hope of finding the appearance of perturbative degrees of freedom in future JLab experiments on $F_\pi(Q^2)$. It is difficult to imagine that in the wide band of nonperturbative theoretical curves there would not be any one that agrees with the experimental data. The large variety of nonperturbative predictions for future data for $F_\pi(Q^2)$ makes it necessary to formulate the problem of detecting of perturbative effects in a slightly different way than it is usually done. We propose to accept one of the theories that describes correctly the existing data and continue the calculations for higher Q^2. If future data would require to adjust the calculations, beginning from some values, by introducing the quark mass dependence on Q^2 to agree the future data, then we would identify these values with the appearance of perturbative effects. In the present article, we use our own approach [6] as an example for the demonstration of the proposed scenario.

The reasons for this choice are as follows. Our approach has already demonstrated its predictive power: without any additional tuning of parameters, we predicted in Ref. [6] the values of $F_\pi(Q^2)$ obtained later in experiments [7–9]. At the same time, the approach gives the correct values of the mean square radius (MSR), the decay constant f_π, and the powerlike asymptotic behavior. Certainly, other criteria of discrimination of the approach may exist. For example, one can consider as "correct" an approach that gives a consistent treatment of the pion form factor in spacelike and timelike regions.

In the present article, we use the approach to the pion form factor $F_\pi(Q^2)$ proposed in our papers of about 10 years ago [6]. Our approach presents one of the versions of the constituent quark model (CQM). The method is based on the dispersion approach to the instant form of Poincaré invariant quantum mechanics [10] (see also the detailed version [11] and the review [12]).

Based on this approach and on the experimental data of the measurement of $F_\pi(Q^2)$ in the range of Q^2 up to 0.26 (GeV)^2 [13], in 1998 we obtained the model function for the pion form factor for the extended range of higher momentum transfers [6]. The experimental data obtained later [7–9] (see also the review of all experimental results in Ref. [2] and references therein) for the range of Q^2 larger by an order of magnitude coincide precisely with our theoretical curve of 1998 [6] with no additional fitting. This means that it is possible to consider our calculations [6] as an accurate prediction of the present experimental data for the pion form factor. The model describes correctly the pion MSR and the decay constant f_π. It is important to notice that the dependence of our results for $F_\pi(Q^2)$ on the form of wave functions is very weak [6]. Moreover, our approach gives the correct powerlike asymptotic behavior of $F_\pi(Q^2)$ at $Q^2 \to \infty$, so the model works well at high as well as low values of Q^2.

Taking into account these advantages of our approach, we hope that the model will continue to give a good description of experimental data at higher momentum transfers, in

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particular for future JLab measurements in the range $2.45 \leqslant Q^2 \leqslant 6$ (GeV)^2 (after having withstood the test of 10-fold increases of Q^2 range, it may withstand another much smaller increase). If the experimental data will not follow our theoretical curve, then we shall adjust the theory by taking into account the quark-mass dependence on Q^2. Within our approach, this dependence is a manifestation of the appearance of perturbative degrees of freedom.

The article is organized in the following way. We start in Sec. II with a brief review of the basic theoretical formalism of our approach. The results of calculations and the comparison with the experimental data and other theoretical models are given in Sec. III. In Sec. IV, the asymptotic behavior of the form factor is considered. Finally, our conclusions are given in Sec. V.

II. THE MODEL

Our method is a version of the instant form of the Poincaré invariant constituent-quark model (PICQM), formulated on the base of a dispersion approach (see, e.g., Refs. [6,10]). As is well known, the dispersion approach is based on the general properties of space and time and therefore is to a certain extent "model independent." That is why the calculation of electromagnetic form factors using the dispersion approach are distinguished from other approaches. This advantage of our method is emphasized in Ref. [14].[1]

The main point of our approach is the construction of the operator of electromagnetic current, which preserves Lorentz covariance and conservation laws in the relativistic invariant impulse approximation (the so-called modified impulse approximation, MIA) [10]. This approximation is constructed using dispersion-relation integrals over composite-particle mass, that is, over the Mandelstam variables s, s' [11]. This variant of dispersion approach was developed in Refs. [15–21] and was fruitfully used to investigate the structure of composite systems.

Let us recall some principal points of our approach [6,10]. In our variant of PICQM, pion electromagnetic form factor in MIA has the form

$$F_\pi(Q^2) = \int d\sqrt{s}\, d\sqrt{s'}\, \varphi(k)\, g_0(s, Q^2, s')\, \varphi(k'). \qquad (1)$$

Here, $\varphi(k)$ is the pion wave function in the sense of PICQM and $g_0(s, Q^2, s')$ is the free two-particle form factor. It may be obtained explicitly by the methods of relativistic kinematics and is a relativistic invariant function.

The wave function in Eq. (1) has the following structure:

$$\varphi(k) = \sqrt[4]{s}\, u(k) k, \quad s = 4(k^2 + M^2).$$

Here M is the mass of the constituent quark. For the function $u(k)$, we use some phenomenological wave functions.

[1]However, our free form factor differs from the free form factor of Ref. [14] because in Ref. [14] the normalizations of one-particle wave vectors and two-particle wave vectors are inconsistent in the basis where of the two-particle center of mass is separated.

The function $g_0(s, Q^2, s')$ is written in terms of the quark electromagnetic form factors in the form

$$g_0(s, Q^2, s') = \frac{(s + s' + Q^2)Q^2}{2\sqrt{(s - 4M^2)(s' - 4M^2)}} \frac{\theta(s, Q^2, s')}{[\lambda(s, -Q^2, s')]^{3/2}} \times \frac{1}{\sqrt{1 + Q^2/4M^2}} \Big\{ (s + s' + Q^2)\big[G_E^q(Q^2) + G_E^{\bar{q}}(Q^2)\big] \cos(\omega_1 + \omega_2) + \frac{1}{M} \xi(s, Q^2, s') \times \big[G_M^q(Q^2) + G_M^{\bar{q}}(Q^2)\big] \sin(\omega_1 + \omega_2) \Big\}. \qquad (2)$$

Here, $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$,

$$\xi = \sqrt{ss'Q^2 - M^2\lambda(s, -Q^2, s')},$$

ω_1 and ω_2 are the Wigner rotation parameters

$$\omega_1 = \arctan \frac{\xi(s, Q^2, s')}{M[(\sqrt{s} + \sqrt{s'})^2 + Q^2] + \sqrt{ss'}(\sqrt{s} + \sqrt{s'})},$$

$$\omega_2 = \arctan \frac{\alpha(s, s')\xi(s, Q^2, s')}{M(s + s' + Q^2)\alpha(s, s') + \sqrt{ss'}(4M^2 + Q^2)},$$

$\alpha(s, s') = 2M + \sqrt{s} + \sqrt{s'}$, $\theta(s, Q^2, s') = \vartheta(s' - s_1) - \vartheta(s' - s_2)$, ϑ is the step function, and

$$s_{1,2} = 2M^2 + \frac{1}{2M^2}(2M^2 + Q^2)(s - 2M^2) \mp \frac{1}{2M^2}\sqrt{Q^2(Q^2 + 4M^2)s(s - 4M^2)}.$$

Note that the magnetic form factor contribution to Eq. (2) is due to the spin rotation effect only [22]. Here, $G_{E,M}^{u,\bar{d}}(Q^2)$ are electric and magnetic form factors of quarks, respectively.

Let us note that we introduce electromagnetic quark form factors, in particular, to obtain a description of the maximal set of experimental data on pions, including the MSR and the decay constant simultaneously, at the same values of the parameters of the model [23].

We use the following explicit forms of the quark form factors:

$$G_E^q(Q^2) = e_q f_q(Q^2),$$
$$G_M^q(Q^2) = (e_q + \kappa_q) f_q(Q^2),$$

where e_q are quark charges and κ_q are anomalous magnetic moments that enter our equations through the sum $s_q = \kappa_u + \kappa_{\bar{d}}$. We use

$$f_q(Q^2) = \frac{1}{1 + \ln\left(1 + \langle r_q^2 \rangle Q^2/6\right)}, \qquad (3)$$

where $\langle r_q^2 \rangle$ is the quark MSR.

Let us discuss in brief the motivation for choosing the explicit form in Eq. (3). One of the features of our approach is that the form factor asymptotic behavior at $Q^2 \to \infty$, $M \to 0$ does not depend on the choice of the wave function in Eq. (1) and is defined by the relativistic kinematics of the two-quark system only [6]. In the pointlike quark approximation ($\kappa_q = 0$, $\langle r_q^2 \rangle = 0$), the asymptotic behavior coincides with that described by quark counting laws [24,25] (see also the recent discussion in Ref. [26]): $F_\pi(Q^2) \sim Q^{-2}$. The asymptotic behavior of the pion form factor was considered in Refs. [27,28] (see also Ref. [29]). The form in Eq. (3) gives logarithmic corrections to the power-law asymptotics, obtained in QCD. So, in our approach, the form in Eq. (3) for the quark form factor gives the same asymptotics as predicted by QCD. Let us note that another choice of the form of quark form factor, for example, the monopole form [30], changes essentially the pion form factor asymptotic behavior so that it does not correspond to QCD asymptotics anymore.

To calculate the pion form factor, we use wave functions of different forms: harmonic oscillator wave functions (analogous to those used in the seminal Ref. [31] and continued recently in Ref. [32]), power-law-type wave functions with the explicit form motivated by perturbative QCD calculations at high Q^2 [33–35], and wave functions with linear confinement and Coulomb-like behavior at small distances [36]. These functions are of the form

$$u(k) = N_{\rm HO}\exp(-k^2/2b^2), \quad N_{\rm HO} = \sqrt{\frac{4}{\sqrt{\pi}\, b^3}}, \tag{4}$$

$$u(k) = N_{\rm PL}\,(k^2/b^2+1)^{-n}, \quad n=3, \quad N_{\rm PL} = 16\sqrt{\frac{2}{7\,\pi\, b^3}}, \tag{5}$$

$$\begin{aligned} u(r) &= N_T\,\exp(-\alpha r^{3/2} - \beta r),\\ \alpha &= \frac{2}{3}\sqrt{2\,M_r\,a}, \quad \beta = M_r\,b, \quad N_T = \frac{3\sqrt{2}\,\alpha}{\sqrt{N(\alpha\,,\beta)}},\\ N(\alpha\,,\beta) &= 9\,\alpha\,\sqrt[3]{2\alpha}\,\Gamma\left(\frac{5}{3}\right)\,{}_1F_1\left(\frac{5}{6},\frac{2}{3},t\right)\\ &\quad - 2\sqrt[3]{4\alpha^2}\,\beta\,\Gamma\left(\frac{1}{3}\right)\,{}_1F_1\left(\frac{7}{6},\frac{4}{3},t\right)\\ &\quad + 6\,\beta^2\,{}_2F_2\left(1,\frac{3}{2},\frac{4}{3},\frac{5}{3},t\right), \quad t = -\frac{8\,\beta^3}{27\,\alpha^2}, \end{aligned} \tag{6}$$

where a , b are the parameters of linear and Coulomb parts of potential, respectively; M_r is the reduced mass of the two-particle system; $b = (4/3)\,\alpha_s$, $\alpha_s = 0.59$ on the scale of the light meson mass; ${}_pF_q$ are hypergeometric functions; and $\Gamma(x)$ is the Euler Γ function.

The parameters of the model are the same as in Ref. [6] where the motivation of the choice is described in detail. Let us note that for the constituent-quark mass $M = 0.22$ GeV, the values of parameters (4)–(6) were chosen in such a way as to ensure the pion MSR within experimental uncertainties $\langle r_\pi^2\rangle_{\rm exp}^{1/2} = 0.657 \pm 0.012$ fm [13] as well as the best description of the decay constant $f_{\pi\,\rm exp} = 0.1317 \pm 0.0002$ GeV [37]. The sum of the quark anomalous magnetic moments is $s_q = 0.0268$, and the quark MSR is $\langle r_q^2\rangle \simeq 0.3/M^2$. The values of other parameters are the following: in model (4), $b = 0.3500$ GeV (the decay constant is $f_\pi = 127.4$ MeV); in model (5), $n = 3$, $b = 0.6131$ GeV ($f_\pi = 131.7$ MeV); and in model (6), $a = 0.1331$ GeV^2 $f_\pi = 131.7$ MeV). An interesting feature of our results is that at the fixed constituent quark mass, the dependence of the pion form factor on the choice of the models (4)–(6) is rather weak. The curves calculated with different wave functions but one and the same quark mass form groups [6]. From the theoretical point of view, this weak dependence of our calculations on the model is the consequence of the dispersion-relation base of the approach.

III. RESULTS OF THE CALCULATIONS

The results of the calculation of the charge pion form factor using the wave functions given by Eqs. (4), (5), and (6) and the value of constituent-quark mass $M = 0.22$ GeV (this parameter was fixed as early as 1998 [6] from the data at $Q^2 \leqslant 0.26$ (GeV)^2 [13]) are shown in Figs. 1 and 2.

Let us note that our relativistic CQM describes well the experimental data for the pion form factor, including the recent points [2]. Our upper curve corresponds to the model (4), and the lower curve corresponds to the models (5), with $n = 3$, and (6), which lie close to one another.

Let us emphasize that the parameters used in our calculations were obtained from the fitting to the experimental data up to $Q^2 \simeq 0.26$ GeV^2 [13]. At that time, the data for higher Q^2 were not correlated in different experiments and had significant uncertainties. The later data for pion form factor in JLab experiments up to $Q^2 = 2.45$ GeV^2 were obtained with rather good accuracy. All experimental points obtained in JLab so far agree very well with our prediction from 1998.

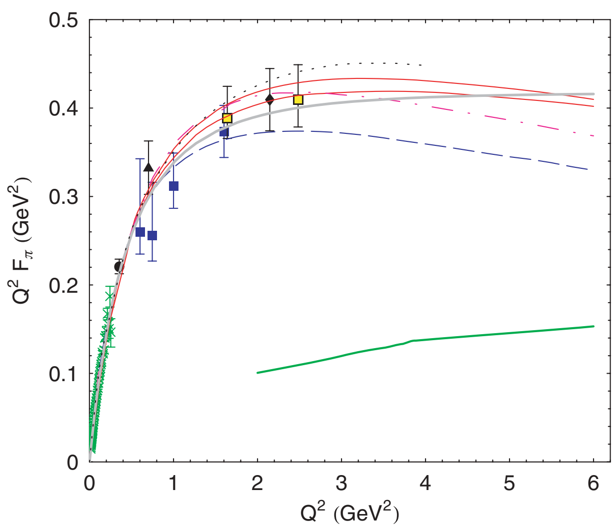

FIG. 1. (Color online) Our predictions for the pion form factor given in 1998 [6] are compared with data and with some other models. Our 1998 predictions are shown as full red (top) lines; the upper line is wave functions from Eq. (4) and the lower line is wave functions from Eqs. (5), with $n = 3$, and (6). The left (green) crosses represent data points of Amendolia *et al.* [13]. Other data points (all taken from Ref. [2]) are reanalyzed points of Ackerman *et al.* [38] (full circles); reanalyzed points of Brauel *et al.* [39] (full triangles); JLab results (full diamonds and squares). Other theoretical curves are the QCD approximation of Maris and Tandy [40] (dotted line), perturbative QCD (leading and next-to-leading order) of Bakulev *et al.* [41] (the lowest solid, green) and Nesterenko and Radyushkin [42] (dash-dotted, magenta), dispersion approach of Donoghue and Na [43] (dashed, blue), and the holographic approach of Grigoryan and Radyushkin [44] (thick, grey) (see also [51]).

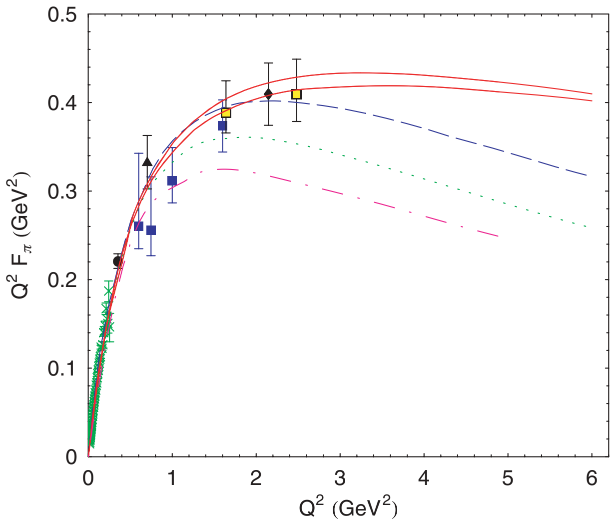

FIG. 2. (Color online) Comparison of our predictions with other CQMs. Data and our curves are the same as in Fig. 1. Other theoretical curves are those by C.-W. Hwang [45] (dashed, blue), Cardarelli *et al.* [30] and (precisely coinciding with it) instant-form predictions [46] (dash-dotted, magenta), and Ref. [47] (dotted, green). Predictions of an upgraded version of a paper [31] (Ref. [32], Fig. 5, $m_q = 0.22$) coincide precisely with our upper curve.

Let us discuss briefly the basic moments that provide good results for the pion form factor in our approach. First, throughout the calculation, the condition of Lorentz covariance and the conservation laws for the operator of electromagnetic current were satisfied. Second, the accurate description of the pion MSR constrains the behavior of the wave functions in momentum representation Eqs. (4), (5) at small relative momentum of quarks or of the wave function in coordinate representation Eq. (6) at large distances because of special properties of the integral representation Eq. (1). Third, the best description of the decay constant defines constraints for the wave function at large relative momenta because the contribution of small relative momenta to the decay constant is suppressed, as can be seen from the relativistic formula (see, e.g. [6,48]):

$$f_\pi = \frac{M\sqrt{3}}{\pi}\int \frac{k^2\,dk}{(k^2+M^2)^{3/4}}\,u(k).$$

So, our way of fixing the model parameters effectively constrains the behavior of wave functions both at small and at large relative momenta. The structure of our relativistic integral representation, Eq. (1), is so that the form factor behavior in the region of small momentum transfers is determined by the wave function at small relative momenta and the behavior of the form factor in the region of high momentum transfer is determined by the wave function at large relative momenta. The constraints for the wave functions provide the limitations for the form factor, and this is seen in the results of the calculation.

IV. ASYMPTOTIC BEHAVIOR

It is worth considering the form factor asymptotic form behavior at $Q^2 \to \infty$ in particular. In our article, Ref. [49], we show that in our approach the pion form factor asymptotic form at $Q^2 \to \infty$ and $M \to 0$ does not depend on the choice of a wave function but is defined by the relativistic kinematics only. We consider the fact that the asymptotic behavior obtained in our nonperturbative approach does coincide with that predicted by QCD as very significant. Our approach is consistent with the asymptotic freedom, and this feature surely distinguishes it from other nonperturbative approaches.

It is obvious that at very high momentum transfers, the quark mass decreases as it goes to zero at infinity. Our approach permits to take into account the dependence $M(Q^2)$ beginning from the range where this becomes necessary to correspond to experimental data. It is possible that this will take place at values of Q^2 less than 6 GeV^2.

The correct asymptotics is the consequence of the fact that the relativism is an intrinsic property of our approach. To demonstrate how it works, let us consider the simple example of pointlike quarks and model wave functions given by Eq. (4). In this case, in Eqs. (1) and (2),

$$G_E^u(Q^2) + G_E^{\bar d}(Q^2) = G_M^u(Q^2) + G_M^{\bar d}(Q^2) = 1.$$

For the model (4), it is easy to obtain the nonrelativistic integral representation of the form factor as the corresponding limit of the Eq. (1). Now the integration can be performed analytically, and the following form for the nonrelativistic pion form factor can be derived:

$$F_\pi(Q^2) = \exp\left(-\frac{Q^2}{16\,b^2}\right).$$

One can see that in the nonrelativistic case, the form factor does not depend on the mass of constituents and its asymptotics cannot agree with those of QCD. The correct asymptotic behavior is provided by relativistic effects.

In the relativistic case, the results for the integrals cannot be obtained analytically. To derive the asymptotic behavior in question, it is possible to use the asymptotic series for double integrals obtained in Ref. [50]. The first two terms give

$$F_\pi(Q^2) \sim \frac{2^{5/2}M}{Q}\,\mathrm{e}^{-(QM/4b^2)}\left(1+\frac{7b^2}{2MQ}\right). \qquad (7)$$

Let us take in relation (7) the limit at $M/b \to 0$. This means that the parameters of the model are such that $M/b \ll 1$. The physical meaning of this limit is that the increase of the momentum transfer is followed by the "undressing" of the constituent quarks and its transformation into current quark of pQCD. In this limit, from Eq. (7) up to logarithmic prefactors, we obtain the powerlike behavior coinciding with that of pQCD [6]:

$$F_\pi(Q^2) \sim \frac{14\sqrt{2}b^2}{Q^2},$$

V. SUMMARY

To conclude, we make some predictions about the results of the future JLab experiments on the pion form factor based on the method proposed in our previous papers. The method is a variant of composite quark model in the instant form of Poincaré invariant quantum mechanics. Our approach has certain advantages as compared with other CQM calculations. From a theoretical point of view, these advantages are the consequence of the fact that our approach has dispersion-relation-motivated foundations. This provides, in particular, weak model dependence of the results of calculations. The approach has demonstrated earlier its predictive power in describing all the data on the pion form factor obtained later in JLab experiments. Our calculations also give accurate values of the pion MSR and of the decay constant f_π and the correct asymptotic behavior at $Q^2 \to \infty$.

We hope that our model will provide a good description of the results of future JLab experiments on the measurement of the pion form factor in the range of momentum transfers up to $Q^2 \approx 6\ (\mathrm{GeV})^2$. If one needs to adjust the calculations beginning from some values by introducing the quark mass dependence on Q^2 to agree the data, then we propose to identify the effect with the appearance of perturbative effects.

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