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Pion-nucleon scattering in Kadyshevsky formalism. II. Baryon exchange sector

J. W. Wagenaar and T. A. Rijken
Imapp, FNWI, Radboud University Nijmegen, Nijmegen, The Netherlands

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In this article, which is the second part in a series of two, we construct tree-level baryon exchange and resonance amplitudes for $\pi N/MB$ scattering in the framework of the Kadyshevsky formalism. We use this formalism to formally implement absolute pair suppression, where we make use of the method of Takahashi and Umezawa. The resulting amplitudes are Lorentz invariant and causal. We continue studying the frame dependence of the Kadyshevsky integral equation using the method of Gross and Jackiw. The invariant amplitudes, including those for meson exchange, are linked to the phase shifts using the partial-wave basis.

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I. INTRODUCTION

In the previous article, referred to as article I [1], we have given a motivation for constructing a pion-nucleon (πN) scattering, or more generally a meson-baryon (MB) scattering, model. We have given the main ingredients of the model and, besides others, the (theoretical) results for meson exchange processes. In this article, referred to as article II, we present the results in the baryon sector. We construct tree-level amplitudes for baryon exchange and resonance or, to put it in other words, u- and s-channel baryon exchange diagrams in the Kadyshevsky formalism [2–5].

The Kadyshevsky formalism is equivalent to the Feynman formalism, because it can be derived using the same S-matrix formula. The main features for exploiting the Kadyshevsky formalism is that all particles are on the mass shell at the cost of an extra quasiparticle, which carries four-momentum only. A three-dimensional Lippmann-Schwinger type of integral equation comes about naturally, without any approximations as, for instance, in Ref. [6,7]. Especially at second order, this formalism provides a covariant, though frame dependent,[1] separation of positive- and negative-energy contributions. In this way it is a natural basis for implementing pair suppression, which may also be interesting for relativistic many-body theories.

Our motivation for studying pair suppression is threefold: (i) In pion-nucleon scattering the Weinberg [8] theorem concerning the smallness of some scattering lengths is a consequence of chiral symmetry. This can be realized in, for example, the linear σ model. However, this implies a strict relation between $g_{\pi NN}$ and $g_{\sigma NN}$ to bring about an almost complete cancellation between the Z graph and the σ exchange graph. This is "unnatural." Therefore, the nonlinear σ model is an attractive alternative and is preferred nowadays. Here, the Z graphs are much smaller. So, the use of the pseudovector $\gamma_\mu\gamma_5$ coupling for the pseudoscalar mesons seems to be superior to the pseudoscalar γ_5 coupling. Now in this particular case the degree of pair suppression is fixed by the use of the pseudovector coupling. (ii) Both baryon-baryon [9] and pion-nucleon [6] models with (absolute) pair suppression are phenomenologically very successful. (iii) Large N, SU(N) theory gives phenomenologically a rather satisfactory picture of hadron dynamics and supports the idea of pair suppression in hadron processes [10]. Considering the possibility that pair suppression is a general dynamical phenomenon at low energies, it is desirable to have a Lorentz-invariant description that enables one to have pair suppression to any degree. Such a description is provided in this article.

In Ref. [6] pair suppression is assumed by considering positive states in the integral equation only. Here we implement pair suppression formally and, to our knowledge for the first time, in a covariant and frame-independent way. This is done by using a method based on the Takahashi-Umezawa (TU) method [11–13]; see also article I. In article I we studied the n dependence of the integral equation using the method of Gross and Jackiw (GJ) [14]. We continue this here.

In Sec. II we start with introducing the concept of pair suppression. After discussing how it can be implemented formally we apply it to πN system. The amplitudes are calculated in Sec. III. In Sec. IV we use the helicity basis and make a partial-wave expansion to introduce the phase shifts. We show how the amplitudes are related to these phase shifts. This is done for the entire model.

II. PAIR SUPPRESSION FORMALISM

To understand the idea of pair suppression at low energy, picture a general meson-baryon (MB) vertex in terms of their constituent quarks (see Fig. 1). As stated in Ref. [15] every time a quark-antiquark ($q\bar{q}$) pair is created from the vacuum the vertex is damped. This idea is supported by results in Ref. [10], where that author considers a vertex creating a baryon-antibaryon ($B\bar{B}$) pair in a large N, SU(N) theory.[2] Such a vertex is comparable to that in Fig. 1(b), but now $N-1$ pairs need to be created. It is claimed in Ref. [10] that such vertices are indeed suppressed. Although it is questionable whether $N=3$ is really large, we assume that pair suppression holds for $\mathrm{SU}_{(F)}(3)$ theories at low energy.

Now, one could imagine that this principle should also apply for the creation of a meson-antimeson ($M\bar{M}$) pair and

[1]By "frame dependent" we mean dependent on a vector n^μ.

[2]In a SU(N) theory a baryon is represented as a q^N state, whereas a meson is always a $q\bar{q}$ state, independent of N.

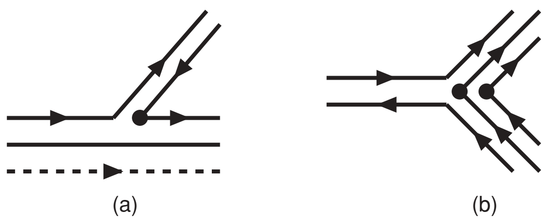

FIG. 1. (a) MMM (MBB) vertex and (b) $MB\bar{B}$ vertex.

therefore pair suppression should be implemented in the meson exchange sector (article I). For the reason why we have not done this one should look again at Fig. 1 and consider the large N, SU(N) theory again. For the creation of a $M\bar{M}$ pair at the vertex only one extra $q\bar{q}$ pair needs to be created instead of the $N-1$ pairs in the $B\bar{B}$ case and it is therefore much likelier to happen. Going back to the real $\mathrm{SU}_{(F)}(3)$ the difference is only one $q\bar{q}$ pair, nevertheless we assume that a $M\bar{M}$ pair creation is not suppressed.

Also from physical point of view it is nonsense to imply pair suppression in the meson sector. To see this one has to realize that an antimeson is also a meson. So, assuming pair suppression in the meson sector means that a triple meson (MMM) vertex is suppressed, which makes it impossible to consider meson exchange in meson-baryon scattering as we did in article I. From Fig. 1(a) we see that the MMM vertex is of the same order [in number of $q\bar{q}$ creations, as compared to Fig. 1(b)] as the meson-baryon-baryon (MBB) vertex in $\mathrm{SU}_{(F)}(3)$. So, suppressing the MMM vertex means that we should also suppress the MBB vertex and no description of MB scattering in terms of MB vertices is possible at all!

This does not mean, however, that there is no pair suppression what so ever in the meson sector. As can be seen from the amplitudes in article I we only considered MBB vertices, whereas in principle also $MB\bar{B}$ vertices could have been included. The latter vertices are not considered using the argument of pair suppression as discussed above. We will come back to this later.

Because we suppressed the $MB\bar{B}$ vertex it means that pair suppression should also be active in the vector-meson dominance (VMD) [16] model describing nucleon Compton scattering ($\gamma N \to \gamma N$). From electron Compton scattering it is well known that the Thomson limit is exclusively due to the negative-energy electron states (see, for instance, section 3-9 of Ref. [17]). However, because the nucleon is composite it may well be that the negative-energy contribution is produced by only one of the constituents [18] and it is not necessary to create an entire antibaryon.

The suppression of negative-energy states may harm the causality and Lorentz invariance condition. Therefore, the question may arise whether it is possible to include pair suppression and still maintain causality and Lorentz invariance. The following example shows that it should in principle be possible: Imagine an infinitely dense medium where all antinucleon states are filled, i.e., the Fermi energy of the antinucleons $\bar{p}_F = \infty$, and that for nucleons $p_F = 0$. An example would be an antineutron star of infinite density. Then, in such a medium pair production in πN scattering is Pauli blocked, because all antinucleon states are filled. Denoting the ground state by $|\Omega\rangle$, one has (see, e.g., Ref. [19]),

$$S_F(x-y) = -i\langle\Omega|T[\psi(x)\bar{\psi}(y)]|\Omega\rangle$$

which gives in momentum space [19]

$$S_F(p; p_F, \bar{p}_F) = \frac{\not{p}+M}{2E_p}\left\{\frac{1-n_F(p)}{p_0-E_p+i\varepsilon} + \frac{n_F(p)}{p_0-E_p-i\varepsilon} - \frac{1-\bar{n}_F(p)}{p_0+E_p-i\varepsilon} - \frac{\bar{n}_F(p)}{p_0+E_p+i\varepsilon}\right\}.$$

At zero temperature $T=0$ the noninteracting fermion functions $n_F, \bar{n}_F$ are defined by

$$n_F = \begin{cases} 1, & |\mathbf{p}| < p_F \\ 0, & |\mathbf{p}| > p_F \end{cases} \quad \bar{n}_F = \begin{cases} 1, & |\mathbf{p}| < \bar{p}_F \\ 0, & |\mathbf{p}| > \bar{p}_F \end{cases}.$$

In the medium sketched above, clearly $n_F(p) = 0$ and $\bar{n}_F(p) = 1$, which leads to a propagator $S_{\rm ret}(p; 0, \infty)$. This propagator is causal and Lorentz invariant. The above (academic) example may perhaps convince a sceptical reader that a perfect relativistic model with "absolute pair suppression" is feasible indeed.

As far as our results are concerned we refer to Sec. III, where we will see that intermediate baryon states are represented by retarded(-like) propagators, which have the nice feature of being causal and n independent. We, therefore, have a theory that is relativistic and yet it does contain (absolute) pair suppression.

A. Equations of motion

Consider a Lagrangian containing not only the free fermion part but also a (simple) coupling between fermions and a scalar

$$\mathcal{L} = \mathcal{L}_{\rm free} + \mathcal{L}_I = \bar{\psi}\left(\frac{i}{2}\overrightarrow{\not{\partial}} - \frac{i}{2}\overleftarrow{\not{\partial}} - M\right)\psi + g\bar{\psi}\Gamma\psi\cdot\phi \tag{1}$$

The Euler-Lagrange equation for the fermion part reads

$$(i\not{\partial} - M)\psi = -g\Gamma\psi\cdot\phi \tag{2}$$

To incorporate pair suppression we pose that the transitions between positive- and negative-energy fermion states vanish in the interaction part of (1), i.e., $\overline{\psi^{(+)}}\Gamma\psi^{(-)} = \overline{\psi^{(-)}}\Gamma\psi^{(+)} = 0$. So, we impose *absolute* pair suppression. From now on, when we speak of pair suppression we mean absolute pair suppression, unless it is mentioned otherwise. Of course it is in principle possible to allow for some pair production. This can be done, for instance, by not eliminating the terms $\overline{\psi^{(+)}}\Gamma\psi^{(-)}$ and $\overline{\psi^{(-)}}\Gamma\psi^{(+)}$ in Eq. (1) but allowing them with some small coupling $g' \ll g$. This, however, makes the situation much more complicated and it is not worked out here.

Because half of the term on the right-hand side of Eq. (2) finds its origin in such vanished terms, it is reduced by a factor of 2 by the pair suppression condition. Making the split up $\psi = \psi^{(+)} + \psi^{(-)}$, which is invariant under orthochronous Lorentz transformations, in Eq. (2) we assume both parts are independent, so we have

$$(i\not{\partial} - M)\psi^{(+)} = -\frac{g}{2}\Gamma\psi^{(+)}\cdot\phi, \tag{3a}$$

$$(i\not{\partial} - M)\psi^{(-)} = -\frac{g}{2}\Gamma\psi^{(-)}\cdot\phi. \tag{3b}$$

One might wonder why we did not consider independent positive- and negative-energy fields from the start in Eq. (1). Although this would not cause any trouble in the interaction part ($\mathcal{L}_I$) it will in the free part. The quantum condition in such a situation would be $\{\psi^{(\pm)}(x), \pi^{(\pm)}(y)\} = i\delta^3(x-y)$. This is in conflict with the important relations between the positive- and negative-energy components

$$\begin{aligned}\{\psi^{(+)}(x), \overline{\psi^{(+)}}(y)\} &= (i\partial\!\!\!/ + M)\Delta^+(x-y)\\ \{\psi^{(-)}(x), \overline{\psi^{(-)}}(y)\} &= -(i\partial\!\!\!/ + M)\Delta^-(x-y)\end{aligned} \tag{4}$$

that we do need. Therefore we do not make the split up in the Lagrangian but in the equations of motion.

The assumption that both parts $\psi^{(+)}$ and $\psi^{(-)}$ are independent means that in addition to the anticommutation relations in Eq. (4) all others are zero.

To incorporate pair suppression in the meson sector (see article I) the only thing to do is to exclude the transitions $\overline{\psi^{(+)}}\Gamma\psi^{(-)}$ and $\overline{\psi^{(-)}}\Gamma\psi^{(+)}$ in the interaction Lagrangians. By doing so, only u and $\bar{u}$ spinors will contribute. Therefore, only these spinors are present in the results for meson exchange (article I). For baryon exchange and resonance diagrams the implications for pair suppression are less trivial. We, therefore, discuss how pair suppression can be implemented in these situations in the following subsections.

B. Takahashi-Umezawa scheme for pair suppression

To obtain the interaction Hamiltonian in case of pair suppression we set up the theory very similar to the TU scheme [11–13] introduced and applied in article I. Because we only make the split-up in the fermion fields, the scalar fields are unaffected and therefore not included in this subsection.

We start with defining the currents

$$\boldsymbol{j}_{\psi^{(\pm)},a}(x) = \left(-\frac{\partial\mathcal{L}_I}{\partial\boldsymbol{\psi}^{(\pm)}(x)}, -\frac{\partial\mathcal{L}_I}{\partial(\partial_\mu\boldsymbol{\psi}^{(\pm)})(x)}\right). \tag{5}$$

Solutions to the equations of motion resulting from a general (interaction) Lagrangian are Yang-Feldman (YF) [20] types of equations

$$\begin{aligned}\boldsymbol{\psi}^{(\pm)}(x) = \psi^{(\pm)}(x) + \frac{1}{2}\int d^4y\; D_a(y)(i\partial\!\!\!/ + M)\theta[n(x-y)]\\ \times\Delta(x-y)\cdot\boldsymbol{j}_{\psi^{(\pm)},a}(y).\end{aligned} \tag{6}$$

Here, we have chosen to use the retarded Green functions again to be close to the treatment of article I.

Furthermore, we introduce the auxiliary fields

$$\begin{aligned}\psi^{(\pm)}(x,\sigma) = \psi^{(\pm)}(x) \mp i\int_{-\infty}^{\sigma} d^4y D_a(y)(i\partial\!\!\!/ + M)\\ \times\Delta^\pm(x-y)\cdot\boldsymbol{j}_{\psi^{(\pm)},a}(y).\end{aligned} \tag{7}$$

Combining these two equations [(6) and (7)] we get

$$\begin{aligned}\boldsymbol{\psi}^{(\pm)}(x) = \psi^{(\pm)}(x/\sigma) + \frac{1}{4}\int d^4y[D_a(y)(i\partial\!\!\!/ + M), \epsilon(x-y)]\\ \times\Delta(x-y)\cdot\boldsymbol{j}_{\psi^{(\pm)},a}(y) \pm \frac{i}{2}\int d^4y\;\theta[n(x-y)]\\ \times D_a(y)(i\partial\!\!\!/ + M)\Delta^{(1)}(x-y)\cdot\boldsymbol{j}_{\psi^{(\pm)},a}(y).\end{aligned} \tag{8}$$

The factor $1/2$ in Eq. (6) is essential. This becomes clear when we decompose $\Delta^\pm(x-y) = \frac{\pm i}{2}\Delta(x-y) + \frac{1}{2}\Delta^{(1)}(x-y)$ in Eq. (7). The first part (Δ) combines with Eq. (6) to the second term on the right-hand side of Eq. (8) and the second part $[\Delta^{(1)}]$ gives a new contribution to $\psi^{(\pm)}$ as compared to ψ in the original treatment. We see that if we add $\psi^{(+)}$ and $\psi^{(-)}$ we get back the ψ in the original treatment again. This makes the factor 1/2 difference in the first part of Eq. (8) easier to understand.

Similar to the treatment in Appendix C of article I, it can be shown that $\psi^{(\pm)}(x)$ and $\psi^{(\pm)}(x,\sigma)$ satisfy the same commutation relation and that the unitary operator connecting the two is related to the S matrix. Following similar steps the defining equation for the interaction Hamiltonian is

$$\begin{aligned}[\psi^{(\pm)}(x), \mathcal{H}_I(y;n)] = U^{-1}[\sigma][D_a(y)(\pm)(i\partial\!\!\!/ + M)\\ \times\Delta^\pm(x-y)\cdot\boldsymbol{j}_{\psi^{(\pm)},a}(y)]U[\sigma].\end{aligned} \tag{9}$$

Having discussed the formalism to implement pair suppression, now we are going to apply it.

C. (Pseudo-)scalar coupling

In the (pseudo-)scalar sector of the theory including pair suppression we start with the following interaction Lagrangian[3]

$$\mathcal{L}_I = g\overline{\boldsymbol{\psi}^{(+)}}\Gamma\boldsymbol{\psi}^{(+)}\cdot\boldsymbol{\phi} + g\overline{\boldsymbol{\psi}^{(-)}}\Gamma\boldsymbol{\psi}^{(-)}\cdot\boldsymbol{\phi}, \tag{10}$$

where $\Gamma = 1$ or $\Gamma = i\gamma^5$. We will not use the specific forms for Γ until the discussion of the amplitudes in Sec. III. This is to be as general as possible.

From Eq. (10) we deduce the currents according to Eq. (5)

$$\begin{aligned}\boldsymbol{j}_{\psi^{(\pm)},a} &= (-g\Gamma\boldsymbol{\psi}^{(\pm)}\cdot\phi, 0)\\ \boldsymbol{j}_{\phi,a} &= (-g\overline{\psi^{(+)}}\Gamma\boldsymbol{\psi}^{(+)} - g\overline{\psi^{(-)}}\Gamma\boldsymbol{\psi}^{(-)}, 0).\end{aligned} \tag{11}$$

The fields in the Heisenberg representation (HR) can be expressed in terms of fields in the interaction representation (IR) using (8)

$$\begin{aligned}\boldsymbol{\psi}^{(\pm)}(x) = \psi^{(\pm)}(x/\sigma) \mp \frac{ig}{2}\int d^4y\;\theta[n(x-y)](i\partial\!\!\!/ + M)\\ \times\Delta^{(1)}(x-y)\Gamma\psi^{(\pm)}(y)\cdot\phi(y)\end{aligned} \tag{12a}$$

$$\begin{aligned}\phi(x) &= \phi(x/\sigma) + \frac{1}{4}\int d^4y[D_a(y), \epsilon(x-y)]\\ &\quad\times\Delta(x-y)\cdot\boldsymbol{j}_{\phi,a}(y)\\ &= \phi(x/\sigma).\end{aligned} \tag{12b}$$

Equation (12a) was found by assuming that the coupling constant is small and considering only contributions up to order g, just as in article I.

With the expressions (12a) and (12b) and the definition of the commutator of the (fermion) fields with the interaction

[3]We note that this interaction Lagrangian (10) is charge invariant.

Hamiltonian (9) we get

$$
\begin{aligned}
&[\psi^{(+)}(x), \mathcal{H}_I(y;n)] \\
&\quad = -g(i\partial\!\!\!/ + M)\Delta^+(x-y)\Gamma\psi^{(+)}(y)\cdot\phi(y) \\
&\qquad + \frac{ig^2}{2}(i\partial\!\!\!/ + M)\Delta^+(x-y) \\
&\qquad \times \int d^4z\Gamma\theta[n(y-z)](i\partial\!\!\!/_y + M)\Delta^{(1)}(y-z) \\
&\qquad \times \Gamma\psi^{(+)}(z)\cdot\phi(z)\phi(y), \\
&[\psi^{(-)}(x), \mathcal{H}_I(y;n)] \\
&\quad = g(i\partial\!\!\!/ + M)\Delta^-(x-y)\Gamma\psi^{(-)}(y)\cdot\phi(y) \\
&\qquad + \frac{ig^2}{2}(i\partial\!\!\!/ + M)\Delta^-(x-y) \\
&\qquad \times \int d^4z\Gamma\theta[n(y-z)](i\partial\!\!\!/_y + M)\Delta^{(1)}(y-z) \\
&\qquad \times \Gamma\psi^{(-)}(z)\cdot\phi(z)\phi(y).
\end{aligned}
\tag{13}
$$

Here, we have not included the commutator of the scalar field ϕ with the interaction Hamiltonian, because (13) already contains enough information to get the interaction Hamiltonian

$$
\begin{aligned}
&\mathcal{H}_I(x;n) \\
&\quad = -g\overline{\psi^{(+)}}\Gamma\psi^{(+)}\cdot\phi - g\overline{\psi^{(-)}}\Gamma\psi^{(-)}\cdot\phi \\
&\qquad + \frac{ig^2}{2}\int d^4y[\overline{\psi^{(+)}}\Gamma\phi]_x\theta[n(x-y)](i\partial\!\!\!/_x + M) \\
&\qquad \times \Delta^{(1)}(x-y)[\Gamma\psi^{(+)}\phi]_y \\
&\qquad - \frac{ig^2}{2}\int d^4y[\overline{\psi^{(-)}}\Gamma\phi]_x\theta[n(x-y)] \\
&\qquad \times (i\partial\!\!\!/_x + M)\Delta^{(1)}(x-y)[\Gamma\psi^{(-)}\phi]_y.
\end{aligned}
\tag{14}
$$

In Eq. (14) we see that the interaction Hamiltonian contains terms proportional to $\Delta^{(1)}(x-y)$ which are of order $O(g^2)$. These terms will be essential to get covariant and n-independent S matrix elements and amplitudes at order $O(g^2)$.

If we would include external quasifields in interaction Lagrangian (10), then the terms of order g^2 in the interaction Hamiltonian (14) would be quartic in the quasifield. Two quasifields can be contracted

$$
\bar{\chi}(x)\underbrace{\chi(x)\bar{\chi}(y)}\chi(y) = \bar{\chi}(x)\theta[n(x-y)]\chi(y). \tag{15}
$$

So the terms of order g^2 get an additional factor $\theta[n(x-y)]$. However, because these terms already contain such a factor, we make the identification $\theta[n(x-y)]\theta[n(x-y)] \to \theta[n(x-y)]$. Therefore, all relevant πN terms in Eq. (14) are quadratic in the external quasifield, just as we want. This argument is valid for all couplings.

D. (Pseudo-)vector coupling

Here, we repeat the steps of the previous subsection (Sec. II C) but now in the case of (pseudo-)vector coupling. The interaction Lagrangian reads

$$
\mathcal{L}_I = \frac{f}{m_\pi}\overline{\boldsymbol{\psi}^{(+)}}\Gamma_\mu\boldsymbol{\psi}^{(+)}\cdot\partial^\mu\boldsymbol{\phi} + \frac{f}{m_\pi}\overline{\boldsymbol{\psi}^{(-)}}\Gamma_\mu\boldsymbol{\psi}^{(-)}\cdot\partial^\mu\boldsymbol{\phi}, \tag{16}
$$

where $\Gamma_\mu = \gamma_\mu$ or $\Gamma_\mu = \gamma_5\gamma_\mu$. From Eq. (16) we deduce the currents

$$
\begin{aligned}
\boldsymbol{j}_{\psi^{(\pm)},a} &= \left(-\frac{f}{m_\pi}\Gamma_\mu\boldsymbol{\psi}^{(\pm)}\cdot\partial^\mu\boldsymbol{\phi}, 0\right) \\
\boldsymbol{j}_{\phi,a} &= \left(0, -\frac{f}{m_\pi}\overline{\boldsymbol{\psi}^{(+)}}\Gamma_\mu\boldsymbol{\psi}^{(+)} - \frac{f}{m_\pi}\overline{\boldsymbol{\psi}^{(-)}}\Gamma_\mu\boldsymbol{\psi}^{(-)}\right).
\end{aligned}
\tag{17}
$$

The fields in the HR are expressed in terms of fields in the IR as follows

$$
\begin{aligned}
\boldsymbol{\psi}^{(\pm)}(x) &= \psi^{(\pm)}(x/\sigma) \mp \frac{if}{2m_\pi}\int d^4y\theta[n(x-y)](i\partial\!\!\!/ + M) \\
&\quad \times \Delta^{(1)}(x-y)\Gamma_\mu\psi^{(\pm)}(y)\cdot\partial^\mu\phi(y) &\text{(18a)} \\
\boldsymbol{\phi}(x) &= \phi(x/\sigma) &\text{(18b)} \\
\partial^\mu\boldsymbol{\phi}(x) &= [\partial^\mu\phi(x,\sigma)]_{x/\sigma} - \frac{f}{m_\pi}n^\mu\overline{\psi^{(+)}}(x)n\cdot\Gamma\psi^{(+)}(x) \\
&\quad - \frac{f}{m_\pi}n^\mu\overline{\psi^{(-)}}(x)n\cdot\Gamma\psi^{(-)}(x). &\text{(18c)}
\end{aligned}
$$

The commutators of the different fields with the interaction Hamiltonian are

$$
\begin{aligned}
&[\psi^{(+)}(x), \mathcal{H}_I(y;n)] \\
&\quad = \frac{f}{m_\pi}(i\partial\!\!\!/ + M)\Delta^+(x-y)\Big[-\Gamma_\mu\psi^{(+)}\cdot\partial^\mu\phi \\
&\qquad + \frac{f}{m_\pi}n\cdot\Gamma\psi^{(+)}\overline{\psi^{(+)}}n\cdot\Gamma\psi^{(+)} + \frac{f}{m_\pi}n\cdot\Gamma\psi^{(+)} \\
&\qquad \times \overline{\psi^{(-)}}n\cdot\Gamma\psi^{(-)}\Big]_y + \frac{if^2}{2m_\pi^2}(i\partial\!\!\!/ + M)\Delta^+(x-y) \\
&\qquad \times \int d^4z\Gamma_\mu\theta[n(y-z)](i\partial\!\!\!/_y + M)\Delta^{(1)}(y-z)\Gamma_\nu\psi^{(+)} \\
&\qquad \times (z)\cdot\partial^\nu\phi(z)\partial^\mu\phi(y) \\
&[\psi^{(-)}(x), \mathcal{H}_I(y;n)] \\
&\quad = -\frac{f}{m_\pi}(i\partial\!\!\!/ + M)\Delta^-(x-y)\Big[-\Gamma_\mu\psi^{(-)}\cdot\partial^\mu\phi \\
&\qquad + \frac{f}{m_\pi}n\cdot\Gamma\psi^{(-)}\overline{\psi^{(+)}}n\cdot\Gamma\psi^{(+)} + \frac{f}{m_\pi}n\cdot\Gamma\psi^{(-)} \\
&\qquad \times \overline{\psi^{(-)}}n\cdot\Gamma\psi^{(-)}\Big]_y - \frac{if^2}{2m_\pi^2}(i\partial\!\!\!/ + M)\Delta^-(x-y) \\
&\qquad \times \int d^4z\Gamma_\mu\theta[n(y-z)](i\partial\!\!\!/_y + M)\Delta^{(1)}(y-z) \\
&\qquad \times \Gamma_\nu\psi^{(-)}(z)\cdot\partial^\nu\phi(z)\partial^\mu\phi(y)
\end{aligned}
\tag{19}
$$

and from these equations we deduce the interaction Hamiltonian

$$
\begin{aligned}
&\mathcal{H}_I(x;n) \\
&\quad = -\frac{f}{m_\pi}\overline{\psi^{(+)}}\Gamma_\mu\psi^{(+)}\cdot\partial^\mu\phi - \frac{f}{m_\pi}\overline{\psi^{(-)}}\Gamma_\mu\psi^{(-)}\cdot\partial^\mu\phi \\
&\qquad + \frac{f^2}{2m_\pi^2}[\overline{\psi^{(+)}}n\cdot\Gamma\psi^{(+)}]^2 + \frac{f^2}{2m_\pi^2}[\overline{\psi^{(-)}}n\cdot\Gamma\psi^{(-)}]^2
\end{aligned}
$$

$$+\frac{f^2}{m_\pi^2}[\overline{\psi^{(+)}}n\cdot\Gamma\psi^{(+)}][\overline{\psi^{(-)}}n\cdot\Gamma\psi^{(-)}]$$
$$+\frac{if^2}{2m_\pi^2}\int d^4y[\overline{\psi^{(+)}}\Gamma_\mu\partial^\mu\phi]_x\theta[n(x-y)]$$
$$\times(i\partial\!\!\!/+M)\Delta^{(1)}(x-y)[\Gamma_\nu\psi^{(+)}\partial^\nu\phi]_y$$
$$-\frac{if^2}{2m_\pi^2}\int d^4y[\overline{\psi^{(-)}}\Gamma_\mu\partial^\mu\phi]_x\theta[n(x-y)]$$
$$\times(i\partial\!\!\!/+M)\Delta^{(1)}(x-y)[\Gamma_\nu\psi^{(-)}\partial^\nu\phi]_y. \quad (20)$$

As in Eq. (14) there are also terms proportional to $\Delta^{(1)}(x-y)$ quadratic in the coupling constant. Also, Eq. (20) contains contact terms, but they do not contribute to πN scattering.

E. $\pi N\Delta_{33}$ coupling

At this point we deviated from Ref. [6] as far as the interaction Lagrangian is concerned. For the description of the coupling of the Δ_{33}, which is a spin-3/2 field, to πN we follow [21,22] by using the gauge-invariant interaction Lagrangian

$$\mathcal{L}_I = g_{gi}\epsilon^{\mu\nu\alpha\beta}(\partial_\mu\overline{\mathbf{\Psi}_\nu^{(+)}})\gamma_5\gamma_\alpha\mathbf{\Psi}^{(+)}(\partial_\beta\boldsymbol{\phi})$$
$$+g_{gi}\epsilon^{\mu\nu\alpha\beta}\overline{\boldsymbol{\psi}^{(+)}}\gamma_5\gamma_\alpha(\partial_\mu\mathbf{\Psi}_\nu^{(+)})(\partial_\beta\boldsymbol{\phi})$$
$$+g_{gi}\epsilon^{\mu\nu\alpha\beta}(\partial_\mu\overline{\mathbf{\Psi}_\nu^{(-)}})\gamma_5\gamma_\alpha\mathbf{\Psi}^{(-)}(\partial_\beta\boldsymbol{\phi})$$
$$+g_{gi}\epsilon^{\mu\nu\alpha\beta}\overline{\boldsymbol{\psi}^{(-)}}\gamma_5\gamma_\alpha(\partial_\mu\mathbf{\Psi}_\nu^{(-)})(\partial_\beta\boldsymbol{\phi}). \quad (21)$$

Here, Ψ_μ represents the spin-3/2 Δ_{33} field. As is mentioned in Ref. [21] the Ψ_μ field contains not only contain spin-3/2 components but also spin-1/2 components. By using the interaction Lagrangian as in Eq. (21) it is assured that only the spin-3/2 components of the Δ_{33} field couple.

From Eq. (21) we deduce the currents

$$\boldsymbol{j}_{\phi,a}(x)=(0,-g_{gi}\epsilon^{\mu\nu\alpha\beta}(\partial_\mu\overline{\mathbf{\Psi}_\nu^{(+)}})\gamma_5\gamma_\alpha\boldsymbol{\psi}^{(+)}-g_{gi}\epsilon^{\mu\nu\alpha\beta}\overline{\boldsymbol{\psi}^{(+)}}$$
$$\times\gamma_5\gamma_\alpha(\partial_\mu\mathbf{\Psi}_\nu^{(+)})-g_{gi}\epsilon^{\mu\nu\alpha\beta}(\partial_\mu\overline{\mathbf{\Psi}_\nu^{(-)}})\gamma_5\gamma_\alpha\boldsymbol{\psi}^{(-)}$$
$$-g_{gi}\epsilon^{\mu\nu\alpha\beta}\overline{\boldsymbol{\psi}^{(-)}}\gamma_5\gamma_\alpha(\partial_\mu\mathbf{\Psi}_\nu^{(-)}))$$
$$\boldsymbol{j}_{\psi^{(\pm)},a}(x)=(-g_{gi}\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\alpha(\partial_\mu\mathbf{\Psi}_\nu^{(+)})(\partial_\beta\boldsymbol{\phi}),0)$$
$$\boldsymbol{j}_{\Psi_\nu^{(\pm)},a}(x)=(0,-g_{gi}\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\alpha\boldsymbol{\psi}^{(+)}(\partial_\beta\boldsymbol{\phi})). \quad (22)$$

To avoid lengthy equations we express the commutators of the various fields with the interaction Hamiltonian in terms of fields in the HR (9)

$$[\phi(x),\mathcal{H}_I(y;n)]$$
$$=U(\sigma)i\Delta(x-y)\overleftarrow{\partial}_\beta^y[-g_{gi}\epsilon^{\mu\nu\alpha\beta}(\partial_\mu\overline{\mathbf{\Psi}_\nu^{(+)}})\gamma_5\gamma_\alpha\boldsymbol{\psi}^{(+)}$$
$$-g_{gi}\epsilon^{\mu\nu\alpha\beta}\overline{\boldsymbol{\psi}^{(+)}}\gamma_5\gamma_\alpha(\partial_\mu\mathbf{\Psi}_\nu^{(+)})$$
$$-g_{gi}\epsilon^{\mu\nu\alpha\beta}(\partial_\mu\overline{\mathbf{\Psi}_\nu^{(-)}})\gamma_5\gamma_\alpha\boldsymbol{\psi}^{(-)}-g_{gi}\epsilon^{\mu\nu\alpha\beta}$$
$$\times\overline{\boldsymbol{\psi}^{(-)}}\gamma_5\gamma_\alpha(\partial_\mu\mathbf{\Psi}_\nu^{(-)})]_yU^{-1}(\sigma)$$
$$[\psi^\pm(x),\mathcal{H}_I(y;n)]$$
$$=U(\sigma)(\pm)(i\partial\!\!\!/_x+M)\Delta^\pm(x-y)$$
$$\times[-g_{gi}\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\alpha\boldsymbol{\psi}^{(+)}(\partial_\beta\boldsymbol{\phi})]_yU^{(-1)}(\sigma)$$
$$\times[\Psi_\mu^\pm(x),\mathcal{H}_I(y;n)]$$
$$=U(\sigma)(\pm)(i\partial\!\!\!/_x+M_\Delta)(-)\left(g_{\mu\nu}-\frac{1}{3}\gamma_\mu\gamma_\nu\right.$$
$$\left.+\frac{2\partial_\mu\partial_\nu}{3M_\Delta^2}-\frac{1}{3M_\Delta^2}(\gamma_\mu i\partial_\nu-i\partial_\mu\gamma_\nu)\right)\Delta^\pm(x-y)\overleftarrow{\partial}_\rho^y$$
$$\times(-g_{gi}\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\alpha\mathbf{\Psi}^{(+)}(\partial_\beta\boldsymbol{\phi}))_yU^{-1}(\sigma), \quad (23)$$

where the fields in the HR are expressed in terms of fields in the IR using (8)

$$\boldsymbol{\psi}^{(\pm)}(x)$$
$$=\psi^{(\pm)}(x/\sigma)\pm\frac{i}{2}\int d^4y\,\theta[n(x-y)](i\partial\!\!\!/+M)$$
$$\times\Delta^{(1)}(x-y)g_{gi}\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\alpha[(\partial_\mu\Psi_\nu^{(\pm)})(\partial_\beta\phi)]_y$$
$$\partial_\rho\boldsymbol{\phi}(x)$$
$$=[\partial_\rho\phi(x,\sigma)]_{x/\sigma}-g_{gi}\epsilon^{\mu\nu\alpha\beta}n_\rho(\partial_\mu\overline{\Psi_\nu^{(+)}})$$
$$\times\gamma_5\gamma_\alpha\psi^{(+)}n_\beta-g_{gi}\epsilon^{\mu\nu\alpha\beta}n_\rho\overline{\psi^{(+)}}\gamma_5\gamma_\alpha(\partial_\mu\Psi_\nu^{(+)})$$
$$\times n_\beta-g_{gi}\epsilon^{\mu\nu\alpha\beta}n_\rho(\partial_\mu\overline{\Psi_\nu^{(-)}})\gamma_5\gamma_\alpha\psi^{(-)}n_\beta$$
$$-g_{gi}\epsilon^{\mu\nu\alpha\beta}n_\rho\overline{\psi^{(-)}}\gamma_5\gamma_\alpha(\partial_\mu\Psi_\nu^{(-)})n_\beta$$
$$\partial_\rho\boldsymbol{\psi}_\mu^{(\pm)}(x)$$
$$=[\partial_\rho\Psi_\mu^{(\pm)}(x,\sigma)]_{x/\sigma}+\frac{g_{gi}}{2}[(i\partial\!\!\!/_x+M_\Delta)n_\rho n_\gamma$$
$$+n\!\!\!/(i\partial_\rho n_\gamma+n_\rho i\partial_\gamma)-2n\!\!\!/n_\rho n_\gamma n\cdot i\partial]$$
$$\times\left(g_{\mu\nu}-\frac{1}{3}\gamma_\mu\gamma_\nu\right)\epsilon^{\rho\nu\alpha\beta}\gamma_5\gamma_\alpha\psi^{(\pm)}(\partial_\beta\phi)$$
$$\mp\frac{ig_{gi}}{2}\int d^4y\theta[n(x-y)](i\partial\!\!\!/_x+M_\Delta)$$
$$\times\left[g_{\mu\nu}-\frac{1}{3}\gamma_\mu\gamma_\nu\right]\partial_\rho\partial_\gamma\Delta^{(1)}(x-y)$$
$$\times[\epsilon^{\rho\nu\alpha\beta}\gamma_5\gamma_\alpha\psi^{(\pm)}(\partial_\beta\phi)]_y. \quad (24)$$

Here, we have already used that $\partial_\rho\mathbf{\Psi}_\mu^{(\pm)}(x)$ always appears in combination with $\epsilon^{\rho\mu\alpha\beta}$. Therefore, we have eliminated terms that are symmetric in ρ and μ.

With these ingredients we can construct the interaction Hamiltonian. Because it contains a lot of terms we only focus on those terms that contribute to πN scattering

$$\mathcal{H}_I(x;n)$$
$$=-g_{gi}\ \epsilon^{\mu\nu\alpha\beta}(\partial_\mu\overline{\Psi_\nu^{(+)}})\gamma_5\gamma_\alpha\psi^{(+)}(\partial_\beta\phi)$$
$$-g_{gi}\ \epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(+)}}\gamma_5\gamma_\alpha(\partial_\mu\Psi_\nu^{(+)})(\partial_\beta\phi)$$
$$-g_{gi}\ \epsilon^{\mu\nu\alpha\beta}(\partial_\mu\overline{\Psi_\nu^{(-)}})\gamma_5\gamma_\alpha\psi^{(-)}\ (\partial_\beta\phi)$$
$$-g_{gi}\ \epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(-)}}\gamma_5\gamma_\alpha(\partial_\mu\Psi_\nu^{(-)})(\partial_\beta\phi)$$
$$-\frac{g_{gi}^2}{2}\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(+)}}\gamma_5\gamma_\alpha\ (\partial_\beta\phi)[(i\partial\!\!\!/_x+M_\Delta)n_\mu n_{\mu'}$$
$$+n\!\!\!/(i\partial_\mu n_{\mu'}+n_\mu i\partial_{\mu'})-2n\!\!\!/n_\mu n_{\mu'}n\cdot i\partial]$$
$$\times\left(g_{\nu\nu'}-\frac{1}{3}\gamma_\nu\gamma_{\nu'}\right)\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(+)}(\partial_{\beta'}\phi)$$

$$
\begin{aligned}
&-\frac{g_{gi}^2}{2}\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(-)}}\gamma_5\gamma_\alpha(\partial_\beta\phi)[(i\overleftrightarrow{\partial}_x+M_\Delta)n_\mu n_{\mu'}\\
&+\not{n}(i\partial_\mu n_{\mu'}+n_\mu i\partial_{\mu'})-2\not{n}n_\mu n_{\mu'}n\cdot i\partial]\\
&\times\left(g_{\nu\nu'}-\frac{1}{3}\gamma_\nu\gamma_{\nu'}\right)\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(-)}(\partial_{\beta'}\phi)\\
&+\frac{ig_{gi}^2}{2}\int d^4y[\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(+)}}\gamma_5\gamma_\alpha(\partial_\beta\phi)]_x\\
&\times\theta[n(x-y)](i\overleftrightarrow{\partial}_x+M_\Delta)\left(g_{\nu\nu'}-\frac{1}{3}\gamma_\nu\gamma_{\nu'}\right)\\
&\times\partial_\mu\partial_{\mu'}\Delta^{(1)}(x-y)[\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(+)}(\partial_{\beta'}\phi)]_y\\
&-\frac{ig_{gi}^2}{2}\int d^4y[\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(-)}}\gamma_5\gamma_\alpha\ (\partial_\beta\phi)]_x\theta[n(x-y)]\\
&\times(i\overleftrightarrow{\partial}_x+M_\Delta)\left(g_{\nu\nu'}-\frac{1}{3}\gamma_\nu\gamma_{\nu'}\right)\partial_\mu\partial_{\mu'}\\
&\times\Delta^{(1)}(x-y)[\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(-)}(\partial_{\beta'}\phi)]_y. \qquad (25)
\end{aligned}
$$

III. *S*-MATRIX ELEMENTS AND AMPLITUDES

Because the Kadyshevsky rules as presented in Appendix A of article I do not contain pair suppression, we are going to derive the amplitudes from the S matrix. We have constructed the basic ingredients, namely the interaction Hamiltonians, in the previous subsection (Secs. II C, II D, and II E) for different couplings. As in article I we also consider here equal initial and final states, i.e., πN (MB) scattering. For the results for general MB initial and final states we refer to Appendix A.

A. (Pseudo-)scalar coupling

For the pseudoscalar coupling case we collect all g^2 contributions to the S matrix [see (14)]

$$
\begin{aligned}
S^{(2)} &= (-i)^2\int d^4xd^4y\theta[n(x-y)]\mathcal{H}_I(x)\mathcal{H}_I(y)\\
&= -g^2\int d^4xd^4y\theta[n(x-y)][\overline{\psi^{(+)}}\Gamma\phi]_x\\
&\quad\times(i\overleftrightarrow{\partial}+M)\Delta^+(x-y)[\Gamma\psi^{(+)}\phi]_y\\
S^{(1)} &= (-i)\int d^4x\mathcal{H}_I(x)\\
&= \frac{g^2}{2}\int d^4xd^4y[\overline{\psi^{(+)}}\Gamma\phi]_x\theta[n(x-y)]\\
&\quad\times(i\overleftrightarrow{\partial}_x+M)\Delta^{(1)}(x-y)[\Gamma\psi^{(+)}\phi]_y, \qquad (26)
\end{aligned}
$$

which need to be added

$$
\begin{aligned}
S^{(2)}+S^{(1)} = -\frac{ig^2}{2}&\int d^4xd^4y[\overline{\psi^{(+)}}\Gamma\phi]_x\theta[n(x-y)]\\
&\times(i\overleftrightarrow{\partial}+M)\Delta(x-y)[\Gamma\psi^{(+)}\phi]_y. \qquad (27)
\end{aligned}
$$

We see here that indeed the $\Delta^{(1)}(x-y)$ propagator in the interaction Hamiltonian (14) is crucial, because it combines with the $\Delta^{(+)}(x-y)$ propagator (26) to form a $\Delta(x-y)$ propagator (27). Together with the $\theta[n(x-y)]$ in Eq. (27) we recognize the causal retarded(-like) character as we mentioned in Sec. II. The S-matrix element is therefore covariant and if we analyze its n dependence using the GJ method [14] as in article I we would see that it is n independent (for vanishing external quasi momenta, of course).

Also we notice that the initial and final states are still positive-energy states. We started with a separation of positive- and negative-energy states in Sec. II and after the whole procedure this is still valid for the end states. However, we have to notice that inside an amplitude, negative energy propagates via the $\Delta(x-y)$ propagator, but this is also the case in our example of the infinite dense antinucleon star of Sec. II. Moreover, in Ref. [6] pair suppression is assumed by considering only positive-energy end states, and this is what we have achieved formally.

All the above observations are also valid in the case of (pseudo-)vector coupling and the $\pi N\Delta_{33}$ coupling of subsection Secs. III B and III C, respectively, as we will see.

The last important observation is that in Eq. (27) it does not matter whether the derivative just acts on the $\Delta(x-y)$ propagator or also on the $\theta[n(x-y)]$ function.[4] Therefore, the $\bar{P}$ method of article I can be applied, although it is not really necessary. This situation is contrary to ordinary baryon exchange, where the $\bar{P}$ method can be applied only for the summed diagrams, as explained in article I. The summed S-matrix elements (27) lead to baryon exchange and resonance Kadyshevsky diagrams, which are exposed in Fig. 2. We treat them separately.

The amplitude for the (pseudo-)scalar baryon exchange and resonance resulting from the S matrix in Eq. (27) are

$$
\begin{aligned}
M_{\kappa'\kappa}(u) &= \frac{g^2}{2}\int\frac{d\kappa_1}{\kappa_1+i\varepsilon}\bar{u}(p's')[\Gamma(\not{P}_u+M_B)\Gamma]u(ps)\Delta(P_u)\\
M_{\kappa'\kappa}(s) &= \frac{g^2}{2}\int\frac{d\kappa_1}{\kappa_1+i\varepsilon}\bar{u}(p's')[\Gamma(\not{P}_s+M_B)\Gamma]u(ps)\Delta(P_s).
\end{aligned}
\qquad (28)
$$

Here $P_i=\Delta_i+n\bar{\kappa}-n\kappa_1$ and $\Delta(P_i)=\epsilon(P_i^0)\delta(P_i^2-M_B^2)$ ($i=u,s$). The Δ_i stand for

$$
\begin{aligned}
\Delta_u &= \tfrac{1}{2}(p'+p-q'-q)\\
\Delta_s &= \tfrac{1}{2}(p'+p+q'+q).
\end{aligned}
\qquad (29)
$$

After expanding the $\delta(P_i^2-M_B^2)$ function the κ_1 integral can be performed

$$
\begin{aligned}
\delta\left(P_i^2-M_B^2\right) &= \frac{1}{|\kappa_1^+-\kappa_1^-|}[\delta(\kappa_1-\kappa_1^+)+\delta(\kappa_1-\kappa_1^-)]\\
\kappa_1^\pm &= \Delta_i\cdot n+\bar{\kappa}\pm A_i.
\end{aligned}
\qquad (30)
$$

The $\epsilon(P_i^0)$ selects both solutions with a relative minus sign. This yields for the amplitudes

$$
\begin{aligned}
M^{\mathrm{S}}_{\kappa'\kappa}(u) = \frac{g_S^2}{2}\,&\bar{u}(p's')[M+M_B-Q+\bar{\kappa}\not{n}]\,u(ps)\\
&\times\frac{1}{(\Delta_u\cdot n+\bar{\kappa})^2-A_u^2+i\varepsilon}
\end{aligned}
$$

[4] This is because $\delta(x^0-y^0)\Delta(x-y)=0$.

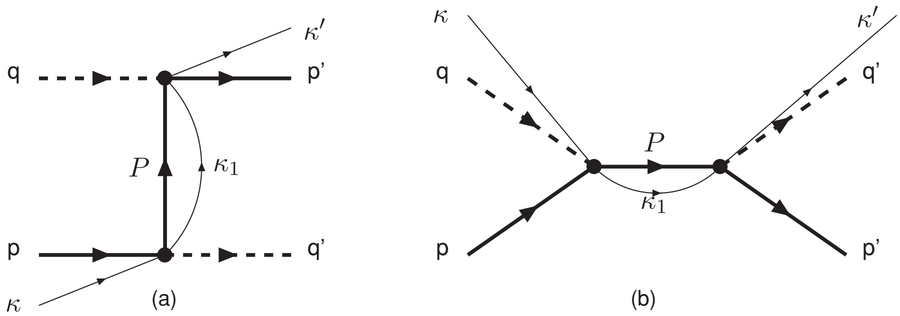

FIG. 2. Baryon exchange (a) and resonance (b) diagrams.

$$M^{\rm PS}_{\kappa'\kappa}(u) = \frac{g^2_{\rm PS}}{2}\,\bar u(p's')[M - M_B - \not{Q} + \bar\kappa\not{n}]\,u(ps) \times \frac{1}{(\Delta_u\cdot n+\bar\kappa)^2 - A_u^2 + i\varepsilon}$$
$$M^{\rm S}_{\kappa'\kappa}(s) = \frac{g^2_{S}}{2}\,\bar u(p's')[M + M_B + \not{Q} + \bar\kappa\not{n}]\,u(ps) \times \frac{1}{(\Delta_s\cdot n+\bar\kappa)^2 - A_s^2 + i\varepsilon}$$
$$M^{\rm PS}_{\kappa'\kappa}(s) = \frac{g^2_{\rm PS}}{2}\,\bar u(p's')[M - M_B + \not{Q} + \bar\kappa\not{n}]\,u(ps) \times \frac{1}{(\Delta_s\cdot n+\bar\kappa)^2 - A_s^2 + i\varepsilon}, \quad (31)$$

where S and PS stand for *scalar* and *pseudoscalar*, respectively. Taking the limit of $\kappa' = \kappa = 0$ in (31) we get

$$M^{\rm S}_{00}(u) = \frac{g^2_{S}}{2}\,\bar u(p's')[M + M_B - \not{Q}]\,u(ps)\frac{1}{u - M_B^2 + i\varepsilon}$$
$$M^{\rm PS}_{00}(u) = \frac{g^2_{\rm PS}}{2}\,\bar u(p's')[M - M_B - \not{Q}]\,u(ps)\frac{1}{u - M_B^2 + i\varepsilon}$$
$$M^{\rm S}_{00}(s) = \frac{g^2_{S}}{2}\,\bar u(p's')[M + M_B + \not{Q}]\,u(ps)\frac{1}{s - M_B^2 + i\varepsilon}$$
$$M^{\rm PS}_{00}(s) = \frac{g^2_{\rm PS}}{2}\,\bar u(p's')[M - M_B + \not{Q}]\,u(ps)\frac{1}{s - M_B^2 + i\varepsilon}, \quad (32)$$

which is a factor 1/2 of the result in Ref. [6]. This factor is because of the fact that we took only the positive-energy contribution. This difference can easily be intercepted by considering an interaction Lagrangian as in Eq. (10) scaled by a factor of $\sqrt{2}$ and eventually identifying $g/\sqrt{2}$ as the physical coupling constant. We stress here that although we have included absolute pair suppression formally, we still get a factor 1/2 of the usual Feynman expression.

In article I we studied the n dependence of the (approximation of the) Kadyshevsky integral equation using the GJ method

$$M_{00} = M^{\rm irr}_{00} + \int d\kappa\, M^{\rm irr}_{0\kappa}\, G'_\kappa\, M_{\kappa 0},$$
$$P^{\alpha\beta}\frac{\partial}{\partial n^\beta}M_{00} = P^{\alpha\beta}\left(\frac{\partial M^{\rm irr}_{00}}{\partial n^\beta}\right) + P^{\alpha\beta}\int d\kappa_1\left[\left(\frac{\partial M^{\rm irr}_{0\kappa_1}}{\partial n^\beta}\right) \times G'_\kappa\, M^{\rm irr}_{\kappa_1 0} + M^{\rm irr}_{0\kappa_1}\, G'_{\kappa_1}\left(\frac{\partial M^{\rm irr}_{\kappa_1 0}}{\partial n^\beta}\right)\right]$$
$$+ P^{\alpha\beta}\int d\kappa_1 d\kappa_2\left[\left(\frac{\partial M^{\rm irr}_{0\kappa_1}}{\partial n^\beta}\right) G'_{\kappa_1}\, M^{\rm irr}_{\kappa_1\kappa_2}\, G'_{\kappa_2} \times M^{\rm irr}_{\kappa_2 0} + M^{\rm irr}_{0\kappa_1}\, G'_{\kappa_1}\left(\frac{\partial M^{\rm irr}_{\kappa_1\kappa_2}}{\partial n^\beta}\right) G'_{\kappa_2}\, M^{\rm irr}_{\kappa_2 0} + M^{\rm irr}_{0\kappa_1}\, G'_{\kappa_1}\, M^{\rm irr}_{\kappa_1\kappa_2}\, G'_{\kappa_2}\left(\frac{\partial M^{\rm irr}_{\kappa_2 0}}{\partial n^\beta}\right)\right] + \cdots = 0. \quad (33)$$

Important was that the integrals on the right-hand side of the second equation of Eq. (33) are all of the form

$$\int d\kappa\;f(\kappa)\,h(\kappa)G'_\kappa, \quad (34)$$

where $f(\kappa)$ is at least linear proportional to κ and $h(\kappa)$ only has poles in the lower half complex κ plane. In some cases a phenomenological "form factor" is needed

$$F(\kappa) = \left(\frac{\Lambda_\kappa^2}{\Lambda_\kappa^2 - \kappa^2 - i\epsilon(\kappa)\varepsilon}\right)^{N_\kappa}. \quad (35)$$

For the details we refer to article I. Whether (34) applies and (35) is necessary we need to check for every exchange and resonance process.

To do so in the case of (P)S baryon exchange or resonance we notice that two contributions are added (30) and that the sum has poles in the lower half complex $\bar\kappa$ plane

$$\frac{1}{2A_i}\cdot\frac{1}{\Delta_i\cdot n+\bar\kappa - A_i + i\varepsilon} - \frac{1}{2A_i}\cdot\frac{1}{\Delta_i\cdot n+\bar\kappa + A_i + i\varepsilon} = \frac{1}{(\Delta_i\cdot n+\bar\kappa)^2 - A_i^2 + 2i\epsilon(\Delta_i\cdot n+\bar\kappa)}, \quad (36)$$

where $i = u, s$. In fact this is valid for all baryon exchange and resonance amplitudes, so we will not repeat this in case of (pseudo-)vector and $\pi N\Delta_{33}$ coupling. Taking a closer look at the denominators in (31)

$$(\Delta_i\cdot n+\bar\kappa)^2 - A_s^2 = \Delta_i^2 - M_B^2 + 2\Delta_i\cdot n\bar\kappa + \bar\kappa^2, \quad (37)$$

we conclude that all n-dependent terms in Eq. (31) are at least linear proportional to either κ or κ' (or both). If we would consider only (P)S baryon exchange or resonance in the Kadyshevsky integral equation, then we indeed would have a situation as in Eq. (34). Looking at the powers of κ, κ' in Eq. (31) we see that $h(\kappa)$ in Eq. (34) will be of the order $O(\frac{1}{\kappa^2})$ and the phenomenological "form factor" (35) would not be necessary.

B. (Pseudo-)vector coupling

The g^2 contributions of (pseudo-)vector coupling in the second and first order of the S matrix are

$$\begin{aligned} S^{(2)} &= (-i)^2\int d^4x d^4y \theta[n(x-y)]\mathcal{H}_I(x)\mathcal{H}_I(y) \\ &= -\frac{f^2}{m_\pi^2}\int d^4x d^4y\theta[n(x-y)][\overline{\psi^{(+)}}\Gamma_\mu(\partial^\mu\phi)]_x(i\not\partial+M) \\ &\quad\times\Delta^+(x-y)[\Gamma_\nu\psi^{(+)}(\partial^\nu\phi)]_y \\ S^{(1)} &= (-i)\int d^4x\mathcal{H}_I(x) \\ &= \frac{f^2}{2m_\pi^2}\int d^4xd^4y[\overline{\psi^{(+)}}\Gamma_\mu(\partial^\mu\phi)]_x\theta[n(x-y)](i\not\partial+M) \\ &\quad\times\Delta^{(1)}(x-y)[\Gamma_\nu\psi^{(+)}(\partial^\nu\phi)]_y. \end{aligned} \tag{38}$$

Adding the two together

$$\begin{aligned} S^{(2)}+S^{(1)} = -\frac{if^2}{2m_\pi^2}\int d^4xd^4y\theta[n(x-y)][\overline{\psi^{(+)}}\Gamma_\mu(\partial^\mu\phi)]_x \\ \times(i\not\partial+M)\Delta(x-y)[\Gamma_\nu\psi^{(+)}(\partial^\nu\phi)]_y, \end{aligned} \tag{39}$$

leads again to a covariant, n-independent result ($\kappa'=\kappa=0$). See the text below (27) about this issue and other important observations.

The two Kadyshevsky diagrams resulting from Eq. (39) are the same as shown in Fig. 2. The amplitudes that go with them, in case of (pseudo-)vector coupling, are

$$\begin{aligned} M_{\kappa'\kappa}(u) &= \frac{f^2}{2m_\pi^2}\int\frac{d\kappa_1}{\kappa_1+i\varepsilon}\bar{u}(p's')[(\Gamma\cdot q)(\not P_u+M_B) \\ &\quad\times(\Gamma\cdot q')]\, u(ps)\Delta(P_u) \\ M_{\kappa'\kappa}(s) &= \frac{f^2}{2m_\pi^2}\int\frac{d\kappa_1}{\kappa_1+i\varepsilon}\bar{u}(p's')[(\Gamma\cdot q')(\not P_s+M_B) \\ &\quad\times(\Gamma\cdot q)]\, u(ps)\Delta(P_s), \end{aligned} \tag{40}$$

where P_i and $\Delta(P_i)$ are defined below (28). As far as the κ_1 integration is concerned we take similar steps as in Eq. (30).

After some (Dirac) algebra the amplitudes in Eq. (40) become

$$\begin{aligned} &M^{\rm V}_{\kappa'\kappa}(u) \\ &= \frac{f_{\rm V}^2}{2m_\pi^2}\bar{u}(p's')\Big[-(M-M_B)\Big(-M^2+\frac{1}{2}(u_{p'q}+u_{pq'}) \\ &\quad +2M\not Q-\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n+\frac{1}{2}(\kappa'-\kappa)[\not n,\not Q] \\ &\quad -\frac{1}{2}(\kappa'-\kappa)^2\Big)-\frac{1}{2}(u_{pq'}-M^2)\Big(\not Q+\frac{1}{2}(\kappa'-\kappa)\not n\Big) \\ &\quad -\frac{1}{2}(u_{p'q}-M^2)\Big(\not Q-\frac{1}{2}(\kappa'-\kappa)\not n\Big) \\ &\quad +\bar\kappa\Big(-(p'-p)\cdot n\,\not Q+2Q\cdot n\,\not Q+M^2\not n \\ &\quad -\frac{\not n}{2}(u_{p'q}+u_{pq'})\Big)\Big]\; u(p)\frac{1}{(\Delta_u\cdot n+\bar\kappa)^2-A_u^2+i\varepsilon}, \end{aligned}$$

$$\begin{aligned} &M^{\rm PV}_{\kappa'\kappa}(u) \\ &= \frac{f_{\rm PV}^2}{2m_\pi^2}\bar{u}(p's')\Big[-(M+M_B)\Big(-M^2+\frac{1}{2}(u_{p'q}+u_{pq'}) \\ &\quad +2M\not Q-\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n+\frac{1}{2}(\kappa'-\kappa)[\not n,\not Q] \\ &\quad -\frac{1}{2}(\kappa'-\kappa)^2\Big)-\frac{1}{2}(u_{pq'}-M^2)\Big(\not Q+\frac{1}{2}(\kappa'-\kappa)\not n\Big) \\ &\quad -\frac{1}{2}(u_{p'q}-M^2)\Big(\not Q-\frac{1}{2}(\kappa'-\kappa)\not n\Big) \\ &\quad +\bar\kappa\Big(-(p'-p)\cdot n\,\not Q+2Q\cdot n\,\not Q+M^2\not n \\ &\quad -\frac{\not n}{2}(u_{p'q}+u_{pq'})\Big)\Big]\; u(p)\frac{1}{(\Delta_u\cdot n+\bar\kappa)^2-A_u^2+i\varepsilon}, \end{aligned}$$

$$\begin{aligned} &M^{\rm V}_{\kappa'\kappa}(s) \\ &= \frac{f_{\rm V}^2}{2m_\pi^2}\bar{u}(p's')\Big[-(M-M_B)\Big(-M^2+\frac{1}{2}(s_{p'q'}+s_{pq}) \\ &\quad -2M\not Q-\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n-\frac{1}{2}(\kappa'-\kappa)[\not n,\not Q] \\ &\quad -\frac{1}{2}(\kappa'-\kappa)^2\Big)+\frac{1}{2}(s_{p'q'}-M^2)\Big(\not Q+\frac{1}{2}(\kappa'-\kappa)\not n\Big) \\ &\quad +\frac{1}{2}(s_{pq}-M^2)\Big(\not Q-\frac{1}{2}(\kappa'-\kappa)\not n\Big) \\ &\quad +\bar\kappa\Big((p'-p)\cdot n\,\not Q+2Q\cdot n\,\not Q+M^2\not n \\ &\quad -\frac{\not n}{2}(s_{p'q'}+s_{pq})\Big)\Big]\; u(p)\frac{1}{(\Delta_s\cdot n+\bar\kappa)^2-A_s^2+i\varepsilon} \end{aligned}$$

$$\begin{aligned} &M^{\rm PV}_{\kappa'\kappa}(s) \\ &= \frac{f_{\rm PV}^2}{2m_\pi^2}\bar{u}(p's')\Big[-(M+M_B)\Big(-M^2+\frac{1}{2}(s_{p'q'}+s_{pq}) \\ &\quad -2M\not Q-\frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n-\frac{1}{2}(\kappa'-\kappa)[\not n,\not Q] \\ &\quad -\frac{1}{2}(\kappa'-\kappa)^2\Big)+\frac{1}{2}(s_{p'q'}-M^2)\Big(\not Q+\frac{1}{2}(\kappa'-\kappa)\not n\Big) \\ &\quad +\frac{1}{2}(s_{pq}-M^2)\Big(\not Q-\frac{1}{2}(\kappa'-\kappa)\not n\Big) \\ &\quad +\bar\kappa\Big((p'-p)\cdot n\,\not Q+2Q\cdot n\,\not Q+M^2\not n \\ &\quad -\frac{\not n}{2}(s_{p'q'}+s_{pq})\Big)\Big]\; u(p)\frac{1}{(\Delta_s\cdot n+\bar\kappa)^2-A_s^2+i\varepsilon}. \end{aligned} \tag{41}$$

Here, (P)V stands for *(pseudo-)vector*. Taking the limit $\kappa'=\kappa=0$

$$\begin{aligned} M^{\rm V}_{00}(u) = \frac{f_{\rm V}^2}{2m_\pi^2}\bar{u}(p's')[-(M-M_B)(-M^2+u+2M\not Q) \\ -(u-M^2)\not Q]\; u(p)\frac{1}{u-M_B^2+i\varepsilon} \end{aligned}$$

$$M_{00}^{\rm PV}(u) = \frac{f_{\rm PV}^2}{2m_\pi^2}\bar{u}(p's')[-(M+M_B)(-M^2+u+2M\not{Q}) - (u-M^2)\not{Q}]\,u(p)\frac{1}{u-M_B^2+i\varepsilon}$$
$$M_{00}^{\rm V}(s) = \frac{f_{\rm V}^2}{2m_\pi^2}\bar{u}(p's')[-(M-M_B)(-M^2+s-2M\not{Q}) + (s-M^2)\not{Q}]\,u(p)\frac{1}{s-M_B^2+i\varepsilon}$$
$$M_{00}^{\rm PV}(s) = \frac{f_{\rm PV}^2}{2m_\pi^2}\bar{u}(p's')[-(M+M_B)(-M^2+s-2M\not{Q}) + (s-M^2)\not{Q}]\,u(p)\frac{1}{s-M_B^2+i\varepsilon}, \quad (42)$$

where we, again, get factor 1/2 from the result in Ref. [6] for the same reason as mentioned in Sec. III A.

Studying the n dependence of the amplitudes (41) in light of the n dependence of the Kadyshevsky integral equation as before (Sec. III A), we see that, again, all n-dependent terms in Eq. (41) are at least linear proportional to either κ or κ'. Therefore, when we would only consider (P)V baryon exchange or resonance in the Kadyshevsky integral equation, we would, again, find ourself in a similar situation as in Eq. (34), when studying the n dependence. However, looking at the powers of κ and κ' in Eq. (41) we notice that the function $h(\kappa)$ in Eq. (34) is of higher order then $O(\frac{1}{\kappa^2})$. Therefore, the phenomenological "form factor" (35) would be necessary.

C. $\pi N\Delta_{33}$ coupling

As far as the $\pi N\Delta_{33}$ coupling is concerned we find the following g_{gi}^2 contribution in the second and first order of the S matrix from (25)

$$\begin{aligned} S^{(2)} &= (-i)^2\int d^4xd^4y\theta[n(x-y)]\mathcal{H}_I(x)\mathcal{H}_I(y) \\ &= -g_{gi}^2\int d^4xd^4y\theta[n(x-y)][\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(+)}}\gamma_5\gamma_\alpha\partial_\beta\phi]_x\partial_\mu^x\partial_{\mu'}^y \\ &\quad\times(i\not{\partial}+M_\Delta)\Lambda_{\nu\nu'}\Delta^+(x-y)[\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(+)}\partial_{\beta'}\phi]_y \\ S^{(1)} &= (-i)\int d^4x\mathcal{H}_I(x) \\ &= \frac{g_{gi}^2}{2}\int d^4xd^4y[\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(+)}}\gamma_5\gamma_\alpha\partial_\beta\phi]_x\theta[n(x-y)]\partial_\mu\partial_{\mu'} \\ &\quad\times(i\not{\partial}+M_\Delta)\left(g_{\nu\nu'}-\frac{1}{3}\gamma_\nu\gamma_{\nu'}\right)\Delta^{(1)}(x-y) \\ &\quad\times[\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(+)}\partial_{\beta'}\phi]_y \\ &\quad+\frac{ig_{gi}^2}{2}[\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(+)}}\gamma_5\gamma_\alpha\partial_\beta\phi] \\ &\quad\times[(i\not{\partial}+M_\Delta)n_\mu n_{\mu'}+\not{n}(n_\mu i\partial_{\mu'}+i\partial_\mu n_{\mu'}) \\ &\quad-2\not{n}n_\mu n_{\mu'}n\cdot i\partial]\left(g_{\nu\nu'}-\frac{1}{3}\gamma_\nu\gamma_{\nu'}\right) \\ &\quad\times[\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(+)}\partial_{\beta'}\phi], \end{aligned} \quad (43)$$

where

$$\Lambda_{\mu\nu} = -\left[g_{\mu\nu}-\frac{1}{3}\gamma_\mu\gamma_\nu+\frac{2\partial_\mu\partial_\nu}{3M^2} -\frac{1}{3M_\Delta}(\gamma_\mu i\partial_\nu-\gamma_\nu i\partial_\mu)\right]. \quad (44)$$

Because of the antisymmetric property of the epsilon tensor all derivative terms in Eq. (44) do not contribute.

On addition of the two contributions in Eq. (43) we find

$$\begin{aligned} S^{(2)}+S^{(1)} &= -\frac{ig_{gi}^2}{2}\int d^4xd^4y[\epsilon^{\mu\nu\alpha\beta}\overline{\psi^{(+)}}\gamma_5\gamma_\alpha\partial_\beta\phi]_x \\ &\quad\times\partial_\mu\partial_{\mu'}(i\not{\partial}+M_\Delta)\left(g_{\nu\nu'}-\frac{1}{3}\gamma_\nu\gamma_{\nu'}\right)\theta[n(x-y)] \\ &\quad\times\Delta(x-y)[\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_5\gamma_{\alpha'}\psi^{(+)}\partial_{\beta'}\phi]_y. \end{aligned} \quad (45)$$

Again, we have a similar situation for the S-matrix element as in Sec. III A. Therefore, we refer for the discussion of Ref. (45) to the text below (27).

A difference of this S-matrix element as compared of those of the forgoing subsections (Secs. III A and III B) is that the derivatives act not only on the $\Delta(x-y)$ propagator in Eq. (45) but also on the $\theta[n(x-y)]$. Therefore, the $\bar{P}$ method of article I can be applied. Of course this is obvious because this method was introduced to incorporate terms like the second term on the right-hand side of $S^{(1)}$ in Eq. (43).

As in the previous subsections (Secs. III A and III B) two amplitudes arise from this S matrix: Δ_{33} exchange and resonance, whose the Kadyshevsky diagrams are shown in Fig. 2. The amplitudes are

$$\begin{aligned} M_{\kappa'\kappa}(u) &= -\frac{g_{gi}^2}{2}\int\frac{d\kappa_1}{\kappa_1+i\varepsilon}\,\epsilon^{\mu\nu\alpha\beta}\bar{u}(p's')\gamma_\alpha\gamma_5q_\beta(\bar{P}_u)_\mu(\bar{P}_u)_{\mu'} \\ &\quad\times(\bar{\not{P}}_u+M_\Delta)\left(g_{\nu\nu'}-\frac{1}{3}\gamma_\nu\gamma_{\nu'}\right)\Delta(P_u)\epsilon^{\mu'\nu'\alpha'\beta'} \\ &\quad\times\gamma_{\alpha'}\gamma_5q'_{\beta'}\,u(ps) \\ M_{\kappa'\kappa}(s) &= -\frac{g_{gi}^2}{2}\int\frac{d\kappa_1}{\kappa_1+i\varepsilon}\,\epsilon^{\mu\nu\alpha\beta}\bar{u}(p's')\gamma_\alpha\gamma_5q'_\beta(\bar{P}_s)_\mu\,(\bar{P}_s)_{\mu'} \\ &\quad\times(\bar{\not{P}}_s+M_\Delta)\left(g_{\nu\nu'}-\frac{1}{3}\gamma_\nu\gamma_{\nu'}\right)\Delta(P_s)\epsilon^{\mu'\nu'\alpha'\beta'} \\ &\quad\times\gamma_{\alpha'}\gamma_5q_{\beta'}\,u(ps), \end{aligned} \quad (46)$$

where $\bar{P}_i = P_i + n\kappa_1$, $i = u, s$. P_i and $\Delta(P_i)$ are as before.

Performing the κ_1 integral is in this situation even simpler than in the previous cases (Secs. III A and III B). As can be seen from Eq. (30) the $\Delta(P_i)$ in Eq. (46) selects two solutions for κ_1 [with a relative minus sign, due to $\epsilon(P_i^0)$], which need to be applied only to the quasiscalar propagator $1/(\kappa_1+i\varepsilon)$. This is because the $\bar{P}_i$ is κ_1 independent. Contracting all the indices in Eq. (46) the amplitudes become

$$\begin{aligned} M_{\kappa'\kappa}(u) &= -\frac{g_{gi}^2}{2}\bar{u}(p's')\Big[(\bar{\not{P}}_u+M_\Delta)\Big(\bar{P}_u^2(q'\cdot q)-\frac{1}{3}\bar{P}_u^2\not{q}\not{q}' \\ &\quad-\frac{1}{3}\bar{\not{P}}_u\not{q}(\bar{P}_u\cdot q')+\frac{1}{3}\bar{\not{P}}_u\not{q}'(\bar{P}_u\cdot q)-\frac{2}{3}(\bar{P}_u\cdot q') \\ &\quad\times\ (\bar{P}_u\cdot q)\Big)\Big]\ u(ps)\frac{1}{(\Delta_u\cdot n+\bar{\kappa})^2-A_u^2+i\varepsilon}\ , \end{aligned}$$

$$
\begin{aligned}
M_{\kappa'\kappa}(s) = -\frac{g_{gi}^2}{2}\bar{u}(p's')\Bigg[(\not{\bar{P}}_s + M_\Delta)\bigg(\bar{P}_s^2(q'\cdot q) - \frac{1}{3}\bar{P}_s^2\not{q}'\not{q} \\
- \frac{1}{3}\not{\bar{P}}_s\not{q}'(\bar{P}_s\cdot q) + \frac{1}{3}\not{\bar{P}}_s\not{q}(\bar{P}_s\cdot q') - \frac{2}{3}(\bar{P}_s\cdot q') \\
\times\ (\bar{P}_s\cdot q)\bigg)\Bigg]\ u(ps)\frac{1}{(\Delta_s\cdot n + \bar{\kappa})^2 - A_s^2 + i\varepsilon},
\end{aligned}
$$

which leads, after some (Dirac) algebra, to

$$
\begin{aligned}
&M_{\kappa'\kappa}(u) \\
&= -\frac{g_{gi}^2}{2}\bar{u}(p's')\bigg[\frac{1}{2}\bar{P}_u^2\,(M + M_\Delta - \not{Q} + \bar{\kappa}\not{n})\,(2m^2 - t_{q'q}) \\
&\quad - \frac{1}{3}\bar{P}_u^2\bigg((M + M_\Delta)\,\not{q}\not{q}' + \frac{1}{2}(u_{pq'} - M^2)\not{q} \\
&\quad + \frac{1}{2}(s_{pq} + t_{q'q} - M^2 - 4m^2)\not{q}' + \bar{\kappa}\not{n}\not{q}\not{q}'\bigg) \\
&\quad - \frac{1}{12}\bigg(\bar{P}_u^2\not{q} + \frac{M_\Delta}{2}(s_{pq} - M^2 - 2m^2) - \frac{M_\Delta}{2}\not{q}'\not{q} \\
&\quad + M_\Delta\bar{\kappa}\not{n}\not{q}\bigg)(-4m^2 + s_{p'q'} - u_{pq'} + t_{q'q} - 2\bar{\kappa}(p'-p) \\
&\quad \cdot n + 4\bar{\kappa}n\cdot Q - (\kappa'^2 - \kappa^2)) + \frac{1}{12}\bigg(\bar{P}_u^2\not{q}' + \frac{M_\Delta}{2}(M^2 \\
&\quad - u_{pq'}) - \frac{M_\Delta}{2}\not{q}\not{q}' + M_\Delta\bar{\kappa}\not{n}\not{q}'\bigg)(-4m^2 + s_{pq} - u_{p'q} \\
&\quad + t_{q'q} + 2\bar{\kappa}(p'-p)\cdot n + 4\bar{\kappa}n\cdot Q + (\kappa'^2 - \kappa^2)) \\
&\quad - \frac{1}{24}(M + M_\Delta - \not{Q} + \bar{\kappa}\not{n})(-4m^2 + s_{p'q'} - u_{pq'} + t_{q'q} \\
&\quad - 2\bar{\kappa}(p'-p)\cdot n + 4\bar{\kappa}n\cdot Q - (\kappa'^2 - \kappa^2))(-4m^2 + s_{pq} \\
&\quad - u_{p'q} + t_{q'q} + 2\bar{\kappa}(p'-p)\cdot n + 4\bar{\kappa}n\cdot Q \\
&\quad + (\kappa'^2 - \kappa^2))\bigg]\ u(ps)\frac{1}{(\Delta_u\cdot n + \bar{\kappa})^2 - A_u^2 + i\varepsilon},
\end{aligned}
$$

$$
\begin{aligned}
&M_{\kappa'\kappa}(s) \\
&= -\frac{g_{gi}^2}{2}\bar{u}(p's')\bigg[\frac{1}{2}\bar{P}_s^2(M + M_\Delta + \not{Q} + \bar{\kappa}\not{n})(2m^2 - t_{q'q}) \\
&\quad - \frac{1}{3}\bar{P}_s^2\bigg((M + M_\Delta)\,\not{q}'\not{q} - \frac{1}{2}(s_{pq} - M^2)\not{q}' \\
&\quad - \frac{1}{2}\left(u_{pq'} + t_{q'q} - M^2 - 4m^2\right)\not{q} + \bar{\kappa}\not{n}\not{q}'\not{q}\bigg) \\
&\quad - \frac{1}{12}\bigg(\bar{P}_s^2\not{q}' + \frac{M_\Delta}{2}(M^2 + 2m^2 - u_{pq'}) + \frac{M_\Delta}{2}\not{q}\not{q}' \\
&\quad + M_\Delta\bar{\kappa}\not{n}\not{q}'\bigg)(4m^2 + s_{pq} - u_{p'q} - t_{q'q} \\
&\quad + 2\bar{\kappa}(p'-p)\cdot n + 4\bar{\kappa}n\cdot Q + (\kappa'^2 - \kappa^2)) \\
&\quad + \frac{1}{12}\bigg(\bar{P}_s^2\not{q} + \frac{M_\Delta}{2}(s_{pq} - M^2) + \frac{M_\Delta}{2}\not{q}'\not{q} + M_\Delta\bar{\kappa}\not{n}\not{q}\bigg) \\
&\quad \times(4m^2 + s_{p'q'} - u_{pq'} - t_{q'q} - 2\bar{\kappa}(p'-p)\cdot n \\
&\quad + 4\bar{\kappa}n\cdot Q - (\kappa'^2 - \kappa^2)) \\
&\quad - \frac{1}{24}(M + M_\Delta + \not{Q} + \bar{\kappa}\not{n})(4m^2 + s_{p'q'} - u_{pq'} - t_{q'q} \\
&\quad - 2\bar{\kappa}(p'-p)\cdot n + 4\bar{\kappa}n\cdot Q - (\kappa'^2 - \kappa^2))(4m^2 + s_{pq} \\
&\quad - u_{p'q} - t_{q'q} + 2\bar{\kappa}(p'-p)\cdot n + 4\bar{\kappa}n\cdot Q \\
&\quad + (\kappa'^2 - \kappa^2))\bigg]u(ps)\frac{1}{(\Delta_s\cdot n + \bar{\kappa})^2 - A_s^2 + i\varepsilon},
\end{aligned}
\tag{47}
$$

where

$$
\begin{aligned}
\bar{P}_u^2 &= \tfrac{1}{2}(u_{p'q} + u_{pq'}) - \tfrac{1}{4}(\kappa' - \kappa)^2 + 2\bar{\kappa}\,\Delta_u\cdot n + \bar{\kappa}^2 \\
\bar{P}_s^2 &= \tfrac{1}{2}(s_{p'q'} + s_{pq}) - \tfrac{1}{4}(\kappa' - \kappa)^2 + 2\bar{\kappa}\,\Delta_s\cdot n + \bar{\kappa}^2
\end{aligned}
\tag{48}
$$

and

$$
\begin{aligned}
\not{q}' &= \not{Q} - \tfrac{1}{2}\not{n}(\kappa' - \kappa) \\
\not{q} &= \not{Q} + \tfrac{1}{2}\not{n}(\kappa' - \kappa) \\
\not{q}'\not{q} &= -2M\not{Q} + \tfrac{1}{2}(s_{p'q'} + s_{pq}) - M^2 - \tfrac{1}{2}(\kappa' - \kappa)(p' - p)\cdot n \\
&\quad + \tfrac{1}{2}(\kappa' - \kappa)\,[\not{Q}, \not{n}] - \tfrac{1}{2}(\kappa' - \kappa)^2 \\
\not{q}\not{q}' &= 2M\not{Q} + \tfrac{1}{2}(u_{p'q} + u_{pq'}) - M^2 - \tfrac{1}{2}(\kappa' - \kappa)(p' - p)\cdot n \\
&\quad - \tfrac{1}{2}(\kappa' - \kappa)\,[\not{Q}, \not{n}] - \tfrac{1}{2}(\kappa' - \kappa)^2 \\
\not{n}\not{q}' &= M\not{n} - (n\cdot p') - \tfrac{1}{2}\,[\not{Q}, \not{n}] + n\cdot Q - \tfrac{1}{2}(\kappa' - \kappa) \\
\not{n}\not{q} &= -M\not{n} + (n\cdot p') - \tfrac{1}{2}\,[\not{Q}, \not{n}] + n\cdot Q + \tfrac{1}{2}(\kappa' - \kappa) \\
\not{n}\not{q}'\not{q} &= -M^2\not{n} + \tfrac{1}{2}(s_{p'q'} + s_{pq})\not{n} - \tfrac{1}{2}(\kappa' - \kappa)n\cdot(p' - p)\not{n} \\
&\quad + (\kappa' - \kappa)(n\cdot Q)\not{n} - (\kappa' - \kappa)\not{Q} - 2n\cdot(p' - p)\not{Q} \\
&\quad - \tfrac{1}{2}(\kappa' - \kappa)^2\not{n} \\
\not{n}\not{q}\not{q}' &= -M^2\not{n} + \tfrac{1}{2}(u_{p'q} + u_{pq'})\not{n} - \tfrac{1}{2}(\kappa' - \kappa)n\cdot(p' - p)\not{n} \\
&\quad - (\kappa' - \kappa)(n\cdot Q)\not{n} + (\kappa' - \kappa)\not{Q} + 2n\cdot(p' - p)\not{Q} \\
&\quad - \tfrac{1}{2}(\kappa' - \kappa)^2\not{n}.
\end{aligned}
\tag{49}
$$

Taking the limit $\kappa' = \kappa = 0$ yields

$$
\begin{aligned}
M_{00}(u) = -\frac{g_{gi}^2}{2}\bar{u}(p's')\bigg[&\frac{u}{2}(M + M_\Delta - \not{Q})(2m^2 - t) \\
&- \frac{u}{3}((M + M_\Delta)(2M\not{Q} + u - M^2) - m^2\not{Q}) \\
&- \frac{1}{6}(u\not{Q} + M_\Delta(M\not{Q} - m^2))(M^2 - m^2 - u) \\
&+ \frac{1}{6}(u\not{Q} + M_\Delta(M^2 - u - M\not{Q}))(M^2 - m^2 - u) \\
&- \frac{1}{6}(M + M_\Delta - \not{Q})(M^2 - m^2 - u)^2\bigg]\ u(ps) \\
&\times\frac{1}{u - M_\Delta^2 + i\varepsilon}
\end{aligned}
$$

$$
\begin{aligned}
M_{00}(s) = -\frac{g_{gi}^2}{2}\bar{u}(p's')\bigg[&\frac{s}{2}(M + M_\Delta + \not{Q})(2m^2 - t) \\
&- \frac{s}{3}((M + M_\Delta)(-2M\not{Q} + s - M^2) + m^2\not{Q}) \\
&- \frac{1}{6}(s\not{Q} + M_\Delta(M\not{Q} + m^2))(s - M^2 + m^2) \\
&+ \frac{1}{6}(s\not{Q} + M_\Delta(s - M^2 - M\not{Q}))(s - M^2 + m^2)
\end{aligned}
$$

$$- \frac{1}{6}(M + M_\Delta + \not{Q})(s - M^2 + m^2)^2 \Big] u(ps) \times \frac{1}{s - M_\Delta^2 + i\varepsilon}. \quad (50)$$

Considering only the Δ_{33} exchange and resonance in the Kadyshevsky integral equation and study its n dependence, we see from Eqs. (47) and (49) that all n-dependent terms in Eq. (47) and (49) are at least linear proportional to either κ or κ' and therefore Eq. (34) applies. The function $h(\kappa)$ is such that the phenomenological "form factor" (35) is necessary.

IV. INVARIANTS AND PARTIAL-WAVE EXPANSION

In elastic-scattering processes important (indirect) observables are the phase shifts. In this section we introduce the phase shifts by introducing the partial-wave expansion, which is particularly convenient for solving the Kadyshevsky integral equation. By also using the helicity basis we are able to link the amplitudes obtained in article I and the previous section (Sec. III) to the phase shifts.

A. Amplitudes and invariants

Following the standard procedure, see, e.g., Ref. [23], the most general form of the parity-conserving amplitude describing πN scattering in Kadyshevsky formalism is

$$M_{\kappa'\kappa} = \bar{u}(p's')[A + B\not{Q} + A'\not{n} + B'[\not{n}, \not{Q}]]\ u(ps), \quad (51)$$

where the invariants A, B, A', and B' are functions of the Mandelstam variables and of κ and κ'. The contribution of the invariants to the various exchange processes is given in Appendix A.

In proceeding we do not keep n^μ general but instead choose it to be [3,5]

$$n^\mu = \frac{(p+q)^\mu}{\sqrt{s_{pq}}} = \frac{(p'+q')^\mu}{\sqrt{s_{p'q'}}}. \quad (52)$$

With this choice, n^μ is no longer an independent variable and the number of invariants is reduced to 2. This is made explicit as follows

$$\begin{aligned} \bar{u}(p's')[\not{n}]\ u(ps) &= \frac{1}{\sqrt{s_{p'q'}} + \sqrt{s_{pq}}}\bar{u}(p's') \\ &\quad \times [M_f + M_i + 2\not{Q}]\ u(ps) \quad (53) \\ \bar{u}(p's')[[\not{n}, \not{Q}]]\ u(ps) &= 0. \end{aligned}$$

As a result of the choice (52) the invariants A and B in Eq. (51) receive contributions from the invariant A'. We, therefore, redefine the amplitude

$$\begin{aligned} M_{\kappa'\kappa} &= \bar{u}(p's')[A'' + B''\not{Q}]\ u(ps) \\ A'' &= A + \frac{1}{\sqrt{s_{p'q'}} + \sqrt{s_{pq}}}(M_f + M_i)A' \\ B'' &= B + \frac{2}{\sqrt{s_{p'q'}} + \sqrt{s_{pq}}}A'. \quad (54) \end{aligned}$$

In addition to the invariants A'' and B'', we also introduce the invariants F and G very similar to [24][5]

$$M_{\kappa'\kappa} = \chi^\dagger(s')[F + G\left(\sigma\cdot\hat{\mathbf{p}}'\right)(\sigma\cdot\hat{\mathbf{p}})]\chi(s), \quad (55)$$

because we will use the helicity basis. Here, $\chi(s)$ is a helicity state vector. In Ref. [6] this expansion was used in combination with the expansion of the amplitude in Pauli spinor space. The connection between the two are also given there.

The relation between the invariants A'', B'' and F, G is given by

$$\begin{aligned} F &= \sqrt{(E' + M_f)(E + M_i)} \\ &\quad \left\{A'' + \frac{1}{2}[(W' - M_f) + (W - M_i)]\ B''\right\} \\ G &= \sqrt{(E' - M_f)(E - M_i)} \\ &\quad \left\{-A'' + \frac{1}{2}[(W' + M_f) + (W + M_i)]\ B''\right\}. \end{aligned} \quad (56)$$

B. Helicity amplitudes and partial waves

In this subsection we want to link the invariants A'' and B'' to experimental observable phase shifts. This is done by using the helicity basis and the partial-wave expansion. The procedure is based on that in Ref. [25] and similar to that in Ref. [7].

The helicity amplitude in terms of the invariants F and G [see Eq. (55)] is

$$M_{\kappa'\kappa}(\lambda_f, \lambda_i) = C_{\lambda_f,\lambda_i}(\theta, \phi)[F + 4\lambda_f\lambda_i G], \quad (57)$$

where

$$C_{\lambda_f,\lambda_i}(\theta, \phi) = \chi^\dagger_{\lambda_f}(\hat{\mathbf{p}}')\cdot\chi_{\lambda_i}(\hat{\mathbf{p}}) = D^{1/2*}_{\lambda_i\lambda_f}(\phi, \theta, -\phi). \quad (58)$$

Here, $D^J_{mm'}(\alpha, \beta, \gamma)$ are the Wigner D matrices [25] and the angles θ and ϕ are defined as the polar angles of the center-of-mass (c.m.) momentum $\mathbf{p}'$ in a coordinate system that has $\mathbf{p}$ along the positive z axis. In the following we take as the scattering plane the xz plane, i.e., $\phi = 0$. Furthermore, we introduce the functions $f_{1,2}$ by

$$F = \frac{f_1}{4\pi} \quad G = \frac{f_2}{4\pi}. \quad (59)$$

Then, with these settings the helicity amplitude (57) is

$$M_{\kappa'\kappa}(\lambda_f, \lambda_i) = \frac{1}{4\pi}d^{1/2}_{\lambda_i\lambda_f}(\theta)(f_1 + 4\lambda_f\lambda_i f_2). \quad (60)$$

Next, we make the partial-wave expansion of the helicity amplitudes in the center-of-mass frame very similar to [24][6]

$$M_{\kappa'\kappa}(\lambda_f\lambda_i) = (4\pi)^{-1}\sum_J(2J+1)M^J_{\kappa'\kappa}(\lambda_f\lambda_i) \times D^{J*}_{\lambda_i,\lambda_f}(\phi, \theta, -\phi)$$

[5]The difference is a normalization factor.

[6]The difference is again a normalization factor. We use the same normalization as that in Refs. [6,7].

$$= (4\pi)^{-1} e^{i(\lambda_i-\lambda_f)\phi} \sum_J (2J+1) M^J_{\kappa'\kappa}(\lambda_f\lambda_i) \times d^J_{\lambda_i,\lambda_f}(\theta). \quad (61)$$

Using the partial-wave expansion as in Eq. (61) we obtain the Kadyshevsky integral equation (article I) in the partial-wave basis. Here, we just show the result; for the details we refer to [7]

$$M^J_{00}(\lambda_f\lambda_i) = M^{\text{irr J}}_{00}(\lambda_f\lambda_i) + \sum_{\lambda_n} \int_0^\infty k_n^2 dk_n\ M^{\text{irr J}}_{0\kappa}(\lambda_f\lambda_n) \times G'_\kappa(W_n;W)\ M^J_{\kappa 0}(\lambda_n\lambda_i). \quad (62)$$

As mentioned in article I, the κ label is fixed after integration.

Because of the summation over the intermediate helicity states the partial-wave Kadyshevsky integral equation (62) is a coupled-integral equation. It can be decoupled using the combinations $f_{(J-1/2)+}$ and $f_{(J+1/2)-}$ defined by

$$\begin{pmatrix} f_{L+} \\ f_{(L+1)-} \end{pmatrix} = \begin{pmatrix} +1 & +1 \\ +1 & -1 \end{pmatrix} \begin{pmatrix} M^J(+1/2\ 1/2) \\ M^J(-1/2\ 1/2) \end{pmatrix}; \quad (63)$$

here we introduced $L \equiv J - 1/2$.[7]

In Eq. (63) and in the following we omit the subscript 00 for the final amplitudes where κ and κ' are put to zero.

A similar expansion as (63) holds for $M^{\text{irr J}}_{\kappa'\kappa}(\lambda_f\lambda_i)$ and what one gets is

$$f_{L\pm}(W',W) = f^{\text{irr}}_{L\pm}(W',W) + \sum_{\lambda_n} \int_0^\infty k_n^2 dk_n\ f^{\text{irr}}_{L\pm}(W',W_n) \times G(W_n;W)\ f_{L\pm}(W_n,W). \quad (64)$$

The two-particle unitarity relation for the partial-wave helicity states reads [24]

$$i[M^J(\lambda_f\lambda_i) - M^{J*}(\lambda_i\lambda_f)] = 2\sum_{\lambda_n} k\ M^{J*}(\lambda_f\lambda_n) M^J(\lambda_i\lambda_n). \quad (65)$$

In a manner similar to that for the partial-wave Kadyshevsky integral equation (62), the unitarity relation (65) also decouples for the combinations (63). One gets

$$\text{Im} f_{L\pm}(W) = k\ f^*_{L\pm}(W) f_{L\pm}(W), \quad (66)$$

which allows for the introduction of the elastic phase shifts

$$f_{L\pm}(W) = \frac{1}{k}\ e^{i\delta_{L\pm}(W)} \sin\delta_{L\pm}(W). \quad (67)$$

From Eq. (67) we see that once we have found the invariants $f_{L\pm}(W)$ by solving the partial-wave Kadyshevsky integral equation (62) we can determine the phase shifts. The relation between the invariants $f_{L\pm}(W)$ and the invariants $f_{1,2}$ is

$$f_{L\pm} = \frac{1}{2}\int_{-1}^{+1} dx\ [P_L(x) f_1 + P_{L\pm1}(x) f_2] = f_{1,L} + f_{2,L\pm1}, \quad (68)$$

where $x = \cos\theta$.

[7] The labels $L+$ and $(L+1)-$ in Eq. (63) and their relation to total angular momentum J come from parity arguments as is best explained in Ref. [25].

C. Partial-wave projection

Via the equations (68), (59), and (56), the partial waves $f_{L\pm}$ can be traced back to the partial-wave projection of the invariant amplitudes A'' and B'', which means that we are looking for the partial-wave projections of the invariants A, B, A', and B'.

Before doing so we include form factors in the same way as in Ref. [6]. As mentioned there, they are needed to regulate the high-energy behavior and to take into account the extended size of the mesons and baryons. We take them to be

$$F(\Lambda) = e^{\frac{-(\mathbf{k}_f-\mathbf{k}_i)^2}{\Lambda^2}} \quad \text{for} \quad t \text{ channel}$$
$$F(\Lambda) = e^{\frac{-(\mathbf{k}_f^2+\mathbf{k}_i^2)}{\Lambda^2}} \quad \text{for} \quad u, s \text{ channel}. \quad (69)$$

The partial-wave projection includes an integration over $\cos\theta = x$. We, therefore, investigate the x dependence of the invariants. The main concern is the propagators. We want to write them in the form $1/(z \pm x)$, which is especially difficult for the propagators in the t channel, because of the square root in A_t. We therefore use the identity

$$\frac{1}{\omega(\omega+a)} = \frac{1}{\omega^2-a^2} + \frac{2a}{\pi}\int_0^\infty \frac{d\lambda}{\lambda^2+a^2}\left[\frac{1}{\omega^2+\lambda^2} - \frac{1}{\omega^2-a^2}\right], \quad (70)$$

which holds for $\omega, a \in \mathbb{R}$. With this identity we write the propagators as

$$\frac{1}{2A_t}\frac{1}{\Delta_t\cdot n+\bar\kappa-A_t+i\varepsilon} = -\frac{1}{2p'p}\left[\frac{1}{2} + \frac{\Delta_t\cdot n+\bar\kappa}{\pi}\int\frac{d\lambda}{f_\lambda(\bar\kappa)}\right]\frac{1}{z_t(\bar\kappa)-x} + \frac{1}{2p'p}\frac{\Delta_t\cdot n+\bar\kappa}{\pi}\int\frac{d\lambda}{f_\lambda(\bar\kappa)}\frac{1}{z_{t,\lambda}-x}$$
$$\frac{1}{2A_t}\frac{1}{-\Delta_t\cdot n+\bar\kappa-A_t+i\varepsilon} = -\frac{1}{2p'p}\left[\frac{1}{2} - \frac{\Delta_t\cdot n-\bar\kappa}{\pi}\int\frac{d\lambda}{f_\lambda(-\bar\kappa)}\right]\frac{1}{z_t(-\bar\kappa)-x} - \frac{1}{2p'p}\frac{\Delta_t\cdot n-\bar\kappa}{\pi}\int\frac{d\lambda}{f_\lambda(-\bar\kappa)}\frac{1}{z_{t,\lambda}-x}$$
$$\frac{1}{(\bar\kappa+\Delta_u\cdot n)^2 - A_u^2} = -\frac{1}{2p'p}\frac{1}{z_u(\bar\kappa)+x}, \quad (71)$$

where $p'p = |\mathbf{p}'||\mathbf{p}|$ and

$$f_\lambda(\bar\kappa) = \lambda^2 + (\Delta_t\cdot n)^2 + \bar\kappa^2 + 2\bar\kappa\Delta_t\cdot n$$
$$z_i(\bar\kappa) = \frac{1}{2p'p}\left[p'+p+M^2-\bar\kappa^2-2\bar\kappa\Delta_i^0-\left(\Delta_i^0\right)^2\right]$$
$$z_{t,\lambda} = \frac{1}{2p'p}[p'+p+M^2+\lambda^2]. \quad (72)$$

The invariants are expanded in polynomials of x, like

$$\begin{aligned}
j^{\pm}(t) &= [X^j(\pm) + xY^j(\pm)]D^{(1)}(\pm\Delta_t, n, \bar{\kappa}) \\
&= \frac{1}{2p'p}\left[\left(X_1^j(\pm) + xY_1^j(\pm)\right)\frac{F(\Lambda_t)}{z_t(\pm\bar{\kappa}) - x}\right. \\
&\quad \left. + \left(X_2^j(\pm) + xY_2^j(\pm)\right)\frac{F(\Lambda_t)}{z_{t,\lambda} - x}\right], \\
j(u) &= \frac{1}{2p'p}(X^j + xY^j + x^2Z^j)\frac{F(\Lambda_u)}{z_u(\bar{\kappa}) + x} \\
j(s) &= (X^j + xY^j + x^2Z^j)\frac{F(\Lambda_s)}{\frac{1}{4}(W' + W + \kappa' + \kappa)^2 - M_B^2},
\end{aligned} \tag{73}$$

where j is an element of the set (A, B, A', B'). Furthermore, there are the relations in the t channel

$$\begin{aligned}
X_1^j(\pm) &= -\left[\frac{1}{2} + \frac{\pm\Delta_t^0 + \bar{\kappa}}{\pi}\int\frac{d\lambda}{f_\lambda(\pm\bar{\kappa})}\right]X^j(\pm) \\
X_2^j(\pm) &= \frac{\pm\Delta_t^0 + \bar{\kappa}}{\pi}\int\frac{d\lambda}{f_\lambda(\pm\bar{\kappa})}X^j(\pm).
\end{aligned} \tag{74}$$

The coefficients X^j, Y^j, and Z^j can easily be extracted from the invariants and they are given for the various exchange processes in Appendix A.

With the partial-wave projection

$$j_L(i) = \frac{1}{2}\int_{-1}^{1} dx\, P_L(x)j(i), \tag{75}$$

where $i = t, u, s$, we find the partial-wave projections of the invariants

$$\begin{aligned}
j_L^{\pm}(t) &= \frac{1}{2p'p}\left[\left(X_1^j(\pm) + z_t(\pm\bar{\kappa})Y_1^j(\pm)\right)U_L(\Lambda_t, z_t(\pm\bar{\kappa}))\right. \\
&\quad + \left(X_2^j(\pm) + z_{t,\lambda}Y_2^j(\pm)\right)U_L(\Lambda_t, z_{t,\lambda}) \\
&\quad \left. - Y_1^j(\pm)R_L(\Lambda_t, z_t(\pm\bar{\kappa})) - Y_2^j(\pm)R_L(\Lambda_t, z_{t,\lambda})\right] \\
j_L(u) &= \frac{(-1)^L}{2p'p}\left[\left(X^j - z_u(\bar{\kappa})Y^j + z_u^2(\bar{\kappa})Z^j\right)U_L(\Lambda_u, z_u(\bar{\kappa}))\right. \\
&\quad - (-Y^j + z_u(\bar{\kappa})Z^j)R_L(\Lambda_u, z_u(\bar{\kappa})) \\
&\quad \left. - Z^jS_L(\Lambda_u, z_u(\bar{\kappa}))\right] \\
j_L(s) &= \left[X^j\delta_{L,0} + \frac{1}{3}Y^j\delta_{L,1} + \frac{1}{3}\left(\frac{2}{5}\delta_{L,2} + \delta_{L,0}\right)Z^j\right] \\
&\quad \times \frac{F(\Lambda_s)}{\frac{1}{4}(W' + W + \kappa' + \kappa)^2 - M_B^2},
\end{aligned} \tag{76}$$

where

$$\begin{aligned}
U_L(\Lambda, z) &= \frac{1}{2}\int_{-1}^{1} dx\frac{P_L(x)F(\Lambda)}{z - x} \\
R_L(\Lambda, z) &= \frac{1}{2}\int_{-1}^{1} dx\, P_L(x)F(\Lambda) \\
S_L(\Lambda, z) &= \frac{1}{2}\int_{-1}^{1} dx\, x\, P_L(x)F(\Lambda).
\end{aligned} \tag{77}$$

V. CONCLUSION AND DISCUSSION

In two articles, article I and this one, we have presented the results for meson-baryon, or, more specifically, πN scattering, in the Kadyshevsky formalism. In article I we have presented the results for meson-exchange amplitudes and a second quantization procedure for the quasifield present in the Kadyshevsky formalism is given. We studied the frame dependence, i.e., the n dependence, of the Kadyshevsky integral equation, which we continued in this article.

Couplings containing derivatives and higher-spin fields may cause differences and problems as far as the results in the Kadyshevsky formalism and the Feynman formalism are concerned. This is discussed in article I by means of an example. After a second glance the results in both formalisms are the same; however, they contain extra frame-dependent contact terms. Two methods are shortly introduced and applied, which discuss a second source extra terms: the TU and the GJ methods. The extra terms coming from this second source cancel the former ones exactly. Both formalisms yield the same results. With the use of (one of) these methods the final results for the S matrix or amplitude are covariant and frame independent (n independent). For practical purposes we have introduced and discussed the $\bar{P}$ method and, last but not least, we have shown that the TU method can be derived from the Bogoliubov-Medvedev-Polivanov theory.

In this article we have presented the results for baryon exchange. It also contains a formal introduction and detail discussion of so-called pair suppression. We have formally implemented "absolute" pair suppression and applied it to the baryon exchange processes, although it is in principle possible to also allow for some pair production. The formalism used is based on the TU method. For the resulting amplitudes, we have shown, to our knowledge for the first time, that they are causal, covariant, and n independent. Moreover, the amplitudes are just a factor 1/2 of the usual Feynman expressions. The amplitudes contain only positive energy (or, if one wishes, only negative energy) initial and final states. This is particularly convenient for the Kadyshevsky integral equation. It should be mentioned that negative energy is present inside an amplitude via the $\Delta(x - y)$ propagator. This is, however, also the case in the academic example of the infinite dense antineutron star.

The last part of this article contains the partial-wave expansion. This is used for solving the Kadyshevsky integral equation and to introduce the phase shifts.

APPENDICES

A. KADYSHEVSKY AMPLITUDES AND INVARIANTS

A. Meson exchange

1. Scalar-meson exchange, diagram (a)

$$M_{\kappa',\kappa}^{(a)} = g_{PPS}g_S[\bar{u}(p')\ u(p)]D^{(1)}(\Delta_t, n, \bar{\kappa}), \tag{A1}$$

where $D^{(1)}(\Delta_t, n, \bar{\kappa}) = \frac{1}{2A_t}\cdot\frac{1}{\Delta_t\cdot n + \bar{\kappa} - A_t + i\varepsilon}$

$$A_S = g_{PPS}g_S D^{(1)}(\Delta_t, n, \bar{\kappa}). \tag{A2}$$

$$X_S^A = g_{PPS}g_S. \tag{A3}$$

2. Scalar-meson exchange, diagram (b)

$$M^{(b)}_{\kappa',\kappa} = g_{PPS}g_S[\bar{u}(p's')\, u(p)]D^{(1)}(-\Delta_t, n, \bar{\kappa}). \tag{A4}$$

$$A_S = g_{PPS}g_S D^{(1)}(-\Delta_t, n, \bar{\kappa}). \tag{A5}$$

$$X^A_S = g_{PPS}g_S. \tag{A6}$$

3. Pomeron exchange

$$M_{\kappa'\kappa} = \frac{g_{PPP}g_P}{M}[\bar{u}(p's')\, u(p)]. \tag{A7}$$

$$A_P = \frac{g_{PPP}g_P}{M}. \tag{A8}$$

The partial-wave projection is obtained by applying (75) straightforwardly.

4. Vector-meson exchange, diagram (a)

$$\begin{aligned} M^{(a)}_{\kappa',\kappa} = &-g_{VPP}\, \bar{u}(p's')\bigg[2g_V \not{Q} - \frac{g_V}{M_V^2}((M_f - M_i) + \kappa'\not{n}) \\ &\times \bigg(\frac{1}{4}(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q}) \\ &- (m_f^2 - m_i^2) + 2\bar{\kappa} n\cdot Q\bigg) \\ &+ \frac{f_V}{2M_V}\bigg(2(M_f + M_i)\not{Q} + \frac{1}{2}(u_{pq'} + u_{p'q}) \\ &- \frac{1}{2}(s_{p'q'} + s_{pq})\bigg) \\ &- \frac{f_V}{2M_V^3}\bigg(\frac{1}{2}(M_f^2 + M_i^2) + \frac{1}{2}(m_f^2 + m_i^2) \\ &- \frac{1}{2}\bigg(\frac{1}{2}(t_{p'p} + t_{q'q}) + u_{pq'} + s_{pq}\bigg) \\ &+ (M_f + M_i)\kappa'\not{n} + \frac{1}{4}(\kappa' - \kappa)^2 - (p' + p)\cdot n\bar{\kappa}\bigg) \\ &\times \bigg(\frac{1}{4}(s_{p'q'} - s_{pq}) + \frac{1}{4}(u_{pq'} - u_{p'q}) \\ &- (m_f^2 - m_i^2) + 2\bar{\kappa} n\cdot Q\bigg)\bigg]\ u(ps) \times D^{(1)}(\Delta_t, n, \bar{\kappa}). \end{aligned} \tag{A9}$$

$$\begin{aligned} A_V = &-g_{VPP}\bigg[-\frac{g_V}{M_V^2}(M_f - M_i)\bigg(\frac{1}{4}(s_{p'q'} - s_{pq} \\ &+ u_{pq'} - u_{p'q}) - (m_f^2 - m_i^2) + 2\bar{\kappa} n\cdot Q\bigg) \\ &+ \frac{f_V}{4M_V}(u_{pq'} + u_{p'q} - s_{p'q'} - s_{pq}) \\ &- \frac{f_V}{2M_V^3}\bigg(\frac{1}{2}(M_f^2 + M_i^2) + \frac{1}{2}(m_f^2 + m_i^2) \\ &- \frac{1}{2}\bigg(\frac{1}{2}(t_{p'p} + t_{q'q}) + u_{pq'} + s_{pq}\bigg) + \frac{1}{4}(\kappa' - \kappa)^2 \\ &- (p' + p)\cdot n\bar{\kappa}\bigg)\bigg(\frac{1}{4}(s_{p'q'} - s_{pq}) + \frac{1}{4}(u_{pq'} - u_{p'q}) \\ &- (m_f^2 - m_i^2) + 2\bar{\kappa} n\cdot Q\bigg)\bigg]\ D^{(1)}(\Delta_t, n, \bar{\kappa}) \\ B_V = &-2g_{VPP}\bigg[g_V + \frac{f_V}{2M_V}(M_f + M_i)\bigg] D^{(1)}(\Delta_t, n, \bar{\kappa}) \\ A'_V = &\frac{g_{VPP}\kappa'}{M_V^2}\bigg[g_V + \frac{f_V}{2M_V}(M_f + M_i)\bigg]\bigg(\frac{1}{4}(s_{p'q'} - s_{pq} \\ &+ u_{pq'} - u_{p'q}) - (m_f^2 - m_i^2) + 2\bar{\kappa} n\cdot Q\bigg) \\ &\times D^{(1)}(\Delta_t, n, \bar{\kappa}). \end{aligned} \tag{A10}$$

$$\begin{aligned} X^A_V = &-g_{VPP}\bigg[-\frac{g_V}{M_V^2}(M_f - M_i)\bigg(\frac{1}{4}(E' + \mathcal{E}')^2 \\ &- \frac{1}{4}(E + \mathcal{E})^2 - \frac{1}{4}(M_f^2 - M_i^2) - \frac{3}{4}(m_f^2 - m_i^2) \\ &+ \frac{1}{2}(E'\mathcal{E} - E\mathcal{E}') + \bar{\kappa}(\mathcal{E}' + \mathcal{E})\bigg) \\ &+ \frac{f_V}{4M_V}\big(M_f^2 + M_i^2 + m_f^2 + m_i^2 - 2(E'\mathcal{E} + E\mathcal{E}') \\ &- (E' + \mathcal{E}')^2 - (E + \mathcal{E})^2\big) - \frac{f_V}{4M_V^3}\bigg(M_f^2 + M_i^2 + m_f^2 \\ &+ m_i^2 + \frac{1}{2}(\kappa' - \kappa)^2 - \frac{1}{2}\big(M_f^2 + 3M_i^2 + 3m_f^2 + m_i^2 \\ &- 2E'E - 2\mathcal{E}'\mathcal{E} - 4\mathcal{E}'E + (E + \mathcal{E})^2\big) \\ &- 2(E' + E)\bar{\kappa}\bigg)\bigg(\frac{1}{4}(E' + \mathcal{E}')^2 - \frac{1}{4}(E + \mathcal{E})^2 \\ &- \frac{1}{4}(M_f^2 - M_i^2) - \frac{3}{4}(m_f^2 - m_i^2) \\ &+ \frac{1}{2}(E'\mathcal{E} - E\mathcal{E}') + \bar{\kappa}(\mathcal{E}' + \mathcal{E})\bigg)\bigg] \\ Y^A_V = &-\frac{g_{VPP}f_V\, p'p}{M_V}\bigg[1 + \frac{1}{4M_V^2}\big((E' + \mathcal{E}')^2 - (E + \mathcal{E})^2 \\ &- (M_f^2 - M_i^2) - 3(m_f^2 - m_i^2) + 2(E'\mathcal{E} - E\mathcal{E}') \\ &+ 4\bar{\kappa}(\mathcal{E}' + \mathcal{E})\big)\bigg] \\ X^B_V = &-2g_{VPP}\bigg[g_V + \frac{f_V}{2M_V}(M_f + M_i)\bigg], \\ X^{A'}_V = &\frac{g_{VPP}\kappa'}{4M_V^2}\bigg[g_V + \frac{f_V}{2M_V}(M_f + M_i)\bigg]\big((E' + \mathcal{E}')^2 \\ &- (E + \mathcal{E})^2 - (M_f^2 - M_i^2) - 3(m_f^2 - m_i^2) \\ &+ 2(E'\mathcal{E} - E\mathcal{E}') + 4\bar{\kappa}(\mathcal{E}' + \mathcal{E})\big). \end{aligned} \tag{A11}$$

5. Vector-meson exchange, diagram (b)

$$M^{(b)}_{\kappa',\kappa} = -g_{VPP}\,\bar{u}(p's')\Bigg[2g_V \not{Q} - \frac{g_V}{M_V^2}((M_f - M_i) - \kappa\not{n}) \times \left(\frac{1}{4}(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q}) - (m_f^2 - m_i^2) - 2\bar{\kappa} n\cdot Q\right) + \frac{f_V}{2M_V}\left(2(M_f+M_i)\not{Q} + \frac{1}{2}(u_{pq'} + u_{p'q}) - \frac{1}{2}(s_{p'q'} + s_{pq})\right) - \frac{f_V}{2M_V^3}\left(\frac{1}{2}(M_f^2 + M_i^2) + \frac{1}{2}(m_f^2 + m_i^2) - \frac{1}{2}\left(\frac{1}{2}(t_{p'p} + t_{q'q}) + u_{pq'} + s_{pq}\right) - (M_f + M_i)\kappa\not{n} + \frac{1}{4}(\kappa' - \kappa)^2 + (p'+p)\cdot n\bar{\kappa}\right) \times \left(\frac{1}{4}(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q}) - (m_f^2 - m_i^2) - 2\bar{\kappa} n\cdot Q\right)\Bigg]\, u(ps) \times D^{(1)}(-\Delta_t, n, \bar{\kappa}). \quad \text{(A12)}$$

$$A_V = -g_{VPP}\Bigg[-\frac{g_V}{M_V^2}(M_f - M_i)\left(\frac{1}{4}(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q}) - (m_f^2 - m_i^2) - 2\bar{\kappa} n\cdot Q\right) + \frac{f_V}{4M_V}(u_{pq'} + u_{p'q} - s_{p'q'} - s_{pq}) - \frac{f_V}{2M_V^3}\left(\frac{1}{2}(M_f^2 + M_i^2) + \frac{1}{2}(m_f^2 + m_i^2) - \frac{1}{2}\left(\frac{1}{2}(t_{p'p} + t_{q'q}) + u_{pq'} + s_{pq}\right) + \frac{1}{4}(\kappa' - \kappa)^2 + (p'+p)\cdot n\bar{\kappa}\right)\left(\frac{1}{4}(s_{p'q'} - s_{pq}) + \frac{1}{4}(u_{pq'} - u_{p'q}) - (m_f^2 - m_i^2) - 2\bar{\kappa} n\cdot Q\right)\Bigg]\, D^{(1)}(-\Delta_t, n, \bar{\kappa})$$

$$B_V = -2g_{VPP}\left[g_V + \frac{f_V}{2M_V}(M_f + M_i)\right] D^{(1)}(-\Delta_t, n, \bar{\kappa})$$

$$A'_V = -\frac{g_{VPP}\kappa}{M_V^2}\left[g_V + \frac{f_V}{2M_V}(M_f + M_i)\right]\left(\frac{1}{4}(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q}) - (m_f^2 - m_i^2) - 2\bar{\kappa} n\cdot Q\right) \times D^{(1)}(-\Delta_t, n, \bar{\kappa}). \quad \text{(A13)}$$

$$X_V^A = -g_{VPP}\Bigg[-\frac{g_V}{M_V^2}(M_f - M_i)\left(\frac{1}{4}(E' + \mathcal{E}')^2 - \frac{1}{4}(E + \mathcal{E})^2 - \frac{1}{4}(M_f^2 - M_i^2) - \frac{3}{4}(m_f^2 - m_i^2) + \frac{1}{2}(E'\mathcal{E} - E\mathcal{E}') - \bar{\kappa}(\mathcal{E}' + \mathcal{E})\right) + \frac{f_V}{4M_V}\Big(M_f^2 + M_i^2 + m_f^2 + m_i^2 - 2(E'\mathcal{E} + E\mathcal{E}') - (E' + \mathcal{E}')^2 - (E + \mathcal{E})^2\Big) - \frac{f_V}{4M_V^3}\Big(M_f^2 + M_i^2 + m_f^2 + m_i^2 + \frac{1}{2}(\kappa' - \kappa)^2 - \frac{1}{2}\big(M_f^2 + 3M_i^2 + 3m_f^2 + m_i^2 - 2E'E - 2\mathcal{E}'\mathcal{E} - 4\mathcal{E}'E + (E + \mathcal{E})^2\big) + 2(E' + E)\bar{\kappa}\Big)\Big(\frac{1}{4}(E' + \mathcal{E}')^2 - \frac{1}{4}(E + \mathcal{E})^2 - \frac{1}{4}(M_f^2 - M_i^2) - \frac{3}{4}(m_f^2 - m_i^2) + \frac{1}{2}(E'\mathcal{E} - E\mathcal{E}') - \bar{\kappa}(\mathcal{E}' + \mathcal{E})\Big)\Bigg]$$

$$Y_V^A = -\frac{g_{VPP} f_V\, p'p}{M_V}\left[1 + \frac{1}{4M_V^2}\Big((E' + \mathcal{E}')^2 - (E + \mathcal{E})^2 - (M_f^2 - M_i^2) - 3(m_f^2 - m_i^2) + 2(E'\mathcal{E} - E\mathcal{E}') - 4\bar{\kappa}(\mathcal{E}' + \mathcal{E})\Big)\right]$$

$$X_V^B = -2g_{VPP}\left[g_V + \frac{f_V}{2M_V}(M_f + M_i)\right],$$

$$X_V^{A'} = -\frac{g_{VPP}\kappa}{4M_V^2}\left[g_V + \frac{f_V}{2M_V}(M_f + M_i)\right]\Big((E' + \mathcal{E}')^2 - (E + \mathcal{E})^2 - (M_f^2 - M_i^2) - 3(m_f^2 - m_i^2) + 2(E'\mathcal{E} - E\mathcal{E}') - 4\bar{\kappa}(\mathcal{E}' + \mathcal{E})\Big). \quad \text{(A14)}$$

B. Baryon exchange/resonance

1. Baryon exchange, scalar coupling

$$M^S_{\kappa',\kappa} = \frac{g_S^2}{2}\,\bar{u}(p's')\left[\frac{1}{2}(M_f + M_i) + M_B - \not{Q} + \not{n}\bar{\kappa}\right] u(ps) D^{(2)}(\Delta_u, n, \bar{\kappa}), \quad \text{(A15)}$$

where the denominator function is $D^{(2)}(\Delta_i, n, \bar{\kappa}) = [(\bar{\kappa} + \Delta_i\cdot n)^2 - A_i^2]^{-1}$, $i = u, s$.

$$A_S = \frac{g_S^2}{2}\left[\frac{1}{2}(M_f + M_i) + M_B\right] D^{(2)}(\Delta_u, n, \bar{\kappa})$$

$$B_S = -\frac{g_S^2}{2}\, D^{(2)}(\Delta_u, n, \bar{\kappa}) \quad \text{(A16)}$$

$$A'_S = \frac{g_S^2}{2}\bar{\kappa}\, D^{(2)}(\Delta_u, n, \bar{\kappa}).$$

$$X_S^A = -\frac{g_S^2}{2}\left[\frac{1}{2}(M_f + M_i) + M_B\right]$$

$$X_S^B = \frac{g_S^2}{2} \quad \text{(A17)}$$

$$X_S^{A'} = -\frac{g_S^2}{2}\bar{\kappa}.$$

2. Baryon exchange, pseudoscalar coupling

The expressions for baryon exchange with pseudoscalar coupling are the same as Eqs. (A15)–(A17) with the substitution $M_B \to -M_B$.

3. Baryon resonance, scalar coupling

$$M^S_{\kappa',\kappa} = \frac{g_S^2}{2}\,\bar{u}(p's')\left[\frac{1}{2}(M_f+M_i) + M_B + \not{Q} + \not{n}\bar{\kappa}\right] u(ps)D^{(2)}(\Delta_s,n,\bar{\kappa}). \quad \text{(A18)}$$

$$A_S = \frac{g_S^2}{2}\left[\frac{1}{2}(M_f+M_i)+M_B\right] D^{(2)}(\Delta_s,n,\bar{\kappa})$$
$$B_S = \frac{g_S^2}{2} D^{(2)}(\Delta_s,n,\bar{\kappa})$$
$$A'_S = \frac{g_S^2}{2}\bar{\kappa} D^{(2)}(\Delta_s,n,\bar{\kappa}). \quad \text{(A19)}$$
$$X_S^A = -\frac{g_S^2}{2}\left[\frac{1}{2}(M_f+M_i)+M_B\right]$$
$$X_S^B = -\frac{g_S^2}{2}$$
$$X_S^{A'} = -\frac{g_S^2}{2}\,\bar{\kappa}. \quad \text{(A20)}$$

4. Baryon resonance, pseudoscalar coupling

The expressions for baryon resonance with pseudoscalar coupling are the same as Eqs. (A18)–(A20) with the substitution $M_B \to -M_B$.

5. Baryon exchange vector coupling

$$\begin{aligned} M^V_{\kappa',\kappa} = \frac{f_V^2}{2m_\pi^2}\bar{u}(p's')\Bigg[&-\left(\frac{1}{2}(M_f+M_i)-M_B\right) \\ &\times\left(-\frac{1}{2}(M_f^2+M_i^2)+\frac{1}{2}(u_{p'q}+u_{pq'}) + (M_f+M_i)\not{Q} - \frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n + \frac{1}{2}(\kappa'-\kappa)[\not{n},\not{Q}] - \frac{1}{2}(\kappa'-\kappa)^2\right) \\ &-\frac{1}{2}(u_{pq'}-M_i^2)\left(\frac{1}{2}(M_f-M_i)+\not{Q} + \frac{1}{2}(\kappa'-\kappa)\not{n}\right) - \frac{1}{2}(u_{p'q}-M_f^2) \\ &+\left(-\frac{1}{2}(M_f-M_i)+\not{Q}-\frac{1}{2}(\kappa'-\kappa)\not{n}\right) \\ &+\bar{\kappa}\Bigg(-\frac{1}{2}(M_f-M_i)(p'-p)\cdot n - (p'-p)\cdot n\,\not{Q} + 2Q\cdot n\,\not{Q} - \frac{1}{2}(M_f-M_i)(\kappa'-\kappa) + \frac{1}{2}(M_f-M_i) \\ &\times[\not{n},\not{Q}] + \frac{1}{2}(M_f^2+M_i^2)\not{n} - \frac{1}{2}(u_{p'q}+u_{pq'})\not{n}\Bigg)\Bigg] \\ &\times u_i(p)D^{(2)}(\Delta_u,n,\bar{\kappa}). \end{aligned} \quad \text{(A21)}$$

$$\begin{aligned} A_V = \frac{f_V^2}{2m_\pi^2}\Bigg[&-\left(\frac{1}{2}(M_f+M_i)-M_B\right) \\ &\times\left(-\frac{1}{2}(M_f^2+M_i^2)+\frac{1}{2}(u_{p'q}+u_{pq'}) - \frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n - \frac{1}{2}(\kappa'-\kappa)^2\right) \\ &-\frac{\bar{\kappa}}{2}(M_f-M_i)(p'-p)\cdot n \\ &+\frac{1}{4}\left(u_{p'q}-u_{pq'}-M_f^2+M_i^2\right)(M_f-M_i) \\ &-\frac{\bar{\kappa}}{2}(M_f-M_i)(\kappa'-\kappa)\Bigg] D^{(2)}(\Delta_u,n,\bar{\kappa}) \end{aligned}$$

$$\begin{aligned} B_V = \frac{f_V^2}{2m_\pi^2}\Bigg[&-\left(\frac{1}{2}(M_f+M_i)-M_B\right)(M_f+M_i) \\ &+\frac{1}{2}\left(M_f^2+M_i^2-u_{p'q}-u_{pq'}\right) - \bar{\kappa}(p'-p)\cdot n \\ &+2\bar{\kappa}n\cdot Q\Bigg] D^{(2)}(\Delta_u,n,\bar{\kappa}) \end{aligned}$$

$$A'_V = \frac{f_V^2}{4m_\pi^2}\left[\left(M_i^2-u_{pq'}\right)\kappa' + \left(M_f^2-u_{p'q}\right)\kappa\right] D^{(2)}(\Delta_u,n,\bar{\kappa})$$

$$B'_V = -\frac{f_V^2}{4m_\pi^2}[\kappa' M_i - \kappa M_f - (\kappa'-\kappa)M_B]D^{(2)}(\Delta_u,n,\bar{\kappa}). \quad \text{(A22)}$$

$$\begin{aligned} X_V^A = -\frac{f_V^2}{2m_\pi^2}\Bigg[&-\left(\frac{1}{2}(M_f+M_i)-M_B\right)\left(\frac{1}{2}\left(m_f^2+m_i^2\right) - E'\mathcal{E} - E\mathcal{E}' - \frac{1}{2}(\kappa'-\kappa)(E'-E) - \frac{1}{2}(\kappa'-\kappa)^2\right) \\ &-\frac{\bar{\kappa}}{2}(M_f-M_i)(E'-E) - \frac{1}{4}\left(m_f^2-m_i^2+2E'\mathcal{E} - 2E\mathcal{E}'\right)(M_f-M_i) - \frac{\bar{\kappa}}{2}(M_f-M_i)(\kappa'-\kappa)\Bigg], \end{aligned}$$

$$Y_V^A = \frac{f_V^2 p'p}{m_\pi^2}\left[\frac{1}{2}(M_f+M_i)-M_B\right]$$

$$\begin{aligned} X_V^B = \frac{f_V^2}{2m_\pi^2}\Bigg[&\left(\frac{1}{2}(M_f+M_i)-M_B\right)(M_f+M_i) \\ &+\frac{1}{2}\left(m_f^2+m_i^2-2E'\mathcal{E}-2E\mathcal{E}'\right) \\ &+\bar{\kappa}(E'-E)\cdot n - \bar{\kappa}(\mathcal{E}'+\mathcal{E})\Bigg] \end{aligned}$$

$$Y_V^B = \frac{f_V^2 p'p}{m_\pi^2}$$
$$X_V^{A'} = \frac{f_V^2}{4m_\pi^2}\left[\kappa'\left(m_f^2 - 2E\mathcal{E}'\right) + \kappa\left(m_i^2 - 2E'\mathcal{E}\right)\right],$$
$$Y_V^{A'} = \frac{f_V^2 \bar{\kappa} p'p}{m_\pi^2},$$
$$X_V^{B'} = \frac{f_V^2}{4m_\pi^2}[\kappa' M_i - \kappa M_f - (\kappa' - \kappa)M_B]. \quad \text{(A23)}$$

6. Baryon exchange, pseudovector coupling

The expressions for baryon exchange with pseudovector coupling are the same as Eqs. (A21)–(A23) with the substitution $M_B \to -M_B$.

7. Baryon resonance, vector coupling

$$\begin{aligned} M^V_{\kappa',\kappa} = \frac{f_V^2}{m_\pi^2}\bar{u}(p's')\Bigg[&-\left(\frac{1}{2}(M_f + M_i) - M_B\right) \\ &\times\Bigg(-\frac{1}{2}\left(M_f^2 + M_i^2\right) + \frac{1}{2}(s_{p'q'} + s_{pq}) - (M_f + M_i)\not{Q} \\ &- \frac{1}{2}(\kappa' - \kappa)(p' - p)\cdot n - \frac{1}{2}(\kappa' - \kappa)[\not{n}, \not{Q}] \\ &- \frac{1}{2}(\kappa' - \kappa)^2\Bigg) + \frac{1}{2}\left(s_{p'q'} - M_f^2\right) \\ &\times\left(\frac{1}{2}(M_f - M_i) + \not{Q} + \frac{1}{2}(\kappa' - \kappa)\not{n}\right) + \frac{1}{2} \\ &\times\left(s_{pq} - M_i^2\right)\left(-\frac{1}{2}(M_f - M_i) + \not{Q} - \frac{1}{2}(\kappa' - \kappa)\not{n}\right) \\ &+ \bar{\kappa}\Bigg(-\frac{1}{2}(M_f - M_i)(p' - p)\cdot n + (p' - p)\cdot n\,\not{Q} \\ &+ 2Q\cdot n\,\not{Q} - \frac{1}{2}(M_f - M_i)(\kappa' - \kappa) - \frac{1}{2}(M_f - M_i) \\ &\times[\not{n}, \not{Q}] + \frac{1}{2}\left(M_f^2 + M_i^2\right)\not{n} - \frac{1}{2}(s_{p'q'} + s_{pq})\not{n}\Bigg)\Bigg] \\ &\times u_i(p)D^{(2)}(\Delta_s, n, \bar{\kappa}). \end{aligned} \quad \text{(A24)}$$

$$\begin{aligned} A_V = \frac{f_V^2}{2m_\pi^2}\Bigg[&-\left(\frac{1}{2}(M_f + M_i) - M_B\right) \\ &\times\left(\frac{1}{2}(s_{p'q'} + s_{pq}) - \frac{1}{2}\left(M_f^2 + M_i^2\right) - \frac{1}{2}(\kappa' - \kappa)\right. \\ &\left.\times(p' - p)\cdot n - \frac{1}{2}(\kappa' - \kappa)^2\right) - \frac{\bar{\kappa}}{2}(M_f - M_i) \\ &\times(p' - p)\cdot n + \frac{1}{4}\left(s_{p'q'} - s_{pq} - M_f^2 + M_i^2\right) \\ &\times(M_f - M_i) - \frac{\bar{\kappa}}{2}(M_f - M_i)(\kappa' - \kappa)\Bigg]D^{(2)}(\Delta_s, n, \bar{\kappa}) \end{aligned}$$

$$\begin{aligned} B_V = \frac{f_V^2}{2m_\pi^2}\Bigg[&\left(\frac{1}{2}(M_f + M_i) - M_B\right)(M_f + M_i) \\ &+ \frac{1}{2}\left(s_{p'q'} + s_{pq} - M_f^2 - M_i^2\right) \\ &+ \bar{\kappa}(p' - p)\cdot n + 2\bar{\kappa}n\cdot Q\Bigg]D^{(2)}(\Delta_s, n, \bar{\kappa}) \end{aligned}$$
$$A'_V = \frac{f_V^2}{4m_\pi^2}\left[\left(M_i^2 - s_{pq}\right)\kappa' + \left(M_f^2 - s_{p'q'}\right)\kappa\right]D^{(2)}(\Delta_s, n, \bar{\kappa})$$
$$B'_V = \frac{f_V^2}{4m_\pi^2}[\kappa' M_i - \kappa M_f - (\kappa' - \kappa)M_B]D^{(2)}(\Delta_s, n, \bar{\kappa}). \quad \text{(A25)}$$

$$\begin{aligned} X_V^A = -\frac{f_V^2}{2m_\pi^2}\Bigg[&-\frac{1}{2}\left(\frac{1}{2}(M_f + M_i) - M_B\right) \\ &\times\Bigg((E' + \mathcal{E}')^2 + (E + \mathcal{E})^2 - \left(M_f^2 + M_i^2\right) \\ &- \frac{1}{2}(\kappa' - \kappa)(E' - E) - \frac{1}{2}(\kappa' - \kappa)^2\Bigg) \\ &- \frac{\bar{\kappa}}{2}(M_f - M_i)(E' - E) + \frac{1}{4}\Big((E' + \mathcal{E}')^2 - (E + \mathcal{E})^2 \\ &- M_f^2 + M_i^2\Big)(M_f - M_i) - \frac{\bar{\kappa}}{2}(M_f - M_i)(\kappa' - \kappa)\Bigg], \end{aligned}$$
$$\begin{aligned} X_V^B = -\frac{f_V^2}{2m_\pi^2}\Bigg[&\left(\frac{1}{2}(M_f + M_i) - M_B\right)(M_f + M_i) \\ &+ \frac{1}{2}(E' + \mathcal{E}')^2 + \frac{1}{2}(E + \mathcal{E})^2 - \frac{1}{2}\left(M_f^2 + M_i^2\right) \\ &+ \bar{\kappa}(E' - E) + \bar{\kappa}(\mathcal{E}' + \mathcal{E})\Bigg], \end{aligned}$$
$$X_V^{A'} = -\frac{f_V^2}{4m_\pi^2}\left[\left(M_i^2 - (E + \mathcal{E})^2\right)\kappa' + \left(M_f^2 - (E' + \mathcal{E}')^2\right)\kappa\right]$$
$$X_V^{B'} = -\frac{f_V^2}{4m_\pi^2}[\kappa' M_i - \kappa M_f - (\kappa' - \kappa)M_B]. \quad \text{(A26)}$$

8. Baryon resonance, pseudovector coupling

The expressions for baryon resonance with pseudovector coupling are the same as Eqs. (A24)–(A26) with the substitution $M_B \to -M_B$.

9. $\frac{3}{2}^+$ Baryon exchange, gauge-invariant coupling

$$\begin{aligned} M_{\kappa',\kappa} = -\frac{g_{gi}^2}{2}\bar{u}(p's')\Bigg[&\frac{1}{2}\bar{P}_u^2\left(\frac{1}{2}(M_f + M_i)\right. \\ &\left.+ M_\Delta - \not{Q} + \bar{\kappa}\not{n}\right)\left(m_f^2 + m_i^2 - t_{q'q}\right) - \frac{1}{3}\bar{P}_u^2 \\ &\times\Bigg(\left(\frac{1}{2}(M_f + M_i) + M_\Delta\right)\not{q}\not{q}' + \frac{1}{2}\left(u_{pq'} - M_i^2\right)\not{q} \\ &+ \frac{1}{2}\left(s_{pq} + t_{q'q} - M_i^2 - m_f^2 - 3m_i^2\right)\not{q}' + \bar{\kappa}\not{n}\not{q}\not{q}'\Bigg) \\ &-\frac{1}{12}\Bigg(\left(\bar{P}_u^2 + \frac{M_\Delta}{2}(M_f - M_i)\right)\not{q} + \frac{M_\Delta}{2} \end{aligned}$$

$$\times \left(s_{pq} - M_i^2 - 2m_i^2\right) - \frac{M_\Delta}{2} \not{q}'\not{q} + M_\Delta \bar{\kappa} \not{n}\not{q}\Big)$$
$$\times (\bar{P}_u \cdot q') + \frac{1}{12}\left(\left(\bar{P}_u^2 + \frac{M_\Delta}{2}(M_f - M_i)\right)\not{q}'\right.$$
$$+ \frac{M_\Delta}{2}\left(M_i^2 - u_{pq'}\right) - \frac{M_\Delta}{2}\not{q}\not{q}' + M_\Delta\bar{\kappa}\not{n}\not{q}'\Big)(\bar{P}_u \cdot q)$$
$$- \frac{1}{24}\left(\frac{1}{2}(M_f + M_i) + M_\Delta - \not{Q} + \bar{\kappa}\not{n}\right)(\bar{P}_u \cdot q')$$
$$\times \left.(\bar{P}_u \cdot q)\right] u(ps) D^{(2)}(\Delta_u, n, \bar{\kappa}). \quad \text{(A27)}$$

Here, $\bar{P}_u^2$ is defined in Eq. (48). All the expressions for the *slashed* terms (i.e., $\not{q}$, $\not{q}'$, etc.) can be found in Eq. (A68). Furthermore,

$$\bar{P}_u \cdot q' = \left(-M_f^2 + M_i^2 - 3m_f^2 - m_i^2 + s_{p'q'} - u_{pq'} + t_{q'q}\right.$$
$$\left. - 2\bar{\kappa}(p' - p)\cdot n + 4\bar{\kappa} n \cdot Q - (\kappa'^2 - \kappa^2)\right)$$
$$\bar{P}_u \cdot q' = \left(M_f^2 - M_i^2 - m_f^2 - 3m_i^2 + s_{pq} - u_{p'q} + t_{q'q}\right.$$
$$\left. + 2\bar{\kappa}(p' - p)\cdot n + 4\bar{\kappa} n \cdot Q + (\kappa'^2 - \kappa^2)\right). \quad \text{(A28)}$$

$$A_\Delta = -\frac{g_{gi}^2}{2}\left\{\frac{1}{2}\bar{P}_u^2\left[\frac{1}{2}(M_f + M_i) + M_\Delta\right]\left(m_f^2 + m_i^2 - t_{q'q}\right)\right.$$
$$- \frac{1}{3}\bar{P}_u^2\left[\left(\frac{1}{2}(M_f + M_i) + M_\Delta\right)\left(\frac{1}{2}(u_{p'q} + u_{pq'})\right.\right.$$
$$- \frac{1}{2}\left(M_f^2 + M_i^2\right) - \frac{1}{2}(\kappa' - \kappa)(p' - p)\cdot n$$
$$\left. - \frac{1}{2}(\kappa' - \kappa)^2\right) + \frac{1}{4}\left(u_{pq'} - M_i^2\right)(M_f - M_i)$$
$$- \frac{1}{4}\left(s_{pq} + t_{q'q} - M_i^2 - m_f^2 - 3m_i^2\right)$$
$$\left.\times (M_f - M_i) - \bar{\kappa}(M_f - M_i) n \cdot Q\right]$$
$$- \frac{1}{12}\left[\frac{1}{2}\left(\bar{P}_u^2 + \frac{M_\Delta}{2}(M_f - M_i)\right)(M_f - M_i)\right.$$
$$+ \frac{1}{2}M_\Delta\left(s_{pq} - M_i^2 - 2m_i^2\right) - \frac{1}{2}M_\Delta$$
$$\times \left(\frac{1}{2}(s_{p'q'} + s_{pq}) - \frac{1}{2}\left(M_f^2 + M_i^2\right)\right.$$
$$\left. - \frac{1}{2}(\kappa' - \kappa)(p' - p)\cdot n - \frac{1}{2}(\kappa' - \kappa)^2\right)$$
$$\left. + \bar{\kappa} M_\Delta\left(n \cdot p' + n \cdot Q + \frac{1}{2}(\kappa' - \kappa)\right)\right](\bar{P}_u \cdot q')$$
$$- \frac{1}{12}\left[\frac{1}{2}\left(\bar{P}_u^2 + \frac{M_\Delta}{2}(M_f - M_i)\right)\right.$$
$$\times (M_f - M_i) - \frac{1}{2}M_\Delta\left(M_i^2 - u_{pq'}\right)$$
$$+ \frac{1}{2}M_\Delta\left(\frac{1}{2}(u_{p'q} + u_{pq'}) - \frac{1}{2}\left(M_f^2 + M_i^2\right)\right.$$
$$\left. - \frac{1}{2}(\kappa' - \kappa)(p' - p)\cdot n - \frac{1}{2}(\kappa' - \kappa)^2\right) - \bar{\kappa} M_\Delta$$
$$\left.\times \left(-n \cdot p' + n \cdot Q - \frac{1}{2}(\kappa' - \kappa)\right)\right](\bar{P}_u \cdot q)$$
$$\left. - \frac{1}{24}\left[\frac{1}{2}(M_f + M_i) + M_\Delta\right](\bar{P}_u \cdot q')(\bar{P}_u \cdot q)\right\}$$
$$\times D^{(2)}(\Delta_u, n, \bar{\kappa}). \quad \text{(A29)}$$

$$B_\Delta = -\frac{g_{gi}^2}{2}\left\{-\frac{1}{2}\bar{P}_u^2\left(m_f^2 + m_i^2 - t_{q'q}\right)\right.$$
$$- \frac{1}{3}\bar{P}_u^2\left[\left(\frac{1}{2}(M_f + M_i) + M_\Delta\right)(M_f + M_i)\right.$$
$$+ \frac{1}{2}\left(u_{pq'} - M_i^2\right) + \frac{1}{2}\left(s_{pq} + t_{q'q} - M_i^2 - m_f^2 - 3m_i^2\right)$$
$$\left. + 2\bar{\kappa}(p' - p)\cdot n + \frac{1}{2}(\kappa'^2 - \kappa^2)\right]$$
$$- \frac{1}{12}\left(\bar{P}_u^2 + M_\Delta M_f\right)(\bar{P}_u \cdot q') + \frac{1}{12}\left(\bar{P}_u^2 - M_\Delta M_i\right)$$
$$\left.\times (\bar{P}_u \cdot q) + \frac{1}{24}(\bar{P}_u \cdot q')\left(\bar{P}_u \cdot q\right)\right\} D^{(2)}(\Delta_u, n, \bar{\kappa}).$$
$$\text{(A30)}$$

$$A'_\Delta = -\frac{g_{gi}^2}{2}\left\{\frac{\bar{\kappa}}{2}\bar{P}_u^2\left(m_f^2 + m_i^2 - t_{q'q}\right)\right.$$
$$- \frac{1}{3}\bar{P}_u^2\left[\frac{1}{4}(\kappa' - \kappa)\left(u_{pq'} - M_i^2\right) - \frac{1}{4}(\kappa' - \kappa)\right.$$
$$\times \left(s_{pq} + t_{q'q} - M_i^2 - m_f^2 - 3m_i^2\right)$$
$$+ \bar{\kappa}\left(-\frac{1}{2}\left(M_f^2 + M_i^2\right) + \frac{1}{2}(u_{p'q} + u_{pq'}) - \frac{1}{2}(\kappa' - \kappa)\right.$$
$$\left.\left.\times (p' - p)\cdot n - (\kappa' - \kappa) Q \cdot n - \frac{1}{2}(\kappa' - \kappa)^2\right)\right]$$
$$- \frac{1}{24}\left[(\kappa' - \kappa)\bar{P}_u^2 - M_\Delta(\kappa M_f + \kappa' M_i)\right]$$
$$\times \left[s_{p'q'} + s_{pq} - u_{p'q} - u_{pq'} + 2t_{q'q} - 4m_f^2\right.$$
$$\left.\left. - 4m_i^2 + 8\bar{\kappa} n \cdot Q\right] + \frac{\bar{\kappa}}{24}(\bar{P}_u \cdot q')(\bar{P}_u \cdot q)\right\}$$
$$\times D^{(2)}(\Delta_u, n, \bar{\kappa}). \quad \text{(A31)}$$

$$B'_\Delta = \frac{g_{gi}^2}{12}\left\{\bar{P}_u^2\left[M_i\kappa' - M_f\kappa + M_\Delta(\kappa' - \kappa)\right] + \frac{M_\Delta\kappa'}{4}\right.$$
$$\left.\times (\bar{P}_u \cdot q') - \frac{M_\Delta\kappa}{4}(\bar{P}_u \cdot q)\right\} D^{(2)}(\Delta_u, n, \bar{\kappa}). \quad \text{(A32)}$$

$$X_\Delta^A = \frac{g_{gi}^2}{2}\left\{\left(\bar{P}_u^2\right)_{\text{c.m.}}\left[\frac{1}{2}(M_f + M_i) + M_\Delta\right]\mathcal{E}'\mathcal{E} - \frac{1}{3}\left(\bar{P}_u^2\right)_{\text{c.m.}}\right.$$
$$\times \left[\left(\frac{1}{2}(M_f + M_i) + M_\Delta\right)\left(\frac{1}{2}\left(M_f^2 + M_i^2 + m_f^2 + m_i^2\right.\right.\right.$$
$$\left. - 2E'\mathcal{E} - 2\mathcal{E}'E\right) - \frac{1}{2}\left(M_f^2 + M_i^2\right) - \frac{1}{2}(\kappa' - \kappa)$$
$$\left.\times (E' - E) - \frac{1}{2}(\kappa' - \kappa)^2\right) + \frac{1}{4}\left(m_f^2 - 2E\mathcal{E}'\right)$$
$$\times (M_f - M_i) - \frac{1}{4}\left((E + \mathcal{E})^2 - 2\mathcal{E}'\mathcal{E} - M_i^2 - 2m_i^2\right)$$

$$\times (M_f - M_i) - \frac{1}{2}\bar{\kappa}(M_f - M_i)(\mathcal{E}' + \mathcal{E})\Big]$$
$$-\frac{1}{12}\left[\frac{1}{2}\left(\left(\bar{P}_u^2\right)_{\text{c.m.}} + \frac{M_\Delta}{2}(M_f - M_i)\right)(M_f - M_i)\right.$$
$$+\frac{1}{2}M_\Delta\big((E+\mathcal{E})^2 - M_i^2 - 2m_i^2\big) - \frac{1}{2}M_\Delta$$
$$\times\left(\frac{1}{2}(E'+\mathcal{E}')^2 + \frac{1}{2}(E+\mathcal{E})^2 - \frac{1}{2}\big(M_f^2 + M_i^2\big)\right.$$
$$\left.-\frac{1}{2}(\kappa'-\kappa)(E'-E) - \frac{1}{2}(\kappa'-\kappa)^2\right) + \bar{\kappa}M_\Delta$$
$$\left.\times\left(E' + \frac{1}{2}(\mathcal{E}'+\mathcal{E}) + \frac{1}{2}(\kappa'-\kappa)\right)\right](\bar{P}_u\cdot q')_{\text{c.m.}}$$
$$-\frac{1}{12}\left[\frac{1}{2}\left(\left(\bar{P}_u^2\right)_{\text{c.m.}} + \frac{M_\Delta}{2}(M_f - M_i)\right)(M_f - M_i)\right.$$
$$-\frac{1}{2}M_\Delta\big(m_f^2 - 2E\mathcal{E}'\big) + \frac{1}{2}M_\Delta\left(\frac{1}{2}\big(m_f^2 + m_i^2 - 2E'\mathcal{E}\right.$$
$$\left.-2\mathcal{E}'E\big) - \frac{1}{2}(\kappa'-\kappa)(E'-E) - \frac{1}{2}(\kappa'-\kappa)^2\right)$$
$$\left.-\bar{\kappa}M_\Delta\left(-E' + \frac{1}{2}(\mathcal{E}'+\mathcal{E}) - \frac{1}{2}(\kappa'-\kappa)\right)\right](\bar{P}_u\cdot q)_{\text{c.m.}}$$
$$\left.-\frac{1}{24}\left[\frac{1}{2}(M_f + M_i) + M_\Delta\right](\bar{P}_u\cdot q')_{\text{c.m.}}(\bar{P}_u\cdot q)_{\text{c.m.}}\right\}, \tag{A33}$$

where

$$\left(\bar{P}_u^2\right)_{\text{c.m.}} = \left[\frac{1}{2}\big(M_f^2 + M_i^2 + m_f^2 + m_i^2 - 2E'\mathcal{E} - 2\mathcal{E}'E\big)\right.$$
$$\left. + \kappa'\kappa + \bar{\kappa}(E' + E - \mathcal{E}' - \mathcal{E})\right]$$
$$(\bar{P}_u\cdot q')_{\text{c.m.}} = \big[-M_f^2 - 3m_f^2 + (E'+\mathcal{E}')^2 + 2E\mathcal{E}' - 2\mathcal{E}'\mathcal{E}$$
$$- 2\bar{\kappa}(E'-E) + 2\bar{\kappa}(\mathcal{E}'+\mathcal{E}) - (\kappa'^2 - \kappa^2)\big]$$
$$(\bar{P}_u\cdot q)_{\text{c.m.}} = \big[-M_i^2 - 3m_i^2 + (E+\mathcal{E})^2 + 2E'\mathcal{E} - 2\mathcal{E}'\mathcal{E}$$
$$+ 2\bar{\kappa}(E'-E) + 2\bar{\kappa}(\mathcal{E}'+\mathcal{E}) + (\kappa'^2 - \kappa^2)\big]. \tag{A34}$$

$$Y_\Delta^A = \frac{g_{gi}^2 p'p}{2}\left\{\left[\frac{1}{2}(M_f + M_i) + M_\Delta\right]2\mathcal{E}'\mathcal{E} - \frac{5}{3}\left(\bar{P}_u^2\right)_{\text{c.m.}}\right.$$
$$\times\left[\frac{1}{2}(M_f + M_i) + M_\Delta\right] - \frac{2}{3}\left[\left(\frac{1}{2}(M_f + M_i) + M_\Delta\right)\right.$$
$$\times\left(\frac{1}{2}\big(M_f^2 + M_i^2 + m_f^2 + m_i^2 - 2E'\mathcal{E} - 2\mathcal{E}'E\big)\right.$$
$$\left.-\frac{1}{2}\big(M_f^2 + M_i^2\big) - \frac{1}{2}(\kappa'-\kappa)(E'-E) - \frac{1}{2}(\kappa'-\kappa)^2\right)$$
$$-\frac{1}{4}\big((E+\mathcal{E})^2 - 2\mathcal{E}'\mathcal{E} - M_i^2 - 2m_i^2\big)(M_f - M_i)$$
$$\left.+\frac{1}{4}\big(m_f^2 - 2E\mathcal{E}'\big)(M_f - M_i) - \frac{1}{2}\bar{\kappa}(M_f - M_i)(\mathcal{E}'+\mathcal{E})\right]$$
$$-\frac{1}{12}\big[-M_f^2 - M_i^2 - 3m_f^2 - 3m_i^2 + (E'+\mathcal{E}')^2$$
$$+ (E+\mathcal{E})^2 + 2E'\mathcal{E} + 2E\mathcal{E}' - 4\mathcal{E}'\mathcal{E} + 4\bar{\kappa}(\mathcal{E}'+\mathcal{E})\big]$$
$$\left.\times[M_f - M_i] - \frac{M_\Delta}{6}\left(\bar{P}_u\cdot q\right)_{\text{c.m.}}\right\}. \tag{A35}$$

$$Z_\Delta^A = -\frac{5g_{gi}^2(p'p)^2}{3}\left[\frac{1}{2}(M_f + M_i) + M_\Delta\right]. \tag{A36}$$

$$X_\Delta^B = \frac{g_{gi}^2}{2}\left\{-\left(\bar{P}_u^2\right)_{\text{c.m.}}\mathcal{E}'\mathcal{E} - \frac{1}{3}\left(\bar{P}_u^2\right)_{\text{c.m.}}\right.$$
$$\times\left[\left(\frac{1}{2}(M_f + M_i) + M_\Delta\right)(M_f + M_i)\right.$$
$$+\frac{1}{2}\big((E+\mathcal{E})^2 - 2\mathcal{E}'\mathcal{E} - M_i^2 - 2m_i^2\big)$$
$$\left.+\frac{1}{2}\big(m_f^2 - 2E\mathcal{E}'\big) + 2\bar{\kappa}(E'-E) + \frac{1}{2}(\kappa'^2 - \kappa^2)\right]$$
$$-\frac{1}{12}\big[\left(\bar{P}_u^2\right)_{\text{c.m.}} + M_\Delta M_f\big](\bar{P}_u\cdot q')_{\text{c.m.}}$$
$$+\frac{1}{12}\big[\left(\bar{P}_u^2\right)_{\text{c.m.}} - M_\Delta M_i\big](\bar{P}_u\cdot q)_{\text{c.m.}}$$
$$\left.+\frac{1}{24}(\bar{P}_u\cdot q')_{\text{c.m.}}(\bar{P}_u\cdot q)_{\text{c.m.}}\right\}. \tag{A37}$$

$$Y_\Delta^B = \frac{g_{gi}^2 p'p}{2}\left\{-2\mathcal{E}'\mathcal{E} + \frac{1}{3}\left(\bar{P}_u^2\right)_{\text{c.m.}}\right.$$
$$-\frac{2}{3}\left[\left(\frac{1}{2}(M_f + M_i) + M_\Delta\right)\right.$$
$$\times(M_f + M_i) + \frac{1}{2}\big((E+\mathcal{E})^2 - 2\mathcal{E}'\mathcal{E} - M_i^2 - 2m_i^2\big)$$
$$\left.+\frac{1}{2}\big(m_f^2 - 2E\mathcal{E}'\big) + \bar{\kappa}\left((\kappa'-\kappa) + 2(E'-E)\right)\right]$$
$$+\frac{1}{6}\big[M_f^2 - M_i^2 + 3m_f^2 - 3m_i^2 - (E'+\mathcal{E}')^2$$
$$+ (E+\mathcal{E})^2 - 2E\mathcal{E}' + 2E'\mathcal{E} + 4\bar{\kappa}(E'-E)$$
$$\left.+2(\kappa'^2 - \kappa^2)\big]\right\}. \tag{A38}$$

$$Z_\Delta^B = \frac{g_{gi}^2(p'p)^2}{3}. \tag{A39}$$

$$X_\Delta^{A'} = \frac{g_{gi}^2}{2}\left\{\bar{\kappa}\left(\bar{P}_u^2\right)_{\text{c.m.}}\mathcal{E}'\mathcal{E} - \frac{1}{3}\left(\bar{P}_u^2\right)_{\text{c.m.}}\left[\frac{1}{4}(\kappa'-\kappa)\right.\right.$$
$$\times\big(m_f^2 - 2E\mathcal{E}' + 2\mathcal{E}'\mathcal{E} - (E+\mathcal{E})^2 + M_i^2 + 2m_i^2\big)$$
$$+\bar{\kappa}\left(\frac{1}{2}\big(m_f^2 + m_i^2\big) - E'\mathcal{E} - \mathcal{E}'E - \frac{1}{2}(\kappa'-\kappa)\right.$$
$$\left.\left.\times(E'-E) - \frac{1}{2}(\kappa'-\kappa)^2 - \frac{1}{2}(\kappa'-\kappa)(\mathcal{E}'+\mathcal{E})\right)\right]$$
$$-\frac{1}{12}\big[(\kappa'-\kappa)\left(\bar{P}_u^2\right)_{\text{c.m.}} - M_\Delta(\kappa M_f + \kappa' M_i)\big]$$
$$\times\big[(E'+\mathcal{E}')^2 + (E+\mathcal{E})^2 + 2E'\mathcal{E} + 2E\mathcal{E}' - 2\mathcal{E}'\mathcal{E}$$
$$- M_f^2 - M_i^2 - 3m_f^2 - 3m_f^2 + 4\bar{\kappa}(\mathcal{E}'+\mathcal{E})\big]$$
$$\left.+\frac{\bar{\kappa}}{24}(\bar{P}_u\cdot q')_{\text{c.m.}}(\bar{P}_u\cdot q)_{\text{c.m.}}\right\}. \tag{A40}$$

$$Y_{\Delta}^{A'} = \frac{g_{gi}^2 p'p}{2}\Big\{2\bar{\kappa}\mathcal{E}'\mathcal{E} - \frac{5\bar{\kappa}}{3}\left(\bar{P}_u^2\right)_{\rm c.m.} - \frac{2}{3}\Big[\frac{1}{4}(\kappa'-\kappa) \times\left(m_f^2 - 2E\mathcal{E}' - (E+\mathcal{E})^2 + 2\mathcal{E}'\mathcal{E} + M_i^2 + 2m_i^2\right) + \bar{\kappa}\left(\frac{1}{2}\left(m_f^2+m_i^2\right) - E'\mathcal{E} - \mathcal{E}'E - \frac{1}{2}(\kappa'-\kappa)(E'-E) - \frac{1}{2}(\kappa'-\kappa)^2 - \frac{1}{2}(\kappa'-\kappa)(\mathcal{E}'+\mathcal{E})\right)\Big] - \frac{(\kappa'-\kappa)}{12} \times\left[-M_f^2 - M_i^2 - 3m_f^2 - 3m_i^2 + (E'+\mathcal{E}')^2 + (E+\mathcal{E})^2 + 2E'\mathcal{E} + 2E\mathcal{E}' - 4\mathcal{E}'\mathcal{E} + 4\bar{\kappa}(\mathcal{E}'+\mathcal{E})\right]\Big\}. \quad \text{(A41)}$$

$$Z_{\Delta}^{A'} = -\frac{5g_{gi}^2 (p'p)^2\bar{\kappa}}{3}. \quad \text{(A42)}$$

$$X_{\Delta}^{B'} = -\frac{g_{gi}^2}{12}\Big\{\left(\bar{P}_u^2\right)_{\rm c.m.}[M_i\kappa' - M_f\kappa + M_\Delta(\kappa'-\kappa)] + \frac{M_\Delta\kappa'}{4}(\bar{P}_u\cdot q')_{\rm c.m.} - \frac{M_\Delta\kappa}{4}(\bar{P}_u\cdot q)_{\rm c.m.}\Big\}. \quad \text{(A43)}$$

$$Y_{\Delta}^{B'} = -\frac{g_{gi}^2 p'p}{6}[M_i\kappa' - M_f\kappa + M_\Delta(\kappa'-\kappa)]. \quad \text{(A44)}$$

10. $\frac{3}{2}^+$ Baryon resonance, gauge-invariant coupling

$$M_{\kappa',\kappa} = -\frac{g_{gi}^2}{2}\bar{u}(p's')\Big[\frac{1}{2}\bar{P}_s^2\left(\frac{1}{2}(M_f+M_i) + M_\Delta + \not{Q} + \bar{\kappa}\not{n}\right)\left(m_f^2+m_i^2-t_{q'q}\right) - \frac{1}{3}\bar{P}_s^2 \times\left(\left(\frac{1}{2}(M_f+M_i)+M_\Delta\right)\not{q}'\not{q} - \frac{1}{2}\left(s_{pq}-M_i^2\right)\not{q}' - \frac{1}{2}\left(u_{pq'}+t_{q'q}-M_i^2-3m_f^2-m_i^2\right)\not{q} + \bar{\kappa}\not{n}\not{q}'\not{q}\right) - \frac{1}{12}\left(\left(\bar{P}_s^2 + \frac{M_\Delta}{2}(M_f-M_i)\right)\not{q}' + \frac{M_\Delta}{2} \times\left(M_i^2+2m_f^2-u_{pq'}\right) + \frac{M_\Delta}{2}\not{q}\not{q}' + M_\Delta\bar{\kappa}\not{n}\not{q}'\right) \times(\bar{P}_s\cdot q) + \frac{1}{12}\left(\left(\bar{P}_s^2 + \frac{M_\Delta}{2}(M_f-M_i)\right)\not{q} + \frac{M_\Delta}{2} \times\left(s_{pq}-M_i^2\right) + \frac{M_\Delta}{2}\not{q}'\not{q} + M_\Delta\bar{\kappa}\not{n}\not{q}\right)(\bar{P}_s\cdot q') - \frac{1}{24}\left(\frac{1}{2}(M_f+M_i)+M_\Delta+\not{Q}+\bar{\kappa}\not{n}\right) \times(\bar{P}_s\cdot q')(\bar{P}_s\cdot q)\Big]\, u(ps)D^{(2)}(\Delta_s,n,\bar{\kappa}), \quad \text{(A45)}$$

where $\bar{P}_s^2$ is defined in Eq. (48) and the slashed terms are, as before, defined in Eq. (A68). The inner products in Eq. (A45) are

$$\bar{P}_s\cdot q' = \left(-M_f^2 + M_i^2 + 3m_f^2 + m_i^2 + s_{p'q'} - u_{pq'} - t_{q'q} - 2\bar{\kappa}(p'-p)\cdot n + 4\bar{\kappa}n\cdot Q - (\kappa'^2-\kappa^2)\right)$$

$$\bar{P}_s\cdot q = \left(M_f^2 - M_i^2 + m_f^2 + 3m_i^2 + s_{pq} - u_{p'q} - t_{q'q} + 2\bar{\kappa}(p'-p)\cdot n + 4\bar{\kappa}n\cdot Q + (\kappa'^2-\kappa^2)\right). \quad \text{(A46)}$$

$$A_\Delta = -\frac{g_{gi}^2}{2}\Big\{\frac{1}{2}\bar{P}_s^2\left[\frac{1}{2}(M_f+M_i)+M_\Delta\right]\left(m_f^2+m_i^2-t_{q'q}\right) - \frac{1}{3}\bar{P}_s^2\Big[\left(\frac{1}{2}(M_f+M_i)+M_\Delta\right) \times\left(\frac{1}{2}(s_{p'q'}+s_{pq}) - \frac{1}{2}\left(M_f^2+M_i^2\right) - \frac{1}{2}(\kappa'-\kappa) \times(p'-p)\cdot n - \frac{1}{2}(\kappa'-\kappa)^2\right) + \frac{1}{4}\left(s_{pq}-M_i^2\right) \times(M_f-M_i) + \frac{1}{4}\left(M_i^2+3m_f^2+m_i^2-u_{pq'}-t_{q'q}\right) \times(M_f-M_i) + \bar{\kappa}(M_f-M_i)n\cdot Q\Big] + \frac{1}{12}\Big[\frac{1}{2}\left(\bar{P}_s^2 + \frac{M_\Delta}{2}(M_f-M_i)\right)(M_f-M_i) - \frac{1}{2}M_\Delta\left(M_i^2+2m_f^2-u_{pq'}\right) - \frac{1}{2}M_\Delta\Big(\frac{1}{2}(u_{p'q} + u_{pq'}) - \frac{1}{2}\left(M_f^2+M_i^2\right) - \frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n - \frac{1}{2}(\kappa'-\kappa)^2\Big) - \bar{\kappa}M_\Delta\Big(-n\cdot p' + n\cdot Q - \frac{1}{2}(\kappa'-\kappa)\Big)\Big](\bar{P}_s\cdot q) + \frac{1}{12}\Big[\frac{1}{2}\Big(\bar{P}_s^2 + \frac{M_\Delta}{2}(M_f-M_i)\Big)(M_f-M_i) + \frac{1}{2}M_\Delta\left(s_{pq}-M_i^2\right) + \frac{1}{2}M_\Delta\Big(\frac{1}{2}(s_{p'q'}+s_{pq}) - \frac{1}{2}\left(M_f^2+M_i^2\right) - \frac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n - \frac{1}{2}(\kappa'-\kappa)^2\Big) + \bar{\kappa}M_\Delta\Big(n\cdot p' + n\cdot Q + \frac{1}{2}(\kappa'-\kappa)\Big)\Big](\bar{P}_s\cdot q') - \frac{1}{24}\left[\frac{1}{2}(M_f+M_i)+M_\Delta\right](\bar{P}_s\cdot q')(\bar{P}_s\cdot q)\Big\} \times D^{(2)}(\Delta_u,n,\bar{\kappa}). \quad \text{(A47)}$$

$$B_\Delta = -\frac{g_{gi}^2}{2}\Big\{\frac{1}{2}\bar{P}_s^2\left(m_f^2+m_i^2-t_{q'q}\right) - \frac{1}{3}\bar{P}_s^2\Big[-\left(\frac{1}{2}(M_f+M_i)+M_\Delta\right)(M_f+M_i) - \frac{1}{2}\left(s_{pq}-M_i^2\right) + \frac{1}{2}\left(M_i^2+3m_f^2+m_i^2-u_{pq'} - t_{q'q}\right) - 2\bar{\kappa}(p'-p)\cdot n - \frac{1}{2}(\kappa'^2-\kappa^2)\Big] - \frac{1}{12}\left(\bar{P}_s^2 + M_\Delta M_f\right)(\bar{P}_s\cdot q) + \frac{1}{12}\left(\bar{P}_s^2 - M_\Delta M_i\right)(\bar{P}_s\cdot q') + \frac{1}{24}(\bar{P}_s\cdot q')(\bar{P}_s\cdot q)\Big\}\, D^{(2)}(\Delta_s,n,\bar{\kappa}). \quad \text{(A48)}$$

$$A'_\Delta = -\frac{g_{gi}^2}{2}\left\{\frac{\bar\kappa}{2}\bar P_s^2\left(m_f^2+m_i^2-t_{q'q}\right)-\frac{1}{3}\bar P_s^2\left[\frac{1}{4}(\kappa'-\kappa)\times\left(s_{pq}-M_i^2\right)+\frac{1}{4}(\kappa'-\kappa)\left(M_i^2+3m_f^2+m_i^2-u_{pq'}-t_{q'q}\right)+\bar\kappa\left(-\frac{1}{2}\left(M_f^2+M_i^2\right)+\frac{1}{2}(s_{p'q'}+s_{pq})-\frac{1}{2}\times(\kappa'-\kappa)(p'-p)\cdot n+(\kappa'-\kappa)Q\cdot n-\frac{1}{2}(\kappa'-\kappa)^2\right)\right]+\frac{1}{24}\left[(\kappa'-\kappa)\bar P_s^2-M_\Delta(\kappa M_f+\kappa' M_i)\right]\left[s_{p'q'}+s_{pq}-u_{p'q}-u_{pq'}-2t_{q'q}+4m_f^2+4m_i^2+8\bar\kappa n\cdot Q\right]-\frac{\bar\kappa}{24}(\bar P_s\cdot q')(\bar P_s\cdot q')\right\}D^{(2)}(\Delta_s,n,\bar\kappa). \quad (A49)$$

$$B'_\Delta = -\frac{g_{gi}^2}{12}\left\{\bar P_s^2[M_i\kappa'-M_f\kappa+M_\Delta(\kappa'-\kappa)]-\frac{\kappa' M_\Delta}{4}(\bar P_s\cdot q)+\frac{\kappa M_\Delta}{4}(\bar P_s\cdot q')\right\}D^{(2)}(\Delta_s,n,\bar\kappa). \quad (A50)$$

$$X_\Delta^A = \frac{g_{gi}^2}{2}\left\{\left(\bar P_s^2\right)_{\rm c.m.}\left[\frac{1}{2}(M_f+M_i)+M_\Delta\right]\mathcal{E}'\mathcal{E}-\frac{1}{3}\left(\bar P_s^2\right)_{\rm c.m.}\left[\left(\frac{1}{2}(M_f+M_i)+M_\Delta\right)\times\left(\frac{1}{2}(E'+\mathcal{E}')^2+\frac{1}{2}(E+\mathcal{E})^2-\frac{1}{2}\left(M_f^2+M_i^2\right)-\frac{1}{2}(\kappa'-\kappa)(E'-E)-\frac{1}{2}(\kappa'-\kappa)^2\right)+\frac{1}{4}\left(m_f^2+2E\mathcal{E}'+2\mathcal{E}'\mathcal{E}\right)(M_f-M_i)+\frac{1}{4}\left((E+\mathcal{E})^2-M_i^2\right)\times(M_f-M_i)+\frac{\bar\kappa}{2}(M_f-M_i)(\mathcal{E}'+\mathcal{E})\right]+\frac{1}{12}\left[\frac{1}{2}\left(\left(\bar P_s^2\right)_{\rm c.m.}+\frac{M_\Delta}{2}(M_f-M_i)\right)(M_f-M_i)-\frac{1}{2}M_\Delta\left(m_f^2+2E\mathcal{E}'\right)-\frac{1}{2}M_\Delta\left(\frac{1}{2}\left(m_f^2+m_i^2\right)-2(E'\mathcal{E}+E\mathcal{E}')-\frac{1}{2}(\kappa'-\kappa)(E'-E)-\frac{1}{2}(\kappa'-\kappa)^2\right)-\bar\kappa M_\Delta\left(-E'+\frac{1}{2}(\mathcal{E}'+\mathcal{E})-\frac{1}{2}(\kappa'-\kappa)\right)\right](\bar P_s\cdot q)_{\rm c.m.}+\frac{1}{12}\left[\frac{1}{2}\left(\left(\bar P_s^2\right)_{\rm c.m.}+\frac{M_\Delta}{2}(M_f-M_i)\right)(M_f-M_i)+\frac{1}{2}M_\Delta\left((E+\mathcal{E})^2-M_i^2\right)+\frac{1}{2}M_\Delta\times\left(\frac{1}{2}(E'+\mathcal{E}')^2+\frac{1}{2}(E+\mathcal{E})^2-\frac{1}{2}\left(M_f^2+M_i^2\right)-\frac{1}{2}(\kappa'-\kappa)(E'-E)-\frac{1}{2}(\kappa'-\kappa)^2\right)+\bar\kappa M_\Delta\left(E'+\frac{1}{2}(\mathcal{E}'+\mathcal{E})+\frac{1}{2}(\kappa'-\kappa)\right)\right](\bar P_s\cdot q')_{\rm c.m.}-\frac{1}{24}\left[\frac{1}{2}(M_f+M_i)+M_\Delta\right](\bar P_s\cdot q')_{\rm c.m.}(\bar P_s\cdot q)_{\rm c.m.}\right\}, \quad (A51)$$

where

$$\left(\bar P_s^2\right)_{\rm c.m.} = \left[\frac{1}{2}(E'+\mathcal{E}')^2+\frac{1}{2}(E+\mathcal{E})^2+\kappa'\kappa+\bar\kappa(E'+E+\mathcal{E}'+\mathcal{E})\right],$$
$$(\bar P_s\cdot q')_{\rm c.m.} = \left[-M_f^2+m_f^2+(E'+\mathcal{E}')^2+2E\mathcal{E}'+2\mathcal{E}'\mathcal{E}-2\bar\kappa(E'-E)+2\bar\kappa(\mathcal{E}'+\mathcal{E})-(\kappa'^2-\kappa^2)\right]$$
$$(\bar P_s\cdot q)_{\rm c.m.} = \left[-M_i^2+m_i^2+(E+\mathcal{E})^2+2E'\mathcal{E}+2\mathcal{E}'\mathcal{E}+2\bar\kappa(E'-E)+2\bar\kappa(\mathcal{E}'+\mathcal{E})+(\kappa'^2-\kappa^2)\right]. \quad (A52)$$

$$Y_\Delta^A = \frac{g_{gi}^2 p'p}{2}\left\{-\left(\bar P_s^2\right)_{\rm c.m.}\left[\frac{1}{2}(M_f+M_i)+M_\Delta\right]+\frac{M_\Delta}{6}\left[-\frac{1}{2}M_f^2+\frac{1}{2}M_i^2+2M_fM_i+\frac{1}{2}\left(3m_f^2+m_i^2\right)-(E'\mathcal{E}-E\mathcal{E}')-\frac{1}{2}(E'+\mathcal{E}')^2-\frac{3}{2}(E+\mathcal{E})^2-4\bar\kappa E'-(\kappa'^2-\kappa^2)\right]+\frac{1}{6}\left[\frac{1}{2}(M_f+M_i)+M_\Delta\right]\times\left[-M_f^2-M_i^2+m_f^2+m_i^2+(E'+\mathcal{E}')^2+(E+\mathcal{E})^2+2E'\mathcal{E}+2E\mathcal{E}'+4\mathcal{E}'\mathcal{E}+4\bar\kappa(\mathcal{E}'+\mathcal{E})\right]\right\}. \quad (A53)$$

$$Z_\Delta^A = -\frac{g_{gi}^2(p'p)^2}{3}\left[\frac{1}{2}(M_f+M_i)+M_\Delta\right]. \quad (A54)$$

$$X_\Delta^B = \frac{g_{gi}^2}{2}\left\{\left(\bar P_s^2\right)_{\rm c.m.}\mathcal{E}'\mathcal{E}+\frac{1}{3}\left(\bar P_s^2\right)_{\rm c.m.}\left[\left(\frac{1}{2}(M_f+M_i)+M_\Delta\right)\times(M_f+M_i)-\frac{1}{2}(m_f^2+2E\mathcal{E}'+2\mathcal{E}'\mathcal{E})+\frac{1}{2}\times\left((E+\mathcal{E})^2-M_i^2\right)+2\bar\kappa(E'-E)+\frac{1}{2}(\kappa'^2-\kappa^2)\right]-\frac{1}{12}\left[\left(\bar P_s^2\right)_{\rm c.m.}+M_\Delta M_f\right](\bar P_s\cdot q)_{\rm c.m.}+\frac{1}{12}\left[\left(\bar P_s^2\right)_{\rm c.m.}-M_\Delta M_i\right](\bar P_s\cdot q')_{\rm c.m.}+\frac{1}{24}(\bar P_s\cdot q')_{\rm c.m.}(\bar P_s\cdot q)_{\rm c.m.}\right\}. \quad (A55)$$

$$Y_\Delta^B = -\frac{g_{gi}^2 p'p}{6}\left\{\left(\bar P_s^2\right)_{\rm c.m.}-M_\Delta(M_f+M_i)+\frac{1}{2}\left[-M_f^2-M_i^2+m_f^2+m_i^2+(E'+\mathcal{E}')^2+(E+\mathcal{E})^2+2E\mathcal{E}'+2E'\mathcal{E}+4\mathcal{E}'\mathcal{E}+4\bar\kappa(\mathcal{E}'+\mathcal{E})\right]\right\}. \quad (A56)$$

$$Z_\Delta^B = \frac{g_{gi}^2 (p'p)^2}{3}. \quad \text{(A57)}$$

$$\begin{aligned} X_\Delta^{A'} &= \frac{g_{gi}^2}{2}\Bigg\{\bar\kappa\big(\bar P_s^2\big)_{\text{c.m.}}\mathcal{E}'\mathcal{E} - \frac{1}{3}\big(\bar P_s^2\big)_{\text{c.m.}}\Bigg[\frac{1}{4}(\kappa'-\kappa) \\ &\quad\times\big((E+\mathcal{E})^2 - M_i^2\big) + \frac{1}{4}(\kappa'-\kappa)(m_f^2 + 2E\mathcal{E}' + 2\mathcal{E}'\mathcal{E}) \\ &\quad+\bar\kappa\Bigg(-\frac{1}{2}\big(M_f^2+M_i^2\big) + \frac{1}{2}(E'+\mathcal{E}')^2 + \frac{1}{2}(E+\mathcal{E})^2 \\ &\quad-\frac{1}{2}(\kappa'-\kappa)(E'-E) + \frac{1}{2}(\kappa'-\kappa)(\mathcal{E}'+\mathcal{E}) \\ &\quad-\frac{1}{2}(\kappa'-\kappa)^2\Bigg)\Bigg] + \frac{1}{24}\big[(\kappa'-\kappa)\big(\bar P_s^2\big)_{\text{c.m.}} \\ &\quad- M_\Delta(\kappa' M_i + \kappa M_f)\big]\big[-M_f^2 - M_i^2 + m_f^2 + m_i^2 \\ &\quad+(E'+\mathcal{E}')^2 + (E+\mathcal{E})^2 + 2E'\mathcal{E} + 2E\mathcal{E}' + 4\mathcal{E}'\mathcal{E} \\ &\quad+4\bar\kappa(\mathcal{E}'+\mathcal{E})\big] - \frac{\bar\kappa}{24}(\bar P_s\cdot q')_{\text{c.m.}}(\bar P_s\cdot q)_{\text{c.m.}}\Bigg\}. \end{aligned} \quad \text{(A58)}$$

$$\begin{aligned} Y_\Delta^{A'} &= -\frac{g_{gi}^2 p'p}{2}\Bigg\{\bar\kappa\big(\bar P_s^2\big)_{\text{c.m.}} - \frac{M_\Delta}{3}[\kappa' M_i + \kappa M_f] - \frac{\bar\kappa}{6} \\ &\quad\times\big[-M_f^2 - M_i^2 + m_f^2 + m_i^2 + (E'+\mathcal{E}')^2 + (E+\mathcal{E})^2 \\ &\quad+2E'\mathcal{E} + 2E\mathcal{E}' + 4\mathcal{E}'\mathcal{E} + 4\bar\kappa(\mathcal{E}'+\mathcal{E})\big]\Bigg\}. \end{aligned} \quad \text{(A59)}$$

$$Z_\Delta^{A'} = -\frac{g_{gi}^2 (p'p)^2}{3}. \quad \text{(A60)}$$

$$\begin{aligned} X_\Delta^{B'} &= \frac{g_{gi}^2}{12}\Bigg\{\big(\bar P_s^2\big)_{\text{c.m.}}[\kappa' M_i - \kappa M_f + (\kappa'-\kappa)M_\Delta] \\ &\quad-\frac{\kappa' M_\Delta}{4}(\bar P_s\cdot q)_{\text{c.m.}} + \frac{\kappa M_\Delta}{4}(\bar P_s\cdot q')_{\text{c.m.}}\Bigg\}. \end{aligned} \quad \text{(A61)}$$

$$Y_\Delta^{B'} = \frac{g_{gi}^2 M_\Delta p'p}{12}(\kappa'-\kappa). \quad \text{(A62)}$$

C. Useful relations

1. Feynman

In the Feynman formalism the following relations are quite useful

$$\begin{aligned} 2(q'\cdot q) &= m_f^2 + m_i^2 - t \\ 2(p'\cdot p) &= M_f^2 + M_i^2 - t \\ 2(p'\cdot q') &= s - M_f^2 - m_f^2 \\ 2(p\cdot q) &= s - M_i^2 - m_i^2 \\ 2(p\cdot q') &= M_i^2 + m_f^2 - u \\ 2(p'\cdot q) &= M_f^2 + m_i^2 - u. \end{aligned} \quad \text{(A63)}$$

$$s + u + t = M_f^2 + M_i^2 + m_f^2 + m_i^2. \quad \text{(A64)}$$

$$\begin{aligned} \not q &= \tfrac{1}{2}(M_f - M_i) + \not Q \\ \not q' &= -\tfrac{1}{2}(M_f - M_i) + \not Q \\ \not q\not q' &= (M_f + M_i)\not Q - \tfrac{1}{2}\big(M_f^2 + M_i^2\big) + u \\ \not q'\not q &= -(M_f + M_i)\not Q - \tfrac{1}{2}\big(M_f^2 + M_i^2\big) + s. \end{aligned} \quad \text{(A65)}$$

2. Kadyshevsky

In the Kadyshevsky formalism there are similar relations

$$\begin{aligned} 2(q'\cdot q) &= m_f^2 + m_i^2 - t_{q'q} \\ 2(p'\cdot p) &= M_f^2 + M_i^2 - t_{p'p} \\ 2(p'\cdot q') &= s_{p'q'} - M_f^2 - m_f^2 \\ 2(p\cdot q) &= s_{pq} - M_i^2 - m_i^2 \\ 2(p\cdot q') &= M_i^2 + m_f^2 - u_{pq'} \\ 2(p'\cdot q) &= M_f^2 + m_i^2 - u_{p'q}. \end{aligned} \quad \text{(A66)}$$

$$\begin{aligned} &s_{p'q'} + s_{pq} + u_{p'q} + u_{pq'} + t_{p'p} + t_{q'q} \\ &\quad= 2\big(M_f^2 + M_i^2 + m_f^2 + m_i^2\big) + (\kappa'-\kappa)^2 \\ &2\sqrt{s_{p'q'}s_{pq}} + u_{p'q} + u_{pq'} + t_{p'p} + t_{q'q} \\ &\quad= 2\big(M_f^2 + M_i^2 + m_f^2 + m_i^2\big). \end{aligned} \quad \text{(A67)}$$

$$\begin{aligned} \not q' &= -\tfrac{1}{2}(M_f - M_i) + \not Q - \tfrac{1}{2}\not n(\kappa'-\kappa) \\ \not q &= \tfrac{1}{2}(M_f - M_i) + \not Q + \tfrac{1}{2}\not n(\kappa'-\kappa) \\ \not q'\not q &= -(M_f + M_i)\not Q + \tfrac{1}{2}(s_{p'q'} + s_{pq}) \\ &\quad- \tfrac{1}{2}\big(M_f^2 + M_i^2\big) - \tfrac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n \\ &\quad- \tfrac{1}{2}(\kappa'-\kappa)\,[\not n, \not Q] - \tfrac{1}{2}(\kappa'-\kappa)^2 \\ \not q\not q' &= (M_f + M_i)\not Q + \tfrac{1}{2}(u_{p'q} + u_{pq'}) \\ &\quad- \tfrac{1}{2}\big(M_f^2 + M_i^2\big) - \tfrac{1}{2}(\kappa'-\kappa)(p'-p)\cdot n \\ &\quad+ \tfrac{1}{2}(\kappa'-\kappa)\,[\not n, \not Q] - \tfrac{1}{2}(\kappa'-\kappa)^2 \\ \not n\not q' &= \tfrac{1}{2}(M_f + M_i)\not n - (n\cdot p') + \tfrac{1}{2}\,[\not n, \not Q] \\ &\quad+ n\cdot Q - \tfrac{1}{2}(\kappa'-\kappa) \\ \not n\not q &= -\tfrac{1}{2}(M_f + M_i)\not n + (n\cdot p') + \tfrac{1}{2}\,[\not n, \not Q] \\ &\quad+ n\cdot Q + \tfrac{1}{2}(\kappa'-\kappa) \\ \not n\not q'\not q &= -\tfrac{1}{2}\big(M_f^2 + M_i^2\big)\not n + \tfrac{1}{2}(s_{p'q'} + s_{pq})\not n \\ &\quad+ \tfrac{1}{2}(M_f - M_i)[\not n, \not Q] + (M_f - M_i)n\cdot Q \\ &\quad- \tfrac{1}{2}(\kappa'-\kappa)n\cdot(p'-p)\not n + (\kappa'-\kappa)(n\cdot Q)\not n \\ &\quad- (\kappa'-\kappa)\not Q - 2n\cdot(p'-p)\not Q - \tfrac{1}{2}(\kappa'-\kappa)^2\not n \\ \not n\not q\not q' &= -\tfrac{1}{2}\big(M_f^2 + M_i^2\big)\not n + \tfrac{1}{2}(u_{p'q} + u_{pq'})\not n \\ &\quad- \tfrac{1}{2}(M_f - M_i)[\not n, \not Q] - (M_f - M_i)n\cdot Q \\ &\quad- \tfrac{1}{2}(\kappa'-\kappa)n\cdot(p'-p)\not n - (\kappa'-\kappa)(n\cdot Q)\not n \\ &\quad+ (\kappa'-\kappa)\not Q + 2n\cdot(p'-p)\not Q - \tfrac{1}{2}(\kappa'-\kappa)^2\not n. \end{aligned} \quad \text{(A68)}$$

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