Pion-nucleon scattering in Kadyshevsky formalism. I. Meson exchange sector

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In a series of two articles we present the theoretical results of πN/meson-baryon scattering in the Kadyshevsky formalism. In this article the results are given for meson exchange diagrams. On the formal side we show, by means of an example how general couplings, i.e., couplings containing multiple derivatives and/or higher spin fields, should be treated. We do this by introducing and applying the Takahashi-Umezawa and the Gross-Jackiw method. For practical purposes we introduce the $\bar{P}$ method. We also show how the Takashashi-Umezawa method can be derived using the theory of Bogoliubov and collaborators and the Gross-Jackiw method is also used to study the n dependence of the Kadyshevsky integral equation. Last but not least we present the second quantization procedure of the quasiparticle in Kadyshevsky formalism.

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I. INTRODUCTION

Over the years the Nijmegen group has constructed very successful baryon-baryon models (NN and YN). As, for instance, in Refs. [1,2], soft-core one-boson exchange NN and YN models are constructed based on Regge-pole theory. The models are linked via $SU_f(3)$ symmetry to have more control on the parameters. Based on the same ideas, the Nijmegen group recently broadened its horizon by also including meson-baryon models [3]. Here, a simultaneous πN and K^+N model is constructed using one-meson and one-baryon exchange potentials.

This work is presented in two articles, referred to as article I (this article) and article II [4], and can be regarded as an extension of Ref. [3], because we also consider meson-baryon scattering or pion-nucleon, more specifically. The reason for considering pion-nucleon scattering is, in addition to the interest in its own, that there is a large amount of experimental data. Using the aforementioned $SU_f(3)$ symmetry the extension to other meson-baryon systems is easily made. Last but not least we mention the connection to photo-/electroproduction models.

Compared to Ref. [3] our focus is more on the theoretical background. For instance, we formally include what is called "pair suppression", whereas this was assumed in Ref. [3]. Pair suppression comes down to the suppression of negative-energy contributions. For the first time, at least to our knowledge, we incorporate pair suppression in a covariant and frame-independent way. This may also be interesting for relativistic many-body theories. The details of the formal incorporation of pair suppression are discussed in article II.

To have this covariant and frame-independent pair suppression, we use the Kadyshevsky formalism [5–8]. This formalism is equivalent to Feynman formalism, because it can be derived from the same S-matrix formula. It covariantly, though frame dependently,[1] separates positive and negative energy contributions. Generally, the number of diagrams increases: $1 \to n!$ at order n as in old-fashioned perturbation theory. Contrary to the Feynman formalism all particles in the Kadyshevsky formalism remain on their mass shell at the cost of the introduction of an extra quasiparticle, which carries four-momentum only. A second quantization formalism of this quasifield is presented in Appendix B. The mass-shell condition has the advantage that it enables one to use covariant on-mass-shell form factors, which is not possible in the Feynman formalism. Another advantage of the Kadyshevsky formalism is that it brings about a three-dimensional Lippmann-Schwinger type of integral equation [8], whereas a three dimensional integral equation was achieved in Ref. [3] only after approximations of the Bethe-Salpeter equation [9]. We study the n dependence of the Kadyshevsky integral equation with tree level amplitudes as input in Sec. II A. As compared to the original Kadyshevsky rules we use a slightly different version, introduced and discussed in Appendix A.

Couplings containing derivatives and higher-spin fields may cause differences and problems as far as the results in the Kadyshevsky formalism and the Feynman formalism are concerned. This is discussed in Sec. IV B by means of an example of simplified vector-meson exchange. After a second glance the results in both formalisms are the same; however, they contain extra frame-dependent contact terms. Two methods are introduced and applied, which discuss a second source of extra terms: the Takahashi-Umezawa (TU) [10–12] and the Gross-Jackiw (GJ) [13] methods. The extra terms coming from this second source cancel the former ones exactly. Both formalisms, however, yield the same results. With the use of (one of) these methods the final results for the S matrix or amplitude are covariant and frame independent (n independent). In Sec. IV B4 we introduce and discuss the $\bar{P}$ method, which is quite useful for practical purposes. We derive the TU method from the BMP [14–16] theory in Appendix C and in light of this TU method we make some remarks about the Haag theorem [17] in Appendix D.

Although we already discussed some content, this article is organized as follows: we start in Sec. II with some meson-baryon scattering kinematics in Kadyshevsky formalism together with the discussion of the n dependence of the integral equation. We start the application of the Kadyshevsky formalism to the πN system by first discussing the ingredients

[1] By frame dependent we mean dependent on a vector n^μ.

of the model in Sec. III. The meson exchange amplitudes are calculated in Sec. IV, which contains the results for equal initial and final states. For the results for general meson-baryon initial and final states we refer to Appendix A of article II. For the results for baryon exchange we refer to article II as well. As mentioned before Sec. IV also contains the discussion of how general couplings, i.e., couplings containing multiple derivatives and/or higher-spin fields, should be treated in the Kadyshevsky formalism.

II. MESON-BARYON SCATTERING KINEMATICS

We consider the pion-nucleon or, more generally, the meson-baryon reactions

$$M_i(q) + B_i(p, s) \to M_f(q') + B_f(p', s'), \tag{1}$$

where M stands for a meson and B is a baryon. For the four-momentum of the baryons and mesons we take, respectively,

$$\begin{aligned} p_c^\mu &= (E_c, \mathbf{p}_c), \quad \text{where} \quad E_c = \sqrt{\mathbf{p}_c^2 + M_c^2} \\ q_c^\mu &= (\mathcal{E}_c, \mathbf{q}_c), \quad \text{where} \quad \mathcal{E}_c = \sqrt{\mathbf{q}_c^2 + m_c^2}. \end{aligned} \tag{2}$$

Here, c stands for either the initial state i or the final state f. In some cases we find it useful to use the definitions (2) for the intermediate meson-baryon states n.

Using the Kadyshevsky formalism (Appendix A) and especially the second quantization procedure (Appendix B), external quasiparticles may occur with initial and final-state momenta $n\kappa$ and $n\kappa'$, respectively. Therefore, the usual overall four-momentum conservation is generally replaced by

$$p + q + \kappa n = p' + q' + \kappa' n. \tag{3}$$

As (3) and (1) make clear, a "prime" notation is used to indicate final-state momenta; no prime means initial-state momenta. We will maintain this notation (also for the energies) throughout these articles unless indicated otherwise.

Furthermore we find it useful to introduce the Mandelstam variables in the Kadyshevsky formalism

$$\begin{aligned} s_{pq} &= (p+q)^2 \quad s_{p'q'} = (p'+q')^2 \\ t_{p'p} &= (p'-p)^2 \quad t_{q'q} = (q'-q)^2 \\ u_{p'q} &= (p'-q)^2 \quad u_{pq'} = (p-q')^2, \end{aligned} \tag{4}$$

where s_{pq} and $s_{p'q'}$, etc., are identical only for $\kappa' = \kappa = 0$. These Mandelstam variables satisfy the relation

$$\begin{aligned} &2\sqrt{s_{p'q'}s_{pq}} + t_{p'p} + t_{q'q} + u_{pq'} + u_{p'q} \\ &\quad = 2\left(M_f^2 + M_i^2 + m_f^2 + m_i^2\right). \end{aligned} \tag{5}$$

The total and relative four-momenta of the initial, final, and intermediate channels ($c = i, f, n$) are defined by

$$P_c = p_c + q_c k_c = \mu_{c,2} p_c - \mu_{c,1} q_c, \tag{6}$$

where the weights satisfy $\mu_{c,1} + \mu_{c,2} = 1$. We choose the weights to be

$$\mu_{c,1} = \frac{E_c}{E_c + \mathcal{E}_c} \qquad \mu_{c,2} = \frac{\mathcal{E}_c}{E_c + \mathcal{E}_c}. \tag{7}$$

Because in the Kadyshevsky formalism all particles are on their mass shell, the choice (7) means that always $k_c^0 = 0$.

In the center-of-mass (c.m.) system $\mathbf{p} = -\mathbf{q}$ and $\mathbf{p}' = -\mathbf{q}'$, therefore

$$\begin{aligned} P_i &= (W, \mathbf{0}) \quad P_f = (W', \mathbf{0}) \\ k_i &= (0, \mathbf{p}) \quad k_f = (0, \mathbf{p}'). \end{aligned} \tag{8}$$

where $W = E + \mathcal{E}$ and $W' = E' + \mathcal{E}'$. Furthermore we take $n^\mu = (1, \mathbf{0})$. Moreoever, we take as the scattering plane the xz plane, where the three-momentum of the initial baryon is oriented in the positive z direction.

In the c.m. system the unpolarized differential cross section is defined to be

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}} = \frac{|\mathbf{p}'|}{2|\mathbf{p}|} \sum \left| \frac{M_{fi}}{8\pi\sqrt{s}} \right|^2, \tag{9}$$

where the amplitude M_{fi} is defined in Appendix A and the sum is over the spin components of the final baryon.

To generate amplitudes at all orders we use the Kadyshevsky integral equation in the c.m. system

$$\begin{aligned} &M(W'\,\mathbf{p}'; W\,\mathbf{p}) \\ &= M_{00}^{\text{irr}}(W'\,\mathbf{p}'; W\,\mathbf{p}) + \int d^3k_n\, M_{0\kappa}^{\text{irr}}(W'\,\mathbf{p}'; W_n\,\mathbf{k}_n) \\ &\quad \times \frac{1}{(2\pi)^3} \frac{1}{4\mathcal{E}_n E_n} \frac{1}{\sqrt{s} - \sqrt{s_n} + i\varepsilon} M_{\kappa 0}(W_n\,\mathbf{k}_n; W\,\mathbf{p}). \end{aligned} \tag{10}$$

Although there are still κ labels in Eq. (10), they're fixed at $\kappa = P_i^0 - P_n^0$. Also we have included the spinors of the projection operator of the fermion propagator

$$\begin{aligned} S^{(+)}(p_n) &= \Lambda^{(1/2)}(p_n)\theta\left(p_n^0\right)\delta\left(p_n^2 - M^2\right) \\ &= \sum_{s_n} u(p_n s_n)\bar{u}(p_n s_n)\theta\left(p_n^0\right)\delta\left(p_n^2 - M^2\right), \end{aligned} \tag{11}$$

in the amplitudes $M_{0\kappa}(p'q'; p_n q_n)$ and $M_{\kappa 0}(p_n q_n; pq)$.

We have put the intermediate negative-energy states $[\Delta^{(-)}(x-y; m_\pi^2)$ and $S^{(-)}(x-y; M_N^2)]$ in $M_{\kappa\kappa'}^{\text{irr}}$, but in principle they could also participate in the integral equation. However, using pair suppression the way we do in article II, these terms vanish.

A. *n* independence of Kadyshevsky integral equation

When generating Kadyshevsky diagrams to random order using the Kadyshevsky integral equation, the (full) amplitude is identical to the one obtained in Feynman formalism when the external quasiparticle momenta are put to zero. It is therefore n independent, i.e., frame independent.

Because an approximation is used to solve the Kadyshevsky integral equation, namely tree-level diagrams as driving terms, it is not clear whether in this case also the amplitude generated by solving the Kadyshevsky integral equation is n independent when the external quasiparticle momenta are put to zero. Below we will formulate the conditions for this to happen, and we have explicitly checked that the tree diagrams used in our work satisfy these conditions.

In examining the n dependence of the amplitude we write the Kadyshevsky integral equation schematically as

$$M_{00} = M_{00}^{\rm irr} + \int d\kappa\, M_{0\kappa}^{\rm irr} G_\kappa' M_{\kappa 0} \tag{12}$$

Because $n^2 = 1$, only variations in a spacelike direction are unrestricted, i.e., $n \times \delta n = 0$ [13]. We therefore introduce the projection operator

$$P^{\alpha\beta} = g^{\alpha\beta} - n^\alpha n^\beta, \tag{13}$$

from which it follows that $n_\alpha P^{\alpha\beta} = 0$. The n dependence of the amplitude can now be studied

$$\begin{aligned} P^{\alpha\beta}\frac{\partial}{\partial n^\beta} M_{00} &= P^{\alpha\beta}\left(\frac{\partial M_{00}^{\rm irr}}{\partial n^\beta}\right) + P^{\alpha\beta}\int d\kappa_1 \left[\left(\frac{\partial M_{0\kappa_1}^{\rm irr}}{\partial n^\beta}\right) G_\kappa' M_{\kappa_1 0}^{\rm irr} \right.\\ &\quad \left. + M_{0\kappa_1}^{\rm irr} G_{\kappa_1}' \left(\frac{\partial M_{\kappa_1 0}^{\rm irr}}{\partial n^\beta}\right)\right] \\ &\quad + P^{\alpha\beta}\int d\kappa_1 d\kappa_2 \left[\left(\frac{\partial M_{0\kappa_1}^{\rm irr}}{\partial n^\beta}\right) G_{\kappa_1}' M_{\kappa_1\kappa_2}^{\rm irr} G_{\kappa_2}' M_{\kappa_2 0}^{\rm irr} \right.\\ &\quad + M_{0\kappa_1}^{\rm irr} G_{\kappa_1}' \left(\frac{\partial M_{\kappa_1\kappa_2}^{\rm irr}}{\partial n^\beta}\right) G_{\kappa_2}' M_{\kappa_2 0}^{\rm irr} \\ &\quad \left. + M_{0\kappa_1}^{\rm irr} G_{\kappa_1}' M_{\kappa_1\kappa_2}^{\rm irr} G_{\kappa_2}' \left(\frac{\partial M_{\kappa_2 0}^{\rm irr}}{\partial n^\beta}\right)\right] + \cdots = 0. \end{aligned} \tag{14}$$

Every term in this expansion has to vanish by itself. If both Kadyshevsky contributions are considered in the first term on the right-hand side of Eq. (14), then it is n independent when $\kappa' = \kappa = 0$, because it yields the Feynman expression. Although a single term in the remaining part of the right-hand side of Eq. (14) consists of a product possibly containing a large number of irreducible amplitudes and quasiparticle propagators, the main ingredient of such a term is the derivative with respect to n^β. It can be written as

$$\frac{\partial M_{\kappa_1\kappa_2}^{\rm irr}}{\partial n^\beta} \propto f(\kappa_1\kappa_2)F(\kappa_1\kappa_2). \tag{15}$$

Here, f is a sum of terms, where each term is at least linear proportional to either κ_1 or κ_2. $F(\kappa_1\kappa_2)$ has poles only in the lower half plane.

Because we take for $M_{\kappa_1\kappa_2}^{\rm irr}$ tree-level amplitudes only, we will demonstrate the asserted features for every single amplitude. It is important to note here that when $M_{\kappa_1\kappa_2}^{\rm irr}$ only has poles in the lower half plane, $F(\kappa_1\kappa_2)$ also has this property.

Because of the properties of f we loose no generality by putting one of the arguments to zero. Therefore the important integral looks like

$$\int d\kappa f(\kappa)F(\kappa)G_\kappa' \, M_{\kappa 0}^{\rm irr}\cdots = \int d\kappa f(\kappa)h(\kappa)G_\kappa', \tag{16}$$

where h only has poles in the lower half plane.

When performing the integral we decompose the G_κ' as follows

$$G_\kappa' \propto \frac{1}{\kappa + i\varepsilon} = P\frac{1}{\kappa} - i\pi\delta(\kappa). \tag{17}$$

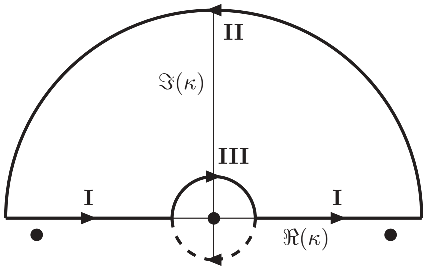

FIG. 1. Principal-value integral.

As far as the $\delta(\kappa)$ part of Eq. (17) is concerned, we immediately see that it gives zero when used in the integral (16) because of the property of $f(\kappa)$. For the principal-valued integral, indicated in Fig. 1 by **I**, we close the integral by connecting the end point ($\kappa = \pm\infty$) via a (huge) semicircle in the upper half, complex κ plane (line **II** in Fig. 1) and by connecting the points around zero via a small semicircle also in the upper half-plane (line **III** in Fig. 1). Because h only has poles in the lower half plane and not within the contour, the contour integral is zero.

Because we have added integrals (**II** and **III** in Fig. 1) we need to know what their contributions are. The easiest part is integral **III**. Its contribution is half the residue at $\kappa = 0$ and because the only remaining integrand part $h(\kappa)$ in Eq. (16) does not contain a pole at zero it is zero.

If we want the contribution of integral **II** to be zero, than the integrand should at least be of order $O(\frac{1}{\kappa^2})$. Unfortunately, this is not (always) the case as we will see in Sec. IV and article II. To this end we introduce a phenomenological "form factor"

$$F(\kappa) = \left[\frac{\Lambda_\kappa^2}{\Lambda_\kappa^2 - \kappa^2 - i\epsilon(\kappa)\varepsilon}\right]^{N_\kappa} \tag{18}$$

where Λ_κ is large and N_κ is some positive integer. In Eq. (18) ε is real and positive, though small, and $\epsilon(\kappa) = \theta(\kappa) - \theta(-\kappa)$.

The effect of the function $F(\kappa)$ [Eq. (18)] on the original integrand in Eq. (16) is small, because for large Λ_κ it is close to unity. However, including this function in the integrand makes sure that it is at least of order $O(\frac{1}{\kappa^2})$ so that integral **II** gives a zero contribution. The $-i\epsilon(\kappa)\varepsilon$ part ensures that there are now poles on or within the closed contour, because they are always in the lower half-plane (indicated by the dots in Fig. 1).

III. APPLICATION: PION-NUCLEON SCATTERING

In the following sections we are going to apply the Kadyshevsky formalism to the pion-nucleon system, although we present it in such a way that it can easily be extended

TABLE I. Exchanged particles in the various channels.

Channel	Exchanged particle
t	f_0, σ, P, ρ
u	$N, N^*, S_{11}, \Delta_{33}$
s	$N, N^*, S_{11}, \Delta_{33}$

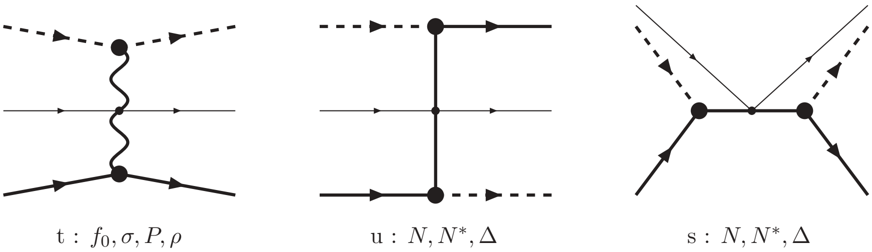

FIG. 2. Tree-level amplitudes as input for integral equation. The inclusion of the quasiparticle lines is schematically. Therefore, the diagrams represent either the (*a*) diagram or the (*b*) diagram.

to other meson-baryon systems. The isospin factors are not included in our treatment; we are concerned only about the Lorentz and Dirac structure. For the details about the isospin factors we refer to Ref. [3].

The ingredients of the model are tree-level, exchange amplitudes as mentioned before. These amplitudes serve as input for the integral equation. Very similar to what is done in [3] we consider for the amplitudes the exchanged particles as in Table I. Graphically, this shown in Fig. 2.

Contrary to Ref. [3] we do not consider the exchange of the tensor mesons, because their contribution is small. The inclusion of the them can be regarded as an extension of this work. For the description of the amplitudes we need the interaction Lagrangians, which in our treatment always serve as the starting points:

(i) Triple meson vertices

$$\mathcal{L}_{SPP} = g_{PPS}\, \phi_{P,a}\phi_{P,b} \cdot \phi_S \tag{19a}$$

$$\mathcal{L}_{VPP} = g_{VPP}(\phi_a i \overleftrightarrow{\partial}_\mu \phi_b) \cdot \phi^\mu, \tag{19b}$$

where S, V, and P stand for *scalar*, *vector*, and *pseudoscalar* to indicate the various mesons. The indices a and b are used to indicate the outgoing and incoming meson, respectively. For the derivative $\overleftrightarrow{\partial}_\mu = \overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu$.

(ii) Meson-baryon vertices

$$\mathcal{L}_{SNN} = g_S\, \bar{\psi}\psi \cdot \phi_S \tag{20a}$$

$$\mathcal{L}_{VNN} = g_V\, \bar{\psi}\gamma_\mu\psi \;\cdot\; \phi^\mu - \frac{f_V}{2M_V} i\partial^\mu \left(\bar{\psi}\sigma_{\mu\nu}\psi\right) \cdot \phi^\nu \tag{20b}$$

$$\mathcal{L}_{PV} = \frac{f_{PV}}{m_\pi}\, \bar{\psi}\gamma_5\gamma_\mu\psi \cdot \partial^\mu\phi_P \tag{20c}$$

$$\mathcal{L}_V = \frac{f_V}{m_\pi}\, \bar{\psi}\gamma_\mu\psi \cdot \partial^\mu\phi_P, \tag{20d}$$

where $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$. The coupling constants f_V of (20b) and (20d) do not necessarily coincide.

We have chosen Eq. (20b) in such a way that the vector meson couples to a current, which may contain a derivative. This is a bit different from Refs. [3,18], where the derivative acts on the vector meson. In Feynman theory this does not make a difference; however, it does in Kadyshevsky formalism, because of the presence of the quasiparticles.

Equation (20c) is used to describe the exchange (u, s channels) of the nucleon and Roper (N^*) and (20d) is used for the S_{11} exchange. This, because of their intrinsic parities. Note, that we could also have chosen the pseudoscalar and scalar couplings for these exchanges. However, because the interactions (20c) and (20d) are also used in Ref. [3] and in chiral symmetry–based models, we use these interactions.

(iii) $\pi N \Delta_{33}$ vertex

$$\mathcal{L}_{\pi N\Delta} = g_{gi}\, \epsilon^{\mu\nu\alpha\beta}(\partial_\mu \bar{\Psi}_\nu)\gamma_5\gamma_\alpha\psi \;\cdot\; (\partial_\beta\phi) + g_{gi}\, \epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_5\gamma_\alpha(\partial_\mu\Psi_\nu) \;\cdot\; (\partial_\beta\phi) \tag{21}$$

The use of this interaction Lagrangian differs from the one used in Ref. [3]. We will come back to this in article II.

The meson exchange processes are discussed in Sec. IV. As mentioned, the discussion of the baryon exchange processes (including pair suppression) is postponed to article II. Another important ingredient of the model is the use of form factors. We also postpone the discussion of them to article II.

IV. MESON EXCHANGE

Here, we proceed with the discussion of the meson exchange processes. We give the amplitudes for meson-baryon scattering or pion-nucleon scattering, specifically meaning that we take equal initial and final states ($M_f = M_i = M$ and $m_f = m_i = m$, where M and m are the masses of the nucleon and pion, respectively). The results for general meson-baryon initial and final states are presented in Appendix A of article II.

A. Scalar-meson exchange

For the description of the scalar-meson exchange processes at tree level, graphically shown in Fig. 3, we use the interaction Lagrangians (19a) and (20a), which lead to the vertices

$$\Gamma_{PPS} = 1 \qquad \Gamma_S = 1 \tag{22}$$

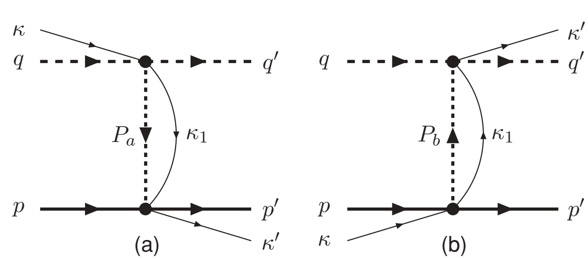

FIG. 3. Scalar-meson exchange.

using $\mathcal{L}_I = -\mathcal{H}_I \to -\Gamma$. For the appropriate propagator we use the first line of Eq. (A2).

Applying the Kadyshevsky rules as discussed in Appendix A, the amplitudes read

$$M^{a,b}_{\kappa'\kappa} = g_{PSS}g_S\int\frac{d\kappa_1}{\kappa_1+i\varepsilon}[\bar{u}(p's')u(ps)]\theta\left(P^0_{a,b}\right)\delta\left(P^2_{a,b}-M^2_S\right), \tag{23}$$

where $P_{a,b} = \pm\Delta_t + \frac{1}{2}(\kappa'+\kappa)n - \kappa_1 n$ (here a corresponds to the $+$ sign and b to the $-$ sign) and $\Delta_t = \frac{1}{2}(p'-p-q'+q)$. For the κ_1 integration we consider the δ function in Eq. (23)

$$(a):\delta\left(P^2_a - M^2_S\right) = \frac{1}{|\kappa_1^+ - \kappa_1^-|}[\delta(\kappa_1-\kappa_1^+)+\delta(\kappa_1-\kappa_1^-)]$$
$$\kappa_1^\pm = \Delta_t\cdot n + \frac{1}{2}(\kappa'+\kappa)\pm A_t$$
$$(b):\delta\left(P^2_b - M^2_S\right) = \frac{1}{|\kappa_1^+ - \kappa_1^-|}[\delta(\kappa_1-\kappa_1^+)+\delta(\kappa_1-\kappa_1^-)]$$
$$\kappa_1^\pm = -\Delta_t\cdot n + \frac{1}{2}(\kappa'+\kappa)\pm A_t, \tag{24}$$

where $A_t = \sqrt{(n\cdot\Delta_t)^2-\Delta_t^2+M_S^2}$. In both cases $\theta(P^0_{a,b})$ selects the κ_1^- solution. Therefore,

$$\begin{aligned} P_a &= \Delta_t - (\Delta_t\cdot n)n + A_t n \\ P_b &= -\Delta_t + (\Delta_t\cdot n)n + A_t n. \end{aligned} \tag{25}$$

With these expression we find for the amplitudes

$$\begin{aligned} M^{(a)}_{\kappa'\kappa} &= g_{PSS}g_S\,[\bar{u}(p's')u(ps)]\frac{1}{2A_t}\cdot\frac{1}{\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}, \\ M^{(b)}_{\kappa'\kappa} &= g_{PSS}g_S\,[\bar{u}(p's')u(ps)]\frac{1}{2A_t}\cdot\frac{1}{-\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}, \end{aligned} \tag{26}$$

where $\bar{\kappa} = \frac{1}{2}(\kappa'+\kappa)$.

Adding the two together and putting $\kappa'=\kappa=0$ we get

$$M_{00} = g_{PSS}g_S\,[\bar{u}(p's')u(ps)]\frac{1}{t-M_S^2+i\varepsilon} \tag{27}$$

which is Feynman result [3].

In Sec. II A we discussed the n dependence of the Kadyshevsky integral equation. For it to be n independent we consider the amplitudes in Eq. (26). Both contributions have poles in the lower half complex $\bar{\kappa}$ plane and therefore also their sum

$$\begin{aligned} &\frac{1}{2A_t}\cdot\frac{1}{\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}+\frac{1}{2A_t}\cdot\frac{1}{-\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon} \\ &\quad = \left(\frac{\bar{\kappa}}{A_t}-1\right)\frac{1}{-(\Delta_t\cdot n)^2+(\bar{\kappa}-A_t)^2+2i\epsilon(\bar{\kappa}-A_t)}. \end{aligned} \tag{28}$$

Furthermore, we notice that when writing out the squares, especially A_t^2, that all n-dependent terms are at least linear proportional to $\bar{\kappa}$. Therefore, if we would consider only scalar-meson exchange in the Kadyshevsky integral equation the integrand would be of the form (16), where $h(\kappa)$ would by itself be of order $O(\frac{1}{\kappa^2})$ as can be seen from Eq. (28), and the phenomenological "form factor" (18) would not be needed.

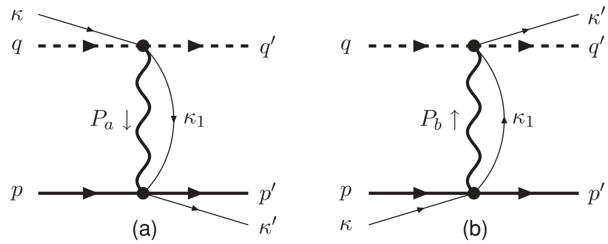

FIG. 4. Vector-meson exchange in Kadyshevsky formalism.

Because there is no propagator as far as Pomeron exchange is concerned, the Kadyshevsky amplitude is the same as the Feynman amplitude for Pomeron exchange [3]

$$M_{\kappa'\kappa} = \frac{g_{PPP}g_P}{M}\,[\bar{u}(p's')u(p)]. \tag{29}$$

B. Vector-meson exchange: example

Before we go on with real vector-meson exchange, we consider simplified vector-meson exchange. We use this as an example to illustrate seaming problems that might occur in the results in the Kadyshevsky formalism, especially when compared to those in the Feynman formalism. We stress that although we consider the example of simplified vector-meson exchange, these peculiarities are generally present when interaction Lagrangians containing derivatives and/or higher-spin fields ($s \geqslant 1$) are considered.

To study simplified vector-meson exchange we take interaction Lagrangians (19b) and (20b), without the $\sigma_{\mu\nu}$ term

$$\mathcal{L}_I = g\,\phi_a i\overleftrightarrow{\partial}_\mu\phi_b\cdot\phi^\mu + g\,\bar{\psi}\gamma_\mu\psi\cdot\phi^\mu. \tag{30}$$

1. *Naive Kadyshevsky approach*

The Kadyshevsky diagrams for the (simplified) vector-meson exchange are shown in Fig. 4.

For the various components of the diagrams we take the following vertex functions

$$\begin{aligned} \Gamma^{\bar{\psi}\psi}_\mu &= \gamma_\mu \\ \Gamma^{\phi\phi}_\mu &= (q'+q)_\mu \end{aligned} \tag{31}$$

following from Eq. (30), and the third line of Eq. (A2) for the propagator.

Applying the Kadyshevsky rules as given in Appendix A straightforwardly we get the following amplitudes

$$\begin{aligned} M^{(a,b)}_{\kappa'\kappa} &= -g^2\int\frac{d\kappa_1}{\kappa_1+i\varepsilon}[\bar{u}(p's')\gamma_\mu u(ps)]\left(g^{\mu\nu}-\frac{P^\mu_{a,b}P^\nu_{a,b}}{M_V^2}\right) \\ &\quad\times\theta\left(P^0_{a,b}\right)\delta\left(P^2_{a,b}-M_V^2\right)(q'+q)_\nu. \end{aligned} \tag{32}$$

The κ_1 integral is discussed in Eqs. (24) and (25). We, therefore, give the results immediately

$$\begin{aligned} M^{(a)}_{\kappa'\kappa} &= -g^2\,\bar{u}(p's')\left[2\not{Q}-\frac{1}{M_V^2}\left((M_f-M_i)+\frac{1}{2}\not{n}(\kappa'-\kappa)\right.\right. \\ &\quad\left.-(\Delta_t\cdot n - A_t)\not{n}\right)\left(\frac{1}{4}(s_{p'q'}-s_{pq})+\frac{1}{4}(u_{pq'}-u_{p'q})\right. \end{aligned}$$

$$-\left(m_f^2 - m_i^2\right) - 2(\Delta_t \cdot n - A_t) n \cdot Q\Big) \Big] u(ps)$$
$$\times \frac{1}{2A_t} \frac{1}{\Delta_t \cdot n + \frac{1}{2}(\kappa' + \kappa) - A_t + i\varepsilon}$$
$$M^{(b)}_{\kappa'\kappa} = -g^2\, \bar{u}(p's') \Bigg[2\not{Q} - \frac{1}{M_V^2} \Bigg((M_f - M_i) + \frac{1}{2}\not{n}(\kappa' - \kappa) - (\Delta_t \cdot n + A_t)\not{n} \Bigg) \Bigg(\frac{1}{4}(s_{p'q'} - s_{pq}) + \frac{1}{4}(u_{pq'} - u_{p'q}) - \left(m_f^2 - m_i^2\right) \; - 2(\Delta_t \cdot n + A_t) n \cdot Q \Bigg) \Bigg] u(ps)$$
$$\times \frac{1}{2A_t} \frac{1}{-\Delta_t \cdot n + \frac{1}{2}(\kappa' + \kappa) - A_t + i\varepsilon}. \tag{33}$$

Adding the two together and putting $\kappa' = \kappa = 0$ we should get back the Feynman expression

$$M_{00} = M^{(a)}_{00} + M^{(b)}_{00} = -g^2 \bar{u}(p's') \left[2\not{Q} + \frac{(M_f - M_i)}{M_V^2} \left(m_f^2 - m_i^2\right) \right] u(ps) \times \frac{1}{t - M_V^2 + i\varepsilon} - g^2 \bar{u}(p's') [\not{n}]\, u(ps)\, \frac{2Q \cdot n}{M_V^2}. \tag{34}$$

From Eq. (34) we see that the first term on the right-hand side is indeed the Feynman result. However, the second term on the right-hand side is an unwanted, n-dependent, contact term.

As mentioned, similar discrepancies are obtained when couplings containing higher-spin fields ($s \geqslant 1$) are used. Therefore, it seems that the Kadyshevsky formalism does not yield the same results in these cases as the Feynman formalism when κ' and κ are put to zero. Because the real difference between Feynman formalism and Kadyshevsky formalism lies in the treatment of the time-ordered product (TOP) or θ function also the difference in results should find its origin in this treatment.

In Feynman formalism derivatives are taken out of the TOP to get Feynman functions, which may yield extra terms. This is also the case in the above example

$$T[\phi^\mu(x)\phi^\nu(y)] = -\left[g^{\mu\nu} + \frac{\partial^\mu \partial^\nu}{M_V^2} \right] i\Delta_F(x - y) - i\delta^\mu_0 \delta^\nu_0\, \delta^4(x - y)$$
$$S_{fi} = (-i)^2 g^2 \int d^4x d^4y [\bar{\psi}\gamma_\mu \psi]_x T[\phi^\mu(x)\phi^\nu(y)] \times [\phi_a\, i\overleftrightarrow{\partial}_\nu \phi_b]_y$$
$$\Rightarrow M_{\text{extra}} = -g^2 \bar{u}(p's') [\not{n}]\, u(ps)\, \frac{2Q \cdot n}{M_V^2}. \tag{35}$$

[2]If we include the extra term of Eq. (35) on the Feynman side we see that both formalisms yield the same result.

Although we have exact equivalence between the two formalisms, the result, though covariant, is still n dependent, i.e., frame dependent. Of course, this is not what we want. As it will turn out, there is another source of extra terms canceling exactly; for instance, the one that pops up in our example [Eqs. (34) and (35)]. As mentioned in the Introduction we present two methods for getting these extra terms to cancel the one in Eqs. (34) and (35): the TU method is more fundamental and the GJ method is more systematic and pragmatic. We will introduce both methods shortly and apply them to the problem in Secs. IV B2 and IV B3, respectively.

2. *Takahahsi and Umezawa solution*

To demonstrate the TU method [10–12], we start with a rewritten version of the Yang-Feldman (YF) equations [19] for a general interaction

$$\mathbf{\Phi}_\alpha(x) = \Phi_\alpha(x) - \int d^4y\; R_{\alpha\beta}(\partial) D_a(y) \Delta_{ret}(x - y) \cdot \mathbf{j}_{\beta;a}(y), \tag{36}$$

where $\mathbf{\Phi}_\alpha(x)$ and $\Phi_\alpha(x)$ are fields in the *Heisenberg representation* (HR) and *interaction representation* (IR), respectively. Furthermore, the vectors $D_a(x)$ and $\mathbf{j}_{\alpha;a}(x)$ are defined to be

$$D_a(x) \equiv (1, \partial_{\mu_1}, \partial_{\mu_1}\partial_{\mu_2}, \ldots)$$
$$\mathbf{j}_{\alpha;a}(x) \equiv \left(-\frac{\partial \mathcal{L}_I}{\partial \mathbf{\Phi}_\alpha(x)} - \frac{\partial \mathcal{L}_I}{\partial(\partial_{\mu_1} \mathbf{\Phi}_\alpha(x))} - \frac{\partial \mathcal{L}_I}{\partial(\partial_{\mu_1}\partial_{\mu_2} \mathbf{\Phi}_\alpha(x))} \cdots \right). \tag{37}$$

Next, a free auxiliary field $\Phi_\alpha(x, \sigma)$ is introduced, where σ is a spacelike surface and x does not necessarily lie on σ. We pose that it has the following form

$$\Phi_\alpha(x, \sigma) \equiv \Phi_\alpha(x) + \int_{-\infty}^{\sigma} d^4y\; R_{\alpha\beta}(\partial) D_a(y) \Delta(x - y) \cdot \mathbf{j}_{\beta;a}(y). \tag{38}$$

Combining (38) with (36) leads to

$$\mathbf{\Phi}_\alpha(x) = \Phi_\alpha(x/\sigma) + \frac{1}{2} \int d^4y [R_{\alpha\beta}(\partial) D_a(y), \epsilon(x - y)] \times \Delta(x - y) \cdot \mathbf{j}_{\beta;a}(y). \tag{39}$$

This equation will be used to express the fields in the HR in terms of fields in the IR.

We explain in Appendix C that the auxiliary fields and the fields in the IR are related by a unitary operator using the BMP theory. Also it is shown how the interaction Hamiltonian should be deduced.

Applying these concepts to our example we determine the "currents" via (37)

$$\begin{aligned}
\boldsymbol{j}_{\phi_a,a} &= (-gi\partial_\mu \boldsymbol{\phi}_b \cdot \boldsymbol{\phi}^\mu, ig\boldsymbol{\phi}_b \cdot \boldsymbol{\phi}^\mu) \\
\boldsymbol{j}_{\phi_b,a} &= (gi\partial_\mu \boldsymbol{\phi}_a \cdot \boldsymbol{\phi}^\mu, -ig\boldsymbol{\phi}_a \cdot \boldsymbol{\phi}^\mu) \\
\boldsymbol{j}_{\psi,a} &= (-g\gamma_\mu \boldsymbol{\psi} \cdot \boldsymbol{\phi}^\mu, 0) \\
\boldsymbol{j}_{\phi^\mu,a} &= (-g\boldsymbol{\phi}_a\, i\overrightarrow{\partial}_\nu \boldsymbol{\phi}_b - g\bar{\boldsymbol{\psi}}\gamma_\mu \boldsymbol{\psi}, 0).
\end{aligned} \tag{40}$$

[2]If we include the n^μ vector in the θ function of the TOP, which would not make a difference, then we can make the replacement $\delta^\mu_0 \to n^\mu$. This would make the result more general.

Using (39) we can express the fields in the HR in terms of fields in the IR, i.e., free fields

$$\begin{aligned}
\boldsymbol{\phi}_a(x) &= \phi_a(x/\sigma)\\
\boldsymbol{\phi}_b(x) &= \phi_b(x/\sigma)\\
\partial_\mu \boldsymbol{\phi}_a(x) &= [\partial_\mu \phi_a(x,\sigma)]_{x/\sigma} + \frac{1}{2}\int d^4y\left[\partial^x_\mu \partial^y_\nu, \epsilon(x-y)\right]\\
&\quad \times \Delta(x-y)\,(ig\phi_b\cdot\phi^\nu)_y\\
&= [\partial_\mu \phi_a(x,\sigma)]_{x/\sigma} + ign_\mu \phi_b n\cdot\phi\\
\partial_\mu \boldsymbol{\phi}_b(x) &= [\partial_\mu \phi_b(x,\sigma)]_{x/\sigma} + \frac{1}{2}\int d^4y\left[\partial^x_\mu \partial^y_\nu, \epsilon(x-y)\right]\\
&\quad \times \Delta(x-y)\,(-ig\phi_a\cdot\phi^\nu)_y\\
&= [\partial_\mu \phi_b(x,\sigma)]_{x/\sigma} - ign_\mu \phi_a n\cdot\phi\\
\boldsymbol{\psi}(x) &= \psi(x/\sigma)\\
\boldsymbol{\phi}^\mu(x) &= \phi^\mu(x/\sigma) + \frac{1}{2}\int d^4y\left[\left(-g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{M_V^2}\right),\right.\\
&\quad \left.\epsilon(x-y)\right]\Delta(x-y)(-g\phi_a\, i\overleftrightarrow{\partial}_\nu\phi_b - g\bar{\psi}\gamma_\nu\psi)_y\\
&= \phi^\mu(x/\sigma) - \frac{g\,n^\mu}{M_V^2}(\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b + \bar{\psi}\not{n}\psi). \qquad (41)
\end{aligned}$$

As can be seen from Eq. (39), the first term on the right-hand side is a free field and the second term contains the current expressed in terms of fields in the HR, which on their turn are expanded similarly. Therefore, one gets coupled equations. In solving these equations we assumed that the coupling constant is small and therefore considered only terms up to first order in the coupling constant in the expansion of the fields in the HR. Practically speaking, the currents on the right-hand side of side (41) are expressed in terms of free fields.

These expansions (41) are used in the commutation relations of the fields with the interaction Hamiltonian [Eq. (C17) of Appendix C]

$$\begin{aligned}
&[\phi_a(x), \mathcal{H}_I(y)]\\
&\quad = iU^{-1}(\sigma)\Delta(x-y)[-gi\partial_\mu\boldsymbol{\phi}_b\cdot\boldsymbol{\phi}^\mu + gi\overleftarrow{\partial_\mu}\boldsymbol{\phi}_b\cdot\boldsymbol{\phi}^\mu]_y U(\sigma)\\
&\quad = i\Delta(x-y)[-g\, i\overleftrightarrow{\partial}_\mu\phi_b\cdot\phi^\mu\\
&\qquad + \frac{g^2}{M_V^2} n\cdot i\overleftrightarrow{\partial}\phi_b(\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b + \bar{\psi}\not{n}\psi) - g^2\phi_a(n\cdot\phi)^2]_y\\
&[\psi(x), \mathcal{H}_I(y)]\\
&\quad = iU^{-1}(\sigma)(i\not{\partial} + M)\Delta(x-y)[-g\gamma_\mu\boldsymbol{\psi}\cdot\boldsymbol{\phi}^\mu]_y U(\sigma)\\
&\quad = i(i\not{\partial} + M)\Delta(x-y)\\
&\qquad \times\left[-g\gamma_\mu\psi\cdot\phi^\mu + \frac{g^2}{M_V^2}\not{n}\psi(\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b + \bar{\psi}\not{n}\psi)\right]_y\\
&[\phi^\mu(x), \mathcal{H}_I(y)]\\
&\quad = iU^{-1}(\sigma)\left(-g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{M_V^2}\right)\Delta(x-y)\\
&\qquad \times[-g\,\boldsymbol{\phi}_a\, i\overleftrightarrow{\partial}_\nu\boldsymbol{\phi}_b - g\,\bar{\boldsymbol{\psi}}\gamma_\nu\boldsymbol{\psi}]_y U(\sigma)\\
&\quad = i\left(-g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{M_V^2}\right)\Delta(x-y)[-g\phi_a\, i\overleftrightarrow{\partial}_\nu\phi_b - g\bar{\psi}\gamma_\nu\psi\\
&\qquad - g^2 n_\nu\,\phi_a^2 n\cdot\phi - g^2 n_\nu\,\phi_b^2 n\cdot\phi]_y. \qquad (42)
\end{aligned}$$

As stated below (C17) these are the fundamental equations from which the interaction Hamiltonian can be determined

$$\begin{aligned}
\mathcal{H}_I &= -g\phi_a\, i\overleftrightarrow{\partial}_\mu\phi_b\cdot\phi^\mu - g\bar{\psi}\gamma_\mu\psi\cdot\phi^\mu - \frac{g^2}{2}\phi_a^2(n\cdot\phi)^2\\
&\quad - \frac{g^2}{2}\phi_b^2(n\cdot\phi)^2 + \frac{g^2}{2M_V^2}[\bar{\psi}\not{n}\psi]^2 + \frac{g^2}{M_V^2}[\bar{\psi}\not{n}\psi]\\
&\quad \times[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b] + \frac{g^2}{2M_V^2}[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b]^2 + O(g^3)\ldots. \qquad (43)
\end{aligned}$$

If Eq. (41) was solved completely, then the right-hand side of Eq. (41) would contain higher orders in the coupling constant and therefore also the interaction Hamiltonian (43). These terms are indicated by the ellipsis.

If we want to include the external quasifields as in Appendix B, then the easy way to do this is to apply Eq. (B7) straightforwardly. However, because we want to derive the interaction Hamiltonian from the interaction Lagrangian we would have to include a $\bar{\chi}(x)\chi(x)$ pair in Eq. (30) similarly to that in Eq. (B7). This would mean that the terms of order g^2 in Eq. (43) are quartic in the quasifield, where two of them can be contracted

$$\bar{\chi}(x)\underbrace{\chi(x)\bar{\chi}}\chi(x)\bar{\chi}(x)\chi(x) = \bar{\chi}(x)\theta[n(x-x)]\chi(x). \qquad (44)$$

Defining the θ function to be 1 in its origin we assure that all terms in the interaction Hamiltonian (43) relevant to πN scattering are quadratic in the external quasifields, even higher-order terms in the coupling constant.

The only term of order g^2 in Eq. (43) that gives a contribution to the first order in the S matrix describing πN scattering is the third term on the second line in the right-hand side of Eq. (43). Its contribution to the first order in the S matrix is

$$\begin{aligned}
S^{(1)}_{fi} &= -i\int d^4x\mathcal{H}_I(x) = \frac{-ig^2}{M_V^2}\int d^4x[\bar{\psi}\not{n}\psi]\\
&\quad \times[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b]_x\\
&= \frac{-ig^2}{M_V^2}\,\bar{u}(p's')\not{n}u(ps)n\cdot(q'+q),\\
\Rightarrow M_{\text{canc}} &= g^2\,\bar{u}(p's')\not{n}u(ps)\frac{2n\cdot Q}{M_V^2}. \qquad (45)
\end{aligned}$$

Indeed we see that this term (45) cancels the extra term in Eq. (34).

From Eq. (43) one can see that the interaction Hamiltonian contains not only terms of order g but also higher-order terms. In our example we see that the g^2 terms in the interaction Hamiltonian is responsible for the cancellation. In this light we also mention the specific example of scalar electrodynamics as described in Ref. [20], section 6-1-4. There the interaction Hamiltonian also contains a term of order g^2, which has the same purpose as in our case. The method described in Ref. [20] is not generally applicable, whereas the above described method, although applied to a specific example, is.

3. Gross and Jackiw solution

The essence of the Gross and Jackiw method [13] is to define a different TOP: the T^* product, which is by definition n independent:

$$T^*(x,y) = T(x,y;n) + \tau(x,y;n). \tag{46}$$

Studying the n dependence is done in the same way as described in Sec. II A

$$P^{\alpha\beta}\frac{\delta}{\delta n^\beta}T^*(x,y) = P^{\alpha\beta}\frac{\delta}{\delta n^\beta}T(x,y;n) + P^{\alpha\beta}\frac{\delta}{\delta n^\beta}\tau(x,y;n) \equiv 0. \tag{47}$$

In our applications we are interested in second-order contributions to πN scattering. Therefore, we analyze the n dependence of the TOP of two interaction Hamiltonians, where we take it to be just $\mathcal{H}_I = -\mathcal{L}_I$

$$P^{\alpha\beta}\frac{\delta}{\delta n^\beta}T(x,y;n) = P^{\alpha\beta}(x-y)_\beta\delta\left[n\cdot(x-y)\right] \times \left[\mathcal{H}_I(x),\mathcal{H}_I(y)\right]. \tag{48}$$

In general one has for equal time commutation relations

$$\begin{aligned}\delta[n(x-y)]\,[\mathcal{H}_I(x),\mathcal{H}_I(y)] &= [C(n) + P^{\alpha\beta}S_\alpha(n)\partial_\beta \\ &\quad + P^{\alpha\beta}P^{\mu\nu}Q_{\alpha\mu}(n)\partial_\beta\partial_\nu + \cdots]\delta^4(x-y),\end{aligned} \tag{49}$$

where the ellipsis stand for higher-order derivatives. We will consider (and encounter) only up to quadratic derivatives. The S^α and $Q^{\alpha\beta}$ terms in Eq. (49) are known in the literature as *Schwinger terms*.

It should be mentioned that in Ref. [13] only the first two terms on the right-hand side of Eq. (49) are considered.

Using the fact that the TOP and therefore also the T^* product appears in the S matrix as an integrand, we are allowed to use partial integration for the $S_\alpha(n)$ and $Q_{\alpha\beta}(n)$ terms. The $C(n)$ always vanishes. Furthermore, we use the fact that $P^{\alpha\beta}$ is a projection operator. With these considerations we find from Eqs. (47) to (49) the extra terms

$$\begin{aligned}\tau(x-y;n) = \int^n dn'\beta[S_\beta(n') + P^{\mu\nu}(Q_{\beta\mu}(n') \\ + Q_{\mu\beta}(n'))\partial_\nu]\ \delta^4(x-y).\end{aligned} \tag{50}$$

In principle, the right-hand side of Eq. (50) can also contain a constant term, i.e., independent of n^μ. But because we are looking for n^μ-dependent terms only, this term is irrelevant.

Now, we are going to apply the method of Gross and Jackiw. The "covariantized" equal time commutator of interaction Hamiltonians is

$$\begin{aligned}&\delta[n(x-y)]\,[\mathcal{H}_I(x),\mathcal{H}_I(y)] \\ &= g^2\Bigg\{\frac{1}{M_V^2}([\bar\psi \not{n}\psi]_x[\phi_a i\overleftrightarrow{\partial}_\mu\phi_b]_y + [\bar\psi n_\mu\psi]_x[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b]_y \\ &\quad + [\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b]_x[\bar\psi n_\mu\psi]_y + [\phi_a i\overleftrightarrow{\partial}_\mu\phi_b]_x[\bar\psi\not{n}\psi]_y \\ &\quad + [\bar\psi\not{n}\psi]_y[\bar\psi\gamma_\mu\psi]_x + [\bar\psi\gamma_\mu\psi]_y[\bar\psi\not{n}\psi]_x \\ &\quad + [\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b]_y[i\overleftrightarrow{\partial}_\mu\phi_b]_x + [\phi_a i\overleftrightarrow{\partial}_\mu\phi_b]_y[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b]_x) \\ &\quad + \phi_a(y)n\cdot\phi(x)\phi_a(x)\phi_\mu(y) + \phi_a(y)\phi_\mu(x)\phi_a(x)n\cdot\phi(y) \\ &\quad + [\phi_b n\cdot\phi]_x[\phi_b\phi_\mu]_y + [\phi_b\phi_\mu]_x[\phi_b n\cdot\phi]_y\Bigg\} \\ &\quad \times P^{\mu\rho}i\partial_\rho\delta^4(x-y).\end{aligned} \tag{51}$$

Comparing this with Eq. (49) we see that the terms between curly brackets coincide with $-iS_\alpha(n)$; the $Q_{\alpha\beta}(n)$ terms are absent. Therefore, the τ function, representing the compensating terms, becomes by means of Eqs. (50) and (51)

$$\begin{aligned}\tau(x-y;n) = ig^2\Bigg[\frac{1}{M_V^2}(2[\bar\psi\not{n}\psi][\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b] + [\bar\psi\not{n}\psi]^2 \\ + [\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b]^2) + \phi_a^2(n\cdot\phi)^2 + \phi_b^2(n\cdot\phi)^2\Bigg] \\ \times\delta^4(x-y).\end{aligned} \tag{52}$$

Its contribution to πN-scattering S matrix and amplitude is

$$\begin{aligned}S^{(2)}_{\text{canc}} &= \frac{(-i)^2}{2!}\int d^4x d^4y\frac{2ig^2}{M_V^2}\ [\bar\psi\not{n}\psi]\,[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b)]\delta^4(x-y) \\ M_{\text{canc}} &= g^2\bar u(p's')\not{n}u(ps)\frac{2n\cdot Q}{M_V^2},\end{aligned} \tag{53}$$

which is the same expression as the canceling amplitude derived from the TU scheme in Eq. (45).

4. $\bar P$ approach

From the forgoing subsections (Secs. IV B3 and IV B2) we have seen that if we add all contributions, results in the Feynman formalism and in the Kadyshevsky formalism are the same (of course we need to put $\kappa' = \kappa = 0$). Also, we have seen from Eq. (35) and the forgoing subsections that if we bring out the derivatives out of the TOP in Feynman formalism not only do we get Feynman functions but also the n-dependent contact terms cancel out.

Unfortunately, this is not the case in Kadyshevsky formalism. There, all n-dependent contact terms cancel out after adding up the amplitudes. So, when calculating an amplitude according to the Kadyshevsky rules in Appendix A one always has to keep in mind the contributions as described in Secs. IV B2 and IV B3. For practical purposes this is not very convenient.

Inspired by the Feynman procedure we could also do the same in Kadyshevsky formalism, namely let the derivatives not only act on the vector-meson propagator[3] but also on the quasiparticle propagator (θ function). In doing so, we know that all contact terms cancel out, just as in Feynman formalism.

We show the above in formula form:

$$\begin{aligned}&\theta[n(x-y)]\partial_x^\mu\partial_x^\nu\Delta^{(+)}(x-y) \\ &\quad + \theta[n(y-x)]\partial_x^\mu\partial_x^\nu\Delta^{(+)}(y-x)\end{aligned}$$

[3] By "propagator" we mean the $\Delta^+(x-y)$ and not the Feynman propagator $\Delta_F(x-y)$.

$$= \partial_x^\mu \partial_x^\nu \theta[n(x-y)]\Delta^{(+)}(x-y) + \partial_x^\mu \partial_x^\nu \theta[n(y-x)] \times \Delta^{(+)}(y-x) + in^\mu n^\nu \delta^4(x-y)$$
$$= \frac{i}{2\pi}\int \frac{d\kappa_1}{\kappa_1+i\varepsilon}\int \frac{d^4P}{(2\pi)^3}\theta(P^0)\delta\left(P^2-M_V^2\right)(-\bar{P}_\mu \bar{P}_\nu) \times (e^{-i\kappa_1 n(x-y)}e^{-iP(x-y)} + e^{i\kappa_1 n(x-y)}e^{iP(x-y)}) + in^\mu n^\nu \delta^4(x-y), \tag{54}$$

where $\bar{P} = P + n\kappa_1$. In this way the second order in the S matrix becomes

$$S_{fi}^{(2)} = -g^2\int d^4x d^4y[\bar{u}(p's')\gamma_\mu u(ps)](q'+q)_\nu e^{-ix(q-q')} \times e^{iy(p'-p)}\frac{i}{2\pi}\int \frac{d\kappa_1}{\kappa_1+i\varepsilon}\int \frac{d^4P}{(2\pi)^3}\theta(P^0)\delta\left(P^2-M_V^2\right) \times \left(-g^{\mu\nu}+\frac{\bar{P}^\mu \bar{P}^\nu}{M_V^2}\right)(e^{-i\kappa_1 n(x-y)}e^{-iP(x-y)}e^{in\kappa' x-in\kappa y} + e^{i\kappa_1 n(x-y)}e^{iP(x-y)}e^{-in\kappa' x+in\kappa y}) + ig^2\int d^4x[\bar{u}(p's') \times \not{n}u(ps)]n\cdot(q'+q)e^{-ix(q-q'-p'+p-n\kappa'+n\kappa)}. \tag{55}$$

We see that the second term on the right-hand side of Eq. (55) brings about an amplitude, which is exactly the same as in Eqs. (34) and (35) and is to be canceled by Eqs. (45) and (53).

Performing the various integrals correctly we get

$$(a) \Rightarrow \begin{cases} \kappa_1 = \Delta_t\cdot n - A_t + \frac{1}{2}\left(\kappa'+\kappa\right)n \\ \bar{P} = \Delta_t + \frac{1}{2}\left(\kappa'+\kappa\right)n \end{cases}$$
$$(b) \Rightarrow \begin{cases} \kappa_1 = -\Delta_t\cdot n - A_t + \frac{1}{2}\left(\kappa'+\kappa\right)n \\ \bar{P} = -\Delta_t + \frac{1}{2}\left(\kappa'+\kappa\right)n \end{cases}. \tag{56}$$

This yields for the invariant amplitudes

$$M_{\kappa'\kappa}^{(a)} = -g^2\bar{u}(p's')\left[2\not{Q} + \frac{1}{M_V^2}\left((M_f-M_i) + \frac{1}{2}(\kappa'-\kappa)\not{n} + \not{n}\bar{\kappa}\right)\left(\left(m_f^2-m_i^2\right) + \frac{1}{4}(s_{pq}-s_{p'q'}+u_{p'q}-u_{pq'}) + 2\bar{\kappa}Q\cdot n\right)\right]u(ps) \times \frac{1}{2A_t}\frac{1}{\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}$$

$$M_{\kappa'\kappa}^{(b)} = -g^2\bar{u}(p's')\left[2\not{Q} + \frac{1}{M_V^2}\left((M_f-M_i) + \frac{1}{2}(\kappa'-\kappa)\not{n} - \not{n}\bar{\kappa}\right)\left(\left(m_f^2-m_i^2\right) + \frac{1}{4}(s_{pq}-s_{p'q'}+u_{p'q}-u_{pq'}) - 2\bar{\kappa}Q\cdot n\right)\right]u(ps) \times \frac{1}{2A_t}\frac{1}{-\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}$$

$$M = M_{00}^{(a)} + M_{00}^{(b)} = -g^2\bar{u}(p's')\left[2\not{Q} + \frac{(M_f-M_i)}{M_V^2}\left(m_f^2-m_i^2\right)\right] \times u(ps)\frac{1}{t-M_V^2+i\varepsilon}, \tag{57}$$

where $\bar{\kappa} = \frac{1}{2}(\kappa'+\kappa)$. As before we get back the Feynman expression for the amplitude if we add both amplitudes obtained in Kadyshevsky formalism and put $\kappa' = \kappa = 0$. The big advantage of this procedure is that we do not need to worry about the contribution n-dependent contact terms because they canceled out when introducing $\bar{P}$.

It should be noted, however, that the $\bar{P}$ method is possible only when both Kadyshevsky contributions at second order are added. This becomes clear when looking at the first two lines of Ref. (54): Letting the derivatives also act on the θ function gives compensating terms for the $\Delta^{(+)}(x-y)$ part and for the $\Delta^{(-)}(x-y)$ part. Only when added together do they combine to form the $\delta^4(x-y)$ part.

Also it becomes clear from Eq. (54) that at least two derivatives are needed to generate the $\delta^4(x-y)$ part. Therefore, when there is only one derivative, for instance, in the case of baryon exchange (no derivatives in coupling, only in the propagator) at second order, the $\delta^4(x-y)$ part is not present and it is not necessary to use the $\bar{P}$ method. In these cases it does not matter for the summed diagrams whether the $\bar{P}$ method is used; however, for the individual diagrams it does make a difference. This ambiguity is absent in Feynman theory, there derivatives are always taken out of the TOP (which is similar to the $\bar{P}$ method, as discussed above) to come to Feynman propagators.

In the forgoing we have demonstrated the $\bar{P}$ method for simplified vector-meson exchange and strictly speaking for $\kappa' = \kappa = 0$. We stress, however, that this method is generally applicable, i.e., for $\kappa', \kappa \neq 0$ and for general couplings containing multiple derivatives and/or higher-spin fields.

C. Real vector-meson exchange

Now that we have discussed how to deal with multiple derivatives and/or higher-spin fields in the Kadyshevsky formalism by means of the simplified vector-meson exchange example, we are prepared to deal with real vector-meson exchange. To do so we use the interaction Lagrangians as in Eqs. (19b) and (20b). From these interaction Lagrangians we distillate the already exposed vertex function in Eq. (31) (second line) and

$$g\,\Gamma_{VNN}^\mu = g_V\,\gamma^\mu + \frac{f_V}{2M_V}(p'-p)_\alpha\sigma^{\alpha\mu}. \tag{58}$$

The Kadyshevsky diagrams representing vector-meson exchange are already exposed in Fig. 4. Applying the Kadyshevsky rules of Appendix A and the $\bar{P}$ method described in Sec. IV B4 we obtain the following amplitudes

$$M_{\kappa'\kappa}^{(a)} = -g_{VPP}\,\bar{u}(p's')\left[2g_V\not{Q} - \frac{g_V}{M_V^2}\kappa'\not{n}\left(\frac{1}{4}(s_{p'q'}-s_{pq}+u_{pq'}-u_{p'q}) + 2\bar{\kappa}Q\cdot n\right) + \frac{f_V}{2M_V}\left(4M\not{Q} + \frac{1}{2}(u_{pq'}+u_{p'q}) - \frac{1}{2}(s_{p'q'}+s_{pq}) - \frac{1}{M_V^2}\left(M^2+m^2-\frac{1}{2}\left(\frac{1}{2}(t_{p'p}+t_{q'q})+u_{pq'}+s_{pq}\right)\right.\right.\right.$$

$$+2M\not{n}\kappa' + \frac{1}{4}(\kappa'-\kappa)^2 - (p'+p)\cdot n\bar{\kappa}\Big)$$
$$\times\left(\frac{1}{4}(s_{p'q'}-s_{pq}) + \frac{1}{4}(u_{pq'}-u_{p'q}) + 2\bar{\kappa}n\cdot Q\right)\Big)\Big]$$
$$\times u(ps)\frac{1}{2A_t}\frac{1}{\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}$$

$$M^{(b)}_{\kappa'\kappa} = -g_{VPP}\,\bar{u}(p's')\Big[2g_V\not{Q}$$
$$+\frac{g_V}{M_V^2}\kappa\not{n}\left(\frac{1}{4}(s_{p'q'}-s_{pq}+u_{pq'}-u_{p'q}) - 2\bar{\kappa}Q\cdot n\right)$$
$$+\frac{f_V}{2M_V}\Big(4M\not{Q} + \frac{1}{2}(u_{pq'}+u_{p'q}) - \frac{1}{2}(s_{p'q'}+s_{pq})$$
$$-\frac{1}{M_V^2}\Big(M^2+m^2-\frac{1}{2}\left(\frac{1}{2}(t_{p'p}+t_{q'q})+u_{pq'}+s_{pq}\right)$$
$$-2M\not{n}\kappa + \frac{1}{4}(\kappa'-\kappa)^2 + (p'+p)\cdot n\bar{\kappa}\Big)$$
$$\times\left(\frac{1}{4}(s_{p'q'}-s_{pq}) + \frac{1}{4}(u_{pq'}-u_{p'q}) - 2\bar{\kappa}n\cdot Q\right)\Big)\Big]$$
$$\times u(ps)\,\frac{1}{2A_t}\,\frac{1}{-\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}. \tag{59}$$

The sum of the two in the limit of $\kappa' = \kappa = 0$ yields

$$M_{00} = -g_{VPP}\bar{u}(p's')\left[2g_V\not{Q} + \frac{f_V}{2M_V}\left((u-s)+4M\not{Q}\right)\right]$$
$$\times u(ps)\,\frac{1}{t-M_V^2+i\varepsilon}, \tag{60}$$

which is, again, the Feynman result [3].

Just as in Sec. IV A we consider the amplitudes (59) in light of the n dependence of the Kadyshevsky integral equation (see Sec. II A). Every n-dependent term in the numerators of $M^{(a)}_{\kappa'\kappa}$ and $M^{(b)}_{\kappa'\kappa}$ is at least linear proportional to either κ or κ' (or both) and the poles of $M^{(a)}_{\kappa'\kappa}$ and $M^{(b)}_{\kappa'\kappa}$ are in the lower complex $\bar{\kappa}$ plane. Those terms in the numerators of $M^{(a)}_{\kappa'\kappa}$ and $M^{(b)}_{\kappa'\kappa}$ that are n independent are added in the same way as in the case of scalar-meson exchange and the same reasoning applies.

The numerator of the summed diagrams $M^{(a)}_{0\kappa}$ and $M^{(b)}_{0\kappa}$ is of higher degree in κ then the denominator. Therefore, the function $h(\kappa)$ in Eq. (16) will not be of order $O(\frac{1}{\kappa^2})$ and the "form factor" (18) is necessary.

In Eq. (59) as well as in Eq. (26) we have taken u and $\bar{u}$ spinors. The reason behind this is pair suppression which we will discuss in article II.

APPENDIX A: KADYSHEVSKY RULES

Just as in Feynman theory Kadyshevsky amplitudes can be represented by Kadyshevsky diagrams. Because the basic starting points are the same as in Feynman theory we take a general Feynman diagram and give the Kadyshevsky rules from there on to construct the amplitude M_{fi}. Here, we define the amplitude as

$$S_{fi} = \delta_{fi} - i(2\pi)^2\delta^4(P_f-P_i)\,M_{fi}, \tag{A1}$$

where $P_{f/i}$ is the sum of the final/initial momenta.

Kadyshevsky rules:

(i) Arbitrarily number the vertices of the diagram.

(ii) Connect the vertices with a quasiparticle line, assigned to it a momentum $n\kappa_s$ ($s = 1\cdots n-1$). Attach to vertex 1 an incoming initial quasiparticle with momentum $n\kappa$ and attach to vertex n an outgoing final quasiparticle with momentum $n\kappa'$.[4]

(iii) Orient each internal momentum such that it leaves a vertex with a lower number than the vertex it enters. If two fermion lines with opposite momentum direction come together in one vertex, assign a + symbol to one line and a − to the other. Each possibility to do this yields a different Kadyshevsky diagram.

(iv) Assign to each internal quasiparticle line a propagator, $\frac{1}{\kappa_s+i\varepsilon}$.

(v) Assign to all other internal lines the appropriate Wightman function of Eq. (A2). Assign to a fermion line with a ± symbol: $S^{(\pm)}(P)$ [see (iii)]

$$\begin{aligned}
\Delta^{(+)}(P) &= \theta(P^0)\delta(P^2-M^2)\\
S^{(\pm)}(P) &= \Lambda^{(1/2)}(\pm P)\theta(P^0)\delta(P^2-M^2),\\
\Delta^{(+)}_{\mu\nu}(P) &= \Lambda^{(1)}_{\mu\nu}(P)\theta(P^0)\delta(P^2-M^2),\\
S^{(\pm)}_{\mu\nu}(P) &= \Lambda^{(3/2)}_{\mu\nu}(\pm P)\,\theta(P^0)\delta(P^2-M^2),
\end{aligned} \tag{A2}$$

where

$$\Lambda^{(1/2)}_{\mu\nu}(P) = (\not{P}+M)$$
$$\Lambda^{(1)}_{\mu\nu}(P) = \left(-g_{\mu\nu} + \frac{P_\mu P_\nu}{M^2}\right)$$
$$\Lambda^{(3/2)}_{\mu\nu}(P) = -(\not{P}+M)\Big(g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2P_\mu P_\nu}{3M^2}$$
$$+\frac{1}{3M}(P_\mu\gamma_\nu - \gamma_\mu P_\nu)\Big). \tag{A3}$$

(vi) There is momentum conservation at the vertices, including the quasiparticle momenta.

(vii) Integrate over the internal quasimomenta: $\int_{-\infty}^{\infty} d\kappa_s$.

(viii) Integrate over those internal momenta not fixed by momentum conservation at the vertices: $\int_{-\infty}^{\infty}\frac{d^4P}{(2\pi)^3}$.

(ix) Include a − sign for every fermion loop.

(x) Include a − sign for identical initial or final fermions.

(xi) Repeat the various steps for all different numberings in (i).

[4]Obviously these quasiparticle may not appear as initial or final states, because they are not physical particles. However, because we use Kadyshevsky diagrams as input for an integral equation we allow for external quasiparticles.

It is clear from (iii) and (xi) that one Feynman diagram leads to several Kadyshevsky diagrams. Generally, one Feynman diagram leads to $n!$ Kadyshevsky diagrams, where n is the number of vertices (or the order). Especially for higher-order diagrams this leads to a dramatic increase of labor. Fortunately, we will only consider second-order diagrams.

A few remarks need to be made about these rules as far as the choice of definition is concerned. In item (iii) we have followed Ref. [5] to orient the internal momenta. Furthermore, we have chosen to use the integral representation of the θ function

$$\theta[n\cdot(x-y)] = \frac{i}{2\pi}\int d\kappa_1 \frac{e^{-i\kappa_1 n\cdot(x-y)}}{\kappa_1 + i\varepsilon} \tag{A4}$$

instead of its complex conjugate. Because the θ function is real, this is also a proper representation, originally used in the articles of Kadyshevsky. To understand why we have chosen to deviate from the original approach, consider the S matrix

$$S = 1 + \sum_{n=1}^{\infty}(-i)^n \int_{-\infty}^{\infty} d^4x_1\cdots d^4x_n\, \theta[n(x_1-x_2)]\cdots\theta \\ \times [n(x_{n-1}-x_n)]\mathcal{H}_I(x_1)\cdots\mathcal{H}_I(x_n). \tag{A5}$$

In each order S_n there is a factor $(-i)^n$ already in the definition. In that specific order there are $(n-1)$ θ functions, each containing a factor i from the integral representation (A4). Therefore, every S_n will, regardless of the order, contain a factor $(-i)$. Hence, the amplitude M_{fi}, defined in Eq. (A1), will no longer contain overall factors of i.

The momentum space $S^{(-)}(P)$ functions differ an overall minus sign by their coordinate space analogs $\langle 0|\bar{\psi}(x)\psi(y)|0\rangle = S^{(-)}(x-y)$. The reason for that is twofold. In many cases the Wightman functions $S^{(-)}(x-y)$, including the overall minus sign, appear in combination with the normal ordered product (NOP): $N(\psi\bar{\psi}) = -N(\bar{\psi}\psi)$. Therefore, the minus signs cancel. In all other cases the Wightman functions $S^{(-)}(x-y)$ appear in fermion loops and are therefore responsible for the fermion loop minus sign in (ix), because every fermion loop will contain an odd number of $S^{(-)}(x-y)$ functions. We stress that this method of defining the Kadyshevsky rules for fermions differs from the original one in Ref. [7].

APPENDIX B: SECOND QUANTIZATION

When discussing the Kadyshevsky rules in Appendix A and the Kadyshevsky integral equation in (10) we allowed for quasiparticles to occur in the initial and final state. To do this properly a new theory needs to be set up containing quasiparticle creation and annihilation operators. It is set up in such a way that external quasiparticles occur in the S matrix as trivial exponentials so that when the external quasimomenta are taken to be zero the Feynman expression is obtained. We, therefore, require that the vacuum expectation value of the quasiparticles is the θ function

$$\langle 0|\chi(nx)\bar{\chi}(nx')|0\rangle = \theta[n(x-x')] \tag{B1}$$

and that a quasifield operator acting on a state with quasimomentum $(n)\kappa$ yields only a trivial exponential

$$\chi(nx)|\kappa\rangle = e^{-i\kappa nx} \\ \langle\kappa|\bar{\chi}(nx) = e^{i\kappa nx}. \tag{B2}$$

Assuming that a state with quasimomentum $(n)\kappa$ is created in the usual way

$$a^\dagger(\kappa)|0\rangle = |\kappa\rangle \\ \langle 0|a(\kappa) = \langle\kappa| \tag{B3}$$

we have from the requirements (B1) and (B2) the following momentum expansion of the fields

$$\chi(nx) = \frac{i}{2\pi}\int\frac{d\kappa}{\kappa+i\varepsilon}e^{-i\kappa nx}a(\kappa) \\ \bar{\chi}(nx') = \frac{i}{2\pi}\int\frac{d\kappa}{\kappa+i\varepsilon}e^{i\kappa nx'}a^\dagger(\kappa) \tag{B4}$$

and the fundamental commutation relation of the creation and annihilation operators

$$[a(\kappa), a^\dagger(\kappa')] = -i2\pi\kappa\delta(\kappa-\kappa'). \tag{B5}$$

From this commutator (B5) it is clear that the quasiparticle is not a physical particle nor a ghost.

Now that we have set up the second quantization for the quasi particles we need to include them in the S matrix. This is done by redefining it

$$S = 1 + \sum_{n=1}(-i)^n\int d^4x_1\cdots d^4x_n\tilde{\mathcal{H}}_I(x_1)\cdots\tilde{\mathcal{H}}_I(x_n), \tag{B6}$$

where

$$\tilde{\mathcal{H}}_I(x) \equiv \mathcal{H}_I(x)\bar{\chi}(nx)\chi(nx). \tag{B7}$$

In this sense contraction of the quasifields causes propagation of this field between vertices, just as in the Feynman formalism. Those quasiparticles that are not contracted are used to annihilate external quasiparticles form the vacuum.

$$\begin{aligned} &S^{(2)}(p's'q'n\kappa'; psqn\kappa) \\ &= (-i)^2\int d^4x_1 d^4x_2\langle\pi N\chi|\tilde{\mathcal{H}}_I(x_1)\tilde{\mathcal{H}}_I(x_2)|\pi N\chi\rangle \\ &= (-i)^2\int d^4x_1 d^4x_2\langle 0|b(p's')a(q')a(\kappa') \\ &\quad\times[\bar{\chi}(nx_1)\mathcal{H}_I(x_1)\chi(nx_1)\bar{\chi}(nx_2)\mathcal{H}_I(x_2)\chi(nx_2)] \\ &\quad\times a^\dagger(\kappa)a^\dagger(q)b^\dagger(ps)|0\rangle \\ &= (-i)^2\int d^4x_1 d^4x_2 e^{in\kappa' x_1}e^{-in\kappa x_2}\langle 0|b(p's')a(q')\mathcal{H}_I(x_1) \\ &\quad\times\theta[n(x_1-x_2)]\mathcal{H}_I(x_2)a^\dagger(q)b^\dagger(ps)|0\rangle. \end{aligned} \tag{B8}$$

For the π and N fields we use the well-known momentum expansion

$$\phi(x) = \int\frac{d^3l}{(2\pi)^3 2E_l}[a(l)e^{-ilx} + a^\dagger(l)e^{ilx}]$$

$$\psi(x) = \sum_r \int \frac{d^3k}{(2\pi)^3 2E_k}[b(k,r)u(k,r)e^{-ikx} + d^\dagger(k,r)v(k,r)e^{ikx}], \tag{B9}$$

where the creation and annihilation operators satisfy the following (anti-)commutation relations

$$[a(k), a^\dagger(l)] = (2\pi)^3\, 2E_k\, \delta^3(k-l)$$
$$b(k,s), b^\dagger(l,r) = (2\pi)^3\, 2E_k\, \delta_{sr}\delta^3(k-l) = d(k,s), d^\dagger(l,r). \tag{B10}$$

Putting $\kappa' = \kappa = 0$ in (B8) we see that we get the second order in the S-matrix expansion for πN scattering as in Feynman formalism. Of course, this is what we required from the beginning: external quasiparticle momenta occur only in the S matrix as exponentials.

So, we know now how to include the external quasiparticles in the S matrix and therefore we also know what their effect is on amplitudes. For practical purposes we will not use the S matrix as in Eq. (B6) but keep the above in mind. In those cases where the (possible) inclusion of external quasifields is less trivial we will make some comments.

APPENDIX C: BMP THEORY

According to Haag's theorem [17] in general there does not exist a unitary transformation that relates the fields in the IR and the fields in the HR. However, there is no objection against the existence of an unitary $U[\sigma]$ relating the TU auxiliary fields and the fields in the IR.

$$\Phi_\alpha(x,\sigma) = U^{-1}[\sigma]\,\Phi_\alpha(x)U[\sigma]. \tag{C1}$$

Here, we follow the framework of Bogoliubov and Collaborators [14–16], to which we refer to as the BMP theory to prove (C1) in a straightforward way (see Appendix (C2)).

The BMP theory was originally constructed to bypass the use of an unitary operator U as a mediator between the fields in the HR and in the IR.

1. Setup

In the description of the BMP theory we will consider only scalar fields. By the assumption of asymptotic completeness the S matrix is taken to be a functional of the asymptotic fields $\phi_{as,\rho}(x)$, where as = *in, out*. In the following we use *in*-fields, i.e., $\phi_\rho(x) = \phi_{\text{in},\rho}(x)$

$$S = 1 + \sum_{n=1}^{\infty}\int d^4x_1\cdots d^4x_n S_n(x_1\alpha_1,\cdots,x_n\alpha_n). \times :\phi_{\alpha_1}(x_1)\cdots\phi_{\alpha_n}(x_n): . \tag{C2}$$

Here, concepts like unitarity and the stability of the vacuum, i.e., $\langle 0|S|0\rangle = 1$, and the one-particle states, i.e., $\langle 0|S|1\rangle = 0$ are assumed. The *Heisenberg current*, i.e., the current in the HR, is defined as[5]

$$\mathbf{J}_\rho(x) = iS^\dagger\frac{\delta S}{\delta\phi_\rho(x)}. \tag{C3}$$

We note that for a Hermitean field $\phi_\rho(x)$ the current is also Hermitean due to unitarity. *Microcausality* takes the form, see Ref. [15], section 17,[6]

$$\frac{\delta\mathbf{J}_\rho(x)}{\delta\phi_\lambda(y)} = 0 \quad \text{for} \quad x \leqslant y. \tag{C4}$$

It can be shown that the notion of microcausality is reflected in the expression of the S matrix as the time-ordered exponential. See Ref. [15] for the details on this point of view. It can also be shown that with the current (C3) the asymptotic fields $\phi_{\text{in/out},\rho}(x)$ satisfy a YF type of equation [as in Eq. (C11)]

$$\boldsymbol{\phi}_\rho(x) = \phi_{\text{in/put},\rho}(x) + \int d^4y\,\Delta_{\text{ret/adv}}(x-y)\,\mathbf{J}_\rho(y) \tag{C5}$$

giving the Heisenberg fields $\boldsymbol{\phi}_\rho(x)$ in terms of the $\phi_{\text{in/out}}(x)$ fields.

Lehmann, Symanzik, and Zimmermann (LSZ) [21] formulated an asymptotic condition utilizing the notion of weak convergence in the Hilbert space of state vectors. See, e.g., Ref. [22] for an detailed exposition of the LSZ formalism. The correspondence of BMP theory with LSZ is obtained by the identification

$$\mathbf{J}_\rho(x) = -iS^\dagger\frac{\delta S}{\delta\phi_\rho(x)} \equiv (\Box + m^2)\,\boldsymbol{\phi}_\rho(x). \tag{C6}$$

As is explained in, for instance, Ref. [16], the local commutivity of the currents follows from microcausality (C4). Using the YF equations one can show that for spacelike separations the fields in the HR commute with the currents and among themselves, as was assumed in the LSZ formalism. For more details and results of BMP, see Refs. [14–16].

2. Application to Takahashi-Umezawa scheme

In this subsection we introduce the auxiliary field similar to (38)

$$\phi(x,\sigma) \equiv \phi(x) - \int_{-\infty}^{\sigma} d^4x'\Delta(x-x')\,\mathbf{J}(x') \tag{C7}$$

and prove that $\phi(x)$ and $\phi(x,\sigma)$ satisfy the same (usual) commutation relations. Such a relation justifies the existence of an unitary operator connecting the two as in Eq. (C1).

[5] Note that in Ref. [16] the *out*-field is used. Then

$$\mathbf{J}_\rho(x) = i\frac{\delta S}{\delta\phi_\rho(x)}S^\dagger.$$

Also, we take a minus sign in the definition of the current.

[6] Here $x \leqslant y$ means either $(x-y)^2 \geqslant 0$ and $x^0 < y^0$ or $(x-y)^2 < 0$. So the point x is in the past of or is spacelike separated from the point y.

The difference of the commutation relations is, using Eq. (C7),

$$\begin{aligned}[]
&[\phi(x,\sigma),\phi(y,\sigma)]-[\phi(x),\phi(y)]\\
&\quad=-\int_{-\infty}^{\sigma}d^4y'\Delta(y-y')[\phi(x),\mathbf{J}(y')]\\
&\qquad+\int_{-\infty}^{\sigma}d^4x'\Delta(x-x')[\phi(y),\mathbf{J}(x')]\\
&\qquad+\int_{-\infty}^{\sigma}\int_{-\infty}^{\sigma}d^4x'd^4y'\ \Delta(x-x')\Delta(y-y')[\mathbf{J}(x'),\mathbf{J}(y')].
\end{aligned}\tag{C8}$$

Because the S operator is an expansion in asymptotic fields, so is $\mathbf{J}(x)$ by means of its definition in terms of this S operator (C3). Now, from the commutation relations of the asymptotic fields one has

$$[\phi_\rho(x),\mathbf{J}_\sigma(y)]=i\int d^4x'\Delta(x-x')\frac{\delta\mathbf{J}_\sigma(y)}{\delta\phi_\rho(x')}.\tag{C9}$$

Using this in Eq. (C8) we have

$$\begin{aligned}[]
&[\phi(x,\sigma),\phi(y,\sigma)]-[\phi(x),\phi(y)]\\
&\quad=-i\int_{-\infty}^{\sigma}d^4y'\int_{-\infty}^{\infty}d^4x'\Delta(x-x')\Delta(y-y')\frac{\delta\mathbf{J}(y')}{\delta\phi(x')}\\
&\qquad+i\int_{-\infty}^{\sigma}d^4x'\int_{-\infty}^{\infty}d^4y'\Delta(x-x')\Delta(y-y')\frac{\delta\mathbf{J}(x')}{\delta\phi(y')}\\
&\qquad-i\int_{-\infty}^{\sigma}d^4x'\int_{-\infty}^{\sigma}d^4y'\Delta(x-x')\Delta(y-y')\left(\frac{\delta\mathbf{J}(x')}{\delta\phi(y')}\right.\\
&\qquad\left.-\frac{\delta\mathbf{J}(y')}{\delta\phi(x')}\right)=0.
\end{aligned}\tag{C10}$$

Cancellation takes place in Eq. (C10) when the second integral of the first two term on the right-hand side in Eq. (C10) is split up: $\int_{-\infty}^{\infty}=\int_{-\infty}^{\sigma}+\int_{\sigma}^{\infty}$. The remaining terms are zero because of the microcausility condition (C4). Although we shown the proof for scalar fields only, the generalization to other types of fields is easily made.

Complementary to what is in Refs. [10–12] we explicitly show that the unitary operator in Eq. (C1) is not any operator but the one connected to the S matrix. We, therefore, consider (general) *in*- and *out*-fields. Their relation to the fields in the HR is

$$\begin{aligned}
\mathbf{\Phi}_\alpha(x)&=\Phi_{\text{in},\alpha}(x)+\int d^4y R_{\alpha\beta}(\partial)\Delta_{\text{ret}}(x-y)\ \mathbf{J}_\beta(y)\\
&=\Phi_{\text{out},\alpha}(x)+\int d^4y R_{\alpha\beta}(\partial)\Delta_{\text{adv}}(x-y)\ \mathbf{J}_\beta(y),
\end{aligned}\tag{C11}$$

where $\Delta_{\text{ret}}(x-y)=-\theta(x^0-y^0)\Delta(x-y)$ and $\Delta_{\text{adv}}(x-y)=\theta(y^0-x^0)\Delta(x-y)$.

Equation (C11) makes clear that the choice of the Green function determines the choice of the free field (*in*- or *out*-field) to be used. In this light we make the following identification: $\Phi_\alpha(x,-\infty)\equiv\Phi_{\text{in},\alpha}(x)$, because we have used the retarded Green function in Sec. IV B2 [text below Eq. (36)]. With Eq. (C11) we can also relate the *out*-field to the auxiliary field $\Phi_\alpha(x,\infty)=\Phi_{\text{out},\alpha}(x)$.

Using these identifications in Eq. (C1) we obtain the relation between $\Phi_{\alpha,\text{in}}(x)$ and $\Phi_{\alpha,\text{out}}(x)$

$$\begin{aligned}
\Phi_{\alpha,\text{in}}(x)&=U^{-1}[-\infty]U[\infty]\Phi_{\alpha,\text{out}}(x)U^{-1}[\infty]U[-\infty]\\
\Phi_{\text{in},\alpha}(x)&=S\Phi_{\text{out},\alpha}S^{-1}.
\end{aligned}\tag{C12}$$

Obviously, the operator connecting the *in*- and *out*-fields is the S matrix, where the relation between $U[\sigma]$ and the S matrix is

$$\begin{aligned}
&U[\sigma]=T\left[\exp\left(-i\int_{-\infty}^{\sigma}d^4x\mathcal{H}_I(x)\right)\right]\\
&U[\infty]=S\qquad U[-\infty]=1.
\end{aligned}\tag{C13}$$

To make contact with the interaction Hamiltonian we follow Refs. [10–12] for completion by realizing that the unitary operator satisfies the Tomonaga-Schwinger equation

$$i\frac{\delta U[\sigma]}{\delta\sigma(x)}=\mathcal{H}_I(x;n)U[\sigma]|_{x/\sigma}=U[\sigma]\,\mathcal{H}_I(x/\sigma;n).\tag{C14}$$

Here, the interaction Hamiltonian will in general depend on the vector $n_\mu(x)$ locally normal to the surface $\sigma(x)$, i.e., $n^\mu(x)d\sigma_\mu=0$. It is Hermitean because of the unitarity of $U[\sigma]$. Then, from (C1) and (C14) one gets that

$$i\frac{\delta\Phi_\alpha(x,\sigma)}{\delta\sigma(y)}=U^{-1}[\sigma]\,[\Phi_\alpha(x),\mathcal{H}_I(y;n)]\ U[\sigma].\tag{C15}$$

However, varying (38) with respect to $\sigma(y)$ gives

$$i\frac{\delta\Phi_\alpha(x,\sigma)}{\delta\sigma(y)}=i\,D_a(y)R_{\alpha\beta}(\partial)\Delta(x-y)\cdot\mathbf{j}_{\beta;a}(y).\tag{C16}$$

Comparing Eqs. (C15) and (C16) gives the relation

$$\begin{aligned}[]
[\Phi_\alpha(x),\mathcal{H}_I(y;n)]=i\ U[\sigma][D_a(y)R_{\alpha\beta}(\partial)\Delta(x-y)\\
\cdot\mathbf{j}_{\beta;a}(y)]U^{-1}[\sigma].
\end{aligned}\tag{C17}$$

This is the fundamental equation by which the interaction Hamiltonian must be determined.

APPENDIX D: REMARKS ON THE HAAG THEOREM

Here, we take a closer look at the connection between the fields in the HR and in the IR in the covariant formulation of Tomonaga and Schwinger [23,24]

$$\mathbf{\Phi}_\alpha(x)=U^{-1}[\sigma]\Phi_\alpha(x)\ U[\sigma].\tag{D1}$$

This is in light of the Haag theorem [17], which states that if there is a unitary operator connecting fields in two representations at some time [as in Eq. (D1)], where the field in one representation is free, both fields are free. This would lead to a triviality.

The question is whether this situation (D1) is applicable to our case. To answer that question we look at the results of the previous section (Appendix C). By introducing the auxiliary field in the scalar case as in Eq. (38) [or for general fields as in Eq. (C7)], we proved Eq. (C1) using BMP theory.

Now, we start with Eq. (36) and use similar arguments[7] to come to

$$\begin{aligned}
\mathbf{\Phi}_\alpha(x) &= \Phi_\alpha(x) + \int_{-\infty}^{\infty} d^4y\; D_a(y)\; R_{\alpha\beta}(\partial)\; \theta[n(x-y)] \\
&\quad \times \Delta(x-y)\cdot \mathbf{j}_{\beta;a}(y) \\
&= \Phi_\alpha(x) + \int_{-\infty}^{\infty} d^4y \theta[n(x-y)]\; D_a(y)\; R_{\alpha\beta}(\partial) \\
&\quad \times \Delta(x-y)\cdot \mathbf{j}_{\beta;a}(y) \\
&\quad + \int_{-\infty}^{\infty} d^4y [D_a(y) R_{\alpha\beta}(\partial), \theta[n(x-y)]] \\
&\quad \times \Delta(x-y)\cdot \mathbf{j}_{\beta;a}(y) \\
\Rightarrow \mathbf{\Phi}_\alpha(x) &= U^{-1}[\sigma]\Phi_\alpha(x)\; U[\sigma]|_{x/\sigma} \\
&\quad + \frac{1}{2}\int_{-\infty}^{\infty} d^4y [D_a(y) R_{\alpha\beta}(\partial), \epsilon(x-y)] \\
&\quad \times \Delta(x-y)\cdot \mathbf{j}_{\beta;a}(y).
\end{aligned} \tag{D2}$$

[7]We have included the n^μ vector in the first line of Eq. (D2), which causes no effect. The reason for this inclusion is that we can keep the surface σ general though spacelike.

The above is different from what is exposed in Ref. [22] (ch. 17.2). The difference is the commutator part in Eq. (D2) and this term is nonzero for theories with couplings containing derivatives and higher-spin fields, carefully excluded in the treatment of Ref. [22]. Therefore (D2) could be seen as an extension of what is written in Ref. [22].

Returning to Haag's theorem we see that if the last term in Eq. (D2) is absent there is a unitary operator connecting $\mathbf{\Phi}_\alpha(x)$ and $\Phi_\alpha(x)$ and therefore they are both free fields. Such theories can then be considered as trivial, although they can still be useful as effective theories.

In our application we use various interaction Lagrangians (for the overview see Sec. III) to be used to describe the various exchange [and resonance (article II)] processes. Whether the nonvanishing commutator part in Eq. (D2) is present depends on the process under consideration. In the vector-meson exchange diagrams (Sec. IV C) and in the spin-3/2 exchange and resonance diagrams (article II) those commutator parts are nonvanishing. If we include pair suppression in the way we do in article II also in the spin-1/2 exchange and resonance diagrams the commutator parts will be nonvanishing. So, if we take the model as a whole (all diagrams) then it is most certainly not trivial in the sense of the Haag theorem.

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