Spin susceptibility of degenerate quark matter

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The expression for spin susceptibility χ of degenerate quark matter is derived with corrections up to $\mathcal{O}(g^4 \ln g^2)$. It is shown that, at low density, χ^{-1} changes sign and turns negative, indicating a ferromagnetic phase transition. To this order, we also calculate sound velocity c_1 and incompressibility K with arbitrary spin polarization. The estimated values of c_1 and K show that the equation of state of the polarized matter is stiffer than that of unpolarized matter. Finally, we determine the finite temperature corrections to the exchange energy and derive corresponding results for the spin susceptibility.

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I. INTRODUCTION

One of the active areas of high-energy physics research is the exploration of the so-called quantum chromodynamics (QCD) phase diagram. In particular, with the advent of ultrarelativistic heavy ion beams at the Relativistic Heavy Ion Collider (RHIC) and CERN, and with the upcoming facilities at GSI where compressed baryonic matter is expected to be produced, such studies assume special importance. In addition to the laboratory experiments, various astrophysical objects, such as neutron stars and quark stars, provide natural sites where many of the theoretical conjectures about the various phases of quark matter can be tested. The latter, in the present context, is more relevant here, as we study the possibility of a para-ferro phase transition in a dense quark system interacting via one-gluon exchange.

The original idea of a para-ferro phase transition in quark matter was proposed recently in Ref. [1] where the possibility of a Bloch-like phase transition [2] was studied; it was shown that spin-polarized quark matter might exist at low density [3]. The underlying mechanism of such a phase transition is analogous to what was originally proposed for the degenerate electron gas [2]. There, for the Coulomb interaction, it was shown that the exchange correction to energy is attractive, which at low density wins over the kinetic energy, thus giving rise to a ferromagnetic state [2]. In Ref. [1], a variational calculation was performed to show that it is indeed possible to have spin-polarized quark matter at low density of a strange quark system, whereas for the light quark matter this never happens [1]. Similar differences between both light and strange quark matter, albeit in a different context, were observed earlier [4]. However, in Ref. [3] it was shown that both the light and heavy flavor systems can exhibit such phase transitions, although the critical density for the strange matter is higher than that for the light quark systems. Such investigations were also performed in Refs. [5–8] and also in Refs. [9,10], where the calculation was extended to include thermal effects. A Bloch-like phase transition for the strange quark was also reconfirmed in Ref. [11].

One shortcoming of all these works, including Ref. [11], is that the calculations are restricted to the Hartree-Fock level and the terms beyond the exchange diagrams, commonly known as the correlation energy [12–16], are ignored. Without such corrections, however, the calculations are known to remain incomplete since the higher order terms are plagued with infrared divergences that arise out of the exchange of massless gluons. This indicates the failure of the naive perturbation series. We know that this problem can be cured by reorganizing the perturbation theory where a particular class of diagrams, that is, the bubbles, are resummed in order to obtain a finite result. Originally, as is well known, this was done by Gell-Mann and Brueckner [17] while calculating the ground-state energy of a degenerate electron gas. The contribution of the bubbles involve terms of $\mathcal{O}(g^4 \ln g^2)$, indicating the nonperturbative nature of the correction [18–21].

In the present work, we calculate the spin susceptibility (χ) of a dense quark system with corrections due to correlations, that is, containing terms up to $\mathcal{O}(g^4 \ln g^2)$. This requires knowledge of the ground-state energy (GSE) of spin-polarized matter with the inclusion of bubble diagrams. The GSE of the polarized quark matter was calculated only recently in Ref. [17], which serves as the starting point of the present article. This work is very similar to that of Brueckner and Swada [22] and that of Refs. [23,24] and is applied to the case of QCD matter. Unlike degenerate electron gas, however, we have both the electric and magnetic interactions; the calculation is performed relativistically, while the nonrelativistic results appear as a limit.

The spin susceptibility χ, for quark matter up to $\mathcal{O}(g^2)$, was already calculated in Ref. [1], which we only briefly discuss. Subsequently, the non-Fermi-liquid corrections to χ were also studied in Refs. [9,10]. These studies provided further motivation for undertaking the present endeavor to include correlation corrections, without which, as mentioned already, the perturbative evaluation of χ remains incomplete. In addition, we also calculate the incompressibility and sound velocity for spin-polarized quark matter with correlation corrections that involve the evaluation of a single-particle energy at the Fermi surface. These quantities are of special interest for applications to astrophysics. Moreover, we also evaluate the exchange energy density at nonzero temperature

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and determine the corresponding corrections to the spin susceptibility.

The plan of the article is as follows. In Sec. II we calculate spin susceptibility with correlation corrections for degenerate quark matter. Analytic expressions are presented both in ultrarelativistic (UR) and nonrelativistic (NR) limits. In Sec. III, we evaluate exchange energy density and spin susceptibility at nonzero temperature. In Sec. IV we summarize and conclude. Detailed expressions of the intermediate expressions from which χ is derived are relegated to the Appendix.

II. SPIN SUSCEPTIBILITY

The spin susceptibility of quark matter is determined by the change in energy of the system as its spins are polarized [22]. We introduce a polarization parameter $\xi = (n_q^+ - n_q^-)/n_q$ with the condition $0 \leqslant \xi \leqslant 1$, where n_q^+ and n_q^- correspond to the densities of spin-up and spin-down quarks, respectively, and $n_q = n_q^+ + n_q^-$ denotes total quark density. The Fermi momenta in the spin-polarized quark matter are then $p_f^+ = p_f(1+\xi)^{1/3}$ and $p_f^- = p_f(1-\xi)^{1/3}$, where $p_f = (\pi^2 n_q)^{1/3}$, is the Fermi momentum of the unpolarized matter ($\xi = 0$). In the small-ξ limit, the GSE behaves as [1]

$$E(\xi) = E(\xi = 0) + \tfrac{1}{2}\beta_s \xi^2 + \mathcal{O}(\xi^4). \tag{1}$$

Here, $\beta_s = \frac{\partial^2 E}{\partial \xi^2}|_{\xi=0}$ is defined as the spin stiffness constant analogous with Refs. [16,21]. The spin susceptibility χ is proportional to the inverse of the spin stiffness; mathematically, $\chi = 2\beta_s^{-1}$ [26]. Note that in Eq. (1), the first term corresponds to unpolarized matter energy.

Now, the leading contributions to the GSE are given by three terms, that is, kinetic, exchange, and correlation energy densities [16]:

$$E = E_{\rm kin} + E_{\rm ex} + E_{\rm corr}. \tag{2}$$

The total kinetic energy density for spin-up and spin-down quarks becomes [1,11]

$$\begin{aligned} E_{\rm kin} = \frac{3}{16\pi^2}\Bigg\{ & p_f(1+\xi)^{1/3} \\ & \times \sqrt{p_f^2(1+\xi)^{2/3} + m_q^2}\,\big[2p_f^2(1+\xi)^{2/3} + m_q^2\big] \\ & - m_q^4 \ln\left(\frac{p_f(1+\xi)^{1/3} + \sqrt{p_f^2(1+\xi)^{2/3} + m_q^2}}{m_q}\right) \\ & + [\xi \to -\xi] \Bigg\}, \end{aligned} \tag{3}$$

where m_q is the quark mass.

The exchange energy density $E_{\rm ex}$ was calculated in Ref. [11] within a Fermi liquid theory approach. One can also directly evaluate the two loop diagrams [1] to obtain

$$E_{\rm ex}^{\rm nf} = \frac{9}{2}\sum_{s=\pm}\iint \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}\theta\big(p_f^s - |p|\big)\theta\big(p_f^s - |p'|\big) f_{pp'}^{\rm nf}, \tag{4}$$

$$E_{\rm ex}^{\rm f} = 9\iint \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}\theta(p_f^+ - |p|)\theta(p_f^- - |p'|) f_{pp'}^{\rm f}, \tag{5}$$

where $f_{pp'}^{\rm nf}$ and $f_{pp'}^{\rm f}$ stand for nonflip ($s = s'$) and flip ($s = -s'$) forward scattering amplitudes given in Refs. [1,11,16]. Here, $E_{\rm ex} = E_{\rm ex}^{\rm nf} + E_{\rm ex}^{\rm f}$ can be estimated numerically. However, an analytical evaluation of these integrals is possible in the UR and NR limits, as reported in Refs. [1,11,16].

The next higher order correction to the GSE beyond the exchange term is the correlation energy $E_{\rm corr}$ [12–15]. A detailed calculation of the correlation energy for spin-polarized matter was derived in Ref. [16], which we quote here:

$$\begin{aligned} E_{\rm corr} \simeq \frac{1}{(2\pi)^3}\frac{1}{2}\int_0^{\pi/2}\sin^2\theta_E d\theta_E \Bigg\{ & \Pi_L^2\left[\ln\left(\frac{\Pi_L}{\varepsilon_f^2}\right) - \frac{1}{2}\right] \\ & + 2\Pi_T^2\left[\ln\left(\frac{\Pi_T}{\varepsilon_f^2}\right) - \frac{1}{2}\right]\Bigg\}, \end{aligned} \tag{6}$$

with $\theta_E = \tan^{-1}(|k|/k_0)$. The relevant Π_L and Π_T are determined to be [16]

$$\Pi_L = \frac{g^2}{4\pi^2}\sum_{s=\pm}\frac{p_f^s \varepsilon_f^s}{\sin^2\theta_E}\left[1 - \frac{\cot\theta_E}{v_f^s}\tan^{-1}\left(v_f^s \tan\theta_E\right)\right], \tag{7}$$

$$\begin{aligned} \Pi_T = & \frac{g^2}{8\pi^2}\sum_{s=\pm} p_f^{s\,2}\cot\theta_E \\ & \times\left[-\frac{\cot\theta_E}{v_f^s} + \left(1 + \frac{\cot^2\theta_E}{v_f^{s\,2}}\right)\tan^{-1}\left(v_f^s\tan\theta_E\right)\right]. \end{aligned} \tag{8}$$

The spin susceptibility is given by Ref. [1] as

$$\chi^{-1} = \frac{1}{2}\frac{\partial^2 E(\xi)}{\partial\xi^2}\Big|_{\xi=0}. \tag{9}$$

We have $\chi^{-1} \equiv \chi_{\rm kin}^{-1} + \chi_{\rm ex}^{-1} + \chi_{\rm corr}^{-1}$. The kinetic and exchange contributions, evaluated in Ref. [1], are given by

$$\chi_{\rm kin}^{-1} = \frac{p_f^5}{6\pi^2\varepsilon_f}, \tag{10}$$

$$\begin{aligned} \chi_{\rm ex}^{-1} = -\frac{g^2 p_f^4}{18\pi^4}\Bigg\{ & 2 - \frac{6p_f^2}{\varepsilon_f^2} - \frac{3p_f}{\varepsilon_f^3}\left[p_f\varepsilon_f - m_q^2\ln\right. \\ & \left.\times\left(\frac{p_f + \varepsilon_f}{m_q}\right)\right] + \frac{2p_f^2}{\varepsilon_f^2}\left[1 + \frac{2m_q}{3(p_f + m_q)}\right]\Bigg\}. \end{aligned} \tag{11}$$

To determine the correlation corrections to spin susceptibility we expand the terms in curly braces from Eq. (6) in powers of the polarization parameter ξ, which gives

$$\begin{aligned} & \Pi_L^2\left[\ln\left(\frac{\Pi_L}{\varepsilon_f^2}\right) - \frac{1}{2}\right] + 2\Pi_T^2\left[\ln\left(\frac{\Pi_T}{\varepsilon_f^2}\right) - \frac{1}{2}\right] \\ & = (\mathcal{A}_{0L} + \mathcal{B}_{0T}) + \xi^2(\mathcal{A}_{1L} + \mathcal{B}_{1T}) + \mathcal{O}(\xi^4). \end{aligned} \tag{12}$$

Here, $\mathcal{A}_{0L}$ and $\mathcal{B}_{0T}$ correspond to unpolarized matter terms and the detailed expressions of $\mathcal{A}_{1L}$ and $\mathcal{B}_{1T}$ are given in the Appendix. $\chi_{\rm corr}^{-1}$ is

$$\chi_{\rm corr}^{-1} = \frac{1}{2}\frac{\partial^2 E_{\rm corr}(\xi)}{\partial\xi^2}\Big|_{\xi=0} \simeq \frac{1}{(2\pi)^3}\frac{1}{2}\int_0^{\pi/2}\sin^2\theta_E d\theta_E(\mathcal{A}_{1L}+\mathcal{B}_{1T}). \quad (13)$$

From this expression, and with the help of the expressions presented in the Appendix, $\chi_{\rm corr}^{-1}$ can be estimated numerically. The results for the two limiting cases, however, can be obtained analytically, as we present in the following two sections.

A. UR limit

In the UR limit, the kinetic, exchange, and correlation energies are [16]

$$\begin{aligned} E_{\rm kin}^{\rm ur} &= \frac{3p_f^4}{8\pi^2}[(1+\xi)^{4/3}+(1-\xi)^{4/3}],\\ E_{\rm ex}^{\rm ur} &= \frac{g^2}{32\pi^4}p_f^4[(1+\xi)^{4/3}+(1-\xi)^{4/3}+2(1-\xi^2)^{2/3}],\\ E_{\rm corr}^{\rm ur} &= \frac{g^4\ln g^2}{2048\pi^6}p_f^4[(1+\xi)^{4/3}+(1-\xi)^{4/3}+2(1-\xi^2)^{2/3}]. \end{aligned} \quad (14)$$

With the help of Eq. (1) each energy contribution to the susceptibility is

$$\begin{aligned} \chi_{\rm kin}^{-1} &= \frac{p_f^4}{6\pi^2},\\ \chi_{\rm ex}^{-1} &= -\frac{g^2p_f^4}{36\pi^4},\\ \chi_{\rm corr}^{-1} &= -\frac{g^4p_f^4}{2304\pi^6}(\ln r_s - 0.286), \end{aligned} \quad (15)$$

with $r_s = g^2(\frac{3\pi}{4})^{1/3}$. From Eq. (15), the sum of all the contributions to the susceptibility can be written as [16]

$$\chi^{\rm ur} = \chi_P\left[1-\frac{g^2}{6\pi^2}-\frac{g^4}{384\pi^4}(\ln r_s - 0.286)\right]^{-1}, \quad (16)$$

where χ_P is the noninteracting susceptibility [23,24].

B. NR limit

Now, we use the NR limit to calculate spin susceptibility in order to compare our results with those of a dense electron gas [22–24,26] interacting via the static Coulomb potential. In this limit, kinetic and exchange energy densities are [1,11,16]

$$\begin{aligned} E_{\rm kin}^{\rm nr} &= \frac{3p_f^5}{20\pi^2 m_q}[(1+\xi)^{5/3}+(1-\xi)^{5/3}],\\ E_{\rm ex}^{\rm nr} &= -\frac{g^2}{8\pi^4}p_f^4[(1+\xi)^{4/3}+(1-\xi)^{4/3}]. \end{aligned} \quad (17)$$

The contribution to the susceptibility from kinetic and exchange energy densities yields

$$\begin{aligned} \chi_{\rm kin}^{-1} &= \frac{p_f^5}{6\pi^2 m_q},\\ \chi_{\rm ex}^{-1} &= -\frac{g^2p_f^4}{18\pi^4}. \end{aligned} \quad (18)$$

We calculate the contribution to the spin susceptibility beyond the exchange correction. For this we must first evaluate the correlation energy in this limit.

The dominant contribution to the correlation energy is found to be

$$\begin{aligned} E_{\rm corr}^{\rm nr} = &-\frac{\lambda^2p_f^5}{\pi^4 m_q}\int_{\lambda^{1/2}}^{k_c}\frac{dk'}{k'}\int_0^\infty x\,dx\sum_{s=\pm}f(s)\\ &\times\left[1-\frac{x^s}{2}\ln\left(\frac{x^s+1}{x^s-1}\right)\right]\sum_{s'=\pm}\theta(1-x^{s'}), \end{aligned} \quad (19)$$

where $\lambda = (g^2m_q)/(8\pi p_f)$, $f(s=\pm) = (1\pm\xi)^{1/3}$, $x = x^s f(s)$, $x^s = (k_0m_q)/(p_f^s k)$, and $k' = k/p_f$. For $s = s'$ we obtain

$$E_{\rm corr}^{{\rm nr},s=s'} \simeq \frac{g^4\ln g^2}{(2\pi)^6}\frac{1}{3}m_q p_f^3(1-\ln 2). \quad (20)$$

Note that, here, the correlation energy is independent of spin polarization ξ. For the spin parallel interactions, ξ-dependent terms contribute to an opposite sign and therefore cancel each other out. For $s=-s'$, the integral on x takes the form

$$\begin{aligned} I = \int_0^\infty x\,dx\Bigg\{&\left(1+\frac{1}{3}\xi\right)\left[1-\frac{1}{2}x\left(1-\frac{1}{3}\xi\right)\right.\\ &\left.\times\ln\left|\frac{x(1-\frac{1}{3}\xi)+1}{x(1-\frac{1}{3}\xi)-1}\right|\right]\theta\left[1-x\left(1+\frac{1}{3}\xi\right)\right]\\ &+(\xi\to-\xi)\Bigg\}. \end{aligned} \quad (21)$$

Expanding the natural logarithm in terms of ξ and retaining up to $\mathcal{O}(\xi^2)$, we have

$$I \simeq \frac{2}{3}\left[(1-\ln 2)-\frac{1}{6}\xi^2\right]. \quad (22)$$

Using Eqs. (19), (21), and (22) we have

$$E_{\rm corr}^{{\rm nr},s=-s'} \simeq \frac{g^4\ln g^2}{128\pi^6}\frac{1}{3}m_q p_f^3\left[(1-\ln 2)-\frac{1}{6}\xi^2\right]. \quad (23)$$

It should be mentioned that similar expressions for a degenerate electron gas interacting via a static Coulomb potential can be found in Ref. [26]. From Eqs. (20) and (23) it is clear that spin antiparallel states are attractive, in contrast to the parallel states obtained by the Pauli exclusion principle. In this limit the correlation contribution to the susceptibility is found to be

$$\chi_{\rm corr}^{-1} = -\frac{g^4\ln g^2}{2304\pi^6}m_q p_f^3. \quad (24)$$

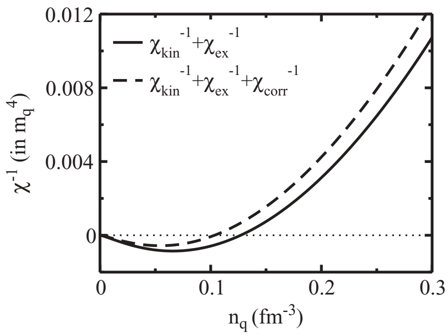

FIG. 1. Density dependence of inverse spin susceptibility.

The total susceptibility is given by

$$\chi^{\rm nr} = \chi_P \left[1 - \frac{g^2}{3\pi^2}\frac{m_q}{p_f} - \frac{g^4 \ln g^2}{384\pi^4}\frac{m_q^2}{p_f^2} \right]^{-1}. \tag{25}$$

In Fig. 1 we plot inverse spin susceptibility, which is valid for all kinematic regimes. It shows that χ^{-1} changes its sign at the density $\sim$0.12 fm^{-3} without a correlation correction and when we include the correlation effect its sign changes at $\sim$0.1 fm^{-3}. This is equivalent to what happens to the GSE as a function of ξ. Needless to say, this change of sign corresponds to the para-ferro phase transition in a dense quark system. The parameter set used here is the same as those of Refs. [1,4,11,16].

C. Incompressibility and sound velocity

Once we have the expressions for the total energy density, the incompressibility (K) and sound velocity (c_1) can be determined. The incompressibility K is defined by the second derivative of the total energy density with respect to the number density n_q, which is given by Ref. [11] as

$$K = 9 n_q \frac{\partial^2 E}{\partial n_q^2}. \tag{26}$$

Since there are two Fermi surfaces corresponding to spin-up (+) and spin-down (−) states, such that $E \equiv E(n_q^+, n_q^-)$, we have [11]

$$\begin{aligned} \frac{\partial E}{\partial n_q} &= \frac{\partial E}{\partial n_q^+}\frac{\partial n_q^+}{\partial n_q} + \frac{\partial E}{\partial n_q^-}\frac{\partial n_q^-}{\partial n_q} \\ &= \frac{1}{2}[(1+\xi)\mu^+ + (1-\xi)\mu^-]. \end{aligned} \tag{27}$$

The single-particle energy at the Fermi surface or the chemical potential of spin-up quark turns out to be

$$\begin{aligned} \mu^{+,\rm ur} &= \mu_{\rm kin}^+ + \mu_{\rm ex}^+ + \mu_{\rm corr}^+ \\ &= p_f^+ + \frac{g^2}{12\pi^2}\left(p_f^+ + \frac{p_f^{+2}}{p_f^-}\right) + \frac{g^4 \ln g^2}{768\pi^4}\left(p_f^+ + \frac{p_f^{+2}}{p_f^-}\right). \end{aligned} \tag{28}$$

Similarly, $\mu^{-,\rm ur}$ can be obtained by replacing $p_f^\pm$ with $p_f^\mp$ in Eq. (28). In Ref. [11], the chemical potential was determined within the Fermi liquid theory approach up to $\mathcal{O}(g^2)$. However, here we calculate $\mu^\pm$ with a different approach up to $\mathcal{O}(g^4 \ln g^2)$.

Using Eqs. (27) and (28), the incompressibility becomes

$$\begin{aligned} K^{\rm ur} = \frac{3}{2} p_f \Big\{ &[(1+\xi)^{4/3} + (1-\xi)^{4/3}] \\ &+ \frac{g^2}{12\pi^2}[(1+\xi)^{4/3} + (1-\xi)^{4/3} + 2(1-\xi^2)^{2/3}] \\ &+ \frac{g^4 \ln g^2}{768\pi^4}[(1+\xi)^{4/3} + (1-\xi)^{4/3} + 2(1-\xi^2)^{2/3}] \Big\}. \end{aligned} \tag{29}$$

Another interesting quantity to calculate is the first sound velocity, which is given by the first derivative of pressure with respect to energy density. Mathematically [11],

$$c_1^2 = \left[\frac{(1+\xi) n_q^+ \frac{\partial \mu^+}{\partial n_q^+} + (1-\xi) n_q^- \frac{\partial \mu^-}{\partial n_q^-}}{(1+\xi)\mu^+ + (1-\xi)\mu^-} \right]. \tag{30}$$

From Eq. (28), we have

$$\begin{aligned} \frac{\partial \mu^+}{\partial n_q^+} = \frac{2\pi^2}{3 p_f^2 (1+\xi)^{2/3}} \Bigg\{ 1 &+ \frac{g^2}{12\pi^2}\left[\frac{(1+\xi)^{2/3} - (1-\xi)^{2/3}}{(1+\xi)^{2/3}}\right] \\ &+ \frac{g^4 \ln g^2}{768\pi^4}\left[\frac{(1+\xi)^{2/3} - (1-\xi)^{2/3}}{(1+\xi)^{2/3}}\right] \Bigg\}. \end{aligned} \tag{31}$$

The second and last terms in the curly braces correspond to the exchange and correlation contributions, respectively. Similarly, $\partial\mu^-/\partial n_q^-$ can be obtained by replacing ξ with $-\xi$. Using $n_q^\pm$, $\mu^\pm$, and $\partial\mu^\pm/\partial n_q^\pm$ we calculate the sound velocity in terms of ξ. Numerically, for unpolarized matter, $c_1 = 0.46$ while for complete polarized matter $c_1 = 0.54$, which is below the causal value of $1/\sqrt{3} = 0.57$ at the high-density limit.

In Fig. 2 we plot the density dependencies of the incompressibility with a correlation correction. This shows that, for a higher value of the order of parameter ξ, the incompressibility becomes higher for the same value of the density. Thus, numerical values for incompressibility and sound velocity show that the equation of state for polarized quark matter is stiffer than for unpolarized matter [11].

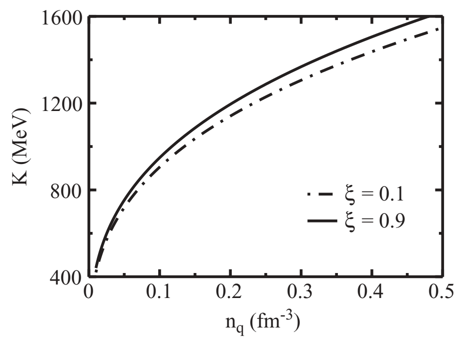

FIG. 2. Incompressibility K in the spin-polarized quark matter.

III. SUSCEPTIBILITY AT NONZERO TEMPERATURE

In this section we calculate the exchange energy density E_{ex} at low temperature ($T \ll \varepsilon_f$), for which we replace $\theta(p_f^{\pm} - |p|)$ of Eqs. (4) and (5) with a proper Fermi distribution function. In the UR limit, the angular averaged interaction parameter is given by [11]

$$f_{pp'}^{\text{ur}} = \frac{g^2}{9pp'} \int \frac{d\Omega_1}{4\pi} \int \frac{d\Omega_2}{4\pi} [1 + (\hat{p} \cdot \hat{s})(\hat{p}' \cdot \hat{s}')]. \quad (32)$$

The spin nonflip contribution to the exchange energy density is

$$\begin{aligned} E_{\text{ex}}^{\text{nf}} &= \frac{9}{2} \sum_{s=\pm} \iint \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} f_{pp'}^{\text{nf}} n_p^s(T) n_{p'}^s(T) \\ &\simeq \frac{g^2}{32\pi^4} p_f^4 [(1+\xi)^{4/3} + (1-\xi)^{4/3}] \\ &\quad + \frac{g^2}{48\pi^2} T^2 p_f^2 [(1+\xi)^{2/3} + (1-\xi)^{2/3}]. \end{aligned} \quad (33)$$

Here $n_{p(p')}^s(T)$ is the Fermi distribution function.

Similarly, E_{ex}^{f} can be evaluated. The total $E_{\text{ex}}^{\text{ur}}$ at low temperature is found to be

$$\begin{aligned} E_{\text{ex}}^{\text{ur}} &\simeq \frac{g^2}{32\pi^4} p_f^4 [(1+\xi)^{4/3} + (1-\xi)^{4/3} + 2(1-\xi^2)^{2/3}] \\ &\quad + \frac{g^2}{24\pi^2} T^2 p_f^2 [(1+\xi)^{2/3} + (1-\xi)^{2/3}]. \end{aligned} \quad (34)$$

The kinetic energy density can be written as

$$\begin{aligned} E_{\text{kin}}^{\text{ur}} &\simeq \frac{3p_f^4}{8\pi^2} [(1+\xi)^{4/3} + (1-\xi)^{4/3}] \\ &\quad + \frac{3T^2 p_f^2}{4} [(1+\xi)^{2/3} + (1-\xi)^{2/3}]. \end{aligned} \quad (35)$$

From Eq. (1) each energy contribution to the susceptibility is

$$\begin{aligned} \chi_{\text{kin}}^{-1} &= \frac{p_f^4}{6\pi^2} \left(1 - \frac{\pi^2 T^2}{p_f^2}\right), \\ \chi_{\text{ex}}^{-1} &= -\frac{g^2 p_f^4}{36\pi^4} \left(1 + \frac{\pi^2 T^2}{3p_f^2}\right). \end{aligned} \quad (36)$$

Note that the T-independent terms of these expressions are identical to those in Eqs. (14) and (15). Thus, the susceptibility at nonzero temperature is given by

$$\chi^{\text{ur}} = \chi_P \left[1 - \frac{g^2}{6\pi^2} \left(1 + \frac{4\pi^2 T^2}{3p_f^2}\right)\right]^{-1}. \quad (37)$$

In the NR limit, the interaction parameter takes the following form [1,11]:

$$f_{pp'}^{\text{nr}} = -\frac{2g^2}{9} \left(\frac{1 + s \cdot s'}{|p - p'|^2}\right). \quad (38)$$

For the spin antiparallel interaction $s = -s'$, then $f_{pp'}^{\text{nr}} = 0$. Thus, the contribution due to the scattering of quarks with unlike spin states vanishes and the dominant contribution to energy density comes from the parallel spin states ($s = s'$). By performing the angular integration of Eq. (4), the exchange energy density up to term $\mathcal{O}(T^2)$ becomes

$$\begin{aligned} E_{\text{ex}}^{\text{nr}} &= -\frac{g^2}{4\pi^4} \sum_{s=\pm} \int p dp n_p^s(T) \int p' dp' n_{p'}^s(T) \ln \left| \frac{p + p'}{p - p'} \right| \\ &\simeq -\frac{g^2}{8\pi^4} p_f^4 [(1+\xi)^{4/3} + (1-\xi)^{4/3}] \\ &\quad - \frac{g^2}{8\pi^2} T^2 m_q p_f [(1+\xi)^{1/3} + (1-\xi)^{1/3}]. \end{aligned} \quad (39)$$

The kinetic energy density is found to be

$$\begin{aligned} E_{\text{kin}}^{\text{nr}} &\simeq \frac{3p_f^5}{20\pi^2 m_q} [(1+\xi)^{5/3} + (1-\xi)^{5/3}] \\ &\quad + \frac{T^2 p_f^2}{2} [(1+\xi)^{2/3} + (1-\xi)^{2/3}]. \end{aligned} \quad (40)$$

A separate contribution from kinetic and exchange energies to susceptibility becomes

$$\begin{aligned} \chi_{\text{kin}}^{-1} &= \frac{p_f^5}{6\pi^2 m_q} \left(1 - \frac{2\pi^2 m_q T^2}{3p_f^3}\right), \\ \chi_{\text{ex}}^{-1} &= -\frac{g^2 p_f^4}{18\pi^4} \left(1 - \frac{\pi^2 m_q T^2}{2p_f^3}\right). \end{aligned} \quad (41)$$

Thus, at low temperature the susceptibility turns out to be

$$\chi^{\text{nr}} = \chi_P \left[1 - \frac{g^2 m_q}{3\pi^2 p_f} \left(1 + \frac{\pi^2 m_q T^2}{6p_f^3}\right)\right]^{-1}. \quad (42)$$

IV. SUMMARY AND CONCLUSION

In this work we derive the spin susceptibility for degenerate quark matter with corrections due to correlation contributions. Analytic expressions for susceptibility are also derived in both the UR and NR limits. It is observed that at low density susceptibility changes sign and becomes negative, thus suggesting the possibility of a ferromagnetic phase transition. In addition, we also derive single-particle energy, sound velocity, and incompressibility up to $\mathcal{O}(g^4 \ln g^2)$. As far as the equation of state is concerned, in the present model, we find that the equation of state for polarized matter is stiffer than that of unpolarized matter. We also determine the exchange energy and susceptibility at nonzero temperature of spin-polarized quark matter.

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APPENDIX

To calculate the correlation contribution to the spin susceptibility we have, from Eq. (12),

$$
\begin{aligned}
\mathcal{A}_{1L} = & -\frac{g^4 p_f^2 \sec^4\theta_E \csc^6\theta_E}{1152\pi^4\varepsilon_f^3\left(m_q^2 + p_f^2\sec^2\theta_E\right)^2} \\
& \times \ln\left\{\frac{g^2\csc^2\theta_E}{2\pi^2\varepsilon_f}[p_f - \varepsilon_f\cot\theta_E\tan^{-1}(v_f\tan\theta_E)]\right\} \\
& \times \left\{64\varepsilon_f^5\cos^2\theta_E\tan^{-1}(v_f\tan\theta_E)\left(p_f^2 + m_q^2\cos^2\theta_E\right)^2\right. \\
& - 2p_f\varepsilon_f^2\sin 2\theta_E\tan^{-1}(v_f\tan\theta_E)\left[12m_q^6 + 51m_q^4p_f^2\right. \\
& + m_q^4\left(4m_q^2 + 5p_f^2\right)\cos 4\theta_E + 68m_q^2p_f^4 \\
& \left.+ 4m_q^2\left(4m_q^4 + 10m_q^2p_f^2 + 7p_f^4\right)\cos 2\theta_E + 32p_f^6\right] \\
& + 4p_f^2\varepsilon_f\sin^2\theta_E\left[6m_q^6 + 29m_q^4p_f^2\right. \\
& + m_q^4\left(2m_q^2 + 3p_f^2\right)\cos 4\theta_E + 36m_q^2p_f^4 \\
& \left.\left.+ 4m_q^2\left(2m_q^4 + 4m_q^2p_f^2 + 3p_f^4\right)\cos 2\theta_E + 16p_f^6\right]\right\},
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
\mathcal{B}_{1T} = & \frac{g^4 p_f^2\cot^2\theta_E\csc^4\theta_E}{1152\pi^4\varepsilon_f^3\left(m_q^2\cos^2\theta_E + p_f^2\right)} \\
& \times \ln\left\{\frac{g^2\cot\theta_E\csc^2\theta_E}{8\pi^2\varepsilon_f^2}\left[2\tan^{-1}(v_f\tan\theta_E)\right.\right. \\
& \left.\left.\times\left(m_q^2\cos^2\theta_E + p_f^2\right) - p_f\varepsilon_f\sin 2\theta_E\right]\right\} \\
& \times\left\{-32\varepsilon_f^3\tan^{-1}(v_f\tan\theta_E)\left(m_q^2\cos^2\theta_E + p_f^2\right)^2\right. \\
& - 8p_f^2\varepsilon_f\left[m_q^4 + p_f^4 + m_q^2p_f^2(1+\cos^2\theta_E)\right]\sin^2 2\theta_E \\
& + 2p_f\tan^{-1}(v_f\tan\theta_E)\sin 2\theta_E \\
& \times\left[8m_q^6 + 31m_q^4p_f^2 + m_q^4p_f^2\cos 4\theta_E + 36m_q^2p_f^4\right. \\
& \left.\left.+ 4m_q^2\left(2m_q^4 + 4m_q^2p_f^2 + 3p_f^4\right)\cos 2\theta_E + 16p_f^6\right]\right\},
\end{aligned}
\tag{A2}
$$

with $v_f = p_f/\varepsilon_f$.

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