

Nuclear structure functions at small x

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I study the nuclear structure function F_2^A and its logarithmic derivative in the high-energy limit (small- x region) using the color glass condensate formalism. In this limit the structure function F_2 depends on the quark-antiquark dipole-target scattering cross section $N_F(x_{bj}, r_t, b_t)$. The same dipole cross section appears in single-hadron and hadron-photon production cross sections in the forward rapidity region in deuteron (proton)-nucleus collisions at high energy, that is, at energies available at the BNL Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC). I use a parametrization of the dipole cross section, which has successfully been used to describe the deuteron-gold data at the RHIC, to compute the nuclear structure function F_2^A and its log Q^2 derivative (which is related to gluon distribution function in the double log limit). I provide a quantitative estimate of the nuclear shadowing of F_2^A and the gluon distribution function in the kinematic region relevant to a future electron-ion collider.

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I. INTRODUCTION

The color glass condensate (CGC) formalism [1] has been quite successful in describing aspects of particle production in high-energy hadronic/nuclear collisions. It has been used to describe data from lepton-proton (nucleus) deeply inelastic scattering (DIS) to hadron multiplicities and transverse momentum spectra in nucleus-nucleus and deuteron-nucleus collisions [2]. The recently observed suppression of single-hadron p_t spectra in the forward rapidity region in deuteron-nucleus collisions at the BNL Relativistic Heavy Ion Collider (RHIC) [3] was predicted by the formalism [4]. Since then, quantitative results that describe the p_t spectrum of single-hadron production in dA collisions have become available [5–7]. Therefore it is desirable to apply the knowledge gained from dA collisions at the RHIC to other processes to further clarify/constrain the region of applicability of CGC formalism to high-energy processes [8].

In this work I consider nuclear shadowing of structure function F_2^A and its log Q^2 derivative using the results of previous work on pion production in dA collisions. In Refs. [6,9], a hybrid approach to dA collisions was used to investigate pion production in dA collisions (for an alternative approach that treats both the projectile and the target using CGC formalism, see Ref. [10]). The main result of that analysis [6] can be summarized roughly as the following: in the midrapidity region, as one goes from low to high p_t , one goes from the saturation region through the scaling region to the pQCD Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) region. The applicability of CGC formalism in the midrapidity region of the RHIC is limited to less than a few GeV in transverse momentum, above which one is in the large- x region. It is worth mentioning that the qualitative conclusion reached in Ref. [7] is different from the conclusions reached in Ref. [6] as a result of using a different parametrization of the dipole cross section that enters the particle production cross section. However, the dipole model in Ref. [7] seems to have an unphysical dependence on transverse momentum at high

p_t . Therefore, I use the parametrization of the dipole cross section in Ref. [6] (referred to as the DHJ model) in this work.

In the most forward rapidity region at the RHIC [and CERN Large Hadron Collider (LHC)], one is in the (target) small- x kinematics (p_t is limited by kinematics) but in the projectile large- x region where the quarks in the projectile contribute more than the projectile gluons (except possibly at very low p_t where they are comparable). Therefore any description of the forward rapidity data must include the effects of the projectile quarks scattering on the gluon field of the target nucleus (or proton).

Using the DHJ model of the dipole cross section employed in Ref. [6], I calculate nuclear structure functions and their shadowing. The main ingredient in the structure function F_2 is the (quark-antiquark) dipole cross section that also appears in single-hadron production cross sections in dA collisions. Specifically, I investigate how the structure function F_2^A computed using the DHJ model of the dipole cross section describes the nuclear shadowing data as measured by the New Muon Collaboration (NMC) [11]. I then make predictions for F_2^A in a broader kinematic region that may be covered by the proposed electron-ion collider (EIC). I then consider shadowing of the nuclear gluon distribution function. In a high-gluon-density environment the gluon distribution function (defined as the leading twist matrix element of gluon field operators) is not what goes into a physical cross section, rather it is n -point functions of Wilson lines. Therefore, we consider the log Q^2 derivative of the F_2^A structure function, which in the high Q^2 (double log) limit is proportional to the gluon distribution function. I then compare the ratio of these log derivatives, for a nucleus and a proton, with the results available in the literature.

II. THE STRUCTURE FUNCTION F_2

In the small- x limit, the structure function F_2 can be written as a convolution of the probability for the splitting of a virtual photon into a quark-antiquark pair with the probability

for the scattering of the quark-antiquark pair on the target so that [12]

$$F_2 = a Q^4 \int dr_t r_t \int dz N_F(x, r_t) \times [f_1(z) K_1^2(\epsilon r_t) + f_0(z) K_0^2(\epsilon r_t)], \quad (1)$$

where $a = \frac{6\sigma_0}{(2\pi)^3} \sum e_f^2$ is an overall constant, $f_1(z) = z(1-z)[z^2 + (1-z)^2]$, and $f_0(z) = 4z^2(1-z)^2$. The sum is over quark flavors (I am taking three flavors of massless quarks) and I have defined $\epsilon^2 = z(1-z)Q^2$. All the QCD dynamics in the small- x limit are encoded in the dipole cross section $N_F(x, r_t)$, which is also the building block of the single-hadron production cross section in the forward rapidity region and satisfies the JIMWLK/BK evolution equations.

It would be ideal to solve the Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner/Balitsky-Kovchegov (JIMWLK/BK) equations to obtain the form of the dipole cross section N_F and use it in Eq. (1) (see Ref. [13] for a recent analysis of the proton structure function using a solution of the BK equation with running coupling). However, the numerical solutions of JIMWLK/BK equations are known to be quite sensitive to the initial condition in the evolution equation unless one is at extremely small x and large corrections from higher order effects are expected. Therefore it is more practical to parametrize the dipole cross section in a form that captures the essential dynamics of the saturation physics. This approach has been employed before to investigate structure functions at HERA [14] and particle production at the RHIC [5,7,15].

In this work we use the parametrization proposed in Ref. [6] that was fit to the single-hadron production data in dA collisions at the RHIC at rapidity $y = 3.2$. It was then used to predict the data at rapidities $y = 0$ and $y = 4$. For a successful description of the midrapidity data one had to exclude the contribution from $x > 0.05$. This is consistent with the lessons learned from HERA where CGC motivated models cannot fit the data (for proton targets) for $x > 0.01$. Therefore, I restrict myself to the region where $x < 0.05$, which puts a severe limit on the number of data points available on nuclear shadowing at reasonably large Q^2 (I consider $Q^2 > 1 \text{ GeV}^2$ only). There was also a need for a K factor in the description of the single-hadron spectra in dA collisions that was large (in midrapidity) but p_t independent (see Ref. [16] for observables that are independent of this K factor). This is usually attributed to higher order corrections that are reduced in magnitude as one goes to the forward rapidity region as was the case in Ref. [6]. Here I do not expect a K factor because I am dealing with a fully inclusive quantity. Furthermore, I do not include the data points measured by the E665 Collaboration because there seems to be a discrepancy between the NMC Collaboration and the E665 Collaboration shadowing results when comparing structure functions for large nuclei (such as gold or lead) with that of deuteron that may be due to the systematic errors of the E665 measurement [17].

A. The dipole cross section and shadowing of F_2^A

In the DHJ parametrization [6], the fundamental (quark-antiquark) dipole scattering cross section is given by (an

overall factor of $\sigma_0 = 23 \text{ mb}$ is factored out in this expression but included in the numerical results)

$$N_F(x, r_t) = 1 - \exp \left\{ -\frac{1}{4} [r_t^2 Q_s^2(x)]^{\gamma(x, r_t)} \right\}, \quad (2)$$

where the anomalous dimension γ is given by (for details, see Ref. [6])

$$\gamma(r_t, y) = \gamma_s + \Delta\gamma(r, y) \\ \Delta\gamma = (1 - \gamma_s) \frac{\log(1/r_t^2 Q_s^2)}{\lambda y + \log(1/r_t^2 Q_s^2) + d\sqrt{y}}, \quad (3)$$

where $y \equiv \log 1/x$ and $d = 1.2$ (see Ref. [6] for details of the parametrization). Because of the anomalous dimension γ , there is a strong “leading twist” shadowing of the target gluons encoded in this formalism [18]. Using this, I compute the minimum bias structure functions for a proton and a nucleus (taken to be gold) in the kinematic region covered by the NMC experiment while keeping $x < 0.05$ and $Q^2 > 1 \text{ GeV}^2$.

My result for nuclear shadowing is shown in Fig. 1. The lower (solid) line is the ratio of F_2^A/F_2^p , where both structure functions for proton and gold are calculated using this formalism. The upper (dashed) line is the ratio F_2^A/F_2^d , where I have included an overall 10% uncertainty to account for the effect of deuteron shadowing (about 2%–3% in this kinematics [19]), the fact that the experimental data is for lead rather than gold, and the fact that my calculation of the proton structure function overestimates the HERA data by about 10% [15]. All these effects would make the calculated shadowing ratio increase. The fact that I am overestimating the proton structure function is not unreasonable because the DHJ parametrization was developed and tested for nuclear rather than proton targets. I emphasize that there is no new parameter introduced for the computation of the structure function. Alternatively, I could have divided the nuclear structure function F_2^A , calculated using this formalism, by the proton structure function measured at HERA. Because I consider the shadowing of the “gluon distribution function” to be defined as the $\log Q^2$ derivative of F_2 , I have decided to divide by the calculated value of F_2^p rather than the measured value and thus have included the upper line as a more realistic estimate of the shadowing effect. As is clear, the difference

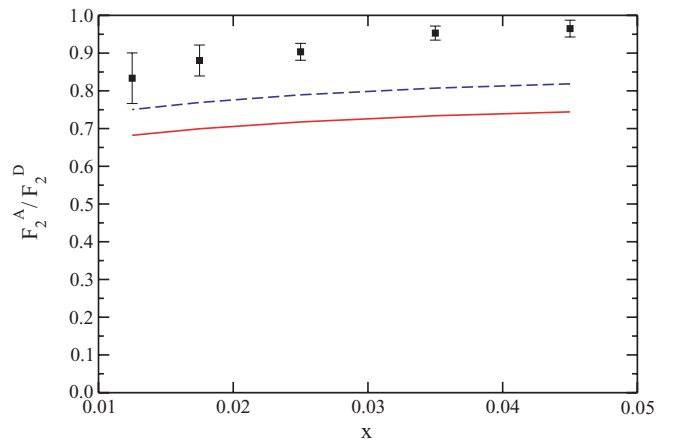
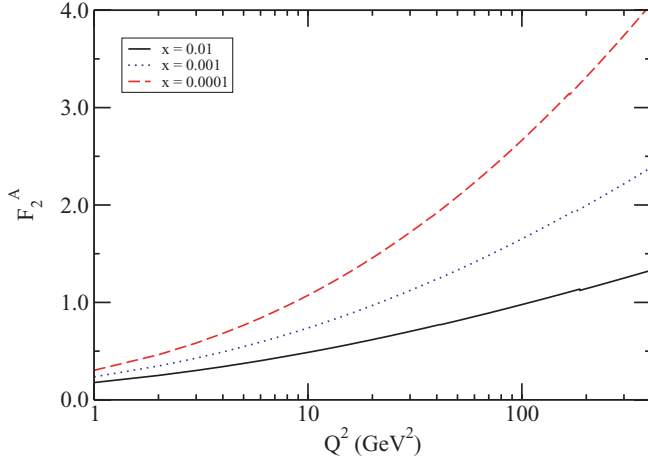


FIG. 1. (Color online) Shadowing of structure function F_2^A compared to NMC measurements.


 FIG. 2. (Color online) Nuclear structure function F_2 .

between the data points is largest (of order of 15%) at the highest x considered ($x = 0.05$) and gets much smaller (a few percent) toward the lowest x in the figure ($x = 0.01$).

In Fig. 2, I show the nuclear structure function F_2^A as a function of Q^2 at different values of x . I show my results for values of Q^2 up to 400 GeV^2 , which was an estimate for the absolute upper limit of the scaling region at HERA [20]. The analysis in Ref. [6] shows that the scaling region may actually be smaller. Furthermore, the calculation of F_2^A is more reliable in my case because I don't have to consider the proton structure function that my calculation overestimates by about 10%.

In Figs. 3 and 4, the shadowing effect for the structure function at different kinematics is shown. In both figures I plotted the ratio of F_2 for a minimum bias gold nucleus and for a proton and ignored the shadowing of deuteron structure function at small x (which may eventually be about 10%) and the fact that my calculation overestimates the proton structure function at higher Q^2 by about 10%. Therefore, the lines in both figures should be understood to be a lower limit that could be pushed up by 10%–15% in a more detailed study.

It is worth noting that one could fit the Q^2 dependence of the lines shown in Fig. 3 by a function of the form $c_0 +$

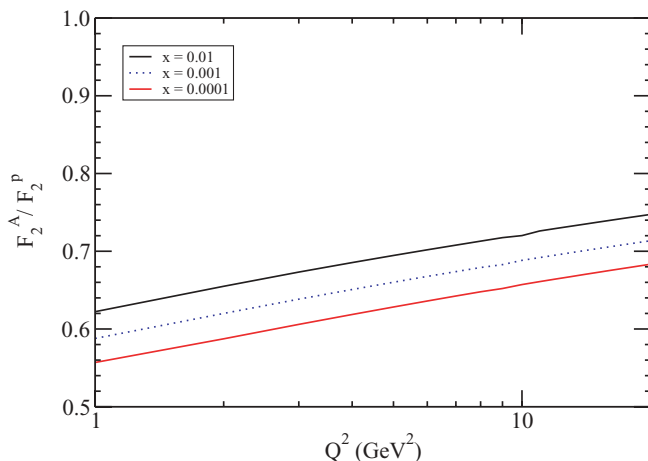
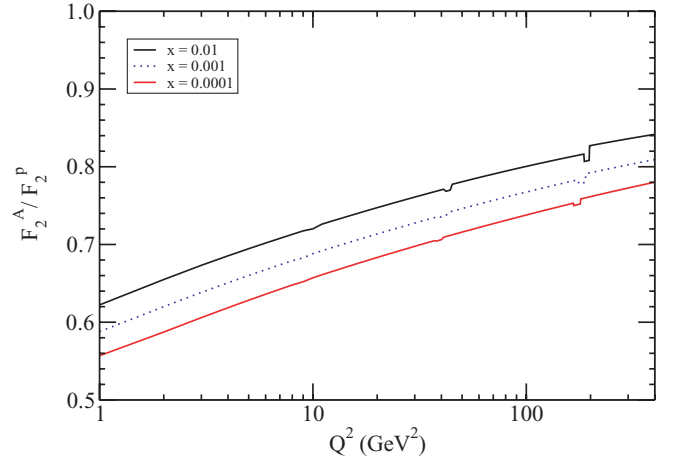

 FIG. 3. (Color online) Nuclear shadowing of structure function F_2^A .


FIG. 4. (Color online) Same as in Fig. 3 but in a larger kinematic region.

$c_1 \text{Log } Q^2/Q_s^2$, with constants c_0 and c_1 being the same (with 2% accuracy) for all three lines shown (the deviation is largest at smallest Q^2 , which may be a sign of reaching the saturation region). Whether the saturation scale appearing in the log is that of a proton or a nucleus cannot be determined from the fit because the constant c_1 is much smaller than the constant c_0 . It is tempting to take this as a sign that the shadowing in this kinematics is leading twist, consistent with the estimates of the scaling region where the dynamics is that of Balitsky-Fadin-Kuraev-Lipatov (BFKL) with a saturation boundary (we note that the saturation scale of a minimum bias gold nucleus is 1.6 GeV at $x = 3 \times 10^{-4}$ and the saturation scale for a proton is 1 GeV). However, in the much larger kinematics shown in Fig. 4, this functional form does not provide a very accurate fit, which may be a sign that we are outside the scaling region.

It would be very interesting to make analytic estimates for the extent of the scaling region in this case; however, this is a bit complicated because of the z integration in Eq. (1). In principle, the value of photon virtuality Q^2 determines whether one is in the saturation region $Q \ll Q_s$, in the scaling region $Q_s \ll Q \ll Q_s^2/\Lambda_{\text{QCD}}$, or in the pQCD region where $Q_s^2/\Lambda_{\text{QCD}} \ll Q$. The integration over the dipole size r_t can be broken up into four distinct regions:

$$\int_0^\infty r_t = \int_0^{\Lambda/Q_s^2} + \int_{\Lambda/Q_s^2}^{1/Q} + \int_{1/Q}^{1/Q_s} + \int_{1/Q_s}^\infty. \quad (4)$$

The first region is that of pQCD where $r_t Q \ll 1$ and $r_t Q_s \ll 1$. Then $N_F \sim r_t^2 Q_s^2$, $K_1^2 \sim 1/\epsilon^2 r_t^2$, and K_0^2 is subleading. In this case the r_t and z integrations can be performed analytically and are finite. A similar thing happens in the saturation region [the last piece in Eq. (4)]. In this case, $r_t Q \gg 1$, $r_t Q_s \gg 1$, and one can approximate $N_F \sim 1$ and $K_0^2 \sim K_1^2 \sim (1/\epsilon r_t) \exp(-2\epsilon r_t Q)$. However, in the other two regions, the z integration diverges as $z \rightarrow 0, 1$ and is not under control (having massive quarks would regulate this divergence). One can still perform the integrals analytically (for fixed γ) but the general form of the result is not very illuminating.

B. Shadowing of gluons

In this section nuclear shadowing of the gluons is considered. In pQCD and at the leading twist level, the gluon distribution function xG is proportional to the $\log Q^2$ derivative of F_2 structure function,

$$xG(x, Q^2) \sim \frac{d}{d \log Q^2} F_2(x, Q^2). \quad (5)$$

This relation can be used to extract the gluon distribution function experimentally in DIS. The CGC formalism, however, is not based on a twist expansion and the relevant degrees of freedom (which appear in physical cross sections) are not gluons but rather are Wilson lines, path-ordered exponentials of gluon fields that take multiple scattering on the target into account. In a typical cross section computed in the CGC formalism, one encounters n -point functions of Wilson lines (fundamental or adjoint), where in the case of the structure function F_2 only the two-point function of (fundamental) Wilson lines appears. Therefore, to provide a comparison with results obtained from other approaches based on leading twist collinear factorization (where shadowing is put in by hand in the initial condition), I consider the $\log Q^2$ derivative of the F_2 structure function and take that to be a measure of shadowing of gluons in the standard language.

Differentiating the structure function F_2 as given in Eq. (1) with respect to $\log Q^2$, one gets two terms: one comes from differentiating the overall Q^4 term and the second contribution comes from the Q^2 dependence of the Bessel functions. One gets

$$\begin{aligned} \frac{d F_2}{d \log Q^2} = & a Q^4 \int dr_t r_t \int dz N_F(x, r_t) \{ 2 f_0(z) K_0^2(\epsilon r_t) \\ & - \epsilon r_t [f_0(z) + f_1(z)] K_0(\epsilon r_t) K_1(\epsilon r_t) \\ & + f_1(z) K_1^2(\epsilon r_t) \}. \end{aligned} \quad (6)$$

Using the expression for N_F given by Eq. (2) one can now calculate the log-derivative of the structure function. In Figs. 5

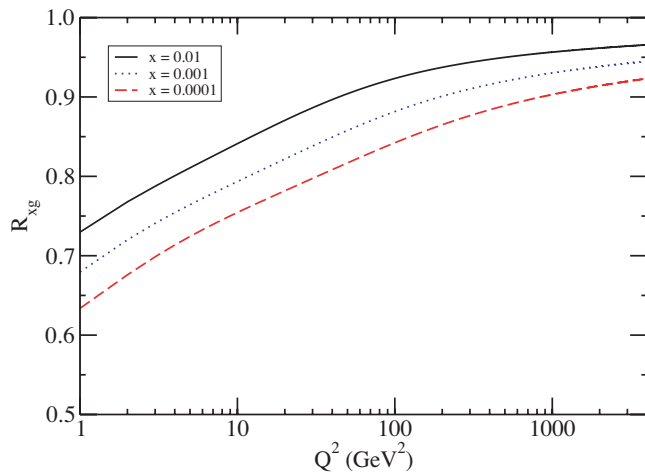


FIG. 5. (Color online) Shadowing of the gluon distribution function as defined in Eq. (7).

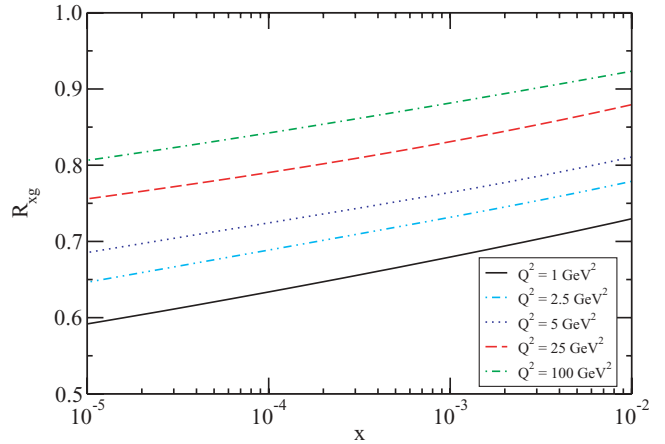


FIG. 6. (Color online) Same as in Fig. 5 but versus x .

and 6, R_{xg} is defined as

$$R_{xg} \equiv \frac{d F_2^A}{d \log Q^2} \bigg/ \frac{d F_2^p}{d \log Q^2}. \quad (7)$$

Here I have ignored the shadowing of gluons in a deuteron. I am also using my formula to calculate this log-derivative for a proton target rather than using xg from available parametrizations such as The Coordinated Theoretical-Experimental Project on QCD (CTEQ) or Martin-Roberts-Stirling-Thorne (MRST) for self consistency. Because of this the amount of shadowing may be underestimated by $\sim 10\%$ – 15% in the small- x but large- Q^2 region. This would mean, for example, in Fig. 5, that this ratio is very close to 1 at the highest Q^2 shown. At the largest x considered ($x = 0.01$), the amount of gluon shadowing is about 10% – 15% at $Q^2 \sim 10 \text{ GeV}^2$ and 25% at $Q^2 = 1 \text{ GeV}^2$. It goes away by Q^2 of several hundred GeV^2 . One should keep in mind that the shadowing results shown are most likely an overestimate (the real ratios are a bit larger) because, first, deuteron shadowing is being ignored and, second, the gluon distribution in a proton is being overestimated by possibly 10% . At smaller x and $Q^2 = 1 \text{ GeV}^2$, gluon shadowing is about 30% and again goes away at higher Q^2 . The Q^2 dependence of shadowing at fixed x is shown in Fig. 6.

III. SUMMARY

I have used the parametrization of the quark-antiquark dipole successfully employed in Ref. [6] to investigate the predictions of the model for shadowing of minimum bias nuclear structure functions. I have compared my results to the available data from the NMC Collaboration in the smallest- x region accessed in that fixed target experiment while keeping the values of Q^2 larger than 1 GeV^2 . My results are within a few percent of the data at the smallest- x bin (~ 0.0125) and are off by about 15% at the highest x considered (~ 0.045). Defining gluon distribution function as the $\log Q^2$ derivative of the F_2 structure function, I calculated the shadowing of gluons and have made predictions for its x and Q^2 dependence that could be tested in a future lepton-nucleus collider experiment.

It is interesting to compare my results with those from other approaches. Typically there are two distinct parton-based approaches (see also Ref. [21] for an alternative approach) to particle production in high-energy collisions and nuclear shadowing of structure functions. The first approach is based on leading twist collinear factorization and DGLAP evolution of distribution functions. In this approach one must put in shadowing by hand as a modification of the distribution functions at some scale Q_0^2 that are then evolved according to DGLAP equations [22–24]. Attempts to simultaneously fit the RHIC data on forward rapidity dA collisions and the nuclear structure functions seem to lead to an almost 100% shadowing of gluons in a gold or lead nucleus at small x , which seems quite unnatural [24] (this ratio seems to be higher in the most recent version EPS09 [24], which uses a NLO analysis). Comparing my results for shadowing of gluon distribution function as defined in Eq. (6) to those in Refs. [23,24], my shadowing ratio is much closer to the results of Ref. [23] than to those of Ref. [24]. For example, I predict $R_{xg} \sim 0.65$ (for a minimum bias gold nucleus) at $x = 10^{-4}$ and $Q^2 \sim 1.7 \text{ GeV}^2$, whereas this ratio in Ref. [23] (LO) is ~ 0.5 , but ~ 0 in Ref. [24] and ~ 0.5 in the most recent version in Ref. [24] (NLO). The difference in the amount of gluon shadowing between Refs. [23] and [24] (LO) is quite interesting. The drastic shadowing of gluons in Ref. [24] (there are almost no gluons in the nucleus) is caused by insisting on describing the RHIC forward rapidity data using the collinear factorization approach where nuclear shadowing is put in the initial gluon distribution by hand. Because having almost no gluons in the nucleus seems a bit unnatural, this may indicate that the leading twist collinear factorization approach to forward rapidity data at the RHIC is not valid. The amount of gluon shadowing in the NLO version is much less than the amount in the LO version but it is not clear whether the NLO version also reproduced the forward rapidity RHIC data. There has also been attempts to include the effect of the higher twist corrections to the DGLAP evolution equations [25] but these effects would break collinear factorization and make the approach practically useless for hadronic/nuclear collisions.

The second approach is based on the concept of gluon saturation and here also one can distinguish between two different approximations to the dipole cross section (see also Ref. [26] in which shadowing using scaling ideas is studied). One is motivated by the original approach of Golec-Biernat and Wusthoff, which is a Glauber-like multiple scattering approach to shadowing but using the parton language. The shadowing generated here is a higher twist (suppressed by powers of Q^2) effect. One such work is Ref. [14] (last article listed), which also investigates the impact parameter dependence of shadowing but does not include the effects of the BFKL

anomalous dimension γ and in this sense does not have leading twist shadowing. A second approach within the saturation picture includes this anomalous dimension but using a different model for the dipole cross section [15]. It is also used to fit the RHIC data in the forward rapidity data but to the best of our knowledge it has not been compared against the NMC data on shadowing of F_2 (it has been compared with the E665 data however) and shadowing of gluons is not considered.

The fact that our parameter-free calculation of the F_2 structure function is so close to the experimental values is another indication that the observed suppression of the single-hadron spectra in the forward rapidity dA collisions is due to gluon saturation dynamics. Nevertheless there are some interesting questions that have not been addressed here and need further investigation. In this context, the first and most important is probably a study of the impact parameter dependence of structure functions including BFKL dynamics. The DHJ parametrization of the dipole cross section was proposed for minimum bias dA collisions. It would be interesting to do a similar analysis for the centrality-dependent hadron spectra in dA collisions and apply the knowledge to predict the centrality dependence of the structure functions that could be measured in a future facility. This would help clarify the dynamics of shadowing, whether is it power or logarithmically suppressed. Because the magnitude of saturation scale is much larger for most central collisions, then the saturation region for central collisions extends to higher values of Q^2 as compared with minimum bias collisions where the value of Q_s^2 is not that large. This work is in progress and will be reported elsewhere [27].

One could also differentiate between different approaches (CGC versus collinear factorization inspired) to shadowing by considering two particle correlations [28]. In the CGC approach where one describes both the projectile and the target using CGC formalism, one expects a decorrelation of the two particles as the rapidity separation between the two observed particles gets larger. This is due to the small- x evolution of the gluon ladder between the produced hadrons. In the standard collinear factorized form of particle production where shadowing is put in by hand, there should be no decorrelation. Therefore, a detailed study of p_t , impact parameter, and rapidity dependence of two-particle production would go a long way toward clarifying the dynamics of shadowing of nuclear structure functions.

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