Determination of nuclear radii for unstable states in ^{12}C with diffraction inelastic scattering

A. N. Danilov,[1] T. L. Belyaeva,[2] A. S. Demyanova,[1] S. A. Goncharov,[3] and A. A Ogloblin[1]
[1]*RRC Kurchatov Institute, Moscow RU-123182, Russia*
[2]*Universidad Autonoma del Estado de Mexico, Toluca, Mexico*
[3]*Skobeltsyn Institute of Nuclear Physics, Vorob'evy Gory, Moscow, Russia*
(Received 13 August 2009; published 12 November 2009)

We propose a method for determining the nuclear radii for the excited states lying above the particle breakup threshold based on the diffraction model of scattering. The method is applied to analyzing the diffraction structure of the elastic and inelastic scattering of ^{2}He, ^{3}He, ^{4}He, ^{6}Li, and ^{12}C ions on ^{12}C at energies below 100 MeV/A. We study the radii of ^{12}C in the excited states up to $E_x \approx 11$ MeV and show that the diffraction radii for the ground and the first 2^+ (4.44 MeV) excited states are approximately the same. The diffraction radii for the 0_2^+ (7.65 MeV) Hoyle state and 3^- (9.64 MeV) states located above the $^{12}\mathrm{C} \to 3\alpha$ threshold are larger by $\approx$0.5 fm. This difference does not depend on the energy or on the kind of projectiles (deuterons are an exception). This fact justifies an application of the proposed method to the determination of the root-mean-square radii of the above-threshold states. We found that the rms radii for the 0_2^+ (7.65 MeV) and 3^- (9.64 MeV) states are a factor of 1.2 larger than the rms radius for the ground state of ^{12}C. Also, we estimated the rms radii for the above-threshold 2^+ (9.9 MeV), 0^+ (10.3 MeV), and 1^- (10.84 MeV) states.

DOI: 10.1103/PhysRevC.80.054603 PACS number(s): 24.10.Ht, 24.10.Eq, 25.55.Ci, 27.20.+n

I. INTRODUCTION

The sizes of nuclei, their charge or nucleon distributions, represent one of the important parameters determining their basic properties and are a consequence of the fundamental features of the strong interaction. To date, a number of reliable methods used to measure the radii of nuclei in the ground states have been developed, among which the most precise and widespread is the elastic scattering of electrons on stable or long-lived targets [1]. Laser spectroscopy, for instance, is used to measure the radii of nuclei in excited states with half-lives of more than 10^{-8}–10^{-9} s, in cases when these nuclei can be obtained in the form of monatomic beams.

Until recently, no method of measuring the radii of nuclear states located above the thresholds of nucleon and cluster emission with half- lives less than 10^{-12} s had been developed. Giant resonances, the excited states of exotic nuclei in the region of the drip lines, and some quasimolecular and cluster states represent numerous examples of these above-threshold states. The investigation of the properties of the short-lived states in nuclei may give rise to the observation of novel phenomena and the development of nuclear structure theory. In addition to its role in nuclear spectroscopy, the study of nuclear radii is of great importance to nuclear astrophysics. Nucleosynthesis mainly occurs by the excitation of intermediate states lying above the thresholds of fusion of the colliding nuclei. Direct measurements of fusion cross sections at low energies of astrophysical interest are extremely difficult or impossible. Corresponding calculations are very sensitive to the precise values of the radii of the nuclei, because their variation can change the result in the order of magnitude.

The problem of measuring the radii of nuclei in the unbound states has attracted plenty of attention in the last decade in connection with a hypothesis of the possible existence of α-particle Bose-Einstein condensation (αBEC) in finite nuclei [2]. Corresponding nuclear states are expected to be dilute systems of almost unperturbed α particles with zero relative angular momentum $L = 0$ and are located closely to the thresholds of complete dissociation to α particles: $A \to n\alpha$. Nuclear radii for the αBEC states are estimated to be a factor of 1.4–1.7 larger than those for the ground state [2].

An intensive study of four-nucleon correlations of the α-cluster type initiated more than 50 years ago [3] established their important role in nuclei. The microscopic α-cluster models [4–8] have succeeded in describing the structure of many states in light nuclei, in particular, around the threshold energy of breakup into constituent clusters. Considerable attention has been drawn to the studies of α-cluster states in ^{12}C, especially the second 0^+ state, located at $E_x = 7.65$ MeV, which is 0.38 MeV above the 3α threshold. As early as 1954, Hoyle showed [9] that this level plays an extremely important role in nucleosynthesis. The properties of the Hoyle state in ^{12}C determine the ratio of carbon to oxygen formed in the stellar helium burning process that strongly affects the future evolution of stars. Detailed analyses of the structure of ^{12}C with the microscopic 3α cluster model [10,11] was made about 30 years ago. The 3α generator coordinate method (GCM) [10] and 3α resonating group method (RGM) [11] calculations showed that the 7.65 MeV 0_2^+ state in ^{12}C has a loosely coupled 3α structure and an enlarged radius. Modern microscopic calculations in the framework of cluster models such as the antisymmetrized molecular dynamics [12] and the fermionic molecular dynamics (FMD) [13] also predict an increased radius of this above-threshold cluster state. Much recent attention has been focused on experimental studies of the α-cluster structure of the excited states in ^{12}C as well as of the neighboring loosely bound ^{14}C and ^{10}Be nuclei [14–17].

In the context of the αBEC hypothesis, the 7.65 MeV level in ^{12}C is considered to be the simplest example of the α-condensed state playing the role of a test for the whole problem. Although the direct measurement of the radius of this short-lived nuclear state is impossible because of its very short half-life [$\tau_{1/2}\,(0^+,\ 7.65\ \text{MeV}) \simeq 2\times 10^{-16}$ s], some information on enhanced dimensions of Hoyle's state could be derived in various ways. Kokalova and collaborators [18] observed enhanced emission of ^{12}C in the 7.65 state from the compound nuclei, which was interpreted as a result of the lowering of the Coulomb barrier due to the larger size of this level. Ohkubo and Hirabayashi [19] analyzed the differential cross sections of ^{3}He and α particles to the 7.65 MeV state in ^{12}C measured at different energies and found that the shifts of rainbow (Airy) minima point to the enlargement of the nuclear radius in this state. Takashina and Sakuragi [20] analyzed the α-particle inelastic scattering on ^{12}C exiting the 0_2^+ state at 7.65 MeV in the framework of the microscopic coupled-channels approach [19] with the density distribution of ^{12}C given by the α-condensate model calculations [21]. The authors found that one can determine the extension of the transition density rather than the nuclear radius of the excited state from the oscillation pattern of the inelastic angular distribution. However, the nuclear radius of the excited state can be deduced from the absolute value of the inelastic differential cross sections through the amplitude of the transition density. Ogloblin *et al.* [22] pointed out that inelastic form factors used to describe the differential cross sections of α scattering at 139 MeV [23] and ^{3}He scattering at 72 MeV [24] to the 7.65 MeV state were taken as a second derivative d^2V/dr^2 of the real part of the optical potentials, because the use of standard form-factors as a first derivative dV/dr failed to get any reasonable agreement with the data. This form factor qualitatively agrees with one predicted by cluster model calculations [11], because both form factors have minima at the same radial distance and a large part of them are located at large distance. Chernykh and collaborators [13] compared the electron scattering data on form factors of the ground state and the transition to the Hoyle state within the three-cluster FMD model. The authors indicated a dilute density of the Hoyle state, which has a large spatial extension estimated to be a factor of ~1.5 larger than that of the ground state. An analysis made by Khoa [25] demonstrated an increase of the absorption in the exit channels for some of the ^{12}C states above the threshold.

As great amounts of existing data suggest a considerable enhancement of the Hoyle state size, it can serve as a good object for testing the methods of measuring the radii of the unstable states. We have proposed two such methods based on the use of inelastic diffraction and rainbow scattering. Preliminary results from the application of the diffraction method to the analysis of the new data of $\alpha + {}^{12}$C inelastic scattering are presented in Refs. [26,27]. This method has demonstrated the vitality and ability to be widely used in view of its clarity and direct relation with the experimental data. A short summary and comparison of the diffraction and rainbow methods in the inelastic α particles and ^{3}He scattering are given in Ref. [28].

In this paper, we present a detailed study of the diffraction structure of the elastic and inelastic scattering of ^{2}H, ^{3}He, ^{4}He, ^{6}Li, and ^{12}C ions on ^{12}C at energies below 100 MeV/A. The analysis is based on the diffraction model of scattering and aims to determine the nuclear radii for the excited states lying above the ^{12}C breakup threshold, mainly the 0_2^+ (7.65 MeV) Hoyle state. We formulate the conditions for the applicability of the method and extract the diffraction and root-mean-square radii of ^{12}C in the excited states by comparing the inelastic scatterings of different light ions.

II. DIFFRACTION MODEL AND DIFFRACTION RADII

A. Input data

We analyzed the $\alpha + {}^{12}$C elastic and inelastic (to the 4.44 MeV 2^+, 7.65 MeV 0^+, and 9.64 MeV 3^- states) scattering data at beam energies of 60 [26], 110 [26,27], 104 [29], 139 [23], 166 [30], 172.5 [31], and 240 [32] MeV. The resolution of the experiments mentioned above (about 700 keV) allowed us to clearly distinguish the 7.65 MeV state. To estimate the sizes of the high-lying states in ^{12}C, we included the data of inelastic α-particle scattering at 240 MeV [32] ($E_x = 10.3$ and 10.84 MeV) and the data obtained in Refs. [33,34] at α-particle energy 388 MeV ($E_x = 9.9$, 10.3, and 10.84 MeV). These three states could give some contributions to a single peak corresponding to the 3^- state at 9.64 MeV. Basing our calculations on the data [33] and taking into account the large widths of the first two levels, we estimated the possible contribution in the maximum of the angular distribution corresponding to the formation of the 9.64 MeV state to be no more than 5–10%.

The available data on ^{3}He + ^{12}C for four lower states in ^{12}C were explored at $E_{\text{lab}}(^3\text{He}) = 34.7$ [35], 50 [36], 60 [36], 72 [24], and 82 [37] MeV.

We also used data of the elastic and inelastic scattering of the ^{2}H + ^{12}C at $E_{\text{lab}} = 52$ [38], 60.6, 77.3, 90 [39], and 200 [40] MeV, the ^{6}Li + ^{12}C scattering at 124 and 169 MeV [41], and the ^{12}C + ^{12}C scattering at 120 [42], 139.5, and 159 MeV [43].

B. Application of the diffraction model of scattering

The experimental differential cross sections of elastically and inelastically scattered light heavy ions at energies 10–100 MeV/A clearly reveal a characteristic region of the small-angle Frauenhofer diffraction. Simple formulas for the inelastic scattering differential cross sections were obtained [44] under the assumption that an adiabatic approximation is valid; that is, the excitation energy of the level is much smaller than the initial energy. The shapes of the angular distributions are determined by combinations of the Bessel functions $J_l(x)$, that is,

$$\begin{gathered}\frac{d\sigma}{d\Omega}(\text{el}) \sim \left|\frac{J_1(x)}{x}\right|^2, \quad \frac{d\sigma}{d\Omega}(0\to 0) \sim J_0^2(x),\\ \frac{d\sigma}{d\Omega}(0\to 2) \sim \left[\frac{1}{4}J_0^2(x) + \frac{3}{4}J_2^2(x)\right],\\ \frac{d\sigma}{d\Omega}(0\to 3) \sim \left[\frac{3}{8}J_1^2(x) + \frac{5}{8}J_3^2(x)\right],\end{gathered} \tag{1}$$

of different order from the argument $x = qR$, where q is the linear transferred momentum and R is the radial parameter.

The Blair phase rule following from Eqs. (1) defines a set of phase relations between the shapes of the elastic and inelastic angular distributions depending on the transferred angular momentum l values: the even l angular distributions are out of phase with the odd $-l$ ones.

In our applications, this method deals with only one parameter, the diffraction radius R_{dif}, extracted directly from the positions of the first small-angle minima (maxima), which are considered as zeros (maxima) of the squared Bessel functions of the $qR = qR_{\mathrm{dif}}$ argument. The diffraction of particles at small angles appears because of the absorption of the incident flux by the nuclear field, and it is defined by the imaginary part of the optical model potential. Because of that, the parameter R_{dif} is close to the strong absorption radius.

Our aim is to establish the connection between the diffraction radius and "real" (say, rms radii $\langle R\rangle$) of the colliding nuclei in the states involved. The diffraction radius R_{dif} for the elastic scattering of the $A_1 + A_2$ system can be defined as a sum of the real radii $\langle R\rangle$ plus some value Δ_{el}, which is determined by the peculiarities of the interaction

$$R_{\mathrm{dif}}(\mathrm{el}) = \langle R_{A_1}\rangle + \langle R_{A_2}\rangle + \Delta_{\mathrm{el}}. \tag{2}$$

For inelastic scattering, we can write the similar relation

$$R_{\mathrm{dif}}(\mathrm{in}) = \langle R_{A_1}\rangle + \langle R^*_{A_2}\rangle + \Delta_{\mathrm{in}}. \tag{3}$$

Assuming that the real radii $\langle R_{A_1}\rangle$ and $\langle R_{A_2}\rangle$ of the colliding nuclei in their ground states are known, which usually is the case, we found that the nuclear radius in the excited state $\langle R^*_{A_2}\rangle$ is

$$\langle R^*_{A_2}\rangle = \langle R_{A_2}\rangle + [R_{\mathrm{dif}}(\mathrm{in}) - R_{\mathrm{dif}}(\mathrm{el})] + [\Delta_{\mathrm{el}} - \Delta_{\mathrm{in}}]. \tag{4}$$

The application of Eqs. (2)–(4) is possible if the following conditions are fulfilled for the particular projectile-target combination:

(i) The incident energy is much greater than the excitation energy of the state under consideration (the adiabatic approximation).

(ii) The diffraction radius R_{dif} for a particular projectile-target combination does not depend on the order of the observed minimum or maximum, which proves a diffraction nature of the observed extremes.

(iii) The difference of the diffraction radii in Eq. (4) does not depend on the energy and concrete projectile-target combination. This fact proves that Eqs. (2)–(4) do not have a formal character and justifies the application of the proposed method for the determination of the rms radii.

(iv) The values of Δ that are measured directly in the elastic scattering (Δ_{el}) can be predicted for the inelastic scattering (Δ_{in}). This statement is crucial to this method of determination of nuclear radii in the excited states. Here we chose the model in which $\Delta_{\mathrm{el}} = \Delta_{\mathrm{in}}$ (see discussion below).

The first condition in our analysis is fulfilled, because even at the lowest energy, 34.7 MeV for the $^3\mathrm{He} + {}^{12}\mathrm{C}$ system, the ratio $E_{\mathrm{c.m.}}/7.65$ is 3.6.

The second condition is fulfilled, because the diffraction radius calculations are realized using well-manifested minima and maxima (not higher than the fourth order) of the angular distributions. As an example, the differential cross sections of the $\alpha + {}^{12}\mathrm{C}$ scattering at 110 and 172 MeV are shown in Fig. 1, and in Table I the minima and maxima and corresponding diffraction radii are displayed.

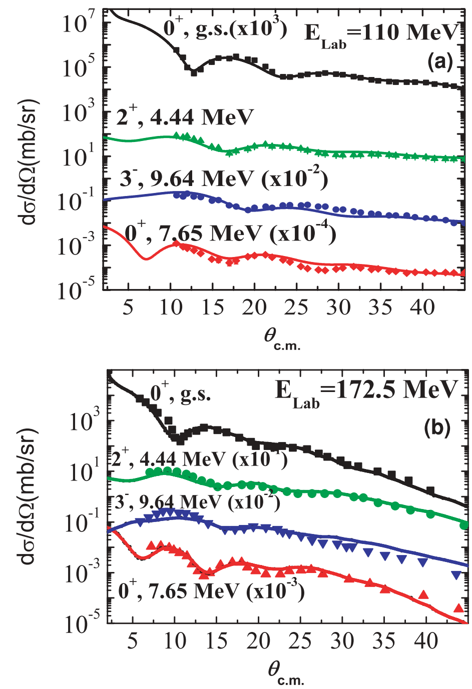

FIG. 1. (Color online) Differential cross sections of the $\alpha + {}^{12}\mathrm{C}$ scattering at (a) 110 [28] and (b) 172.5 MeV [31]. The solid lines represent corresponding distorted-wave Born approximation calculations. (Angles in units of degrees.)

We did not find any substantial or systematic dependence of R_{dif} values from the sequence number of the extremum, and the observed variations of R_{dif} are fully connected with the accuracy of the angles determination.

The justification of the third and fourth conditions is discussed in the following section.

C. Diffraction radii for the $^{12}\mathrm{C}$ states

Diffraction radii pertaining to the first four states of $^{12}\mathrm{C}$ are shown in Fig. 2 ($\alpha + {}^{12}\mathrm{C}$), Fig. 3 ($^3\mathrm{He} + {}^{12}\mathrm{C}$ and $^2\mathrm{H} + {}^{12}\mathrm{C}$), and Fig. 4 ($^6\mathrm{Li} + {}^{12}\mathrm{C}$ and $^{12}\mathrm{C} + {}^{12}\mathrm{C}$). The errors indicated in Figs. 2–4 are the result of averaging the diffraction radii for minima and maxima of various orders. In most cases, deviations from the average values do not exceed 0.10–0.15 fm. Let us note the characteristic features of the diffraction radii.

First, they smoothly (almost linearly) decrease with energy. The energy dependences of the diffraction radii for the elastic scattering as well as for the inelastic one with the formation of the four states surveyed are approximately the same.

Second, for all combinations of nuclei and all energies, the diffraction radii of the ground and first excited (4.44 MeV) states are practically the same (the only deviation is observed for the level 2^+ at the maximum energy of α particles).

Third, and this is the main result, the diffraction radii for the Hoyle state in all cases are greater than R_{dif} for the ground

TABLE I. Positions of minima and maxima of the differential cross sections of $\alpha + {}^{12}C$ scattering at 110 and 172.5 MeV and the corresponding diffraction radii.

E_x (MeV), J^π	θ_{lab} (deg)	R_{dif} (fm)	$\langle R_{dif}\rangle$ (fm)
At 110 MeV:			
0.00, 0^+	12.95	4.87	4.87 ± 0.02
	17.68	4.82	
	24.0	4.90	
	28.74	4.90	
4.44, 2^+	17.0	5.16	4.98 ± 0.07
	22.4	5.12	
	30.0	4.77	
	34.2	4.89	
	41.0	4.89	
7.65, 0^+	16.9	5.49	5.38 ± 0.09
	21.1	5.61	
	27.8	5.31	
	32.4	5.39	
	40.0	5.09	
9.64, 3^-	19.2	5.80	5.64 ± 0.16
	25.5	5.48	
At 172.5 MeV:			
0.00, 0^+	10.0	5.03	4.82 ± 0.14
	20.6	4.55	
	13.9	4.89	
4.44, 2^+	15.3	4.56	4.71 ± 0.06
	24.1	4.70	
	9.0	4.72	
	18.7	4.86	
7.65, 0^+	8.5	5.96	5.45 ± 0.10
	13.6	5.42	
	17.2	5.45	
	21.1	5.52	
	25.5	5.40	
9.64, 3^-	16.2	5.46	5.48 ± 0.02
	20.1	5.49	

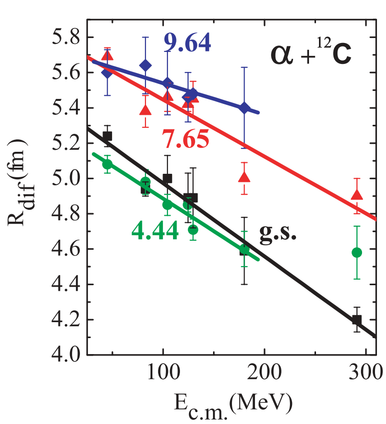

FIG. 2. (Color online) Energy dependence of diffraction radii extracted from the $\alpha + {}^{12}C$ elastic and inelastic to the 2^+ (4.44 MeV), 0^+ (7.65 MeV), and 3^- (9.64 MeV) scattering. The error bars reflect uncertainties in determination of the positions of the corresponding minima (maxima). The solid curves represent linear approximations of the data.

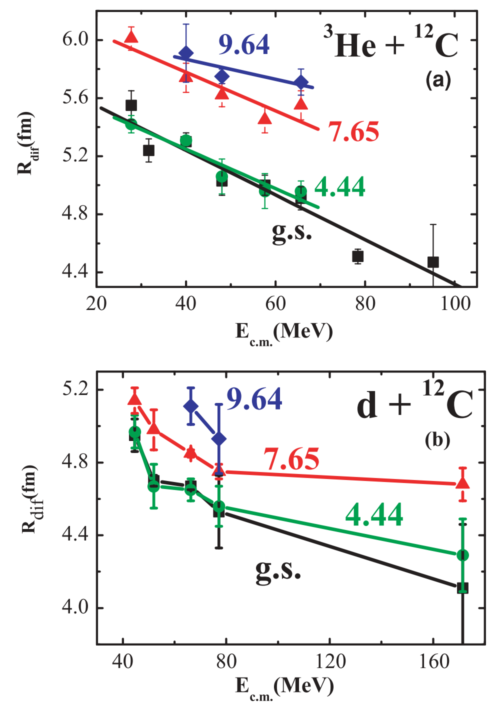

FIG. 3. (Color online) Same as Fig. 2, but for the (a) $^{3}He + {}^{12}C$ and (b) $d + {}^{12}C$ scattering.

state. At the same time, as shown in Figs. 2–4, the differences $R_{dif}(7.65)$–$R_{dif}(0.00)$, within the errors, do not depend on the energy up to $E(\alpha) \sim 100$ MeV/nucleon, where the use of the diffraction model is not quite adequate because of the increasing transparency of the nuclei.

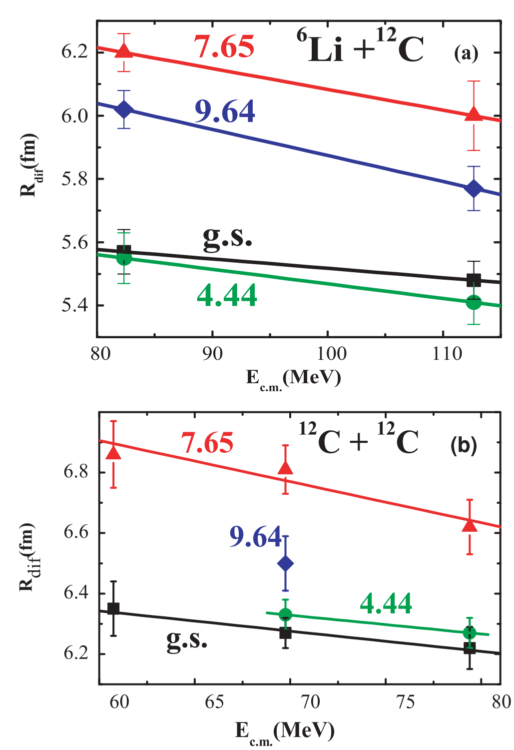

FIG. 4. (Color online) Same as Fig. 2, but for the (a) $^{6}Li + {}^{12}C$ scattering and (b) $^{12}C + {}^{12}C$ scattering.

TABLE II. Differences of the diffraction radii between the 7.65 MeV 0^+ state and the ground state obtained from the different reactions and averaged on energies.

	^{3}He + ^{12}C	α + ^{12}C	^{6}Li + ^{12}C	^{12}C + ^{12}C	d + ^{12}C
$\langle R_{\rm dif}(7.65) - R_{\rm dif}({\rm g.s.})\rangle$ (fm)	0.56 ± 0.06	0.57 ± 0.03	0.58 ± 0.06	0.47 ± 0.05	0.29 ± 0.07

Table II shows the energy-averaged values of these differences for all five systems of nuclei studied. One can see that the errors are small. Nevertheless, one should bear in mind that the overall accuracy of the results obtained from the data on ^{6}Li and ^{12}C scattering is lower than for the ^{3}He and α-particle scattering because of the small number of energy values available for analysis. The absolute values of the differences do not depend on the type of reaction. The only exception may be the data obtained from the scattering of deuterons.

The diffraction radius for the 9.64 MeV state in all the cases also proved to be larger than that of the ground state (see Figs. 2–4). For the scattering of light particles (^{4}He, ^{3}He, and ^{2}H), $R_{\rm dif}$ (9.64) is approximately equal to $R_{\rm dif}$ (7.65), or even a little bit bigger (within the errors). $R_{\rm dif}$(9.64) extracted from the ^{6}Li + ^{12}C and ^{12}C + ^{12}C scattering is slightly less than $R_{\rm dif}$(7.65). However, because of the small number of data for the latter, one cannot say this definitively.

III. RESULTS AND DISCUSSION: "REAL" RADII

A. Model determination of rms radii of ^{12}C

The transition from the diffraction radii of excited states to the "real" radii requires the use of certain models. Our model is defined by Eqs. (2)–(4). In accordance with the condition $\Delta_{\rm el} - \Delta_{\rm in} = 0$, the values of Δ disappear in expression (4). Nevertheless, their erratic behavior would make a procedure of the real radii determination unreliable. From the physical point of view, the Δ values are determined by the finite-range nuclear forces, the dynamics of interaction between the colliding nuclei, and their nucleon distributions. Theoretical calculations of Δ, especially for inelastic scattering, is hardly possible.

For elastic scattering, we have determined $\Delta_{\rm el}$ energy dependencies, according to Eq. (2), for various combinations of colliding nuclei. The Coulomb corrections [45]

$$R_{\rm dif} = \frac{\eta}{k} + \left[(R'_{\rm dif})^2 + \left(\frac{\eta}{k}\right)^2\right]^{1/2} \qquad (5)$$

were included in the calculation of the diffraction radii. In Eq. (5), $\eta/k = Z_1 Z_2 e^2/(2E)$, and $R_{\rm dif}$ and $R'_{\rm dif}$ are, respectively, the corrected diffraction radii and the extracted ones directly from the observable minima and maxima.

We found that the values of $\Delta_{\rm el}$ behave quite systematically and smoothly (for ^{3}He and α particles almost linearly) decrease with energy, as shown in Fig. 5, where these data and the data for some other combinations of colliding nuclei are included. This behavior of $\Delta_{\rm el}$ is not surprising, since the absorption depends mostly on the number of nucleons in the zone of interaction and on the nucleon-nucleon cross section. The latter decreases with energy. There is also some dependence on the mass (size) of the colliding nuclei. For loosely bound nuclei (deuterons, ^{6}Li), $\Delta_{\rm el}$ values at high energies become negative. This means that in these cases, a diffraction occurs at a distance between the colliding nuclei less than the sum of their rms radii.

There is no reason to expect that in inelastic scattering these dependencies will be significantly different. The almost the same energy dependence of diffraction radii of the ground state and as that of the 4.44 and 7.65 MeV states confirms this (see, for instance, Figs. 2–4). Based on the empirical result obtained in the previous section, which shows the independence of the diffraction radii differences from the specific reaction and energies, we have chosen the simplest model, which assumes that in inelastic scattering, Δ remains the same as in elastic scattering, namely, $\Delta_{\rm el} = \Delta_{\rm in}$. A rigorous theoretical justification of this hypothesis is impossible; however, it can have rather wide practical application. Here are some additional arguments in its favor.

A good description of experimental data by Eq. (1) and, in particular, the implementation of the Blair phase rule well established for a large number of different projectile-target

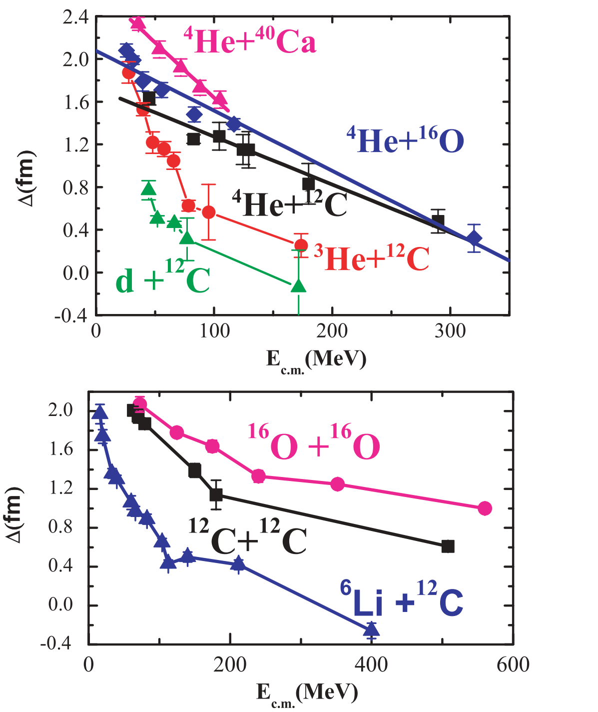

FIG. 5. (Color online) Dependence of Δ on energy in a center-of-mass system for various combinations of colliding nuclei.

combinations in the reactions with excitation of the low-lying states (see, for example, Ref. [45]), implicitly assumes equal diffraction radii for the reference states. In these studies, there was no reason to suggest that the real radius of an excited state differs significantly from the radius of the ground state. Therefore, in all these cases, for each pair of nuclei, it could be considered that $\Delta_{el} = \Delta_{in}$. Our analysis, which demonstrates the approximate equality of diffraction radii for the ground and first excited (4.44 MeV) states, also confirms this conclusion.

The situation could change in the case of the high-lying and above-threshold excited state, which can have enlarged radii. Strong absorption and, consequently, diffraction structure of the cross sections arise from the relatively small overlap of nuclear densities (e.g., a few percent in the scattering of $^{16}O + ^{208}Pb$ [45]). For a given form of the radial nucleon density distribution, there is a rigid connection between the real radius and the distance at which the density reaches a certain value. Thus, the distance between the half-density radius $R_{1/2}$ and a radius of 10% density for the Fermi distribution is $2.2a$, where a is a diffusion parameter. However, if a large change in radius is accompanied with changes in the radial distribution, the value of Δ might not remain constant at the transition from elastic to inelastic scattering. This situation cannot be excluded for the states in which the nucleon density is very different from the normal one, as, for example, the Hoyle state, or for nuclei with neutron halos.

Because the density distribution in the excited state is not known *a priori*, the feasibility of a hypothesis of equality of $\Delta_{el} = \Delta_{in}$ can be tested only empirically. The proximity of the results obtained in the scattering of nuclei with very different density distributions (^{3}He, ^{4}He, ^{6}Li, ^{12}C) shows that in these cases, the equality $\Delta_{el} = \Delta_{in}$ is at least approximately is fulfilled. A somewhat different result was obtained for the scattering of deuterons (Fig. 3, Table II), although it almost does not go beyond the errors. The measured diffraction radius for deuteron scattering was found to be on an average of 0.2 fm less than in other reactions. This difference is naturally attributed to the inequality $\Delta_{el} > \Delta_{in}$ in Eq. (4). If so, the diffraction inelastic scattering of deuterons occurs with a stronger overlap of nuclear densities than for the elastic one.

A radical experimental test of the applicability of the diffraction method in general would be the measurement of the radius of the Hoyle state using other independent models. The above-mentioned rainbow scattering can serve this purpose. The first comparison [28] led to very similar results. A more detailed description of the rainbow method will be provided in a separate publication [46].

Determination of the radii of nuclear states by diffraction scattering is a more direct method for estimating the real radii of nuclei than the calculations based on the optical model. The results of optical model analysis, of course, contain information about the size of colliding nuclei, but only implicitly. Under favorable conditions, it can identify the radii of form factors and potentials but not the radii of nuclei. In addition, the determined values strongly depend on the completeness of experimental data and the quality of fit using several free parameters. In using the diffraction model, the radii of the excited states are extracted directly from experimental data, that is, from the minima and maxima of angular distributions in the forward angles. The model includes only one parameter, $\Delta_{el} = \Delta_{in}$, which is determined from experimental data on elastic scattering. At the same time, this approach does not pretend to reproduce the entire curve of the differential cross section for either elastic or inelastic scattering, which makes it very convenient for quick estimations.

TABLE III. Root-mean-square radii (in fm) of ^{12}C in the excited states obtained from the $^3He + ^{12}C$, $\alpha + ^{12}C$, $^6Li + ^{12}C$, and $^{12}C + ^{12}C$ scattering data.

	E^*(MeV), $I^{\pi}_{^{12}C}$		
	4.44, 2^+	7.65, 0^+	9.64, 3^-
$^3He + ^{12}C$	2.38 ± 0.04	2.90 ± 0.06	3.10 ± 0.09
$\alpha + ^{12}C$	2.38 ± 0.07	2.91 ± 0.03	3.00 ± 0.06
$^6Li + ^{12}C$	2.30 ± 0.03	2.92 ± 0.06	2.71 ± 0.08
$^{12}C + ^{12}C$	2.37 ± 0.03	2.81 ± 0.05	2.57 ± 0.10
$\langle R_{rms} \rangle$	2.36 ± 0.04	2.89 ± 0.04	2.88 ± 0.11

Substituting measured diffraction radii (Table II and Figs. 2–4) and the rms radius of ^{12}C in the ground state, $\langle R_{g.s.} \rangle = 2.34$ fm, in Eq. (4), we determine the rms radii of the excited states of ^{12}C averaged for all reactions. The resulting radii are shown in Table III. Data on deuteron scattering were not used here.

Thus, the rms radius of ^{12}C in the first excited state 2^+ is the same, $\langle R(2^+) \rangle = 2.36$ fm, as in the ground state, $\langle R_{g.s.} \rangle = 2.34$ fm. This result is in good agreement with the calculations made in different works, as shown in Table IV. The radius of ^{12}C in the Hoyle state was found to be 1.2 times larger than that for the ground state. It is less than the estimations provided by various theoretical models.

B. Radii of ^{12}C in high-lying above-threshold states

We tested the proposed diffraction method by determining the radius of the unstable 7.65 MeV Hoyle level in ^{12}C and confirmed its enlarged size in accordance with theoretical predictions and existing indirect experimental data. Thus, this result can be considered as a justification of the chosen method. Now we apply it to the determination of the radii of some other high-lying above-threshold states.

TABLE IV. Root-mean-square radii (in fm) of the nucleus ^{12}C obtained in this work and predicted by different theoretical models.

E^*(MeV), $I^{\pi}_{^{12}C}$	This work	[2]	[11]	[12]	[13]	[47]
0.00, 0^+_1	2.34	2.40	2.40	2.50	2.39	2.44
4.44, 2^+	2.36 ± 0.04	2.38	2.38	2.70	2.50	2.45
7.65, 0^+_2	2.89 ± 0.04	3.83	3.47	3.3	3.38	4.31
9.64, 3^-	2.88 ± 0.11		2.78		3.02	2.96

As already mentioned, the radius of ^{12}C in the 9.64 MeV 3^- state has also been abnormally large. Averaged over all investigated combinations of nuclei shown in Table III, the rms radius was found to be $\langle R(3^-)\rangle = 2.88 \pm 0.11$ fm. The error bar is larger than in the case of the Hoyle state. Because of the small amount of data on the scattering of ^{6}Li + ^{12}C and ^{12}C + ^{12}C systems, it is premature to assume that there exists weak dependence of the radius of the 3^- state on a particular nuclear reaction. In addition, some of the cross sections of the inelastic scattering on the 3^- state can contain contaminants from the transitions to the unresolved states, as noted in Sec. II A. As shown in Table IV, our empirical results agree with the microscopic calculations of Kamimura [11] and Yamada and Schuck [47], as well as with the molecular dynamics calculations in Ref. [13], all of which predicted some enlargement of the radius of the 3^- state. According to Refs. [48,49], this state has the equilateral triangular 3α spatial configuration in contrast to the Hoyle state.

Of great interest is the question of the radii of other above-threshold states of ^{12}C, the structure and sizes of which have attracted much attention in the last few years. The theoretical calculations based on the microscopic 3α cluster models [7,10,11] suggested the existence of a 2_2^+ state of ^{12}C at around $E_{3\alpha} \sim 3$ MeV above the 3α threshold ($E_x \sim 10$ MeV). The interest was enhanced by the suggestion that this state is a member of the β-oscillation band based on the 0_2^+, 7.65 MeV state. It was suspected that the structure of the 2_2^+ state is similar to the 0_2^+ state, and that this state also would have an enlarged radius similar to that of the 7.65 MeV state. Recently the 2_2^+ state was observed in ^{12}C at $E_x = 9.9 \pm 0.3$ MeV with the α decay width $\Gamma_\alpha = 1.0 \pm 0.3$ MeV in the high-resolution measurements of α + ^{12}C inelastic scattering at 386 MeV [33,34]. "Condensate" calculations [47] suggested that the 2_2^+ level should have a very large rms radius, $\langle R\rangle = 6.12$ fm. The differential cross sections of the inelastic scattering of α particles on ^{12}C with excitation of the 0^+, 10.3 ± 0.3 MeV and the 1^-, 10.84 MeV states were measured at $E_\alpha = 240$ [32] and 386 [33] MeV.

Unfortunately, the existing data are insufficient for conducting the same detailed analysis as for the lower states in ^{12}C. Because the data were obtained at only a few energies and the analysis includes only a limited number of minima (maxima), the diffraction radii shown in Table V should be regarded only as estimations (especially those obtained from $E_{\mathrm{lab}} = 388$ MeV). This remark also applies to the rms radii. It appears that all three levels given in Table V have roughly the same radii as the 7.65 and 9.64 MeV states.

TABLE V. Diffraction radii R_{dif} and rms radii $\langle R\rangle$ of ^{12}C for the states at $E_x = 9.9$, 10.3, and 10.84 MeV obtained from data on α particles scattering at $E_{\mathrm{lab}} = 240$ and 388 MeV. The "No. min/max" is the number of minima (maxima) following from the oscillation pattern of the inelastic angular distribution used for the determination of the diffraction radii. Errors are defined by the precision of identification of their positions.

E_x (MeV)	$I^\pi_{^{12}\mathrm{C}}$	240 MeV			388 MeV		
		No. min/max	R_{dif} (fm)	$\langle R\rangle$ (fm)	No. min/max	R_{dif} (fm)	$\langle R\rangle$ (fm)
9.9	2^+				4	5.17 ± 0.23	3.2
10.3	0^+	4	4.84 ± 0.12	2.7	1	4.66 ± 0.13	2.6
10.84	1^-	1	5.13 ± 0.61	3.1	1	5.05 ± 0.22	2.9

IV. CONCLUSION

A method for determining the nuclear radii for the excited states lying above the particle breakup threshold is proposed. The main idea of the method is to directly determine from the observable minima and maxima of the diffraction angular distributions the radial parameter of the interaction, the diffraction radius, and associate it with the rms radii of the nuclei involved in the reaction. The second 0^+ state in ^{12}C at $E_x = 7.65$ MeV, the so-called Hoyle state, was chosen as the main object of the study. For this state, many theoretical works, in accordance with some of the indirect experimental data, predict a significant increase in size compared to other states of this nucleus.

We analyzed the diffraction structure of the differential cross sections of the elastic and inelastic scattering of ^{2}H, ^{3}He, α, ^{6}Li, and ^{12}C on ^{12}C with formation of the excited states up to $E_x \lesssim 11$ MeV at the incident energies below 100 MeV/nucleon. The first three or four minima and maxima of the angular distributions were identified as the diffraction ones and then used to determinate the diffraction radii for different excited states. We found that the diffraction radii smoothly (almost linearly) decrease with energy both for elastic and inelastic scattering. The absolute values of the diffraction radii corresponding to formation of the ground and the first excited state (2_1^+, 4.44 MeV) practically coincide. The diffraction radii corresponding to the formation of the Hoyle state in all the cases are found to be larger than those of the ground state. The difference $R_{\mathrm{dif}}(7.65) - R_{\mathrm{dif}}(0.00)$ does not depend on either the energy or the specific reaction (possibly except for the scattering of deuterons) and is equal to 0.5 fm with good accuracy. Similar features were found for the diffraction radii of the 3^-, 9.64 MeV state.

The persistence of the diffraction radius differences justifies the assumption that the rms radius for the excited state differs from the rms radius for the ground state by the difference of the diffraction radii. Under this assumption, we determined the rms radii for the excited states of ^{12}C averaged for all analyzed reactions and energies. The rms radius of ^{12}C in the first excited state 2_1^+ is found to be $\langle R(2_1^+)\rangle = 2.36 \pm 0.04$ fm, almost equal to the ground state ($\langle R_{\mathrm{g.s.}}\rangle = 2.34$ fm) in accordance with expectations. The rms radius of ^{12}C in the Hoyle state was found to be equal to $\langle R(0_2^+)\rangle = 2.89 \pm 0.04$ fm. This value is 1.2 times larger than that for the ground state but smaller than predicted by most of the theoretical models. The rms radius of the 9.64 MeV 3^-state was found to be $\langle R(3^-)\rangle = 2.88 \pm 0.11$ fm, which is as large as for the Hoyle state. A comparable increase in the size of ^{12}C has been found for all other levels located above the threshold of $^{12}\mathrm{C} \rightarrow 3\alpha$. These findings require a new theoretical analysis. At this stage, we can only reiterate the view expressed previously [26]:

the increased radii of the above-threshold nuclear states may be associated not so much with the peculiarities of their structure, but with the very fact of their location above the breakup threshold of the ^{12}C nucleus.

The results obtained in the present work show that the diffraction structure of the angular distributions of inelastic scattering can be directly connected with the radii of nuclei in the excited states, including those lying above the threshold of particle emission. The developed method, in view of its simplicity, can be widely used. Therefore, an independent confirmation of its functionality and a more detailed study of its applicability remain important tasks.

ACKNOWLEDGMENTS

We are indebted to T. Kawabata for providing some numerical data on α scattering at 388 MeV. This work is partially supported by Grant Nos. 08-02-00924 and 09-02-00456 (RFFI, Russia) and Project 2428/2007 (UAEMex, Mexico).

[1] B. Hahn, D. G. Ravenhall, and R. Hofstadter, Phys. Rev. **101**, 1131 (1956).
[2] A. Tohsaki, H. Horiuchi, P. Schuck, and G. Ropke, Phys. Rev. Lett. **87**, 192501 (2001); P. Schuck, Y. Funaki, H. Horiuchi, G. Ropke, A. Tohsaki, and T. Yamada, Nucl. Phys. **A738**, 94 (2004); Y. Funaki, H. Horiuchi, G. Ropke, P. Schuck, A. Tohsaki, and T. Yamada, Phys. Rev. C **77**, 064312 (2008).
[3] H. Morinaga, Phys. Rev. **101**, 254 (1956); M. Moshinsky, Nucl. Phys. **13**, 104 (1959); V. V. Balashov, V. G. Neudachin, Yu. F. Smirnov, and N. P. Yudin, Zh. Eksp. Teor. Fiz. **37**, 1358 (1959) [Sov. Phys. JETP **37**, 983 (1960)].
[4] D. M. Brink, in *Many-Body Description of Nuclear Structure and Reactions*, *Proceedings of the International School of Physics "Enrico Fermi,"* Course 36, edited by C. Bloch (Academic, New York, 1966).
[5] V. G. Neudachin and Yu. F. Smirnov, *Nucleon Associations in Light Nuclei* (Nauka, Moscow, 1969), in Russian; in *Progress in Nuclear Physics*, edited by D. M. Brink and J. Mulvey (Elsevier, 1969), Vol. 10, p. 273.
[6] K. Wildermuth and Y. C. Tang, *A Unified Theory of the Nucleus* (Vieweg, Braunschweig, 1977).
[7] Y. Fujiwara, H. Horiuchi, K. Ikeda, M. Kamimura, K. Kato, Y. Suzuki, and E. Uegaki, Prog. Theor. Phys. Suppl. **68**, 29 (1980).
[8] H. Horiuchi and K. Ikeda, in *Cluster Models and Other Topics*, International Review of Nuclear Physics (World Scientific, Singapore, 1987), Vol. 4, p. 1.
[9] F. Hoyle, Astrophys. J. Suppl. **1**, 121 (1954).
[10] E. Uegaki, S. Okabe, Y. Abe, and H. Tanaka, Prog. Theor. Phys. **57**, 1262 (1977); E. Uegaki, Y. Abe, S. Okabe, and H. Tanaka, *ibid.* **59**, 1031 (1978); **62**, 1621 (1979).
[11] Y. Fukushima and M. Kamimura, in *Proceedings of the International Conference on Nuclear Structure, Tokyo, September 5–10, 1977*, edited by T. Marumori (Physical Society of Japan, Tokyo, 1978); M. Kamimura, Nucl. Phys. **A351**, 456 (1981).
[12] Y. Kanada-En'yo, Phys. Rev. C **75**, 024302 (2007).
[13] M. Chernykh, H. Feldmeier, T. Neff, P. von Neumann-Cosel, and A. Richter, Phys. Rev. Lett. **98**, 032501 (2007).
[14] W. von Oertzen, M. Freer, and Y. Kanada-En'yo, Phys. Rep. **432**, 43 (2006).
[15] M. Freer *et al.*, Phys. Rev. C **71**, 047305 (2005); **76**, 034320 (2007).
[16] P. J. Haigh, N. I. Ashwood, T. Bloxham, N. Curtis, M. Freer, P. McEwan, D. Price, V. Ziman, H. G. Bohlen, Tz. Kokalova, Ch. Schulz, R. Torabi, W. von Oertzen, C. Wheldon, W. Catford, C. Harlin, R. Kalpakchieva, and T. N. Massey, Phys. Rev. C **78**, 014319 (2008).
[17] H. G. Bohlen, T. Dorsch, Tz. Kokalova, W. von Oertzen, Ch. Schulz, and C. Wheldon, Phys. Rev. C **75**, 054604 (2007).
[18] Tz. Kokalova *et al.*, Eur. Phys. J. A **23**, 19 (2005); Tz. Kokalova, N. Itagaki, W. von Oertzen, and C. Wheldon, Phys. Rev. Lett. **96**, 192502 (2006).
[19] S. Ohkubo and Y. Hirabayashi, Phys. Rev. C **70**, 041602(R) (2004); **75**, 044609 (2007).
[20] M. Takashina and Y. Sakuragi, Phys. Rev. C **74**, 054606 (2006); M. Takashina, *ibid.* **78**, 014602 (2008).
[21] Y. Funaki, A. Tohsaki, H. Horiuchi, P. Schuck, and G. Röpke, Phys. Rev. C **67**, 051306(R) (2003).
[22] A. A. Ogloblin, S. A. Goncharov, T. L. Belyaeva, A. S. Demyanova, Phys. At. Nucl. **69**, 1149 (2006).
[23] S. M. Smith *et al.*, Nucl. Phys. **A207**, 273 (1973).
[24] A. S. Demyanova *et al.*, Nucl. Phys. **A542**, 208 (1992).
[25] D. T. Khoa, Int. J. Mod. Phys. E **17**, 2055 (2008).
[26] A. S. Demyanova *et al.*, Preprint IAE-6470/2, Kurchatov Institute, Moscow, 2007 (unpublished).
[27] A. S. Demyanova *et al.*, in *Proceedings of the International Nuclear Physics Conference, Tokyo, 3–8 June 2007*, edited by S. Nagamiya, T. Motobayashi, M. Oka, R. S. Hayano, and T. Nagae (Physical Society of Japan, Tokyo, 2008), Vol. 2, p. 489.
[28] A. S. Demyanova, A. A. Ogloblin, S. Goncharov, and T. Belyaeva, Int. J. Mod. Phys. E **17**, 2118 (2008).
[29] J. Specht *et al.*, Nucl. Phys. **A171**, 65 (1971).
[30] J. Brissaud *et al.*, Phys. Lett. **B30**, 324 (1969).
[31] S. Wiktor, C. Mayer-Böricke, A. Kiss, M. Rogge, P. Turek, and H. Dabrowski, Acta Phys. Pol. B **12**, 491 (1981).
[32] B. John, Y. Tokimoto, Y. W. Lui, H. L. Clark, X. Chen, and D. H. Youngblood, Phys. Rev. C **68**, 014305 (2003).
[33] M. Itoh *et al.*, Nucl. Phys. **A738**, 268 (2004).
[34] M. Itoh *et al.*, in *Proceedings of the International Nuclear Physics Conference, Tokyo, 3–8 June 2007*, edited by S. Nagamiya, T. Motobayashi, M. Oka, R. S. Hayano, and T. Nagae (Physical Society of Japan, Tokyo, 2008), Vol. 2, p. 371.
[35] T. Fujisawa *et al.*, J. Phys. Soc. Jpn. **34**, 5 (1973).
[36] N. T. Burtebaev *et al.*, Preprint NPI 88-01, Nuclear Physics Institute, Kazakhstan, 1988 (unpublished); V. V. Adodin *et al.*, Preprint NPI 92-01, Nuclear Physics Institute, Kazakhstan, 1992 (unpublished).
[37] T. Tanabe, J. Phys. Soc. Jpn. **41**, 361 (1976).
[38] F. Hinterberger *et al.*, Nucl. Phys. **A115**, 570 (1968).
[39] O. Aspelund *et al.*, Nucl. Phys. **A253**, 263 (1975).
[40] T. Kawabata *et al.*, Phys. Lett. **B646**, 6 (2007).
[41] K. Katori *et al.*, Nucl. Phys. **A480**, 323 (1988).

[42] V. A. Maslov *et al.*, Phys. At. Nucl. (to be published).
[43] S. Kubono *et al.*, Phys. Lett. **B163**, 75 (1985).
[44] J. S. Blair, in *Lectures in Theoretical Physics*, Vol. VIII, edited by F. D. Kunz, D. A. Lind, and W. E. Britten (University of Colorado Press, Boulder, 1966).
[45] G. R. Satchler, *Introduction to Nuclear Reactions* (Macmillan, New York, 1990), p. 165.
[46] A. A. Ogloblin *et al.*, Phys. At. Nucl. (to be published).
[47] T. Yamada and P. Schuck, Phys. Rev. C **69**, 024309 (2004); Eur. Phys. J. A **26**, 185 (2005).
[48] C. Kurokawa and K. Katö, Nucl. Phys. **A792**, 87 (2007).
[49] T. Neff and H. Feldmeier, Int. J. Mod. Phys. E **17**, 2005 (2008).