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Glauber model for α-nucleus total reaction cross section

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The Coulomb-modified Glauber model is employed to calculate the total reaction cross section (σ_R) for α particles from ^{9}Be, ^{12}C, ^{16}O, ^{28}Si, ^{40}Ca, ^{58,60}Ni, ^{112,116,120,124}Sn, and ^{208}Pb at 117.2, 163.9, and 192.4 MeV and from the lighter nuclei also at 69.6 MeV. Our main focus in this work is to assess the suitability of semiphenomenological parametrization of the *NN* amplitude (SP*NN*), used recently [Deeksha Chauhan and Z. A. Khan, Eur. Phys. J. A **41**, 179 (2009)], in the analysis of σ_R at the energies under consideration. Using the realistic form factors for the colliding nuclei, it is found that the SP*NN* works reasonably well and we have quite a satisfactory account of the σ_R data in all the cases. Moreover, our analysis suggests that the SP*NN* could be taken as fairly stable to describe simultaneously the elastic angular distribution and the σ_R for a wide range of target nuclei in the relatively low-energy region.

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I. INTRODUCTION

Over about the past three decades, we have witnessed an increasing interest in experimental [1–18] as well as theoretical [19–37] studies of the total nuclear reaction cross section (σ_R), which is one of the most important physical quantities characterizing nuclear reactions [5,7,38,39]. The total nuclear reaction cross section is very useful for extracting information about nuclear sizes, and the Glauber model has been quite successful in getting the radii of radioactive nuclei from the measured values of σ_R [40]. It also has applications in diverse research areas such as radiobiology and space radiation [41,42]. Keeping this in mind, Charagi and Gupta [22] and Alvi and Abdulmomen [36] have provided closed-form analytic expressions that can be used for a quick determination of σ_R for nucleus-nucleus and α-nucleus collisions within the framework of the Coulomb-modified Glauber model. From a theoretical point of view, the studies of σ_R may not only be helpful in minimizing the different ambiguities in optical model calculations, but may also be helpful in obtaining a better picture of the reaction mechanisms when different models provide equivalent descriptions of the elastic angular distribution data.

Working within the framework of the Glauber multiple scattering model, many authors have applied this model to study nucleus-nucleus total reaction cross section data [7,20,22,43,44]. The results of these studies show that the model works reasonably well at intermediate and high energies. In addition, the Glauber model is found to give fairly good results at relatively lower energies provided it is suitably corrected to account for the Coulomb effects [45]. Unfortunately, these studies involve the so-called optical-limit approximation (OLA) of the full Glauber elastic *S* matrix, which is found to be a rather poor approximation because the series for the Glauber *S* matrix, whose first (leading) term corresponds to the optical-limit result, in the studies of nucleus-nucleus elastic angular distribution shows slow convergence. This shows that one really needs to go beyond the OLA to get a better understanding of σ_R [23,46] and the elastic angular distribution [47–49]. However, keeping in mind the problems encountered in the analytic evaluation of even the leading term of the Glauber S matrix for realistic description of nuclei, efforts have also been made to invoke other approximation schemes for analyzing the nucleus-nucleus scattering within the framework of the Glauber model. Among these schemes the phase expansion approach of Franco and Varma [50] and the effective profile function approach of Ahmad [51] are found to give better approximations of the full Glauber *S* matrix.

Recently [52] we have studied the elastic angular distribution and σ_R for the ^{12}C-^{12}C system at 1.016, 1.449, and 2.4 GeV within the framework of the Coulomb-modified Glauber model in which the effective profile expansion approach of Ahmad [51] has been used to obtain the correlation expansion for the Glauber amplitude. In this work we have laid emphasis on the parametrization of the basic (input) *NN* amplitude that may be used for a wide range of angles. By retaining the first two terms of the correlation expansion and using the realistic densities for the colliding nuclei, it has been shown that the consideration of higher momentum transfer components, and hence the nondiffractive behavior, of the *NN* amplitude [53] provides a more satisfactory account of the data than does the usually parametrized one-term Gaussian *NN* amplitude [54,55]. As pointed out in Ref. [56], although the *NN* amplitude used in Ref. [52] predicts the experimental values of the *NN* total cross section [57] and the ratio of the real to the imaginary parts of the forward *NN* amplitude [58], the consideration of its higher momentum transfer components may not predict the same low q behavior as that obtained from the one-term Gaussian form [54,55] of the *NN* amplitude at the desired energies. Keeping this in mind, we consider it worthwhile to make use of that form of the *NN* amplitude that may preserve the low q behavior and whose higher momentum transfer components may be treated phenomenologically. Such an *NN* amplitude has been used in our recent work [56] to analyze the α-nucleus elastic scattering in the energy range of 25–70 MeV/nucleon. Assuming the effect of nuclear correlations to be fairly small [26,59], the Glauber *S* matrix

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has been replaced by the first term of its correlation expansion [52], which considers no correlations. After correcting for the deviation in the straight line trajectory of the Glauber model due to the Coulomb field [45], the authors have shown [56] that the low- and high-q behaviors of the (free) *NN* amplitude could be assessed separately, and the data on α-α elastic angular distribution have been reproduced satisfactorily well, covering a fairly large value of momentum transfer. Moreover, the trend of the slope parameter of the *NN* amplitude and the values of its high-q components demonstrate the need for nondiffractive behavior of the *NN* amplitude in any realistic study of the nucleus-nucleus collision at relatively lower energies.

To test the usefulness of the *NN* amplitude that takes care of its higher momentum transfer components, the best course of study could have been the simultaneous description of the elastic angular distribution and σ_R, as they complement each other. Unfortunately, the experimental data on the measurements of σ_R are so sparse compared to those for the elastic angular distribution, one may not provide a simultaneous description of the elastic angular distribution and the σ_R at similar incident energies. Keeping in view the status of σ_R, the Uppasala/Redlands Collaboration [8,12,13] has performed measurements for σ_R for light ions in the energy range of 17–50 MeV/nucleon.

Motivated by the success of the Glauber model at projectile energies as low as 25 MeV/nucleon [56], we, in this work, consider the analysis of α particle reaction cross-section data [12] for a variety of target nuclei at 69.6, 117.2, 163.9, and 192.4 MeV. Like in Ref. [52], the present analysis also is based upon the Coulomb-modified correlation expansion for the Glauber amplitude, the first term of which contains all orders of scattering with no correlations, while the others depend successively upon the two-, three-, and many-body densities (correlations) of the colliding nuclei. As demonstrated in Ref. [26], the effect of two-body density (correlation) terms is insignificant at energies of our interest; we expect that the (leading) first term in the correlation expansion [52] could not only suffice for the study of σ_R but also would provide a better (microscopic) understanding of the subject in the present analysis. In addition, our calculation for σ_R may be as quick as the one performed with the closed (analytic) expressions for σ_R [22,36]. To be more specific, our aim in this work is to see how far the *NN* amplitude used in Ref. [56] could be helpful in the analysis of α-nucleus total reaction cross sections at energies under consideration and to see what can be said about the behavior of the *NN* amplitude from the point of view of providing the simultaneous description of the elastic angular distribution and σ_R at relatively lower energies.

The formulation of the problem to calculate σ_R is given in Sec. II. In Sec. III, we present our results for the α-nucleus total reaction cross section. A summary and conclusions are given in Sec. IV.

II. FORMULATION

According to the correlation expansion for the Glauber amplitude [52], the elastic S matrix element S_{00} for nucleus-nucleus collision is written as

$$S_{00}(\vec{b}) = (1 - \Gamma_{00})^{AB} + \text{correlation terms}, \qquad (1)$$

with

$$\Gamma_{00}(\vec{b}) = \langle \psi_0\phi_0 | \Gamma(\vec{b} - \vec{s}_i + \vec{s}'_j) | \phi_0\psi_0 \rangle \qquad (2)$$

$$= \frac{1}{ik}\int dq\, q\, J_0(qb) F_A(\vec{q}) F_B(\vec{q}) f_{NN}(\vec{q}), \qquad (3)$$

where $A(B)$ is the mass number of the target (projectile) nucleus; $\phi_0[F_B(\vec{q})]$ and $\psi_0[F_A(\vec{q})]$ are the intrinsic ground-state wave functions (form factors) of the projectile and target nuclei, respectively; $\vec{b}$ is the impact parameter vector perpendicular to the beam direction; $\vec{s}_i(\vec{s}'_j)$ are the projections of the target (projectile) nucleon coordinates on the impact parameter plane; k is the momentum of the projectile nucleon; q is the momentum transfer; and $\Gamma_{NN}(\vec{b})$ is the *NN* profile function, which is related to the *NN* scattering amplitude $f_{NN}(\vec{q})$ as follows

$$\Gamma_{NN}(\vec{b}) = \frac{1}{2\pi ik}\int e^{-i\vec{q}.\vec{b}} f_{NN}(\vec{q}) d^2q. \qquad (4)$$

Here it may be noted that Eq. (1) with $B = 4$ gives S_{00} for α-nucleus elastic scattering.

As observed in Ref. [59], the optical limit of the correlated Glauber model works reasonably well for studying the nucleus-nucleus elastic and inelastic scattering data, which cover a wide range in the projectile energy (30–350 MeV/nucleon) and the mass number of the colliding nuclei. This suggests that the nuclear correlations may not play a significant role in nucleus-nucleus collisions at the energies under consideration. However, in the present context, the study of Abdulmomen and Ahmad [26] is quite useful. In this work, the authors have specifically shown that the effect of the two-body density term in the analysis of α-nucleus reaction cross sections is small, and if we look into the trend of the results for $\alpha-{}^{16}$O (Fig. 1 in Ref. [26]), we could say that the effects of the two-body density term may be ignored in the energy range considered in this work. Thus the results of Ref. [26] show that in the analysis of α-nucleus reaction cross sections, the consideration of the first term in Eq. (1) seems to be a good approximation to the full Glauber S matrix at relatively low-incident energies:

$$S_{00}(\vec{b}) \simeq [1 - \Gamma_{00}]^{AB}. \qquad (5)$$

Equation (5) has, however, been modified to account for the deviation in the straight line trajectory of the Glauber model because of the Coulomb field. Following Fäldt and Pilkuhn [45], this deviation can be incorporated by replacing b in $S_{00}(\vec{b})$ by b', which is the distance of the closest approach in Rutherford orbits and is given by

$$kb' = \eta + (\eta^2 + k^2b^2)^{1/2}, \qquad (6)$$

where $\eta = Z_A Z_B e^2/\hbar v$ is the Sommerfeld parameter with $Z_A(Z_B)$ as the target (projectile) atomic number and v as the projectile velocity.

Finally it must be pointed out that the distinction between $pp(nn)$ and $pn(np)$ amplitudes has also been incorporated in Eq. (5). This modification leads to the following expression for the Glauber S matrix:

$$S_{00}(\vec{b}) \simeq \left[1-\Gamma_{00}^{\prime pp}\right]^{Z_A Z_B}\left[1-\Gamma_{00}^{\prime np}\right]^{N_B Z_A} \times\left[1-\Gamma_{00}^{\prime pn}\right]^{Z_B N_A}\left[1-\Gamma_{00}^{\prime nn}\right]^{N_A N_B}, \quad (7)$$

with

$$\Gamma_{00}^{\prime mn}=\frac{1}{ik}\int dq q J_0(qb) F_A(\vec{q}) F_B(\vec{q}) f_{mn}(\vec{q}), \quad (8)$$

where $N_A(N_B)$ is the number of neutrons in the target (projectile) nucleus, and m and n stand for a proton and a neutron.

With these considerations, the σ_R for a nucleus-nucleus collision is given by

$$\sigma_R = 2\pi\int dbb[1-|S_{00}(\vec{b})|^2]. \quad (9)$$

III. RESULTS AND DISCUSSION

Following the approach outlined in Sec. II, we have analyzed the α-nucleus reaction cross section data of Ingemarsson *et al.* [12] at 69.6, 117.2, 163.9, and 192.4 MeV. The inputs needed in the calculation are the *NN* amplitude and the form factors of the colliding nuclei. The nuclei involved in the analysis are ^{4}He, ^{9}Be, ^{12}C, ^{16}O, ^{28}Si, ^{40}Ca, ^{58,60}Ni, ^{112,116,120,124}Sn, and ^{208}Pb.

For computational simplicity, we parametrize the required nuclear form factors as a sum of Gaussians:

$$F_\nu(\vec{q})=\sum_j a_j e^{-b_j q^2}; \quad \nu = A, B, \quad (10)$$

where a_j and b_j are parameters, whose values for ^{4}He, ^{12}C, ^{16}O, ^{40}Ca, and ^{58}Ni are taken from Refs. [51,60,61]. The parameter values for ^{9}Be, ^{28}Si, and ^{60}Ni are given in Table I. These values are determined by fitting the proton form factors as obtained from the charge densities after correcting for the finite size of the proton. For this, the charge densities of Sick and McCarthy [62] for ^{12}C and ^{16}O, Chaumeaux *et al.* [63] for ^{40}Ca, and de Vries *et al.* [64] for ^{58,60}Ni have been used. To obtain the parameter values for ^{4}He, ^{9}Be, and ^{28}Si, the electron scattering form factors of Frosch *et al.* [65] for ^{4}He, Bernheim *et al.* [66] for ^{9}Be, and Whitner *et al.* [67] for ^{28}Si have been used. Moreover, we also assume that the proton and the neutron density distributions for the aforesaid nuclei are same. For nuclei ^{112,116,120,124}Sn and ^{208}Pb, we use different density distributions for protons and neutrons as calculated in the relativistic mean-field (RMF) [68] framework; the corresponding values of a_j and b_j in Eq. (10) for proton and neutron form factors are given in Table II.

TABLE I. Parameter values of the sum of the Gaussian parametrization of the nuclear form factor.

Nucleus	a_j	b_j (fm^2)
^{9}Be	0.1044	1.0379
	1.3814	1.0327
	0.3106	1.3066
	0.1876	0.4083
	−0.9841	1.0100
^{28}Si	1.4917	0.5025
	2.8506	0.8148
	3.3808	0.8722
	0.3924	0.5031
	−7.1156	0.6714
^{60}Ni	−7.4461	1.2996
	1.0975	0.4033
	0.0084	0.5399
	−6.0326	0.4789
	8.2669	1.4485
	5.1059	0.4986

TABLE II. Parameter values of the sum of the Gaussian parametrization of the proton and neutron form factors.

Nucleus	Proton form factor		Neutron form factor	
	a_j	b_j (fm^2)	a_j	b_j (fm^2)
^{112}Sn	10.5600	1.6553	5.9223	1.8925
	1.4070	0.5671	1.5986	0.6769
	−0.1704	0.0576	−0.2179	0.2065
	−10.7960	1.3780	−6.3029	1.3769
^{116}Sn	9.6062	1.6930	4.7149	2.0568
	1.4189	0.5797	1.7090	0.8218
	−0.1699	0.0632	−0.0743	0.0611
	−9.8552	1.3842	−5.3496	1.3816
^{120}Sn	9.2067	1.7130	4.2097	2.1793
	1.4533	0.5717	1.5536	0.7980
	−0.1910	0.0698	−0.1184	0.1569
	−9.4691	1.3849	−4.6448	1.4116
^{124}Sn	9.3930	1.7239	3.7008	2.2962
	1.4652	0.5799	1.4236	0.7225
	−0.1822	0.0592	−0.2060	0.2270
	−9.6760	1.3971	−3.9183	1.4100
^{208}Pb	−17.4270	3.4151	−22.0360	4.1771
	−35.6380	4.2002	−22.3500	4.6581
	56.0680	3.7733	47.5880	4.3196
	19.6070	1.4906	20.0000	1.5195
	60.6730	2.2117	61.7430	2.2104
	−78.2590	2.0780	−77.8620	2.0761
	−4.0939	1.0800	−4.1462	1.0896
	2.2820	5.9619	1.0663	5.9886
	−2.1265	3.4265	−2.8810	3.7646
	−0.0341	1.1062	−0.0368	0.9676
	−0.0722	0.5773	−0.1205	0.9570
	0.0196	0.0142	0.0361	0.1395

Because our interest in this work is to establish the suitability of the *NN* amplitude [56] in different situations, the present analysis of α-nucleus reaction cross section also considers the similar form of the *NN* amplitude as used in Ref. [56]:

$$f_{NN}(\vec{q})=\frac{ik\sigma}{4\pi}(1-i\rho)e^{-(\beta+i\gamma)q^2/2}[1+T(\vec{q})], \quad (11)$$

with

$$T(\vec{q})=\sum_{n=1,2\ldots}\lambda_n q^{2(n+1)}, \quad (12)$$

where σ is the NN total cross section, ρ is the ratio of the real to the imaginary parts of the forward NN amplitude, β is the slope parameter, γ is the phase of the NN amplitude [69], and the parameters λ_n take care of the higher momentum transfer components of the NN amplitude. The values of the parameters of $f_{NN}(\vec{q})$, namely, σ, ρ, β, λ_1, and λ_2, should be the same as those for the NN scattering at one-fourth of the kinetic energy of the incident α particle. In the present analysis, we need their values at 17.4, 29.3, 40.9, and 48.1 MeV. The values of σ are obtained using the parametrizations of the NN total cross sections [22], which nicely reproduce the experimentally determined NN total cross sections [57], $\sigma_{pp(nn)}$ and σ_{pn}, in the energy range of 10 MeV to 1.0 GeV:

$$\sigma_{pp(nn)} = 13.73 - 15.04v_0^{-1} + 8.76v_0^{-2} + 68.67v_0^4 \quad (13)$$

$$\sigma_{pn} = -70.67 - 18.18v_0^{-1} + 25.26v_0^{-2} + 113.85v_0, \quad (14)$$

where $\sigma_{pp(nn)}$ and σ_{pn} are expressed in mb and v_0 is the incident nucleon velocity in units of c. To calculate ρ we use the parametrizations of Ahmad *et al.* [70], which reproduce the values of $\rho_{pp(nn)}$ and ρ_{pn} obtained from the phase shifts and Coulomb interference measurements [58]:

$$\rho_{pp(nn)} = -0.386 + 1.224e^{-\frac{1}{2}\left(\frac{k-0.427}{0.178}\right)^2} + 1.01e^{-\frac{1}{2}\left(\frac{k-0.592}{0.638}\right)^2} \quad (15)$$

$$\rho_{pn} = -0.666 + 1.437e^{-\frac{1}{2}\left(\frac{k-0.412}{0.196}\right)^2} + 0.617e^{-\frac{1}{2}\left(\frac{k-0.797}{0.291}\right)^2}, \quad (16)$$

where the incident nucleon laboratory momentum k is expressed in GeV/c. To obtain the values of the other parameters of $f_{NN}(\vec{q})$, namely, β, λ_1, and λ_2, we proceed as follows.

It is well known that the Glauber model calculations are physically meaningful only when one could have consistently a satisfactory account of the available scattering data for different target nuclei at the same incident energy/nucleon, using the similar description for the (input) NN amplitude. Keeping this in mind, we first calibrate the parameters (β, λ_1, and λ_2) of the NN amplitude at the required incident energies/nucleon. For this, we analyzed the σ_R for the α-^{9}Be system at 69.6, 117.2, 163.9, and 192.4 MeV. The values of the parameters of $f_{NN}(\vec{q})$ obtained in this way are reported in Table III; the corresponding values of the phase variation parameter (γ) are given in Table IV. We then undertook the second part of the present work, in which the calculations were performed for σ_R for α particles on ^{12}C, ^{16}O, ^{28}Si, ^{40}Ca, ^{58,60}Ni, ^{112,116,120,124}Sn, and ^{208}Pb at 117.2, 163.9, and 192.4 MeV and on light nuclei also at 69.6 MeV using the same values of the parameters of $f_{NN}(\vec{q})$ as reported in

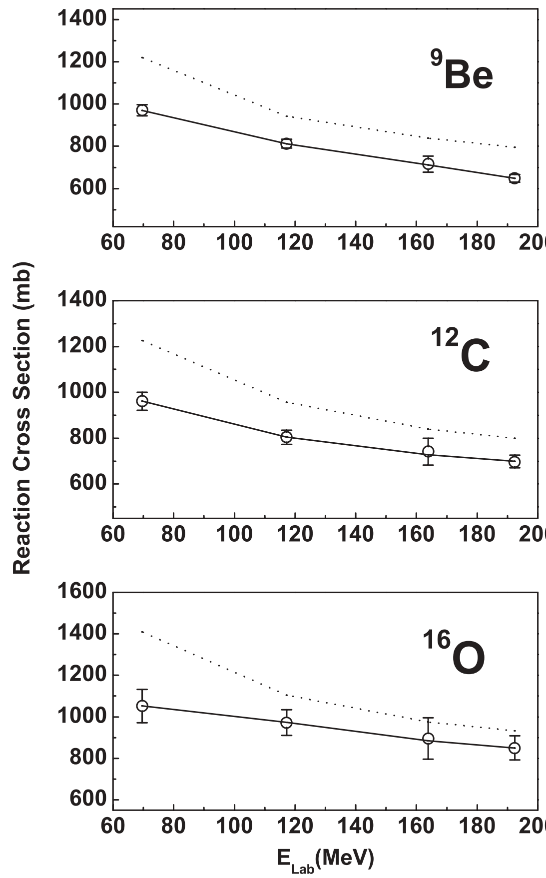

FIG. 1. Total reaction cross section for α particles on ^{9}Be, ^{12}C, and ^{16}O using the parameters of the NN amplitude as reported in Table III. The solid curves include the phase variation of the NN amplitude, whose values are given in Table IV. The dotted curves ignore the phase variation of the NN amplitude. The data are taken from Ref. [12].

TABLE III. Values of the NN amplitude parameters obtained from the analysis of α-^{9}Be total reaction cross section (except for σ and ρ, which are taken from Refs. [57,58]).

Energy of α particle (MeV)	Energy/nucleon (MeV)	NN	σ (fm^2) Ref. [57]	ρ Ref. [58]	β (fm^2)	λ_1 (fm^4)	λ_2 (fm^6)
69.6	17.4	$pp(nn)$	17.74	0.9080	0.3609	$0.6356 + i1.6053$	$0.0495 - i0.3368$
		$pn(np)$	55.51	0.1196	0.4660	$0.1730 + i0.6568$	$0.0135 - i0.1715$
117.2	29.3	$pp(nn)$	9.92	1.1680	0.5180	$0.3131 + i0.5223$	$0.0225 - i0.1501$
		$pn(np)$	30.61	0.3918	0.5939	$0.0512 + i0.3807$	$0.0079 - i0.1094$
163.9	40.9	$pp(nn)$	6.85	1.3817	0.7665	$0.1194 + i0.1692$	$0.0059 - i0.0732$
		$pn(np)$	20.66	0.6080	0.8610	$0.0113 + i0.1565$	$0.0049 - i0.0571$
192.4	48.1	$pp(nn)$	5.76	1.4912	0.8184	$0.0797 + i0.1042$	$0.0045 - i0.0715$
		$pn(np)$	17.09	0.7165	0.9428	$0.0059 + i0.1267$	$0.0038 - i0.0572$

TABLE IV. Values of the *NN* amplitude phase variation parameter that account satisfactorily for the α-nucleus total reaction cross section (σ_R), keeping the values of σ, ρ, β, λ_1, and λ_2 the same as quoted in Table II. The predicted values of σ_R with and without the phase variation (γ) of the *NN* amplitude are represented by σ_R^γ and σ_R^0, respectively. $\sigma_R^{\rm opt}$ is the predicted value of σ_R without γ using the OLA. The last column gives the corresponding experimental values of σ_R [12].

Target nucleus	Energy/nucleon (MeV)	$\gamma_{pp(nn)}$ (fm^2)	$\gamma_{pn(np)}$ (fm^2)	σ_R^γ (mb)	σ_R^0 (mb)	$\sigma_R^{\rm opt}$ (mb)	$\sigma_R^{\rm exp}$ (mb)
^{9}Be	17.4	1.1590	1.0710	970.0	1220.2	1218.5	970 ± 26
	29.3	−0.7894	−0.6862	812.0	942.9	941.8	812 ± 21
	40.9	−0.8120	−0.6900	711.5	838.0	837.3	716 ± 38
	48.1	−0.8957	−0.8000	648.0	796.2	795.9	648 ± 18
^{12}C	17.4	−1.1908	−0.6002	961.0	1226.0	1230.9	961 ± 39
	29.3	−1.1697	−0.4137	804.0	956.0	956.1	804 ± 31
	40.9	−0.8533	−0.3530	727.9	839.1	838.9	741 ± 58
	48.1	−0.8031	−0.3035	699.8	800.1	800.0	698 ± 28
^{16}O	17.4	−1.3325	−0.9138	1052.0	1409.6	1410.1	1052 ± 80
	29.3	−0.8546	−0.4609	973.0	1104.0	1103.8	973 ± 62
	40.9	−0.6333	−0.3031	884.3	974.7	974.4	895 ± 100
	48.1	−0.5788	−0.2681	850.0	932.4	932.3	850 ± 58
^{28}Si	17.4	−1.2287	−0.8688	1400.0	1670.7	1671.0	1400 ± 70
	29.3	−0.6300	−0.3010	1269.9	1362.9	1362.8	1270 ± 60
	40.9	−0.2033	−0.1031	1175.6	1207.1	1206.9	1190 ± 100
	48.1	−0.1832	−0.0831	1132.4	1160.7	1160.6	1110 ± 60
^{40}Ca	17.4	−1.4577	−1.2886	1610.0	1946.6	1946.4	1610 ± 120
	29.3	−0.7530	−0.5824	1470.0	1640.9	1640.6	1470 ± 60
	40.9	−0.4833	−0.2331	1404.1	1486.8	1486.6	1410 ± 120
	48.1	−0.3899	−0.1799	1371.0	1438.3	1438.2	1370 ± 70
^{58}Ni	29.3	−0.7949	−0.5693	1640.0	1836.0	1835.8	1640 ± 80
	40.9	−0.2210	−0.1514	1614.5	1663.4	1663.2	1670 ± 150
	48.1	−0.1820	−0.1230	1572.0	1614.4	1614.4	1550 ± 90
^{60}Ni	29.3	−1.2714	−0.6207	1670.0	1924.0	1923.7	1670 ± 85
	40.9	−0.3189	−0.2129	1700.0	1767.3	1767.1	1700 ± 160
	48.1	−0.2820	−0.1830	1656.0	1718.4	1718.3	1610 ± 90
^{112}Sn	29.3	−0.8835	−0.6991	2140.7	2402.1	2401.9	2140 ± 160
	40.9	−0.4070	−0.3270	2131.1	2253.0	2252.9	2190 ± 240
	48.1	−0.3670	−0.2830	2094.9	2210.3	2210.2	2020 ± 160
^{116}Sn	29.3	−0.7070	−0.4837	2311.1	2463.9	2463.8	2340 ± 150
	40.9	−0.5577	−0.3314	2175.0	2318.4	2318.2	2175 ± 240
	48.1	−0.4501	−0.2799	2149.9	2275.5	2275.4	2150 ± 160
^{120}Sn	29.3	−0.7276	−0.5236	2360.4	2521.3	2521.2	2360 ± 150
	40.9	−0.3770	−0.2550	2281.3	2377.4	2377.3	2380 ± 250
	48.1	−0.2510	−0.1630	2268.2	2334.0	2333.9	2300 ± 170
^{124}Sn	29.3	−0.8835	−0.6991	2343.8	2574.3	2574.1	2340 ± 160
	40.9	−0.5168	−0.2796	2309.9	2431.3	2431.2	2310 ± 240
	48.1	−0.4270	−0.2590	2272.5	2387.3	2387.2	2200 ± 160
^{208}Pb	29.3	−0.7630	−0.4690	2944.8	3141.3	3141.2	2990 ± 180
	40.9	−0.5870	−0.3517	2808.3	2997.9	2997.8	2720 ± 250
	48.1	−0.2530	−0.1330	2893.4	2965.9	2965.8	2900 ± 190

Table III, but we varied only the phase of the *NN* amplitude that could possibly be different for different target nuclei [56]. The results of such calculations are shown by the solid lines in Figs. 1–4. The values of the phase variation parameter, so obtained, are given in Table IV. The dotted lines in Figs. 1–4 depict the corresponding results without any phase variation ($\gamma = 0$) of the *NN* amplitude. The open circles are the experimental data of Ingemarsson *et al.* [12]. Table IV also reports the experimental values of σ_R ($\sigma_R^{\rm exp}$) and their corresponding predicted values with (σ_R^γ) and without (σ_R^0) the phase of the *NN* amplitude for different target nuclei at different incident energies of the α particle. The quantity $\sigma_R^{\rm opt}$ in Table IV represents the predicted values of σ_R, without the phase of the *NN* amplitude, which are calculated

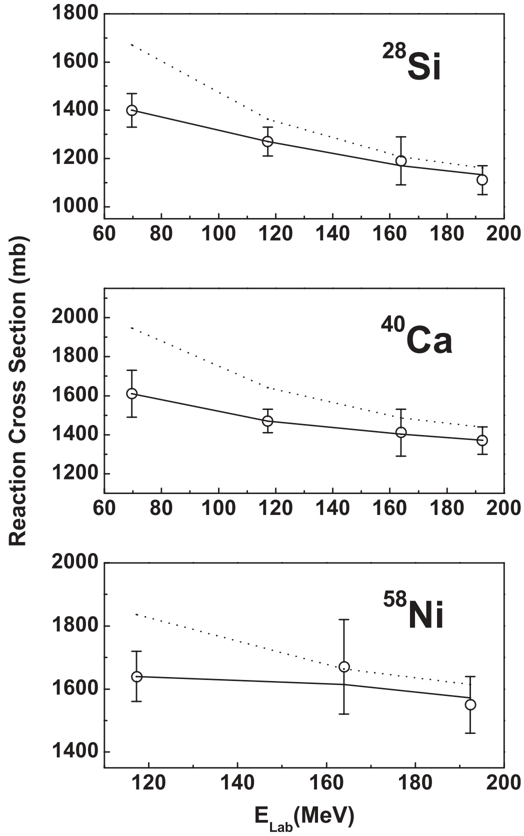

FIG. 2. Same as Fig. 1, but for α particles on ^{28}Si, ^{40}Ca, and ^{58}Ni.

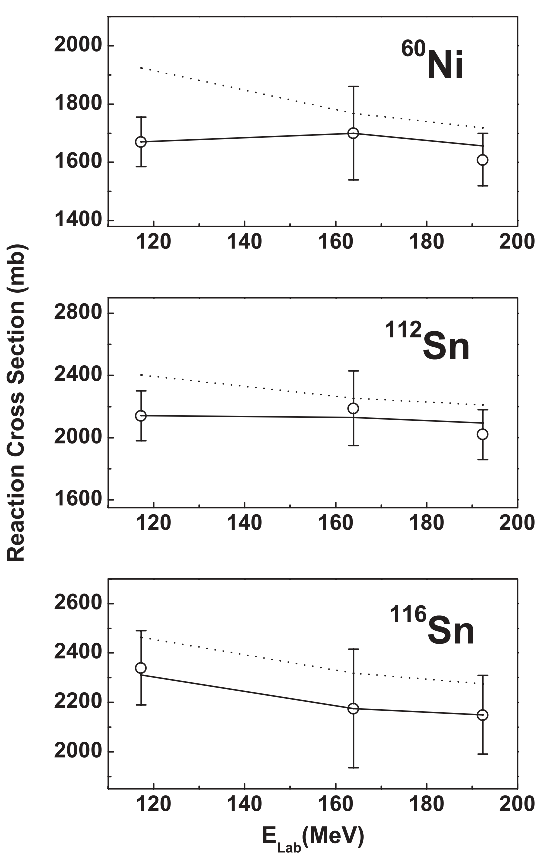

FIG. 3. Same as Fig. 1, but for α particles on ^{60}Ni, ^{112}Sn, and ^{116}Sn.

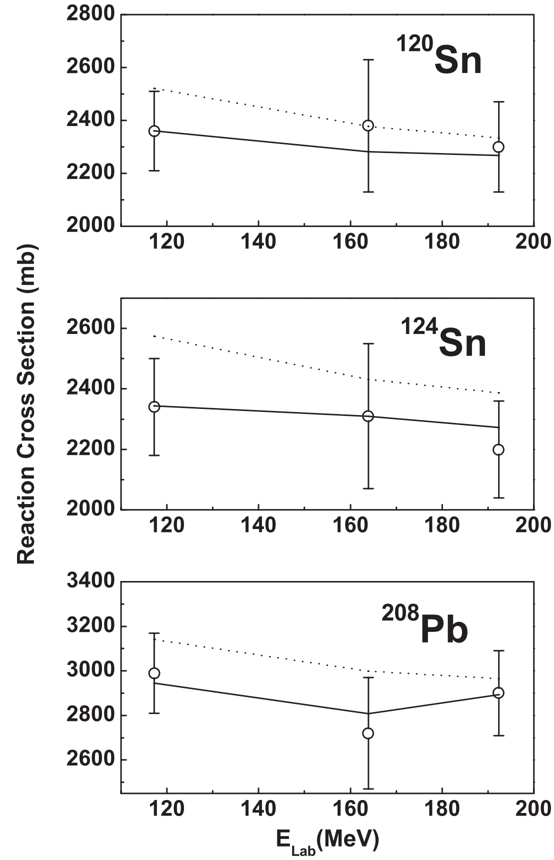

FIG. 4. Same as Fig. 1, but for α particles on ^{120}Sn, ^{124}Sn, and ^{208}Pb.

using the OLA, in which the Glauber S matrix is written as [50]

$$S_{00}(\vec{b}) \simeq e^{-AB\Gamma_{00}(\vec{b})}. \tag{17}$$

Like in Eq. (7), the above equation, with different $pp(nn)$ and $pn(np)$ amplitudes, may be expressed as

$$S_{00}(\vec{b}) \simeq e^{-[Z_A Z_B \Gamma_{00}'^{pp} + N_B Z_A \Gamma_{00}'^{np} + Z_B N_A \Gamma_{00}'^{pn} + N_A N_B \Gamma_{00}'^{nn}]}, \tag{18}$$

where Γ'_{00} has the same form as given in Eq. (8).

The comparison between the predicted values of σ_R in Figs. 1–4 with (solid lines) and without (dotted lines) the phase of the NN amplitude shows that the phase of the NN amplitude pushes the theory closer to the experiment and we have quite a satisfactory account of the data for all the target nuclei at the energies under consideration. The values of the phase variation parameter, reported in Table IV, show a systematic change with the incident energy/nucleon for a given target nucleus. This supports our recent findings [56] in which we have shown that the phase of the NN amplitude gets modified in different ways at different incident energies even if the interacting nucleons move in the same target nucleus.

Regarding the phase variation of the NN amplitude, we further add that because the phase of the NN amplitude does not alter the basic physics of the NN amplitude, it seems that the NN amplitude, as obtained in this work (Table III), is fairly stable over a wide range of target nuclei. Moreover, we find that the values of the parameters (β, λ_1, and λ_2) of $f_{NN}(\vec{q})$ follow the trend of their corresponding values reported in Ref. [56]. This shows that the *NN* amplitude, as obtained in Ref. [56], works well in accounting for the α-nucleus total reaction cross section, and one further expects that the same *NN* amplitude may also be used to provide a satisfactory explanation of both the elastic angular distribution and σ_R at matching incident energies, if available, in the energy range under consideration.

Finally, we compare our predicted values of σ_R using Eqs. (7) and (18) for the Glauber *S* matrix without the phase variation of the *NN* amplitude, represented by σ_R^0 and $\sigma_R^{\rm opt}$, respectively, in Table IV. It is found that although the OLA and the first term of the correlation expansion (1) are two different ways of evaluating the Glauber *S* matrix, the calculations show that they predict nearly similar values of σ_R. To look into the possible causes of such findings, we have calculated the values of $|S_{00}(\vec{b})|^2$ in two situations. Our results show that the values of $|S_{00}(\vec{b})|^2$ are almost similar whether we calculate it using the OLA or the first term of the correlation expansion (1) for the Glauber *S* matrix. This shows that the OLA does not lead to substantial changes in the uncorrelated Glauber model, and hence the OLA and the first term of the correlation expansion (1) may be taken as equivalent choices for calculating the σ_R at the energies under consideration. Moreover, we find that the consideration of the phase of the *NN* amplitude (γ) in the OLA could predict the σ_R as good as the one (σ_R^γ) obtained using the uncorrelated part of the expansion (1) with γ. But, we have noticed that this exercise leads to another set of γ (results not shown) that is different from the one quoted in Table IV. In this connection, it may be emphasized that because there is no way to connect γ with the existing *NN* scattering observables, it is not possible to assess which one of the two sets of γ values corresponds to the exact behavior of the *NN* amplitude in a given situation. Thus it seems that once we compromise with the phase of the *NN* amplitude, the OLA and the uncorrelated part of the expansion (1) may be considered on equal footings to providing a satisfactory account of the σ_R data at the energies considered in this work.

IV. SUMMARY AND CONCLUSIONS

In summary, we have presented a theoretical study of the total reaction cross-section data [12] of α particles from target nuclei ranging from ^{9}Be to ^{208}Pb at 69.6, 117.2, 163.9, and 192.4 MeV using the leading (first) term of the Coulomb-modified correlation expansion for the Glauber *S* matrix for nucleus-nucleus collisions [52]. Our main focus in this work was to assess the suitability of the *NN* amplitude, used in Ref. [56], from the point of view of providing a simultaneous description of the elastic angular distribution and σ_R in the energy range under consideration.

In this work, we first calibrated the parameters of the *NN* amplitude by analyzing the α-^{9}Be reaction cross section at the energies under consideration. The *NN* amplitude parameters so obtained were then used to analyze the σ_R for other target nuclei in which we considered the sole variation of the phase of the *NN* amplitude, which could be different not only for different target nuclei but also for different incident energies/nucleon. We also predicted the values of σ_R using the OLA and the first term of the correlation expansion for the Glauber *S* matrix without considering the phase of the *NN* amplitude.

The comparison of the predicted values of σ_R with and without the phase of the *NN* amplitude shows that the consideration of the phase of *NN* amplitude brings the predictions closer to the experiment and we have quite a satisfactory account of the data in all the cases. The values of the phase variation parameter show a consistent change with the incident energy/nucleon for a given target nucleus, suggesting that the phase of the *NN* amplitude could be different at different incident energies even if the interacting nucleons move in the same target nucleus. In this context, it may be added that because the phase variation of the *NN* amplitude does not change the basic physics of the *NN* amplitude, we find that the *NN* amplitude, as obtained in this work, seems to be fairly stable over a wide range of target nuclei. Moreover, we notice that the values of the parameters of $f_{NN}(\vec{q})$ (Table III) follow the trend of the corresponding values quoted in Ref. [56]. This suggests the usefulness of the *NN* amplitude, as obtained in Ref. [56], in reproducing the α-nucleus total reaction cross section, and one further hopes that the same *NN* amplitude could also be used to provide the simultaneous description of both the elastic angular distribution and σ_R at the energies under consideration. Here, it is important to add that, despite the fact that the SP*NN* amplitude [56] seems to work reasonably well in different situations, it is still desirable to have more precise data on elastic angular distribution and σ_R for nucleus-nucleus collisions at matching incident energies/nucleon, so that one may undertake the analysis of the said experimental data with a motive of having a better understanding of the *NN* amplitude especially at high-q values. Further, we argued that the OLA and the first term of the correlation expansion (1) may be considered as equivalent choices for providing an independent description of σ_R at relatively lower energies. Finally, we conclude that if we look into the simultaneous description of the elastic angular distribution and σ_R, it seems to be the nondiffractive behavior of the *NN* amplitude whose consideration may push down the Glauber model at relatively lower energies.

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