PHYSICAL REVIEW C **80**, 051903(R) (2009)

Color-singlet clustering of partons and recombination model for hadronization of quark-gluon plasma

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(Received 29 May 2009; published 25 November 2009)

SU(N_c) color-singlet restriction, along with flavor and spin symmetry, on a thermal partonic ensemble is shown to recombine the partons with internal color structure into color-singlet multiquark clusters, which can be identified with various hadronic modes at a given temperature. This can provide a possible basis for a recombination model for hadronization of quark-gluon plasma. This also leads to a natural explanation for the ratio of (anti)protons to pions and the quark number scaling of the elliptic flow coefficient in relativistic heavy-ion collisions.

DOI: 10.1103/PhysRevC.80.051903 PACS number(s): 12.38.Mh, 05.30.Ch, 25.75.Dw

By now, it is generally believed that the deconfinement of strongly interacting matter has been achieved in relativistic heavy-ion collisions (RHIC) [1]. Remarkable confirmations for this premise come from the elliptic flow of hadrons and its scaling with the number of valence quarks [2], jet quenching [3], and radiation of thermal photons [4]. However, the results from RHIC experiments also revealed some interesting facts that the nuclear suppression factor depends on the hadron species [3], and the proton-to-pion ratio [5] has a plateau around unity in the transverse momentum range (2–4) GeV$/c$. Recently, it has been argued in Refs. [6,7] that the hadron production at low momenta in a dense medium takes place through the recombination of partons, which explains some of the surprising results of RHIC experiments.

Generally, hadronization by recombination is a so-called two-to-one process in a medium in which the (anti)quarks are effective degrees of freedom and gluons are dynamical ones that disappear at hadronization. This simply boils down to the correct counting of quantum states and momenta within a dynamical theory. However, it is an extremely difficult task within the dynamical quantum chromodynamics (QCD), and various models [6–10] have been formulated to describe the hadron production in heavy-ion collisions.

Recently, under a sudden approximation, the recombination has been considered with a perturbative quark, that is, a minijet and a thermal (anti)quark [7], whereas in Ref. [6], it is with thermal (anti)quarks. In Ref. [7], it is argued that an additional contribution is necessary for the transverse momentum spectra of hadrons at the transition region between the thermal recombination and the individual fragmentation. Such a contribution comes from the recombination of a minijet and a thermal quark. In Ref. [6], on the other hand, it is argued that for momenta below 5 GeV$/c$, the thermal recombination dominates, whereas beyond 5 GeV$/c$, the fragmentation of the independent minijet dominates hadron production. Also, competition between the recombination and fragmentation pushes the onset of fragmentation to a relatively higher transverse momentum of 5–6 GeV$/c$. This indicates that the quark recombination phenomena for hadronization of quark-gluon plasma remain an open as well as an interesting problem.

In this Rapid Communication, we show that the recombination phenomena arise spontaneously on application of a color-singlet projection operator on the partition function for an assembly of quarks and antiquarks having internal color structures. Such recombination phenomena naturally explain the baryon-to-meson ratio and the azimuthal anisotropy of hadron distribution, which scales with the number of valence quarks.

The statistical behavior of a quantum gas in thermal equilibrium is usually studied through an appropriate ensemble. In general, one defines a density matrix for the system as

$$\rho(\beta) = \exp(-\beta \hat{H}), \tag{1}$$

where $\beta = 1/T$ is the inverse of temperature and $\hat{H}$ is the Hamiltonian of the physical system. The corresponding partition function for a quantum gas having a finite volume can be written as

$$\mathcal{Z} = \mathrm{Tr}\,(\hat{\mathcal{P}} e^{-\beta \hat{H}}) = \sum_n \langle n|\hat{\mathcal{P}} e^{-\beta \hat{H}}|n\rangle, \tag{2}$$

where $|n\rangle$ is a many-particle state in the Hilbert space $\mathcal{H}$ and $\hat{\mathcal{P}}$ is the projection operator for any desired configuration. We propose to consider the statistical thermodynamical description of a quantum gas consisting of quarks and antiquarks such that the underlying symmetry amounts to a reordering of the partition function in terms of the color-singlet multiquark modes at a given temperature. We also assume here that the gluons are in thermal background and that the partonic matter is mainly composed of quarks and antiquarks.

Now, for a symmetry group $\mathcal{G}$ (compact Lie group) having unitary representation $\hat{U}(g)$ in a Hilbert space $\mathcal{H}$, the projection operator can be written as [11,12]

$$\hat{\mathcal{P}}_j = d_j \int_{\mathcal{G}} d\mu(g) \chi_j^\star(g) \hat{U}(g), \tag{3}$$

where d_j and χ_j are, respectively, the dimension and the character of the irreducible representation j of $\mathcal{G}$ and $d\mu(g)$ is the normalized Haar measure in the group $\mathcal{G}$. The

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symmetry group associated with the color-singlet configuration of the system is SU(N_c), where N_c is the number of the color corresponding to the fundamental representation, and for the SU(N_c) color-singlet configuration $d_j = 1$ and $\chi_j = 1$. Now, the color-singlet partition function for the system becomes

$$\mathcal{Z}_S = \mathrm{Tr}\left[\int_{\mathrm{SU}(N_c)} d\mu(g)\ \hat{U}(g)\exp(-\beta\hat{H})\right], \quad (4)$$

where $\hat{U}(g)$ can be thought of as a local link variable that links the (anti)quarks in a given state of the physical system. Now, the trace in a Fock space results in a product of the fermionic determinant over momentum modes and color degrees of freedom, which we shall see later. Thus the interchange of the Tr and the integration will lead to an overall product over momentum and color states, and one can write Eq. (4) as

$$\mathcal{Z}_S = \prod\int_{\mathrm{SU}(N_c)} d\mu(g)\mathrm{Tr}[\hat{U}(g)\exp(-\beta\hat{H})]. \quad (5)$$

We neglect the mutual interactions among the constituents, although they must interact to come to a thermal equilibrium. One can imagine a situation in which they are first allowed to come to a thermal equilibrium and then the interactions are slowly turned off [13]. Such a simple thermodynamic description is often useful for various physical systems (e.g., electrons in metal, black-body photons in a heated cavity, phonons at low temperature, neutron matter in neutron stars, etc.). The full Hamiltonian is then the sum of the Hamiltonians for each species, that is, quarks and antiquarks, as $\hat{H} = \hat{H}_q + \hat{H}_{\bar{q}}$ with $\hat{H}_i = \hat{h}_i - \mu_i\hat{N}_i$, in the grand canonical ensemble, with the usual meaning of μ_i and $\hat{N}_i$.

Now, the Hilbert space $\mathcal{H}$ of the composite system has a structure of a tensor product of the individual Fock spaces of quarks and antiquarks as $\mathcal{H} = \mathcal{H}_q \otimes \mathcal{H}_{\bar{q}}$, where the subscripts q and $\bar{q}$ denote the quark and antiquark, respectively. Because of this, the partition function in Eq. (5) in Hilbert space $\mathcal{H}$ decomposes into the product of two traces [12,14–16] in their respective Fock spaces as

$$\mathcal{Z}_S = \prod\int_{\mathrm{SU}(N_c)} d\mu(g)\ \mathrm{Tr}[\hat{U}_q(g)e^{-\beta\hat{H}_q}]\mathrm{Tr}[\hat{U}_{\bar{q}}(g)e^{-\beta\hat{H}_{\bar{q}}}]. \quad (6)$$

For simplicity, we approximate the local link variable $U(g)$ to be a diagonal matrix related only to the diagonal generators in color space associated with the maximal abelian subgroup (Cartan subgroup) [11] of SU(N_c), as, for example, the SU(3) color gauge group has only two parameter abelian subgroups associated with the two diagonal generators that would characterize $U(g)$, including its diagonalization, as can be seen later. Under this approximation, the Haar measure corresponding to SU(N_c) can now be written in the Weyl reduced [11,12] form as

$$\int_{\mathrm{SU}(N_c)} d\mu(g) = \frac{1}{N_c!}\left(\prod_{l=1}^{N_c}\int_{-\pi}^{\pi}\frac{d\theta_l}{2\pi}\right)\delta\left(\sum_l^{N_c}\theta_l\right) \times J(e^{i\theta_1}\cdots e^{i\theta_{N_c}}), \quad (7)$$

where J is the Jacobian of transformation (also known as the Vandermonde determinant [17]). Variable θ_l is a class parameter characterizing the group element g such that $U(g)$ can be diagonalized. It also obeys the periodicity condition $\sum_{l=1}^{N_c}\theta_l = 0\ (\mathrm{mod}2\pi)$, which ensures that the group element is SU(N_c). Thus it is obvious that the product of two Fock space traces in Eq. (6) has to be a class function, which can be obtained, as shown later, using the diagonalization condition in the maximal abelian subspace of SU(N_c).

Now, in each Fock space there exists a basis that diagonalizes both operators as long as $\hat{H}_i$ and $\hat{U}_i(g)$ commute. Let $|\alpha,\sigma\rangle$ be the one-particle states in such a basis, where α labels the eigenvalues of $\hat{H}_i$ (including a possible degeneracy besides the one associated with the symmetry group $\mathcal{G}$) and σ labels the eigenvalues of $\hat{U}_i(g)$. One can write the diagonalized eigenstate as

$$\langle\alpha',\sigma'|\hat{U}_i(g)\hat{A}_i|\alpha,\sigma\rangle = \delta_{\alpha\alpha'}\delta_{\sigma\sigma'}R_{i\sigma\sigma}A_{i\alpha}, \quad (8)$$

where $A_{i\alpha}$ and $R_{i\sigma\sigma}(g)$ are the eigenvalues of $\hat{A}_i$ and $\hat{U}_i(g)$, respectively. Then, following standard procedure, one can obtain [12,13]

$$\mathrm{Tr}\,(\hat{U}_i e^{-\beta\hat{A}_i}) = \exp\{\mathrm{tr}_\alpha\mathrm{tr}_c\ln[1 + R_i(g)e^{-\beta A_{i\alpha}}]\}. \quad (9)$$

Note that tr_α is the trace over the momentum state α, whereas tr_c is the trace over the color degrees of freedom in the same momentum state α. In fundamental representation, $\mathrm{tr}_c R(g^k) = \sum_{l=1}^{N_c}\exp(ik\theta_l)$ along with $R^k(g) = R(g^k)$. This can be related to the Polyakov loop [18,19] in a Polyakov gauge as $\mathcal{L} = \mathrm{tr}_c(L)/N_c = \mathrm{tr}_c R(g^k)$, where L are the thermal Wilson lines defined by the temporal gluons in Euclidian time. This correspondence is because of the choice of diagonal $U(g)$, which resembles the Polyakov loop matrix in a Polyakov gauge, supplemented with the diagonalization condition in Eq. (8).

Now, one can write the product of the two traces in Eq. (6) as

$$\mathrm{Tr}[\hat{U}_q(g)e^{-\beta\hat{H}_q}]\mathrm{Tr}[\hat{U}_{\bar{q}}(g)e^{-\beta\hat{H}_{\bar{q}}}] = \exp(\Theta), \quad (10)$$

where it is easy to show that

$$\Theta = \sum_\alpha\sum_l^{N_c}\sum_s^{N_s}\sum_f^{N_f}[\ln(1 + e^{i\theta_l}e^{-\beta(\epsilon_{q\alpha}-\mu_q)}) + \ln(1 + e^{-i\theta_l}e^{-\beta(\epsilon_{q\alpha}+\mu_q)})]. \quad (11)$$

Here the flavor (f) and spin (s) summations are introduced, where N_f and N_s are the number of flavor and spin degrees of freedom, respectively. Equation (11) clearly indicates the superposition of two Fock spaces (fermionic and antifermionic determinants) with particles having the same momentum and internal color structure and obeying the quantum statistics. The momentum states that do not satisfy this are automatically eliminated by the diagonalization condition in Eq. (8). This essentially amounts to a stacking of the same momentum particles. It is also evident that even if we had started by considering a free gas of quarks and antiquarks, the color-singlet restriction related to the Polyakov loop in a Polyakov gauge links the effective degrees of freedom of (anti)quarks with the surrounding thermal bath through the temporal gluons. As we will see, this allows an interesting scenario of recombinations. Now, the single-particle energy eigenvalues in a given state are the same for q and $\bar{q}$ as $\epsilon_{i\alpha} = \sqrt{p_{i\alpha}^2 + m_i^2}$. However, their occupation energies in a given state differ by their chemical potentials as $\mu_{\bar{q}} = -\mu_q = -\mu$. For convenience [15], we make a substitution $\xi = -i\beta\mu$ and $\epsilon_{q\alpha} = \epsilon_\alpha$ in Eq. (11), which then becomes

$$\Theta = \ln \prod_\alpha \prod_l^{N_c} \mathcal{D},$$
$$\mathcal{D} = \{e^{-\beta\epsilon_\alpha}[2\cosh\beta\epsilon_\alpha + 2\cos(\theta_l + \xi)]\}^{N_f N_s}, \quad (12)$$

where the determinant $\mathcal{D}$ is a class function. We would like to note that the class parameters in color space get associated with the imaginary chemical potential ξ due to the choice of diagonal $U(g)$; as a consequence, we will see later that the color factor N_c appears with the chemical potentials, indicating the number of valence (anti)quarks.

The partition function in Eq. (6) can now be written as

$$\mathcal{Z}_S = \prod_\alpha \int_{\mathrm{SU}(N_c)} d\mu(g)\, \mathcal{D}. \quad (13)$$

The integrations on class parameters in Eq. (13) are now performed exactly by using the properties of the Jacobian and an orthonormal polynomial method [17]. The logarithm of the partition function [16] for $N_c = 3$ and two massless quarks ($N_f = 2$) in the infinite volume, V, limit reads as

$$\frac{\ln \mathcal{Z}_S}{V} = \int \frac{d^3p}{(2\pi)^3} \ln[1 + S], \quad (14)$$

where S is the sum of the Boltzmann factors of the color-singlet multiquark clusters, allowed by the symmetries

$$S = \sum_{b=0}^{2N_f} \sum_{m=\delta_{0b}}^{6N_f - N_c b} C_{mb} \exp\{-[2m\epsilon + N_c b(\epsilon \mp \mu)]\beta\}. \quad (15)$$

In doing so, we have replaced the $\sum_\alpha$ by the integration over phase space volume $d^3x d^3p/(2\pi)^3$. It is interesting to note that because of the color-singlet restriction, the color factor, $N_c = 3$, and the baryon number, b, are always associated with the Boltzmann factor and thus with the partonic chemical potential, indicating the excess number of quarks or antiquarks in addition to m number of quarks and antiquarks. This happens just because the diagonal matrix, $R_{\sigma\sigma}$, in color space or the Polyakov loop readjusts the (anti)quarks of different momenta to be in the same momentum state by allowing them to exchange their momentum in the thermal bath. As seen, in general, a given color-singlet mode in Eq. (15) has energy $E_{mb} = (2m + N_c b)\epsilon$ and parton content $[(m + N_c b)q, m\bar{q}]$ for a hadron, whereas that for an antihadron is $[(m + N_c b)\bar{q}, mq]$. The pure mesonic modes ($b = 0$) can be identified with a parton content $(mq, m\bar{q})$ having total energy $E_{m0} = 2m\epsilon$. Similarly, the pure baryonic modes ($m = 0$) have the energy $E_{0b} = N_c b\epsilon$ with parton content $[N_c bq]$ for baryon, whereas that for antibaryons is $[N_c b\bar{q}]$. In the preceding, C_{mb} is the weight factor because of flavor and spin symmetry, appearing with each color-singlet mesonic/baryonic/antibaryonic mode because of color-singlet restriction. Their values for various modes are listed in Ref. [16]. For low-lying mesons ($m = 1$ and $b = 0$), $C_{10} = 16$, whereas for low-lying baryons and antibaryons ($m = 0$ and $b = 1$), $C_{01} = 20$, respectively. These are exact for SU(2) flavor and SU(2) spin symmetry [20] of the quark model, in which nucleons and Δs are degenerate. On the other hand, $m > 1$ and $b = 0$ correspond to excited mesonic modes, whereas $m \geqslant 1$ and $b \geqslant 1$ are pentaquark and excited baryonic/antibaryonic modes, which are the Hagedron states [21].

Equation (14) clearly exhibits a nontrivial result: SU(3) color-singlet restriction, along with flavor and spin symmetry, on the quark-antiquark ensembles reorders the partition function in terms of Boltzmann factors of the color-singlet multiquark (mesonic/baryonic/antibaryonic) modes at any temperature. These are, however, not the bound states, but they can be regarded as a precursor to the confinement because of recombination [6,7,22]. Under a suitable confining mechanism (e.g., a Polyakov loop model [18,19]), one can hope that these multiquark structures could evolve into color-singlet hadrons in the low-temperature limit.

The probability of finding a single (anti)quark in the system in the energy interval ϵ and $\epsilon + d\epsilon$ follows from Eq. (14) as [23]

$$\mathcal{P}(\epsilon)d\epsilon = \frac{Vd^3p}{(2\pi)^3} \ln[1 + S]. \quad (16)$$

The logarithm is expanded with $S < 1$, which yields a solution $\epsilon > \zeta T$, where $\zeta \sim 1.7$ for $N_f = 2$. This provides an energy cutoff $\geqslant 2\zeta T$ for mesonic modes and $\geqslant 3\zeta T$ for baryonic modes. One can write the preceding as

$$\mathcal{P}(\epsilon)d\epsilon = \sum_{b=0}^{\infty} \sum_{m=\delta_{0b}}^{\infty} C_{mb}^* \exp\{-[2m\epsilon + 3b(\epsilon \mp \mu)]\beta\} \frac{Vd^3p}{(2\pi)^3}$$
$$= \sum_{b=0}^{\infty} \sum_{m=\delta_{0b}}^{\infty} \mathcal{P}_{mb}^q(\epsilon)d\epsilon, \quad (17)$$

in which all higher-order terms are accumulated in C_{mb}^*, where $C_{mb}^* = C_{mb}$ only for low-lying hadronic modes. In the preceding, $\mathcal{P}_{mb}^q(\epsilon)d\epsilon$ is the probability of a single parton, with energy $\epsilon > \zeta T$, in the interval ϵ and $\epsilon + d\epsilon$ in a given mbth mode, which reads as

$$\mathcal{P}_{mb}^q(\epsilon)d\epsilon = C_{mb}^* \exp\{-[2m\epsilon + 3b(\epsilon \mp \mu)]\beta\} \frac{Vd^3p}{(2\pi)^3}. \quad (18)$$

Now, the distribution of a parton within a given mbth mode in the fluid in terms of the momentum of the parton ($p^\mu u_\mu = \epsilon$, u_μ is the four-velocity of the fluid) follows directly from Eq. (18) as

$$\frac{dN_{mb}^q}{d^3x\, d^3p} = \frac{C_{mb}^*}{(2\pi)^3} \exp\{-[2m\epsilon + 3b(\epsilon \mp \mu)]\beta\}. \quad (19)$$

One can easily invert the parton momentum distribution in the preceding equation into the momentum $[P^\mu u_\mu = E_{mb} = (2m + 3b)\epsilon]$ distribution of a mbth hadronic mode as

$$\frac{dN_{mb}}{d^3xd^3P} = n^3 \frac{dN_{mb}^q}{d^3xd^3P} = \frac{C_{mb}^*}{(2\pi)^3} e^{-(E_{mb} \mp 3b\mu)\beta}, \quad (20)$$

where $n = (2m + 3b)$ and the preceding is the Cooper-Frye distribution [24] at the freeze-out in the rest frame of the fluid. We assume that this distribution describes the corresponding hadrons after freeze-out. The preceding distribution also has an important consequence: The entropy would remain conserved because the number of quarks in the quark's phase space is equal to that of the hadrons in the hadron's phase space.

Now, we can obtain the differential proton-to-pion ratio at central rapidity for a given transverse momentum $P_\perp \gg 3\zeta T$ as

$$\frac{dN_{01}}{dN_{10}} = \frac{dN_p}{dN_\pi} = \frac{C_{01}^*}{C_{10}^*} e^{3\mu/T} = \frac{5}{4} e^{\mu_B/T}, \tag{21}$$

where $\mu_B = 3\mu$ is the baryonic chemical potential and the preceding equation is independent of $P_\perp$. PHENIX data show [5] that the ratio has a plateau around unity in the $P_\perp$ range, $2\ \text{GeV}/c \leqslant P_\perp \leqslant 4\ \text{GeV}/c$. Our estimation shows that it is in good agreement with RHIC data [5,25]. Recall that in the recombination model by Fries *et al.* [6], it was found to be $(5/3)\exp(\mu_B/T)$. Similarly, the antiproton-to-pion ratio is $(5/4)\exp(-\mu_B/T)$ and the antiproton-to-proton ratio is $\exp(-2\mu_B/T)$, both of which are again in good agreement with the RHIC data [26]. Thus the color-singlet projection of the thermal parton ensemble based on symmetry considerations of the underlying theory also provides a strong basis for the hadronization by recombination of quark-gluon plasma at intermediate $P_\perp$ ($2\ \text{GeV}/c \leqslant P_\perp \leqslant 4\ \text{GeV}/c$), even though the ingredients are different from the recombination models [6,7] with a sudden approximation.

Now, the total number of a given hadron emitted by a fluid can be obtained from Eq. (19) as

$$N_{mb} = \frac{VC_{mb}^*}{(2\pi)^3} T^3 e^{-n\zeta \pm 3b\mu/T}\left(1 + n\zeta + \frac{n^2\zeta^2}{2}\right), \tag{22}$$

which simply depends on the number of partonic degrees of freedom that form the color-singlet hadronic modes with $\epsilon/T > \zeta$. The ratio of protons to pions becomes ∼0.6, whereas it is ∼0.3 for antiprotons to pions with $\mu_B/T \sim 0.33$ in RHIC. This is consistent with the RHIC data [5].

We now consider noncentral collisions in which the fluid has larger velocity on the x-axis (semiminor) than on the y-axis (semimajor), leading to elliptic flow [27]. The flow coefficient is defined as

$$v_2 = \langle \cos 2\phi \rangle. \tag{23}$$

This requires that the distribution in Eq. (19) have a nontrivial dependence on azimuthal angle ϕ. It is introduced through $\epsilon = p^\mu u_\mu = p_\perp u_0(\phi) - p_\perp u(\phi)$, where u_μ is the four-velocity of the fluid and $u(\phi)$ can be parametrized [27] in the following form:

$$u(\phi) = u + 2\alpha \cos 2\phi, \tag{24}$$

where u is ϕ-averaged over the maximum fluid four-velocity in the ϕ direction and α specifies the magnitude of the elliptic flow, which is about 4% in noncentral Au-Au collisions at RHIC. Considering $u^\mu u_\mu = u_0^2(\phi) - u^2(\phi) = 1$ and expanding it to a first order in α, one can obtain

$$u_0(\phi) = u_0 + 2\alpha v \cos 2\phi, \tag{25}$$

where $v \equiv u/u_0$. Now, the elliptic flow coefficient for the mbth mode in Eq. (23) is found to scale perfectly with the partonic $p_\perp$ as

$$(v_2)_{mb} = \frac{\alpha}{T}(1 - v)(2m + 3b)p_\perp. \tag{26}$$

We note that this scaling is strictly valid in the recombination region ($2\ \text{GeV}/c \leqslant P_\perp \leqslant 4\ \text{GeV}/c$), where the mass dependence of the hadrons is irrelevant. However, for $P_\perp < 1.5\ \text{GeV}/c$, the scaling deviates [2] because of the mass ordering of the hadrons. To demonstrate this fact, one needs a dynamical mass-generating term of the quarks, which is, however, beyond the scope of this calculation.

We propose that the recombination phenomenon occurs readily when one constructs a color-singlet partition function from a thermal ensemble of quarks and antiquarks having internal symmetries, namely, color, spin, and flavor. This color-singlet projection is shown to be related to the Polyakov loop in the Polyakov gauge, which plays an important role in recombination. We also show that such recombination of thermal partons naturally describes some of the puzzling results in RHIC experiments, which, in turn, strongly supports that recombination of thermal (anti)quarks could be one of the dominant mechanisms of hadronization in a dense medium in the intermediate transverse momentum of hadrons ($2\ \text{GeV}/c \leqslant P_\perp \leqslant 4\ \text{GeV}/c$). Nevertheless, how to test the various quark recombination models for hadronization from dynamical QCD still remains an open question. We tried to address this question based on the symmetry consideration of the theory, which may be useful for the eventual solution of this important problem.

Valuable discussions with B. Müller, D. K. Srivastava, S. Bass, and R. Ray are acknowledged.

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