High-resolution study of 0^+ states in ^{170}Yb

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Recently, 0^+ excitations, especially in the rare-earth region, were studied extensively. We extend this work by studying the excited 0^+ states in ^{170}Yb using the ^{172}Yb$(p,t)^{170}$Yb reaction. Eighteen excited 0^+ states, 14 of which are new, are observed up to an energy of 3.5 MeV. The results are analyzed using the *sd* and *spdf* interacting boson models.

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I. INTRODUCTION

In rare-earth nuclei various collective modes appear, from transitional nuclei in the Gd region to well-deformed nuclei in the Yb region and γ-soft nuclei in the Pt region. Therefore, the rare-earth nuclei are an ideal testing ground for studies of collective motions. In recent years 0^+ excitations in this region were studied in unprecedented detail with the (p,t) reaction. Large numbers of 0^+ excitations were identified in ^{152,154,158}Gd, ^{162}Dy, ^{168}Er, ^{176}Hf, ^{180,184}W, and ^{190}Os [1–3]. On average, about 15 0^+ states were found up to excitation energies of roughly 4 MeV. It seems that the high density of 0^+ states is a common feature in this region. Similar high abundances of 0^+ states can be found in the actinide region [4]. Calculations within the *sd* interacting boson model (IBM) [5] failed to reproduce the large number of 0^+ states while the *spdf* IBM predicts numbers of 0^+ states comparable to the numbers observed although, at the same time, there are presumably many noncollective 0^+ states below 4 MeV as well. The origin of the large number of detected 0^+ states is still under discussion [3,6–9].

An extensive mapping of excited 0^+ states in nuclei of this region is necessary to study the evolution of the abundance of 0^+ states as a function of N and Z. In the present work, the (p,t) reaction was used to determine the excited 0^+ states in ^{170}Yb. Up to an excitation energy of 3.5 MeV, 18 states, of which 14 are new, were determined. In Sec. II the experimental details are given. Section III describes the data analysis and compares our results with those from the literature. A comparison with the *sd* IBM and the *spdf* IBM is then given in Sec. IV. Finally, in Sec. V conclusions are drawn.

II. EXPERIMENTAL DETAILS

Two (p,t) experiments were performed at the Maier-Leibnitz Laboratory (MLL) of LMU Munich and TU Munich Tandem accelerator laboratory using the Q3D magnetic spectrograph [10] with procedures similar to those of Ref. [4]. A beam of 25-MeV unpolarized protons was incident on a 95.9% enriched ^{172}Yb target with a thickness of 102 μg/cm^2 on a 9 μg/cm^2 carbon backing. The outgoing tritons were measured at laboratory angles of 5°, 17.5°, and 30° with respect to the beam axis. The data at 5° are particularly crucial for assigning 0^+ states since only $L = 0$ transfers are large at that angle. The ratio $R(5°/17.5°) \equiv \sigma(5°)/\sigma(17.5°)$ provides the essential information. Values of $R(5°/17.5°) \gg 1$ can be firmly assigned as 0^+ states, whereas values $\sim$1 or lower denote higher angular momentum transfers. The energy resolution was 4–6 keV full width at half maximum for 15–20 MeV tritons. With the high-resolution 1-m-long focal plane detector [11] four magnetic settings were necessary to cover an excitation energy range from 0 to 4 MeV. Very clean cross-section measurements down to a few μb/sr, which is a few tenths of a percent of the ground-state cross sections, were possible with little or no background, particularly at 5°. The entire spectrum obtained at 5°, where $L = 0$ transfer cross sections are large, is shown in Fig. 1. Energy calibrations were obtained using known levels, and peaks arising from target impurities were identified from their appropriate reaction Q values.

III. ANALYSIS OF THE DATA

Spin-parity values of 0^+ states can be easily assigned since the (p,t) angular distribution for the $L = 0$ transfer strongly peaks at forward angles. The threshold to assign levels to 0^+ states was defined at $R(5°/17.5°) > 3$ (see the dashed line in Fig. 2). With this conservative assumption we risk overlooking a few 0^+ states. However, it allows us to exclude incorrect assignments since, to our best knowledge, no states with $R(5°/17.5°) > 3$ ratios were found to have an $L > 0$ angular momentum transfer in (p,t) reactions. Table I lists the observed intensities for the $L = 0$ transfer after normalization for the 5° magnetic field settings. It also gives the ratio of intensities at 5° and 17.5°. The ground-state intensity is given a value of 1000. Although absolute cross sections were not measured, it is known (see, e.g., Ref. [12]) that, in this mass region, (p,t)

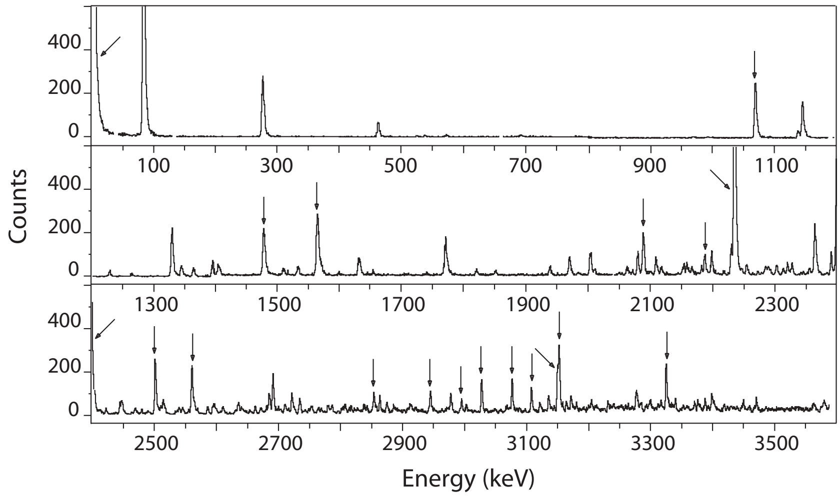

FIG. 1. Entire energy spectrum at 5°. The different magnetic settings are normalized to that in which the ground state was measured. All measured 0^+ states are marked with an arrow.

ground-state cross sections at 5° are invariably very close to 1 b/sr, and thus the 5° entries in the table are roughly in units of mb/sr. Figure 2 shows the measured ratio of the cross sections at 5° and 17.5° for the 0^+ states in comparison to those of other L transfers. We identified 18 excited 0^+ states of which 14 are new. We cannot confirm the tentative assignment in Ref. [13] of the 2328-keV level since the ratio $\frac{I(5^\circ)}{I(17.5^\circ)} = 0.221(17)$ is well below 1. The 1229-keV level presents a complex situation (see Fig. 3). It is firmly assigned as 0^+-based in β decay [14] and Coulomb excitation [15] studies. It was not observed with an $L = 0$ angular distribution in Ref. [16]. However, in that work, no measurements were taken at 5°. In the present study, the angular distribution for the 1229-keV region is shown in Fig. 3. The intensity at 17.5° is higher than at 5°, but also a clear doublet structure can be seen in the 17.5° spectrum near 1229 keV. Clearly, the 0^+ state cross section at 1229 keV is much weaker than for the other 0^+ states, if it is at all populated. We set an upper limit for the expected cross section at 17.5° for a 0^+ assignment in Fig. 3 and give the value of the cross section at 5° as an upper limit for the population of this state in Table I. At 1566 keV, a state was populated with an $L > 0$ angular distribution in Ref. [16]. However, the present data at 1566 keV at 5° confirm the 0^+ assignment.

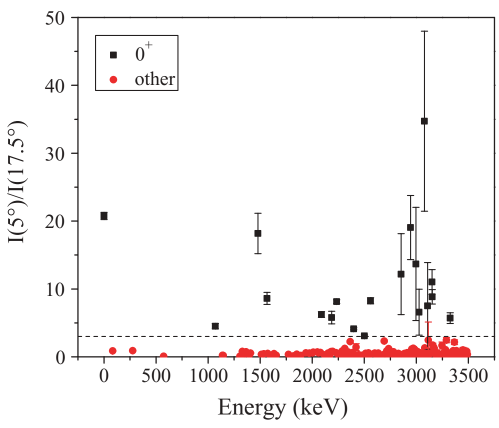

FIG. 2. (Color online) Measured ratio of the cross sections at 5° and 17.5°. The dashed line gives the limit above which levels are assigned to be 0^+ states.

IV. DISCUSSION

The results in Table I are generally consistent with those of other recent studies of two-neutron transfer reaction studies in well-deformed nuclei [2,3,12]. Specifically, the ground state collects the dominant cross section and those to excited 0^+ states are typically only a few percent as large. Also, typical of recent high-resolution studies is a very large increase in the number of known excited 0^+ states, in this case, from the previously known 4, to 18. Such data, therefore, provide a wealth of spectroscopic information and a challenge that is being

TABLE I. Observed intensities at 5° and 17.5° for states that are assigned to be 0^+ states. Previously known states are marked with an asterisk.

E (keV)	I (5°)	$\frac{I(5^\circ)}{I(17.5^\circ)}$
0*	1000.0(35)	20.76(54)
1069*	21.62(52)	4.52(40)
1229*	< 1.24(39)	–
1479*	16.61(46)	18.2(30)
1566*	19.36(54)	8.61(88)
2088	14.05(27)	6.23(43)
2186	8.1(10)	5.79(92)
2234	107.4(10)	8.13(27)
2399	48.07(84)	4.13(15)
2501	20.90(35)	3.08(13)
2560	14.44(31)	8.26(45)
2854	6.32(30)	12.2(60)
2945	7.00(40)	19.1(47)
2995	3.29(22)	13.7(83)
3027	7.86(36)	6.6(34)
3077	9.24(27)	35(13)
3108	8.27(26)	7.5(64)
3150	13.68(69)	11.0(18)
3153	16.82(71)	8.8(11)
3325	11.80(32)	5.70(78)

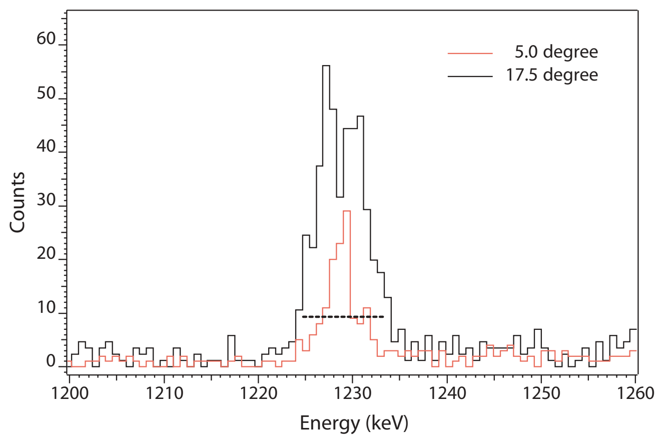

FIG. 3. (Color online) Comparison between normalized triton energy spectra taken at 5° and 17.5°. The population of the 1229-keV states decreases at 5° in contrast to what is expected for a 0^+ state, as firmly assigned in Refs. [14,15]. The dashed line gives the approximate upper limit of the expected spectrum in the 17.5° setting for a 0^+ state at 1229 keV.

taken up by theory [6–8,17]. As in some other nuclei, there are occasional 0^+ states (here, those at 2234 and 2399 keV) with large cross sections relative to most others (5%–15% of the ground-state cross section). The mechanisms of such cross sections are not clear but may involve the transfer of particles in special Nilsson orbitals. It is useful to try to understand the origin of such a large number of 0^+ states. It is well known that there are a number of mechanisms that can produce 0^+ states. These range from quadrupole vibrations, to pairing modes, spin-quadrupole excitations, modes resulting from the combination of monopole and quadrupole pairing interactions in regions with nucleons moving in both prolate and oblate Nilsson orbits near the Fermi surface, and multiphonon states of both positive and negative parity vibrational modes such as double octupole excitations. It is difficult to incorporate all these degrees of freedom in a single model, but one can obtain evidence for their presence by inspecting whether simpler models can account for the observed 0^+ states. To this end, we will consider IBM calculations both in the original *sd* boson form and in an expanded form that includes negative-parity $p(L=1)$ and $f(L=3)$ bosons. This follows Ref. [6].

The IBM [5] describes collective nuclear excitations as an N_B boson problem, where N_B is the number of bosons (pairs of valence fermions). In the simplest version of the model, which we denote as *sd* IBM, the bosons are of s $(L=0)$ and d $(L=2)$ type. We use the simple Hamiltonian

$$\hat{H}(N,\eta,\chi) = c\left[\eta\,\hat{n}_d + \frac{\eta-1}{N_B}\,\hat{Q}_\chi\cdot\hat{Q}_\chi\right], \qquad (1)$$

where $\hat{n}_d = d^\dagger\cdot\tilde{d}$ is the d-boson number operator and $\hat{Q}_\chi = [s^\dagger\tilde{d} + d^\dagger s]^{(2)} + \chi[d^\dagger\times\tilde{d}]^{(2)}$ is the quadrupole operator. In the denominator N_B stands for the total number of bosons (integral of motion) and ensures a convenient scaling. Control parameters η and χ vary within the range $\eta\in[0,1]$ and $\chi\in[-\sqrt{7}/2,0]$. Finally, c is introduced here as a scaling factor needed for comparison with experimental data. The

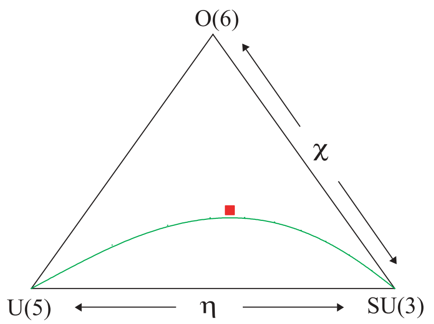

FIG. 4. (Color online) Symmetry triangle of the *sd* IBM. The curve shows the locus of the most regular part [21] inside the symmetry triangle. Also shown are the three dynamical symmetries at the vertices and the χ and η coordinates of the triangle. The location of ^{170}Yb is also marked.

parameter space can be represented by the standard Casten triangle [18] (see Fig. 4) whose $\eta=1$ vertex corresponds to the U(5) dynamical symmetry (spherical shape), while the dynamical symmetries SU(3) (prolate rotor) and O(6) (γ-soft) are located on the $\eta=0$ side: SU(3) at the $\chi=-\sqrt{7}/2$ vertex and O(6) at $\chi=0$.

Figure 5 shows the fit of ^{170}Yb obtained in Refs. [19,20] using the Hamiltonian of Eq. (1) with the parameters $\eta=0.631$ and $\chi=-0.75$. The values correspond to a fit that does not take into account the third experimental 0^+ state in Refs. [19,20] since this was the state at 1229 keV whose 0^+ assignment is not confirmed with the present data. Figure 4 shows the location of the nearly regular region in the Casten triangle and of ^{170}Yb. The regular region is spanned by Hamiltonians that are characterized by almost regular dynamics rather than chaotic behavior outside the three dynamical symmetries of the IBM [21]. Clearly an excellent fit is obtained; $E2$ transitions rates are also well reproduced in Refs. [19,20]. In addition to the level scheme shown in Fig. 5, we also calculate two-neutron (p,t) transfer cross sections using the IBM wave functions for ^{170}Yb as well as those for the target, ^{172}Yb, using the parameters from Refs. [19,20]. The results show excellent agreement with the data—that is, a very large ground-state cross section and almost negligible cross sections to all excited states. The largest cross section to any excited 0^+ state is 3.4% of the ground-state cross section, which is in agreement with the data, except the large cross sections to the 2234- and 2399-keV states, which are anomalous in the spectrum and do not seem amenable to collective model calculations. However, it is well known [22] that two-neutron cross sections to 0^+ states are highly sensitive to the orbit in which the two nucleons are transferred. In a transferred configuration of the type $|j^2J\rangle$, the two nucleons can couple their angular momenta to $J=0,2,4,\ldots,(2j-1)$, and the $L=0$ cross section depends on the probability that they are coupled to $J=0$. This is clearly much larger for low-j orbits than high-j orbits. Therefore, for example, the transfer cross section of two particles in a $3p_{3/2}$ orbit coupled to $J=0$ is more than an order of magnitude larger at 5° than transfer of two particles in a $1i_{13/2}$ orbit [3]. This can suggest that the 2234- and 2399-keV states are not collective, but are rather dominated

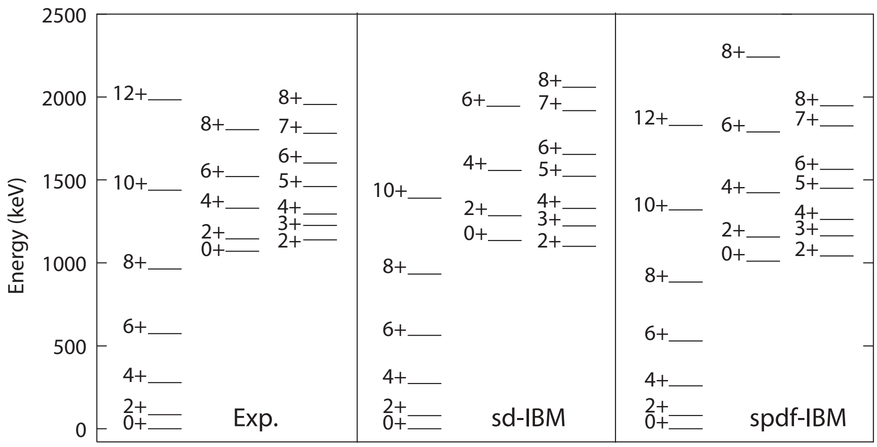

FIG. 5. Comparison of experimental and theoretical levels for ^{170}Yb using the parameter of Refs. [19,20] for the *sd* IBM. The parameters for the *spdf* IBM are given in Sec. IV.

by two-quasiparticle structures involving low-j orbits. Likely candidates will be the Nilsson orbit 1/2[510] or the 3/2[512] or 3/2[501] orbits, which lie well above the Fermi surface in ^{170}Yb.

The 0^+ levels predicted by the *sd* IBM model are compared with the experimental data in Fig. 6. Clearly, these calculations do not at all reproduce the observed number of states. To test the effects of multiphonon octupole modes we added up to two p and f bosons, along the line of the work done in Ref. [6]. In the *spdf* IBM [23], up to two p and f bosons are coupled to the usual *sd* IBM configurations. The Hamiltonian used is

$$\hat{H} = \hat{H}^{sd} + \epsilon_p \hat{n}_p + \epsilon_f \hat{n}_f + 2\kappa \hat{Q}^{(2)}_{sd} \cdot \hat{Q}^{(2)}_{pf} + \kappa' \hat{L}^{(1)}_{sd} \cdot \hat{L}^{(1)}_{pf}, \tag{2}$$

with

$$\hat{H}_{sd} = \epsilon_d \hat{n}_d + a_0 \hat{P}^\dagger \cdot \hat{P} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_5 \hat{n}_d(\hat{n}_d + 4), \tag{3}$$

to describe the $\hat{H}^{sd}_i$ part, and

$$\hat{Q}^{(2)}_{sd} = [s^\dagger \tilde{d} + d^\dagger s]^{(2)} - \tfrac{1}{2}\sqrt{7}[d^\dagger \tilde{d}]^{(2)}, \tag{4}$$

$$\hat{Q}^{(2)}_{pf} = \tfrac{3}{5}\sqrt{7}[p^\dagger \tilde{f} + f^\dagger \tilde{p}]^{(2)} - \tfrac{9}{10}\sqrt{3}[p^\dagger \tilde{p}]^{(2)} - \tfrac{3}{10}\sqrt{42}[f^\dagger \tilde{f}]^{(2)}, \tag{5}$$

$$\hat{L}^{(1)}_{sd} = \sqrt{10}[d^\dagger \tilde{d}]^{(1)}, \tag{6}$$

$$\hat{L}^{(1)}_{pf} = \sqrt{2}[p^\dagger \tilde{p}]^{(1)} + 2\sqrt{7}[f^\dagger \tilde{f}]^{(1)}, \tag{7}$$

where $\hat{H}^{sd}$ is the usual *sd* IBM Hamiltonian in the multipole expansion. A fit of the known negative-parity states of ^{170}Yb, using the simple fit done in Refs. [19,20] for the *sd* part yielded the following parameters: $\epsilon_d = 459.0$, $a_0 = 31$, $a_1 = 1.6$, $a_2 = -10.1$, $a_3 = 11.0$, $a_5 = 2, 17$, $\epsilon_p = 1158.0$, $\epsilon_f = 1187.0$, $\kappa = 4.20$, and $\kappa' = -10.1$, (all in keV). This fit is able to reproduce the overall density of the 0^+ states up to 4 MeV (see Fig. 6) and without destroying the agreement of the other known low-lying excited states (see Fig. 5). This, of course, does not imply that the extra states are of double octupole character. Some are undoubtedly of two-quasiparticle character, but these calculations suggest that degrees of freedom beyond s and d bosons are needed to consider the full spectrum of 0^+ states, and these calculations give an example of a source of at least some of them.

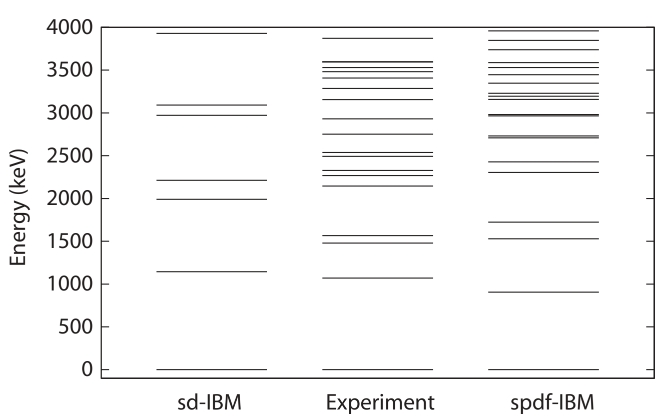

FIG. 6. Comparison between the experimental data and theoretical 0^+ states in ^{170}Yb. Note that the 1229-keV state that was not seen in this experiment is not plotted in the experimental spectrum.

V. CONCLUSION

To summarize, we identify 18 excited 0^+ states in ^{170}Yb in a high-resolution, high-dynamic-range study of the (p,t) reaction using the Q3D magnetic spectrometer and associated focal plane detector at the MLL in Munich. Of these states, 14 are new, greatly expanding the number of such states known. Typical of most well-deformed nuclei, nearly all cross sections to excited 0^+ states are a few percent of the ground-state cross section. Large cross sections ($\sim$10% of the ground-state cross section) are observed for the 2234- and 2399-keV levels. Although it is clear that many of these 0^+ states may not be collective, we carried out test *sd* IBM and *spdf* IBM calculations of 0^+ states in ^{170}Yb to determine how many 0^+ states, and with what overall energy distribution, can be obtained. The results are similar to those of Ref. [6] in that the *sd* IBM can account for only a small fraction of the observed states. The model does qualitatively account for the generic feature that the cross sections to excited states are small. However, it does not reproduce the large cross sections to the 2234- and 2399-keV levels, which are likely to be noncollective states whose structure and cross section are dominated by two-quasiparticle configurations involving primarily low-j orbits that carry large two-neutron transfer strength. Incorporating two negative-parity bosons into the IBM calculations significantly increases the number of 0^+ states below 3 MeV since coupling of two identical negative-parity bosons can give positive-parity angular momentum zero excitations. This result, however, is more an existence proof of the possibility of generating a large number of low-lying 0^+ states than an explicit interpretation of the large number observed, either in ^{170}Yb or in other similar nearby nuclei, and it remains a challenge to develop a convincing theoretical interpretation of these states by simultaneously incorporating both collective and noncollective degrees of freedom. Finally, the study of ^{170}Yb, which lies near the arc of regularity in the IBM triangle [24], does not seem to produce features of the 0^+ states or their cross sections that appear to be qualitatively different from those of other nearby nuclei whose structure places them away from the arc. Further study of the unique structure along the arc and of its spectroscopic consequences are clearly needed.

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