Mass of ^{23}Al for testing the isobaric multiplet mass equation

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The mass excess of the proton-rich nucleus ^{23}Al has been measured with the JYFLTRAP Penning trap setup. As a result of our experiment we obtain a mass excess of 6748.07(34) keV, and by combining the value to existing experimental data we have tested the validity of the isobaric multiplet mass equation (IMME) for the $T = 3/2$ quartet in the $A = 23$ isobar. The fit to the IMME results in a vanishing cubic term equivalent to zero with high precision [0.22(42) keV].

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I. INTRODUCTION

Isospin T is considered to be a good quantum number in the absence of any charge-dependent nucleon-nucleon interactions. As a mathematical concept, the isospin is analogous to angular momentum; its z-projection in the isospin space is defined as $T_3 = \frac{1}{2}(N - Z)$, where T_3 can have values $-T, -T + 1, -T + 2, \ldots, T$. It is shown with the so-called isobaric multiplet mass equation (IMME) that the mass excesses of the $2T + 1$ nuclear states belonging to a given isospin multiplet are related:

$$M(A, T, T_3) = a(A, T) + b(A, T)T_3 + c(A, T)T_3^2. \quad (1)$$

This explicit form of the IMME is attributed to Wigner [1] based on MacDonald's work [2–4] on nuclear electrostatic energies. The mathematical formalism in the context of Coulomb energies and masses of isobaric analog states (IAS) is discussed by Weinberg and Treiman [5] and Jänecke [6]. Here, the basic assumption is that any charge-dependent effects and the Coulomb force between the nucleons are of a two-body nature, i.e., they are of a tensorial rank two or less in isospin space and they can be treated by first-order perturbation theory. Possible higher-order terms (dT_3^3 or higher) may arise, for example, if second-order perturbation theory has to be used or if there is isospin mixing between the isobaric analog state and neighboring states. The IMME can be used to predict energies of the isobaric analog states in regions where experimental data are lacking. Applications include probing both the excitation energies and ground-state masses of nuclei, e.g., along the rapid proton capture process (rp-process) path and the location of the proton drip line. The IMME parameters are also applied to improve shell-model calculations for theoretical corrections needed to test the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [7,8].

The latest compilation of the IMME coefficients includes 430 identified multiplets, and correlations of the IMME coefficients are given [9]. The most notable deviations from Eq. (1), being statistically significant, are found in the $A = 8$, $T = 2$ quintet and the $A = 9$, $T = 3/2$ quartet. They are considered to be the only real deviations from the first-order perturbation treatment. Since this compilation, the validity of IMME has been of particular interest among *sd*-shell nuclei [10–16]. The most notable controversy so far was found in the $A = 33, T = 3/2$ quartet. A possible breakdown of Eq. (1) was suggested after the mass measurement of the ground state of ^{33}Ar [10]. However, this was solved by a better measurement of the excitation energy of the IAS in ^{33}Cl [11]. Triambak *et al.* [13] report a small violation in the $A = 32$, $T = 2$ quintet, determined with high precision. However, there seem to be some controversies in the members of the multiplet [16]. A recent determination of the masses of ^{35,36}K, belonging to the $A = 35$ and $A = 36$ multiplets [14], indicates a possible breakdown of the IMME in the $A = 35$ quartet. Another addition toward testing the IMME in the upper *sd*-shell is provided by the $A = 37$ quartet [15], when the mass of the ground state of ^{37}Ca is measured, but still the experimental data for the analog state of ^{37}Ar are not precise enough for a meaningful test of the quadratic IMME.

The focus of this work is on the $A = 23, T = 3/2$ multiplet, which consists of the ground states of ^{23}Al and ^{23}Ne and the isobaric analog states in ^{23}Mg and ^{23}Na. Mass excesses of two members of this multiplet, the $5/2^+$ ground state of ^{23}Ne and the lowest $5/2^+$, $T = 3/2$ state in ^{23}Na, are known to be better than 100 and 150 eV, respectively [17,18]. The least-known member of this multiplet is ^{23}Al. Its most recent evaluated mass excess is 6770(19) keV [17], derived from two spectroscopic studies [19,20] with results of 6767(25) and 6773(28) keV, respectively. This multiplet has rather large uncertainties in the IMME coefficients [9]. The quadratic fit yields $\chi^2/n = 1.48$ and the possible cubic term is found to be large: −6.6(55) keV. The substantial uncertainty here arises from the uncertainties in the ground-state mass of ^{23}Al and the IAS energy in ^{23}Mg.

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The proton-rich nucleus ^{23}Al has attracted a lot of interest in recent years for a variety of reasons. It has a rather small proton separation energy, $S_p = 122(19)$ keV [17], making it a candidate for a proton-halo nucleus. The halo nature has been studied in reaction experiments [21–23], where the first shows "an abnormal increase" in the reaction cross section, indicating the valence proton to be of an *s*-wave nature, whereas the latter experiments are indicating the valence proton to be dominantly *d* wave. However, the spin and parity of the ^{23}Al ground state was found to be $J^\pi = 5/2^+$ by recent β-NMR [24] and β-decay measurements [25], in agreement with the measurement of its ^{23}Ne mirror. ^{23}Al may also play a crucial role in solving the depletion of the NeNa cycle in ONe novae. The key astrophysical reactions related to ^{23}Al, ^{22}Mg$(p,\gamma)^{23}$Al, and ^{22}Na$(p,\gamma)^{23}$Mg are discussed in Refs. [20,25–27], and references therein.

II. EXPERIMENT AND RESULTS

In this article, we report on a high-precision mass measurement of ^{23}Al using the JYFLTRAP setup at the Ion Guide Isotope Separator On-Line (IGISOL) facility in the Accelerator Laboratory of the University of Jyväskylä. A light-ion fusion ion guide [28] was used to produce the studied isotopes by bombarding a 4.3 mg/cm^2 $^{\text{nat}}$Mg target with 40 MeV protons from the K-130 cyclotron. With the IGISOL technique [29], a fraction of the reaction products are stopped in helium gas. The majority of the stopped reaction products thermalize and end up in a 1^+ charge state from the high ionization potential of helium. The thermalized ions are extracted from the gas cell by gas flow and electric fields through a differentially pumped electrode system before acceleration to 30 keV. The accelerated ions are separated by their A/q ratio with a 55° dipole magnet with typical mass-resolving power (MRP) $\frac{M}{\Delta M} = 500$. The yields of the activities produced with the IGISOL system were measured in the focal plane by using a Si-detector setup (33% absolute efficiency). The typical observed β-counting rate for $A = 23$ was 5×10^4 counts/s with a proton-beam intensity of 8–10 μA. The majority of the activity was identified to be ^{23}Mg, produced via ^{24}Mg$(p,pn)^{23}$Mg. The identification was done by a half-life measurement, giving a yield of about 1.5×10^5 ^{23}Mg ions/s. The yield of ^{23}Al activity, produced via ^{24}Mg$(p,2n)^{23}$Al, was observed to be about 1/200 of the total activity produced [30], resulting in a yield of approximately 700 ^{23}Al ions/s.

After the dipole magnet, the ions having the same $A/q = 23$ were sent into a gas-filled radiofrequency quadrupole (RFQ) cooler-buncher [31] to prepare the samples for injection into the JYFLTRAP Penning trap setup, consisting of two identical cylindrical traps inside the same superconducting 7 T magnet [32]. The first trap works as a purification trap with an MRP up to a few 10^5. This trap is filled with low-pressure helium gas to cool the captured ions. Consequently, the magnetron radius of the ions is increased with an electric dipole excitation and followed by an electric quadrupole excitation to center the ions mass selectively [33]. The isobarically pure sample is transferred into the second trap, that is used to determine the mass of the sample via the standard time-of-flight ion cyclotron resonance (TOF-ICR) method [34,35]. The isobarically pure sample of ions is excited with a phase-locked dipole excitation to increase their magnetron radius [36], followed by a quadrupole excitation with the frequency $\nu_c = \frac{1}{2\pi}\frac{q}{m}B$. This converts the initial magnetron motion of the ions into the reduced cyclotron motion, resulting in a maximum increase in the radial energy for resonantly excited ions. This increase is detected as a reduction in the time of flight when ions travel to a microchannel plate detector (Fig. 1). The mass of the ions of interest is determined from the ratio of the measured cyclotron frequency of the sample and a well-known reference case, i.e.,

$$m = \frac{\nu_{c,\text{ref}}}{\nu_c}(m_{\text{ref}} - m_e) + m_e. \quad (2)$$

Because the true cyclotron frequency ν_c is not a trap eigenfrequency it cannot be directly determined. Instead, the sideband frequency $\nu_+ + \nu_-$ is measured and, although systematic frequency shifts might arise due to misalignment and harmonic distortion, their contributions are lower than the uncertainties typically quoted for unstable ions [37,38].

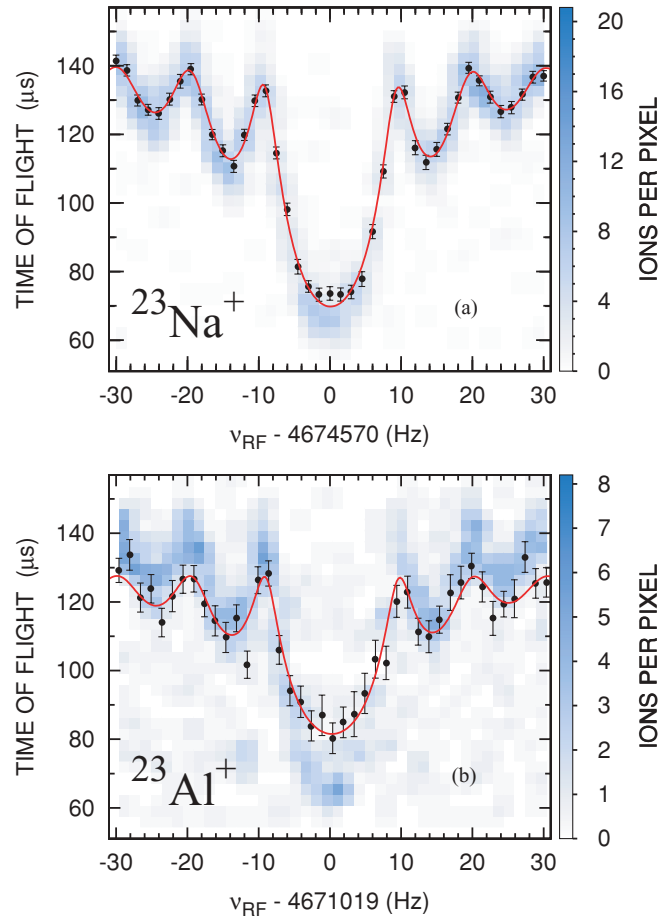

FIG. 1. (Color online) Typical cyclotron resonance curves of ^{23}Na$^+$ (a) and ^{23}Al$^+$ (b) from this experiment. The RF-excitation time was 100 ms.

TABLE I. Measured frequency ratios of given nuclei to reference nucleus ^{23}Na and deduced mass excesses.

Nuclide	$T_{1/2}$	Frequency ratio $\nu_{c,ref}/\nu_c$	ME_{Exp} (keV)	ME_{Lit} (keV)
^{23}Mg	11.317(11) s [18]	1.000189428(36)	−5473.38(77)	−5473.8(13) [17]
^{23}Al	446(6) ms [25]	1.000760142(16)	6748.07(34)	6770(19) [17]

In this experiment, the masses of ^{23}Al and ^{23}Mg were measured by using stable ^{23}Na as a reference. Because all ions used in this study have the same $A/q = 23$, the precision is enhanced from the absence of mass-dependent frequency shifts. The main emphasis of the measurement was to determine the mass excess of ^{23}Al as precisely as possible. Nevertheless, a short period of time was dedicated to determine the mass of ^{23}Mg. The purification trap of the JYFLTRAP setup was used to prepare a clean sample of ions—either ^{23}Al, ^{23}Mg, or ^{23}Na—which were all simultaneously present in the ion beam from the IGISOL. To avoid systematic frequency shifts from high numbers of simultaneously stored ions in the trap, the number of stored ^{23}Mg and ^{23}Na ions were limited to a maximum of two to three ions per bunch. The excitation time of the ions was limited to 100 ms because of a rather strong damping from ion-atom collisions in the precision trap. Because of the lower relative yield of ^{23}Al than other species (especially ^{23}Na) in the isobar and the long accumulation time in the RFQ, some background ions were seen, in particular, in resonances of ^{23}Al. A count-rate class analysis [39] was used in fitting resonances of ^{23}Mg and ^{23}Na. Because the amount of ^{23}Al ions available was rather low, it was not possible to perform a proper count-rate analysis for the resonances of ^{23}Al. Instead, to take into account the possible systematic shift, the final mass uncertainty of ^{23}Al was increased to an averaged value determined by comparing the differences of the uncertainties in fitting the reference resonances with and without the count-rate class analysis. Because the resonances of ^{23}Al were influenced by nonresonant background ions, the shape of the fitted resonance function was taken from the fits of the reference ion resonances; only frequency and constant background were fitted. The shapes of the resonance curves can be assumed to be similar because the ion of interest and the reference have the same A/q; the ions were also recentered in the purification trap after the cleaning process to obtain identical conditions at the beginning of the TOF-ICR procedure. In addition, only bunches with single ions were included in the analysis of ^{23}Al resonances because the majority of the bunches having more than one ion detected consisted of background ions. The measured frequency ratios of ^{23}Mg$^+$/^{23}Na$^+$ and ^{23}Al$^+$/^{23}Na$^+$ with deduced mass excesses are presented in Table I. In total, three resonances were recorded for ^{23}Mg and 29 were recorded for ^{23}Al, interleaved by reference measurements to account for temporal changes in the magnetic field.

III. DISCUSSION

Because our new value for the mass excess of ^{23}Al, 6748.07(34) keV, not only deviates from the previous values but is about two orders of magnitude more precise (see Fig. 2 for visualization), it is worthwhile to use it to test the validity of the IMME by combining it with the existing data. The ^{23}Ne and ^{23}Na mass excesses are taken as given in the latest Atomic Mass Evaluation (AME) [17]. The $T = 3/2$ level energy for ^{23}Na is the adopted value from the latest Nuclear Data Sheets [18]. The AME03 value for ^{23}Mg, −5473.8(13) keV, is deduced from a combination of two spectrometric studies of (p,n) and (p,d) reaction Q values [40,41]. This evaluated value is combined with our value from this measurement,

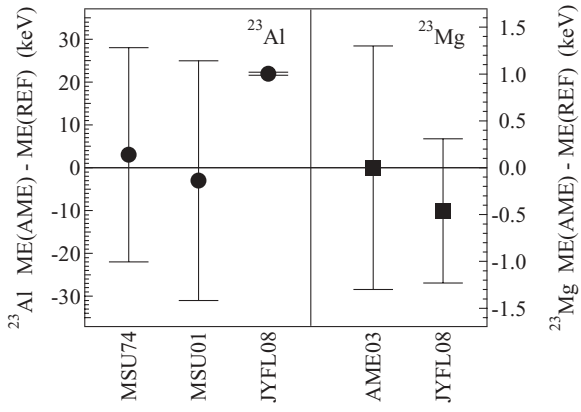

FIG. 2. Differences between the different measurements and the latest AME value (AME03) [17]. MSU74 refers here to Ref. [19], MSU01 refers to Ref. [20], and JYFL08 is from the work presented in this article. The AME value for ^{23}Mg is a combination of two spectrometric studies [40,41].

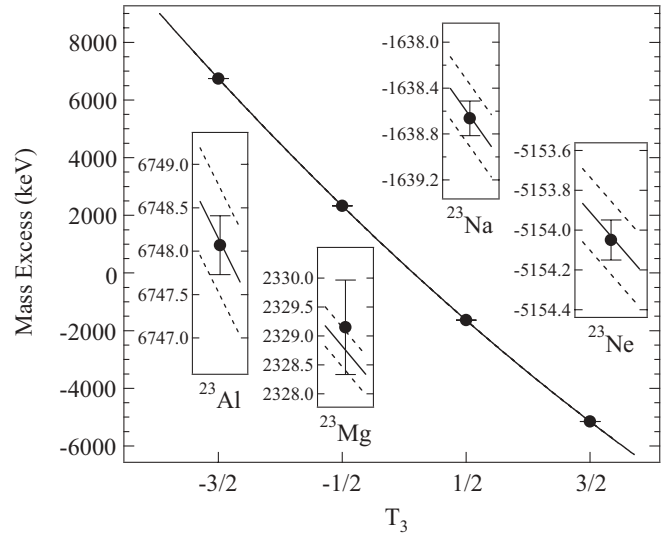

FIG. 3. The $A = 23, T = 3/2$ multiplet and the fit to the quadratic IMME with 1σ error bands.

TABLE II. The members of the $A = 23$, $T = 3/2$ quartet.

Nuclide	$T_{1/2,\mathrm{g.s.}}$	T_3	$ME_{\mathrm{g.s.}}$ (keV)	E_{ex} (keV)	$ME_{T=3/2}$ (keV)
^{23}Ne	37.24(12) s [18]	+3/2	−5154.05(10)[a]	–	−5154.05(10)
^{23}Na	Stable	+1/2	−9529.8536(27)[a]	7891.19(15)[d]	−1638.66(15)
^{23}Mg	11.317(11) s [18]	−1/2	−5473.49(67)[b]	7802.64(48)[e]	2329.15(82)
^{23}Al	446(6) ms [25]	−3/2	6748.07(34)[c]	–	6748.07(34)

[a]From Ref. [17].
[b]Weighted average of Ref. [17] and this work (Table I).
[c]This work.
[d]From Ref. [18].
[e]Weighted average of Refs. [18,42].

−5473.38(77) keV, as a weighted average yielding the new ground-state mass excess of −5473.49(67) keV for ^{23}Mg. The $T = 3/2$ level energy in ^{23}Mg, 7802.64(48) keV, is a weighted average of the adopted level energy 7802.2(14) keV [18] and 7802.7(5) keV from a recent measurement performed at Texas A&M University [25,42]. The adopted value is based mostly on work reported in Ref. [30], and the new, more precise, value is from a refined analysis [42] of the data originally published in Ref. [25].

The fit to the IMME was done by using a standard nonlinear least-squares fitting routine where the uncertainties in the energies of the isobaric analog states were used as the weights. The fit to the quadratic IMME using the values given in Table II is illustrated in Fig. 3. The results for both quadratic and cubic fit are tabulated in Table III and compared to the values from the previous compilation [9]. The obtained fit to the quadratic form is remarkably good ($\chi^2/n = 0.28$) and all the parameters are determined with uncertainties <200 eV. A fit to the cubic form can be used to test how well the quadratic form reproduces the experimental data. Because there are as many parameters as data points, the fit trivially reproduces the experimental data. To estimate the 1σ error bars for the cubic fit, each parameter was locked to the value from the best fit and then varied one by one so that one gets $\chi^2 = 1$ for the fit in each case. The values extracted from the cubic fit are also in Table III. All the lower-order terms (a,b,c) are close to those from a quadratic fit and the cubic term 0.22(42) keV is zero within the uncertainty. Therefore, it is safe to say that IMME seems to hold very well for the $J^\pi = 5/2^+$, $T = 3/2$ quartet in $A = 23$.

Figure 4 compiles recent precision tests for the d-coefficients in the sd-shell. In three cases out of five, the d-coefficient is equal to zero within the uncertainties, the most precise quartet being $A = 23$ following this work. A significant deviation from zero has only been obtained in the $A = 35$ quartet [14] and in the $A = 32$ quintet [13]. However, the deviations in the $A = 35$ case might be from problems with the IAS in ^{35}Cl as discussed in Ref. [14]. The $A = 32$ deviation was reported in Ref. [13] in which the IAS in ^{32}S was remeasured, appearing to break the IMME. It was also speculated that the ^{32}Si mass excess could be wrong and this has been confirmed by a Penning trap mass measurement [16]. Interestingly, it was found to deviate in such a way that does not restore the validity of the IMME in this case. However, some of the ground-state masses in this multiplet are derived from indirect measurements so further studies are called for before any conclusions can be made.

A new value for the ^{23}Al proton separation energy, $S_p(^{23}\mathrm{Al}) = 141.11(43)$ keV, can be calculated by combining our result for the mass excess of ^{23}Al, 6748.07(34) keV, and the mass excesses of ^{22}Mg, −399.79(25) keV [43], and ^{1}H, 7288.97050(11) keV [17]. The resultant S_p value is higher than the previous value indicating a reduced halo nature. This new value has influence on the calculated astrophysical S-factor for the proton capture reaction $^{22}\mathrm{Mg}(p,\gamma)^{23}\mathrm{Al}$ and its corresponding reaction rate in the stellar environments.

TABLE III. IMME coefficients from fit to data in Table II and comparison to existing data. All fit parameters are given in keV.

	Quadratic			Cubic	
	This work	From Ref. [9]		This work	From Ref. [9]
a	288.54(18)	288.3(20)	a	288.77(47)	283.7(43)
b	−3967.40(12)	−3965.8(54)	b	−3967.90(94)	−3959(8)
c	225.99(10)	225.2(27)	c	225.88(23)	232(7)
χ^2/n	0.28	1.48	d	0.22(42)	−6.6(55)

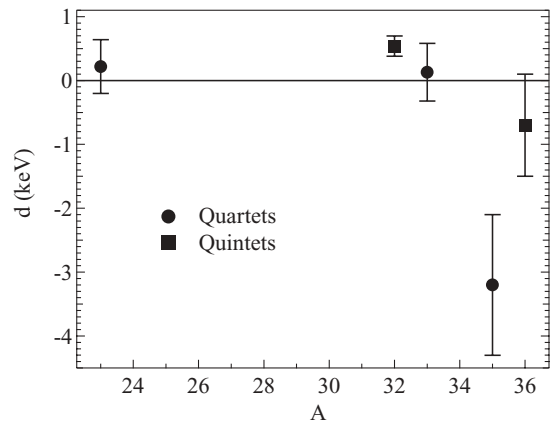

FIG. 4. The most recent and precisely determined d-coefficients from different isobaric multiplets in the sd-shell. The $A = 23$ case is from this work, the $A = 32$ cases are from Refs. [12,13], $A = 33$ is from Ref. [12], and $A = 35$ and $A = 36$ are from Ref. [14].

It also shows ^{23}Al to be more resilient to destruction through photodissociation, making this isotope a more important player in the reaction networks in the explosive H-burning processes, such as novae and x-ray bursts.

In summary, we have presented high-precision mass measurements of ^{23}Al and ^{23}Mg ground states and combined these new values with existing data. Our results show that the quadratic IMME holds well for the $A = 23$, $T = 3/2$ multiplet and also presents, to our knowledge, the most stringent test for the zero cubic term of an isospin quartet so far.

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