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Wobbling mode in ^{167}Ta

D. J. Hartley,[1] R. V. F. Janssens,[2] L. L. Riedinger,[3] M. A. Riley,[4] A. Aguilar,[4,*] M. P. Carpenter,[2] C. J. Chiara,[2,5,6]
P. Chowdhury,[7] I. G. Darby,[3] U. Garg,[8] Q. A. Ijaz,[9] F. G. Kondev,[5] S. Lakshmi,[7] T. Lauritsen,[2] A. Ludington,[1,†] W. C. Ma,[9]
E. A. McCutchan,[2] S. Mukhopadhyay,[8] R. Pifer,[1] E. P. Seyfried,[1] I. Stefanescu,[2,6] S. K. Tandel,[7] U. Tandel,[7] J. R. Vanhoy,[1]
X. Wang,[4] S. Zhu,[2] I. Hamamoto,[10] and S. Frauendorf[8]
[1]*Department of Physics, U.S. Naval Academy, Annapolis, Maryland 21402, USA*
[2]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[3]*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*
[4]*Department of Physics, Florida State University, Tallahassee, Florida 32306, USA*
[5]*Nuclear Engineering Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[6]*Department of Chemistry and Biochemistry, University of Maryland, College Park, Maryland 20742, USA*
[7]*Department of Physics, University of Massachusetts Lowell, Lowell, Massachusetts 01854, USA*
[8]*Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556, USA*
[9]*Department of Physics, Mississippi State University, Mississippi State, Mississippi 39762, USA*
[10]*Department of Mathematical Physics, LTH, University of Lund, Lund, Sweden*

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The collective wobbling mode, the strongest signature for the rotation of a triaxial nucleus, has previously been seen only in a few Lu isotopes in spite of extensive searches in nearby isotopes. A sequence of transitions in the $N = 94$ ^{167}Ta nucleus exhibiting features similar to those attributed to the wobbling bands in the Lu nuclei has now been found. This band feeds into the $\pi i_{13/2}$ band at a relative energy similar to that seen in the established wobbling bands and its dynamic moment of inertia and alignment properties are nearly identical to the $i_{13/2}$ structure over a significant frequency range. Given these characteristics, it is likely that the wobbling mode has been observed for the first time in a nucleus other than Lu, making this collective motion a more general phenomenon.

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Our understanding of the wobbling mode in nuclei (and the associated stable triaxial deformation) has evolved quickly over the past decade. Bohr and Mottelson [1] first proposed that the rotation of a stable triaxially deformed nucleus would result in the presence of wobbling excitations. These excitations occur because the rotational angular momentum is not aligned with any of the body-fixed axes; rather it precesses and wobbles around one of these axes in a manner similar to that of an asymmetric top.

In 1995, Schnack-Petersen *et al.* [2] first suggested that rotational bands based on proton $i_{13/2}$ excitations in ^{163,165}Lu are associated with a triaxial strongly deformed (TSD) potential well. The large deformation is mainly due to the occupation of the intruder $i_{13/2}$ orbital, and the triaxial deformation ($\gamma = 20^\circ$) results from an $N = 94$ shell gap that develops with enhanced quadrupole deformation ($\epsilon_2 \approx 0.37$). No direct experimental evidence for triaxiality was observed until the wobbling mode was confirmed in ^{163}Lu by Ødegård *et al.* [3]. This seminal work established the existence of a band feeding into the $\pi i_{13/2}$ structure where the two sequences have nearly identical moments of inertia and alignments over a large frequency range. The similarities of the moments of inertia and alignments are a predicted feature for a wobbling band as the intrinsic structure for both bands should be the same; the only difference between the two is the degree to which the rotational angular momentum vector lies off axis. The collective wobbling behavior can thus be described within a phonon model, where the energy of each band is equal to $E = \frac{\hbar^2}{2\mathcal{J}} I(I+1) + \hbar\omega_w(n_w + 1/2)$, where $\hbar\omega_w = \hbar\omega_{\text{rot}}\sqrt{(\mathcal{J}_x - \mathcal{J}_y)(\mathcal{J}_x - \mathcal{J}_z)/(\mathcal{J}_y \mathcal{J}_z)}$ [1]. The $n_w = 0$ phonon number is assigned to the energetically lowest band in the family, as its angular momentum vector lies closest to a body axis, and in the case of the Lu isotopes, this is associated with the $\pi i_{13/2}$ band. Wobbling excitations with $n_w = 1$, 2, 3, etc. then follow, each lying successively higher in energy as the rotational angular momentum vector progressively lies farther from the body axis with increasing n_w. Indeed, Jensen *et al.* [4] not only confirmed the $n_w = 1$ sequence in ^{163}Lu but also observed the $n_w = 2$ structure. Perhaps most importantly, Ødegård *et al.* [3] obtained polarization results for the linking transitions from the $n_w = 1$ to the $n_w = 0$ structure which confirmed that the $\Delta I = 1$ transitions are primarily of $E2$ character. This signature cannot be accounted for in normal cranking calculations, but it is expected for the wobbling mode [3].

Only a few years later, examples of wobbling were found in the $N = 94$ ^{165}Lu [5] and $N = 96$ ^{167}Lu [6] isotopes. This led to the assertion that "the wobbling mode is a general phenomenon in this region" [5]. However, subsequent studies in many neighboring Hf [7], Tm [8,9], and Ta [10,11] nuclei failed to generate any further examples of wobbling. These

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[*] Present address: Department of Radiation Oncology, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA.

[†] Present address: Department of Nuclear Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA.

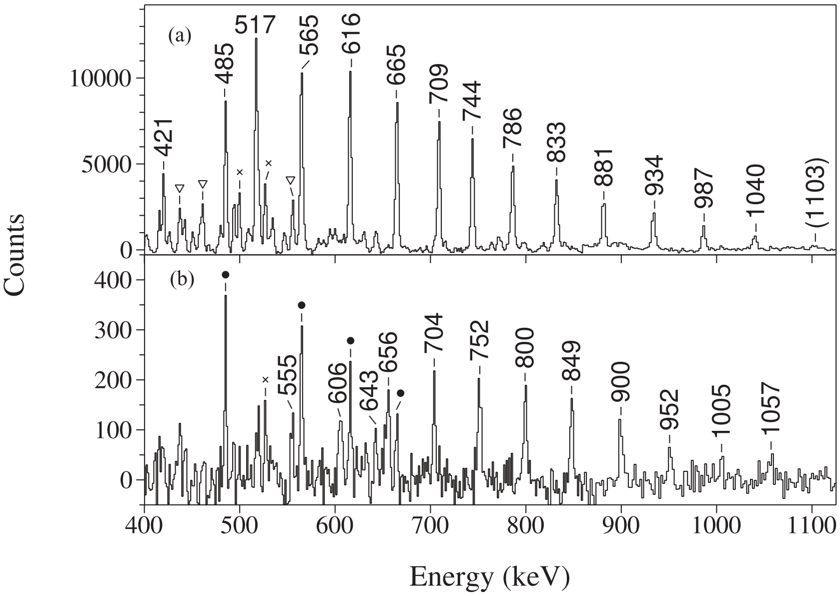

FIG. 1. (a) Spectrum of band TSD 1 ($\pi i_{13/2}$) in ^{167}Ta created by summing all possible combinations of triple gates on inband transitions. Peaks denoted with triangles are transitions from known states in ^{167}Ta; those labeled with a cross are linking transitions. (b) Spectrum of band TSD 2 ($n_w = 1$ sequence) resulting from a sum of all combinations of the 517-keV γ ray in the $i_{13/2}$ band with any two inband transitions. Peaks labeled with a filled circle are from the $i_{13/2}$ band.

observations led to the suggestion that wobbling is not a general phenomenon to be observed in this region, but rather that the Lu isotopic chain is perhaps the only one suited to exhibit this mode. Indeed, the tilted-axis cranking calculations by Frauendorf, given in Ref. [8], indicate that the density of states in the Lu TSD minimum is sufficiently low that the high-excitation energy of the wobbling mode can be observed. In contrast, a much higher density of states is found in TSD minima of Ta and other nuclei, which could produce many TSD bands based on alternative particle-hole configurations. Thus, it is not that wobbling bands are absent in these nuclei; rather, the competition between this collective mode and the particle-hole excitations greatly favors the population of the latter.

This provided the motivation for an experiment producing ^{167}Ta$_{94}$ to search for either wobbling bands based on the $\pi i_{13/2}$ orbital or multiple particle-hole TSD bands. The ^{120}Sn(^{51}V,4n) reaction was used with a ^{51}V beam accelerated to 235 MeV by the ATLAS facility at Argonne National Laboratory. Two self-supporting ^{120}Sn targets were stacked, each with a thickness of 500 μg/cm^2. A total of 101 Compton-suppressed germanium detectors was included in the Gammasphere array [12]. Approximately 2×10^9 four-fold or greater coincidence events were recorded in five days of beam time. The data were sorted into a Blue database [13] such that RADWARE [14] coincidence cubes and hypercubes could be generated. In addition, angular-correlation ratios were obtained from the database using the background-subtraction method of Starosta *et al.* [15]. The ratios were determined by $R_{\text{ang}} = W(\theta_f, \phi)/W(\theta_{90^\circ}, \phi)$, where $W(\theta_f, \phi)$ is the intensity observed in the forward detectors ($\theta = 122^\circ$, 130°, 143°, 148°, and 163°) and $W(\theta_{90^\circ}, \phi)$ is the intensity observed in the Gammasphere rings near 90° ($\theta = 79^\circ$, 81°, 90°, 99°, and 101°). Normalized ratios of approximately 0.6 and 1.0 were observed for known stretched $E1$ and $E2$ transitions, respectively.

If wobbling exists in ^{167}Ta, it would likely be based on the $\pi i_{13/2}$ orbital (in analogy with the Lu isotopes), but this structure had not yet been identified [16]. Figure 1(a) displays the spectrum of a new band in ^{167}Ta. Coincidence triple gates were placed on all possible combinations of the marked peaks in a hypercube to generate the spectrum. Transitions decaying to previously known states in ^{167}Ta were identified, and a partial level scheme is presented in Fig. 2 where this new band is labeled as TSD 1. The angular-correlation analysis indicates that the 500-, 527-, and 772-keV transitions feeding into the [404]7/2 and [402]5/2 structures are of $E2$ character with ratio values of 1.04(4), 0.96(3), and 1.06(8), respectively. Therefore, the spins and parity of this new band have been firmly established as given in Fig. 2. The fragmented decay to the [402]5/2, [411]1/2, and [404]7/2 structures is reminiscent of the $\pi i_{13/2}$ bands observed in heavier Ta nuclei [10,11] as well as in the $N \approx 94$ Lu nuclei [3,5,6]. In addition, the large dynamic moment of inertia (with respect to the other sequences in ^{167}Ta) of this band and its large initial alignment of $\sim 6\hbar$ provide strong arguments that this structure is associated with the $\pi i_{13/2}$ orbital. The excitation energy of this band is also consistent with the theoretical estimates for the $i_{13/2}$ structure of Ref. [17].

As previously stated, an $n_w = 1$ wobbling sequence is expected to have a moment of inertia nearly identical to that of the $n_w = 0$ band (i.e., the $i_{13/2}$ band). Therefore, a search was performed for bands with γ-ray energy spacings similar to those of band TSD 1. A sample spectrum, obtained as a sum of coincidence spectra requiring the presence of the 517-keV transition in TSD 1 together with any two inband transitions in the new sequence, is presented in Fig. 1(b). This new band was added to Fig. 2, where it is labeled TSD 2. It is noteworthy that this TSD 2 sequence decays solely into band TSD 1 through linking transitions for which the angular-correlation measurements indicate $\Delta I = 1$ character, with values of 0.50(8) and 0.71(8) for the 643- and 632-keV transitions, respectively. The positive parity for the TSD 2 levels is proposed based on the exclusive feeding of the $i_{13/2}$ sequence. Band TSD 2 will be associated with the wobbling mode based on arguments outlined below. In addition, another sequence

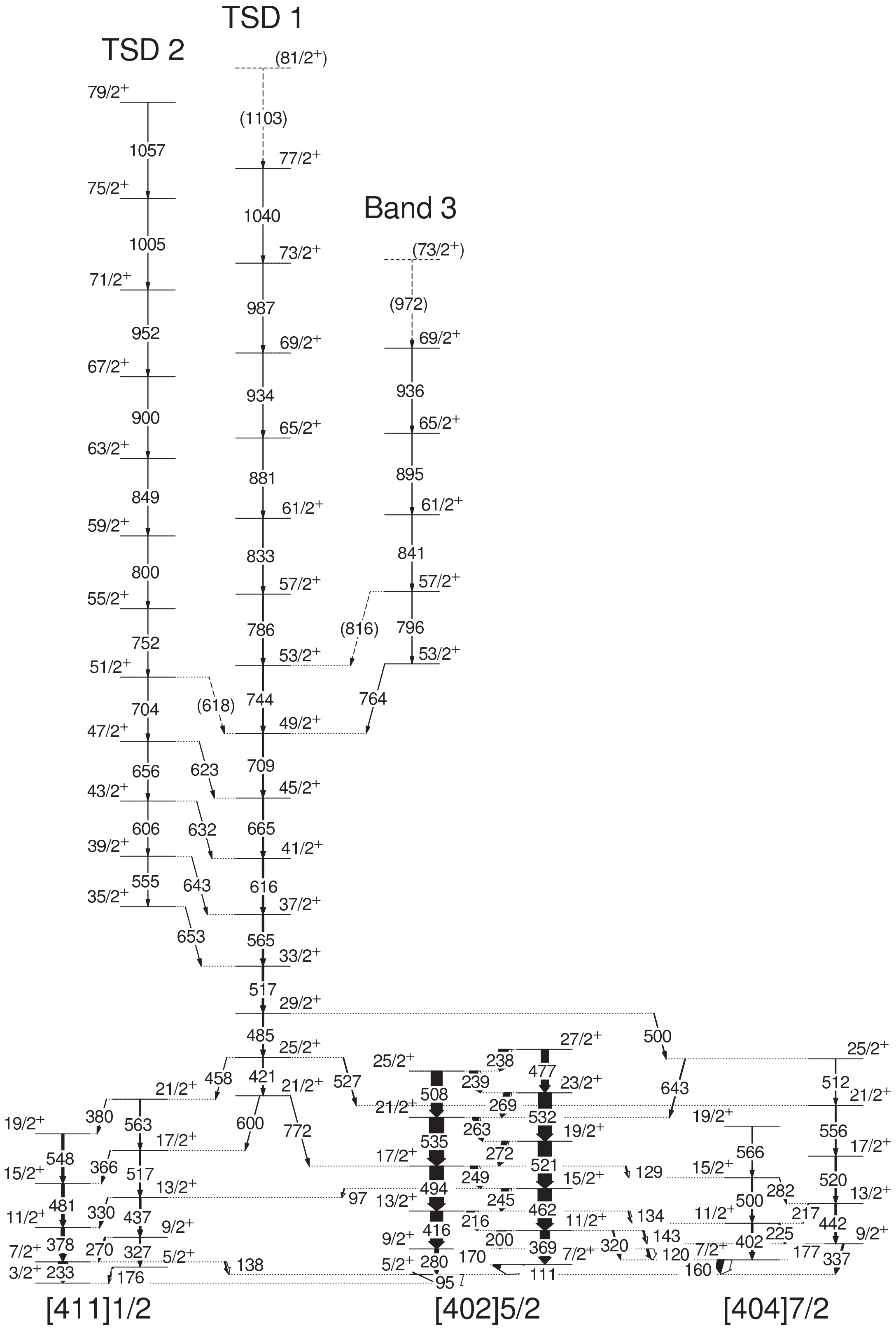

FIG. 2. Partial level scheme for ^{167}Ta displaying the newly observed $\pi i_{13/2}$ (TSD 1) band and the proposed $n_w = 1$ wobbling structure (TSD 2).

(Band 3) was observed to feed the $i_{13/2}$ band, as displayed in Fig. 2. The spin of this sequence was also determined by angular correlation ratios of 1.03(6) and 1.05(6) for the 764- and 796-keV transitions, respectively. This structure is nearly degenerate with TSD 1 and appears to be strongly related to the $i_{13/2}$ configuration as well.

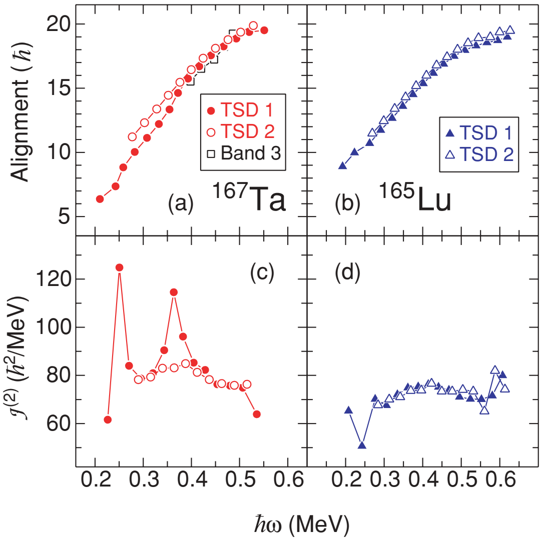

FIG. 3. (Color online) Alignment vs rotational energy for bands TSD 1 ($\pi i_{13/2}$) and TSD 2 ($n_w = 1$) in (a) ^{167}Ta and (b) ^{165}Lu [5]. Harris parameters [18] of $\mathcal{J}_0 = 25\,\hbar^2$/MeV and $\mathcal{J}_1 = 40\,\hbar^4$/MeV^3 were used as a reference. Dynamic moments of inertia for the same bands in (c) ^{167}Ta and (d) ^{165}Lu.

The multiple linking transitions from band TSD 2 to the $i_{13/2}$ band (TSD 1) lead to the suggestion that the two bands likely share a similar structure. Thus, two interpretations must be considered for the former sequence. First, it is possible that this band is the unfavored signature partner of the $i_{13/2}$ sequence. However, it is well known that unfavored signatures of high-j orbitals are characterized by significantly smaller alignments than those seen in the favored partners. Clearly, this is not the case, as can be seen in Fig. 3(a): Band TSD 2 has a slightly *higher* alignment than the $i_{13/2}$ band (TSD 1). Ultimate cranker (UC) calculations also indicate that the unfavored signature of the $i_{13/2}$ band has an excitation energy in the rotating frame (Routhian) approximately 1 MeV higher than that of the favored signature partner. Conversely, band TSD 2 has a Routhian that lies at most ∼350 keV above the TSD 1 sequence. For these reasons, an interpretation in terms of an unfavored signature partner does not properly describe the observations.

The second interpretation for TSD 2 is that it is a wobbling sequence. Inspection of Fig. 3(a) indicates that these two bands have alignments that are quite similar above 0.4 MeV and are only separated by ∼1ħ below this frequency. The moments of inertia are very close as well at high frequency [Fig. 3(c)]. These similarities compare favorably with the properties of the previously established TSD bands in ^{165}Lu [5] displayed in Fig. 3(b) and 3(d), where TSD 2 is the $n_w = 1$ wobbling structure. In addition, the energy separation between the bands in ^{167}Ta mirrors that seen between the wobbling excitation and the $i_{13/2}$ structure in ^{165}Lu, as can be seen in Fig. 4. Therefore, band TSD 2 is a strong candidate for the first $n_w = 1$ wobbling band observed in a nucleus other than those in the Lu chain.

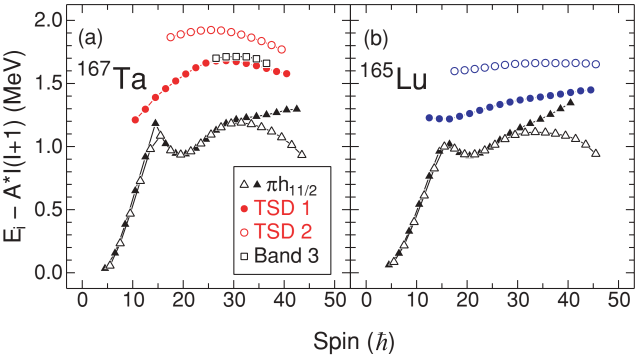

FIG. 4. (Color online) Excitation energy minus a rigid-rotor reference for denoted bands in (a) ^{167}Ta and its isotone (b) ^{165}Lu [5]. The inertia parameter A was set to 0.007 MeV/ħ^2. The $\pi h_{11/2}$ bands are shown for both nuclei as they are the energetically lowest structures.

However, an intriguing feature is observed in the dynamic moment of inertia plot displayed in Fig. 3(c). An interaction is found just above 0.35 MeV in TSD 1 that is absent in TSD 2. This is the same frequency region where band TSD 1 gains approximately 1ħ with respect to TSD 2, as seen in Fig. 3(a). This effect is magnified in Fig. 5, which displays the relative alignments between the $n_w = 1$ wobbling bands and the $i_{13/2}$ structures upon which they are based for ^{163,165,167}Lu and ^{167}Ta. No such alignment gain is observed in ^{163,165}Lu, as the relative alignments are nearly constant at approximately $-0.4\hbar$. Yet, in ^{167}Lu and ^{167}Ta, a similar gain of approximately 1ħ is noted and in ^{167}Ta a constant relative alignment near $-0.4\hbar$ is achieved at the highest frequencies, a value similar to that seen in ^{163,165}Lu.

It is possible that the interactions found in the $A = 167$ nuclei are the result of the $i_{13/2}$ orbital transitioning from a $\gamma = 0^\circ$ strongly deformed (SD) prolate minimum to the TSD minimum. The frequency and rate at which this transition occurs are sensitive to the detailed structure of the individual nuclei; therefore, it is not unexpected that the relative

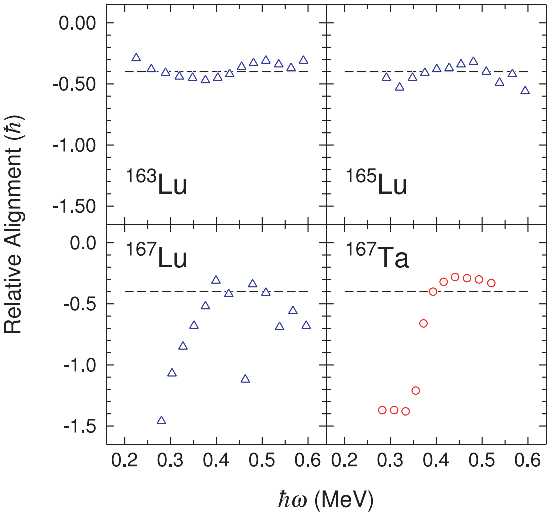

FIG. 5. (Color online) Relative alignment [$\Delta i_x = i_x$(TSD 1) $-i_x$(TSD 2)] vs the rotational frequency of band TSD 1 in the four nuclei denoted. Dashed lines are placed at $\Delta i_x = -0.4\hbar$.

alignments for the Lu and Ta nuclei would behave somewhat differently. Band TSD 1 will display such an interaction (since this is the sequence in which the change is occurring), whereas TSD 2 is not affected as it only exists in the TSD minimum. If this hypothesis is correct, Band 3 in Fig. 2 may well represent the continuation of the $i_{13/2}$ sequence in the $\gamma = 0^\circ$ SD minimum. Its alignment values [Fig. 3(a)] and excitation energy [Fig. 4(a)] indicate that bands TSD 1 and 3 lie along a similar trajectory before the interaction occurs near 0.35 MeV ($I = 51/2$). A detailed lifetime experiment is required to validate this interpretation as the quadrupole moments of the individual states should decrease in TSD 1 as the spin increases through the interaction region, owing to the $\cos(30^\circ + \gamma)$ dependence [1].

Perhaps the most rigorous means to identify a wobbling sequence is to determine the nature of the transitions from the $n_w = 1$ structure to the $n_w = 0$ band. The wobbling interpretation requires these transitions to be of the $\Delta I = 1$ type, with a dominant $E2$ character, as found in ^{163}Lu [3]. Experimental proof requires polarization measurements, which could not be accomplished in the present experiment. Nevertheless, because of the similarity between the linking transitions in ^{167}Ta and those in ^{163,165,167}Lu (in terms of energy and branching ratio), it is reasonable to assume that they also share similarities in their electromagnetic characteristics. Thus, a mixing ratio where the linking transitions are approximately 90% $E2$ (the value measured in ^{163}Lu [3]) has been assumed for those found in ^{167}Ta. The corresponding $B(E2)_{\text{out}}/B(E2)_{\text{in}}$ ratios were then determined for the 39/2, 43/2, and $47/2$ states to be 0.37(4), 0.32(4), and 0.36(4), respectively. These experimental values are large, indeed much larger than possible with any cranking interpretation [$B(E2)_{\text{out}}/B(E2)_{\text{in}} \leqslant 0.01$]. They also exceed the predictions of the particle-rotor model (which decrease from 0.267 to 0.224 in this spin region) and the values found in the Lu isotopes (∼0.2). The differences may result from the fact that, in the spin region of the linking transitions, the two sequences are possibly in different potential minima (as described here). It is also possible that the triaxial deformation is larger in ^{167}Ta compared to its Lu neighbors, as the ratio is proportional to $[\sin(30^\circ + \gamma)/\cos(30^\circ + \gamma)]^2$ [19]. However, UC calculations suggest that both Lu and Ta TSD minima are located near $\gamma = 20^\circ$. Finally, the assumption that the linking transitions are 90% $E2$ in character may be an overestimate and a reduction in this value would decrease the $B(E2)_{\text{out}}/B(E2)_{\text{in}}$ ratio.

In summary, high-spin rotational band structures have been identified in the $N = 94$ nucleus ^{167}Ta, including the $\pi i_{13/2}$ sequence. A new band with nearly identical dynamic moment of inertia and alignment properties above a frequency of 0.4 MeV has been observed that feeds into the latter. These features along with branching ratio measurements of the inband to out-of-band transitions are characteristic of the well-established wobbling bands in Lu isotopes. Thus, we suggest that the wobbling mode has been observed for the first time in a nucleus other than Lu.

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