Continuum-state and bound-state β^--decay rates of the neutron

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For the β^--decay of the neutron we analyze the continuum-state and bound-state decay modes. We calculate the decay rates, the electron energy spectrum for the continuum-state decay mode, and angular distributions of the decay probabilities for the continuum-state and bound-state decay modes. The theoretical results are obtained for the new value for the axial coupling constant $g_A = 1.2750(9)$, obtained recently by H. Abele [Prog. Part. Nucl. Phys. **60**, 1 (2008)] from the fit of the experimental data on the coefficient of the correlation of the neutron spin and the electron momentum of the electron energy spectrum of the continuum-state decay mode. We take into account the contribution of radiative corrections and the scalar and tensor weak couplings. The calculated angular distributions of the probabilities of the bound-state decay modes of the polarized neutron can be used for the experimental measurements of the bound-state β^--decays into the hyperfine states with total angular momentum $F = 1$ and scalar and tensor weak coupling constants.

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I. INTRODUCTION

The continuum-state β^--decay of the neutron $n \to p + e^- + \tilde{\nu}_e$ is experimentally well measured [1,2] (see also [3] and [4]) and investigated theoretically [5–8]. A theoretical analysis of the bound-state β^--decay rate has been carried out in [9,10]. Recently [11,12] Schott *et al.* have reported new experimental data on the bound-state β^--decay of the neutron $n \to \mathrm{H} + \tilde{\nu}_e$.

In this paper we recalculate the continuum-state β^--decay rate of the neutron, the electron energy spectrum, and angular distribution taking into account the contributions of $V - A$, scalar S and tensor T weak interactions, and radiative corrections [13] (see also [14]). Such a recalculation is required by the new precise experimental data on the lifetime of the neutron $\tau_{\beta^-_c} = 878.5(8)\,\mathrm{s}$ [1] and the value of the axial coupling constant $g_A = 1.2750(9)$ [3]. Using recent experimental data on the lifetime of the neutron [1] and correlation coefficients [3] we estimate the values of the scalar g_S and tensor g_T coupling constants. For the experimental analysis of the contributions of the scalar and tensor weak interactions we give angular distributions of the probabilities of the bound-state β^--decay rates of the polarized neutron. For the calculation of the bound-state β^--decay rates we use the technique applied to the analysis of the weak decays of the H-like, He-like, and bare heavy ions and mesic hydrogen in [15–17]. In the conclusion we discuss the obtained results.

II. $V - A$ WEAK HADRONIC INTERACTIONS

The Hamiltonian of the weak interaction we take in the form [15–17]

$$\mathcal{H}_W(x) = \frac{G_F}{\sqrt{2}} V_{ud}\, [\bar{\psi}_p(x)\gamma_\mu(1 - g_A\gamma^5)\psi_n(x)] \times [\bar{\psi}_e(x)\gamma^\mu(1-\gamma^5)\psi_{\nu_e}(x)], \qquad (1)$$

where $G_F = 1.1664 \times 10^{-11}\,\mathrm{MeV}^{-2}$ is the Fermi weak constant, V_{ud} and g_A are the CKM matrix element and the axial coupling constant [4], $\psi_p(x)$, $\psi_n(x)$, $\psi_e(x)$, and $\psi_{\nu_e}(x)$ are operators of the interacting proton, neutron, electron, and antineutrino, respectively. Here and below we use the Hamiltonian weak interaction operator invariant under time reversal. This means that we work with the real axial coupling constant g_A, which is positive in our approach, and real scalar g_S and tensor g_T coupling constants [see Eq. (11)].

For numerical calculations we will use the most precise values $|V_{ud}| = 0.97419(22)$ [4] and $g_A = 1.2750(9)$ [3], where $g_A = 1.2750(9)$ has been obtained from the fit of the neutron spin-electron correlation coefficient $A^{\exp} = -0.11933(34)$, defined in terms of the axial coupling g_A in Eq. (28), of the electron energy spectrum for the continuum-state β^--decay of the neutron [3].

The value of the CKM matrix element $|V_{ud}| = 0.97419(22)$ [4] agrees well with $|V_{ud}| = 0.9738(4)$ [3,18], measured from the superallowed $0^+ \to 0^+$ nuclear β^--decays, which are pure Fermi transitions [18]. It also satisfies the unitarity condition $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0000(6)$ for the CKM matrix elements [4] well.

Another value of the axial coupling constant $g_A = 1.2696(29)$, given in [4] (see also [19]), is determined with the uncertainty by a factor 3 larger compared with the uncertainty

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of $g_A = 1.2750(9)$ [3]. (The theoretical results are adduced in Table III and discussed in the conclusion.)

The amplitudes of the continuum-state and bound-state β^--decays of the neutron are defined by

$$\begin{aligned} M(n \to p + e^- + \tilde{\nu}_e) &= -\langle \tilde{\nu}_e e^- p|\mathcal{H}_W(0)|n\rangle, \\ M(n \to \mathrm{H} + \tilde{\nu}_e) &= -\langle \tilde{\nu}_e \mathrm{H}|\mathcal{H}_W(0)|n\rangle, \end{aligned} \tag{2}$$

where the interacting particles have four-momenta k_a with $a = \tilde{\nu}_e, e, p$, H, and n, respectively.

III. BOUND-STATE AND CONTINUUM-STATE β^--DECAY RATES OF NEUTRON IN $V - A$ THEORY OF WEAK INTERACTIONS

In the final state of the bound-state β^--decay of the neutron hydrogen can be produced only in the ns states, where n is a *principal* quantum number $n = 1, 2, \ldots$ [16,17]. The contribution of excited $n\ell$ states with $\ell > 0$ is negligibly small. Due to hyperfine interactions [20,21] hydrogen can be in two hyperfine states $(ns)_F$ with $F = 0$ and $F = 1$.

The wave function of hydrogen H in the ns state we take in the form [22–24]

$$\begin{aligned} |\mathrm{H}^{(ns)}(\vec{q}\,)\rangle = {} & \frac{1}{(2\pi)^3}\sqrt{2E_{\mathrm{H}}(\vec{q}\,)} \\ & \times \int \frac{d^3k_e}{\sqrt{2E_e(\vec{k}_e)}}\frac{d^3k_p}{\sqrt{2E_p(\vec{k}_p)}}\delta^{(3)}(\vec{q} - \vec{k}_e - \vec{k}_p) \\ & \times \phi_{ns}\left(\frac{m_p\vec{k}_e - m_e\vec{k}_p}{m_p + m_e}\right) a^\dagger_{ns}(\vec{k}_e, \sigma_e) a^\dagger_p(\vec{k}_p, \sigma_p)|0\rangle, \end{aligned} \tag{3}$$

where $E_{\mathrm{H}}(\vec{q}) = \sqrt{M^2_{\mathrm{H}} + \vec{q}^{\,2}}$ and $\vec{q}$ are the total energy and the momentum of hydrogen, $M_{\mathrm{H}} = m_p + m_e + \epsilon_{ns}$ and ϵ_{ns} are the mass and the binding energy of hydrogen H in the $(ns)_F$ hyperfine state, $\phi_{ns}(\vec{k}\,)$ is the wave function of the ns state in the momentum representation [20] (see also [22–24]). For the calculation of the bound state β^--decay rate we can neglect the hyperfine splitting of the energy levels of the ns states [20,21].

For the amplitude of the bound-state β^--decay we obtain the following expression:

$$\begin{aligned} & M(n \to \mathrm{H}^{(ns)} + \tilde{\nu}_e) \\ & = G_F V_{ud}\sqrt{2m_n 2E_{\mathrm{H}} 2E_{\tilde{\nu}_e}} \int \frac{d^3k}{(2\pi)^3}\,\phi^*_{ns}\left(\vec{k} - \frac{m_e}{m_p + m_e}\,\vec{q}\,\right) \\ & \quad \times \left\{\left[\varphi^\dagger_e \chi_{\tilde{\nu}_e}\right][\varphi^\dagger_p \varphi_n] - g_A\left[\varphi^\dagger_e \vec{\sigma}\,\chi_{\tilde{\nu}_e}\right]\cdot[\varphi^\dagger_p \vec{\sigma}\,\varphi_n]\right\}, \end{aligned} \tag{4}$$

where $\varphi_p, \varphi_n, \varphi_e$, and $\chi_{\tilde{\nu}_e}$ are spinorial wave functions of the proton, neutron, electron, and antineutrino. The integral over $\vec{k}$ of the wave function $\phi^*_{ns}(\vec{k}\,)$ defines the wave function $\psi^*_{ns}(0)$ in the coordinate representation, equal to $\psi^*_{ns}(0) = \sqrt{\alpha^3 m^3_e/n^3\pi}$, where m_e is the electron mass and $\alpha = 1/137.036$ is the fine-structure constant. This gives

$$\begin{aligned} & M(n \to \mathrm{H}^{(ns)} + \tilde{\nu}_e) \\ & \quad = G_F V_{ud}\sqrt{2m_n 2E_{\mathrm{H}} 2E_{\tilde{\nu}_e}} \\ & \qquad \times \left\{\left[\varphi^\dagger_e \chi_{\tilde{\nu}_e}\right][\varphi^\dagger_p \varphi_n] - g_A\left[\varphi^\dagger_e \vec{\sigma}\,\chi_{\tilde{\nu}_e}\right]\cdot[\varphi^\dagger_p \vec{\sigma}\,\varphi_n]\right\}\psi^*_{(ns)_F}(0). \end{aligned} \tag{5}$$

The bound-state β^--decay rate of the neutron is

$$\begin{aligned} \lambda_{\beta^-_b} = {} & \frac{1}{2m_n}\int \frac{1}{2}\sum_{n=1}^{\infty}\sum_{\sigma_n,\sigma_p,\sigma_e}|M(n \to \mathrm{H}^{(ns)} + \tilde{\nu}_e)|^2 \\ & \times (2\pi)^4\delta^{(4)}\left(k_{\tilde{\nu}_e} + q - p\right)\frac{d^3q}{(2\pi)^3 2E_{\mathrm{H}}}\frac{d^3k_{\tilde{\nu}_e}}{(2\pi)^3 2E_{\tilde{\nu}_e}}. \end{aligned} \tag{6}$$

Summing over the *principal* quantum number and polarizations we get

$$\begin{aligned} \lambda_{\beta^-_b} = {} & \left(1 + 3g^2_A\right)\zeta(3)\, G^2_F|V_{ud}|^2\frac{\alpha^3 m^3_e}{\pi^2} \\ & \times \sqrt{(m_p + m_e)^2 + Q^2_{\beta^-_c}}\,\frac{Q^2_{\beta^-_c}}{m_n}, \end{aligned} \tag{7}$$

where $\zeta(3) = 1.202$ is the Riemann function, coming from the summation over the *principal* quantum number n, and $Q_{\beta^-_c} = 0.782$ MeV is the Q-value of the continuum-state β^--decay [14]. In the literature [9,10] the bound-state β^--decay rate of the neutron is defined relative to the continuum-state β^--decay rate of the neutron.

The theoretical value of the continuum-state β^--decay rate of the neutron is

$$\begin{aligned} \lambda_{\beta^-_c} &= \left(1 + 3g^2_A\right)\frac{G^2_F|V_{ud}|^2}{2\pi^3}\, f(Q_{\beta^-_c}, Z = 1) \\ &= 1.0931(14) \times 10^{-3}\,\mathrm{s}^{-1}, \end{aligned} \tag{8}$$

where the error of the decay rate is fully defined by the experimental error of the axial coupling constant $g_A = 1.2750(9)$ and the CKM matrix element $|V_{ud}| = 0.97419(22)$. The numerical value of the continuum-state β^--decay rate of the neutron is calculated for the experimental masses of the interacting particles [4] and the Fermi integral $f(Q_{\beta^-_c}, Z = 1)$ equal to

$$\begin{aligned} f\left(Q_{\beta^-_c}, Z = 1\right) &= \int_{m_e}^{Q_{\beta^-_c}+m_e}\left(Q_{\beta^-_c} + m_e - E_e\right)^2 \\ & \quad \times E_e\sqrt{E^2_e - m^2_e}\,F(E_e, Z = 1)\,dE_e \\ &= 0.0588\,\mathrm{MeV}^5, \end{aligned} \tag{9}$$

where $F(E_e, Z = 1)$ is the Fermi function [14]. The theoretical value of the lifetime of the neutron, defined by $\tau_{\beta^-_c} = 1/\lambda_{\beta^-_c}$, is $\tau_{\beta^-_c} = 914.8(1.2)$ s.

Taking into account the radiative corrections [13], we get $f^{(\gamma)}(Q_{\beta^-_c}, Z = 1) = 0.0611\,\mathrm{MeV}^5$ and $\lambda^{(\gamma)}_{\beta^-_c} = 1.1359(14) \times 10^{-3}\,\mathrm{s}^{-1}$ [14]. This reduces the lifetime to the value $\tau^{(\gamma)}_{\beta^-_c} = 880.4.(1.1)$ s. It agrees well with the experimental value

$\tau^{\rm exp}_{\beta^-_c} = 878.5(8)$ s [1] and differs from the world averaged experimental value $\tau^{\rm exp}_{\beta^-_c} = 885.7(8)$ s [4] by a few seconds (-5.3 ± 1.4) s.

For the ratio $R_{b/c} = \lambda_{\beta^-_b}/\lambda^{(\gamma)}_{\beta^-_c}$ of the bound-state and continuum-state β^--decay rates of the neutron we get the following expression:

$$R_{b/c} = \zeta(3)2\pi\,\frac{\alpha^3 m_e^3 Q^2_{\beta^-_c}}{m_n}\,\frac{\sqrt{(m_p+m_e)^2+Q^2_{\beta^-_c}}}{f^{(\gamma)}\big(Q_{\beta^-_c}, Z=1\big)} = 3.92\times 10^{-6}. \qquad (10)$$

Our value for the ratio of the decay rates agrees with the results obtained in [9] (see also [11,12]): $R_{b/c} = 4.20 \times 10^{-6}$.

IV. CONTINUUM-STATE AND BOUND-STATE β^--DECAY RATES OF NEUTRON IN $V - A$, SCALAR, AND TENSOR THEORY OF WEAK INTERACTIONS

In this section we consider the continuum-state and bound-state β^--decays of the neutron by taking into account scalar and tensor weak interactions [5,8]. The effective low-energy Hamiltonian of these interactions can be taken in the following form:

$$\tilde{\mathcal{H}}_W(x) = \frac{G_F}{\sqrt{2}}\,V_{ud}\,\Big\{g_S\,[\bar{\psi}_p(x)\psi_n(x)][\bar{\psi}_e(x)(1-\gamma^5)\psi_{\nu_e}(x)] + \frac{1}{2}\,g_T\,[\bar{\psi}_p(x)\sigma_{\mu\nu}\gamma^5\psi_n(x)] \times[\bar{\psi}_e(x)\sigma^{\mu\nu}(1-\gamma^5)\psi_{\nu_e}(x)]\Big\}, \qquad (11)$$

where g_S and g_T are the constants of scalar and tensor weak interactions and $\sigma_{\mu\nu} = \frac{1}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$ is the Dirac matrix.

In the nonrelativistic approximation for the neutron and the proton, the contribution of the scalar and tensor weak interactions to the amplitude of the continuum-state β^--decay is

$$\tilde{M}(n\to p+e^-+\tilde{\nu}_e) = -\frac{G_F}{\sqrt{2}}\,V_{ud}\,\sqrt{4m_p m_n} \times\Big\{g_S\Big[\bar{u}_e(\vec{k}_e,\sigma_e)\,(1-\gamma^5)v_{\tilde{\nu}_e}\Big(\vec{k}_{\tilde{\nu}_e},+\frac{1}{2}\Big)\Big][\varphi^\dagger_p\varphi_n] + g_T\Big[\bar{u}_e(\vec{k}_e,\sigma_e)\gamma^0\vec{\gamma}\,(1-\gamma^5)v_{\tilde{\nu}_e}\Big(\vec{k}_{\tilde{\nu}_e},+\frac{1}{2}\Big)\Big] \cdot[\varphi^\dagger_p\vec{\sigma}\,\varphi_n]\Big\}. \qquad (12)$$

The total amplitude of the continuum-state β^--decay of the neutron, containing the contributions of $V - A$, S, and T interactions, is

$$M(n\to p+e^-+\tilde{\nu}_e) = -\frac{G_F}{\sqrt{2}}\,V_{ud}\,\sqrt{4m_p m_n}\,\Big\{\Big[\bar{u}_e(\vec{k}_e,\sigma_e)\,(\gamma^0+g_S)\,(1-\gamma^5)v_{\tilde{\nu}_e} \times\Big(\vec{k}_{\tilde{\nu}_e},+\frac{1}{2}\Big)\Big][\varphi^\dagger_p\varphi_n] + \Big[\bar{u}_e(\vec{k}_e,\sigma_e)(g_A+g_T\gamma^0)\,\vec{\gamma}\,(1-\gamma^5)v_{\tilde{\nu}_e} \times\Big(\vec{k}_{\tilde{\nu}_e},+\frac{1}{2}\Big)\Big]\cdot[\varphi^\dagger_p\vec{\sigma}\,\varphi_n]\Big\}. \qquad (13)$$

The theoretical value of the continuum-state β^--decay rate of the neutron, accounting for the contributions of scalar and tensor weak interactions, is

$$\tilde{\lambda}_{\beta^-_c} = \frac{G_F^2|V_{ud}|^2}{2\pi^3}\big\{\big((1+3g_A^2) + (g_S^2+3g_T^2)\big)\,f^{(\gamma)}(Q_{\beta^-_c}, Z=1) + 2\,(g_S+3g_Ag_T)\,\tilde{f}^{(\gamma)}(Q_{\beta^-_c}, Z=1)\big\}, \qquad (14)$$

where $\tilde{f}(Q_{\beta^-_c}, Z=1)$ is the Fermi integral equal to

$$\tilde{f}(Q_{\beta^-_c}, Z=1) = \int_{m_e}^{Q_{\beta^-_c}+m_e} m_e E_e^2(Q_{\beta^-_c}+m_e-E_e)^2F(E_e,Z=1) \times\Big(1+\frac{\alpha}{2\pi}\,g(E_e)\Big)\,dE_e = 0.0404\,\mathrm{MeV}^5, \qquad (15)$$

where we have taken into account the contribution of the radiative corrections [13]. The function $g(E_e)$ is calculated in [13] (see also [14]).

Neglecting the contribution of quadratic values of the scalar g_S and tensor g_T coupling constants, the continuum-state β^--decay rate of the neutron is

$$\tilde{\lambda}^{(\gamma)}_{\beta^-_c} = \lambda^{(\gamma)}_{\beta^-_c}\,(1+b\,\Delta_F), \qquad (16)$$

where $\Delta_F = \tilde{f}^{(\gamma)}(Q_{\beta^-_c}, Z=1)/f^{(\gamma)}(Q_{\beta^-_c}, Z=1) = 0.6612$ and b is the Firz term [3] [see Eqs. (26)–(28)] equal to

$$b = 2\,\frac{g_S+3g_Ag_T}{1+3g_A^2} = 0.0032(23). \qquad (17)$$

The numerical value of the Fierz term is obtained from the fit of the experimental value $\tau^{\rm exp}_{\beta^-_c} = 878.5(8)$ s [2]. For the linear combination $g_S + 3g_Ag_T$ of the scalar and tensor coupling constant we get

$$g_S + 3g_Ag_T = 0.0094(70). \qquad (18)$$

The contribution of the scalar and tensor weak interactions changes the amplitude of the bound-state β^--decay as follows:

$$M(n\to \mathrm{H}^{(ns)}+\tilde{\nu}_e) = G_F V_{ud}\sqrt{2m_n 2E_{\rm H}2E_{\tilde{\nu}_e}} \times\big\{(1+g_S)\,[\varphi^\dagger_e\chi_{\tilde{\nu}_e}][\varphi^\dagger_p\varphi_n] - (g_A+g_T) \times\big[\varphi^\dagger_e\vec{\sigma}\,\chi_{\tilde{\nu}_e}\big]\cdot[\varphi^\dagger_p\vec{\sigma}\,\varphi_n]\big\}\,\psi^*_{(ns)_F}(0). \qquad (19)$$

The bound-state β^--decay rate of the neutron is equal to

$$\tilde{\lambda}_{\beta^-_b} = ((1+g_S)^2+3(g_A+g_T)^2)\,\zeta(3)G_F^2|V_{ud}|^2 \times\frac{\alpha^3m_e^3}{\pi^2}\,\sqrt{(m_p+m_e)^2+Q^2_{\beta^-_c}}\,\frac{Q^2_{\beta^-_c}}{m_n}. \qquad (20)$$

Neglecting the contribution of the quadratic coupling constants of the scalar and tensor weak interactions we get

$$\tilde{\lambda}_{\beta^-_b} = (1+b)\,\lambda_{\beta^-_b} = \lambda_{\beta^-_b}. \qquad (21)$$

Thus, the ratio $\tilde{R}_{b/c} = \tilde{\lambda}_{\beta^-_b}/\tilde{\lambda}_{\beta^-_c}$ of the bound-state and continuum-state β^--decay rates of the neutron is practically not changed $\tilde{R}_{b/c} = 3.92 \times 10^{-6}$.

V. HELICITY AMPLITUDES AND ANGULAR DISTRIBUTIONS OF BOUND-STATE β^--DECAY RATES OF NEUTRON

If the axis of the antineutrino-spin quantization is inclined relative to the axis of the neutron-spin quantization with a polar angle ϑ, the wave function $\chi_{\tilde{\nu}_e}$ can be taken in the following form:

$$\chi_{\tilde{\nu}_e} = \begin{pmatrix} -e^{-i\varphi}\sin\frac{\vartheta}{2} \\ \cos\frac{\vartheta}{2} \end{pmatrix}, \qquad (22)$$

where φ is an azimuthal angle. The contributions of different spinorial states to the helicity amplitudes of the bound-state β^--decay as functions of the angles ϑ and φ are adduced in Table I.

Using the results in Table I we get the helicity amplitudes $M(n \to H_{FM_F} + \tilde{\nu}_e)_{\sigma_n, +\frac{1}{2}}$:

$$\begin{aligned}
&M(n \to H_{00} + \tilde{\nu}_e)_{+\frac{1}{2},+\frac{1}{2}} \\
&\quad = M_0 \frac{1 + 3g_A + g_S + 3g_T}{\sqrt{2}} \cos\frac{\vartheta}{2}, \\
&M(n \to H_{1,+1} + \tilde{\nu}_e)_{+\frac{1}{2},+\frac{1}{2}} \\
&\quad = -M_0(1 - g_A + g_S - g_T)e^{-i\varphi}\sin\frac{\vartheta}{2}, \\
&M(n \to H_{10} + \tilde{\nu}_e)_{+\frac{1}{2},+\frac{1}{2}} \\
&\quad = M_0 \frac{1 - g_A + g_S - g_T}{\sqrt{2}} \cos\frac{\vartheta}{2}, \\
&M(n \to H_{1,-1} + \tilde{\nu}_e)_{+\frac{1}{2},+\frac{1}{2}} = 0, \\
&M(n \to H_{00} + \tilde{\nu}_e)_{-\frac{1}{2},+\frac{1}{2}} \\
&\quad = M_0 \frac{1 + 3g_A + g_S + 3g_T}{\sqrt{2}} e^{-i\varphi}\sin\frac{\vartheta}{2}, \\
&M(n \to H_{1,+1} + \tilde{\nu}_e)_{-\frac{1}{2},+\frac{1}{2}} = 0, \\
&M(n \to H_{10} + \tilde{\nu}_e)_{-\frac{1}{2},+\frac{1}{2}} \\
&\quad = -M_0 \frac{1 - g_A + g_S - g_T}{\sqrt{2}} e^{-i\varphi}\sin\frac{\vartheta}{2}, \\
&M(n \to H_{1,-1} + \tilde{\nu}_e)_{-\frac{1}{2},+\frac{1}{2}} \\
&\quad = M_0(1 - g_A + g_S - g_T)\cos\frac{\vartheta}{2}. \qquad (23)
\end{aligned}$$

TABLE I. The contributions of different spinorial states of the interacting particles to the amplitudes of the bound-state β^--decay of the neutron and the antineutrino in the state with the wave function Eq. (22); f is defined by $f = (1 + g_S)[\varphi_e^\dagger \chi_{\tilde{\nu}_e}][\varphi_p^\dagger \varphi_n] - (g_A + g_T)[\varphi_e^\dagger \vec{\sigma}\, \chi_{\tilde{\nu}_e}] \cdot [\varphi_p^\dagger \vec{\sigma}\, \varphi_n]$.

σ_n	σ_p	σ_e	$\sigma_{\tilde{\nu}_e}$	f
$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$(1 + g_S + g_A + g_T)\cos\frac{\vartheta}{2}$
$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-(1 + g_S - g_A - g_T)e^{-i\varphi}\sin\frac{\vartheta}{2}$
$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0
$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-2(g_A + g_T)\cos\frac{\vartheta}{2}$
$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$2(g_A + g_T)e^{-i\varphi}\sin\frac{\vartheta}{2}$
$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0
$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$(1 + g_S - g_A - g_T)\cos\frac{\vartheta}{2}$
$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-(1 + g_S + g_A + g_T)e^{-i\varphi}\sin\frac{\vartheta}{2}$

The angular distributions of the probabilities of the bound-state β^--decays of the polarized neutron are equal to

$$\begin{aligned}
4\pi \frac{dR^{(+)}_{F=0}}{d\Omega} &= \frac{1}{8}\frac{(1+3g_A)^2}{1+3g_A^2}\frac{1}{1+b} \times \left(1 + 2\frac{g_S + 3g_T}{1+3g_A}\right)(1 + \cos\vartheta), \\
4\pi \frac{dR^{(-)}_{F=0}}{d\Omega} &= \frac{1}{8}\frac{(1+3g_A)^2}{1+3g_A^2}\frac{1}{1+b} \times \left(1 + 2\frac{g_S + 3g_T}{1+3g_A}\right)(1 - \cos\vartheta), \\
4\pi \frac{dR^{(+)}_{F=1}}{d\Omega} &= \frac{1}{8}\frac{(1-g_A)^2}{1+3g_A^2}\frac{1}{1+b} \times \left(1 + 2\frac{g_S - g_T}{1-g_A}\right)(3 - \cos\vartheta), \\
4\pi \frac{dR^{(-)}_{F=1}}{d\Omega} &= \frac{1}{8}\frac{(1-g_A)^2}{1+3g_A^2}\frac{1}{1+b} \times \left(1 + 2\frac{g_S - g_T}{1-g_A}\right)(3 + \cos\vartheta),
\end{aligned} \qquad (24)$$

where $R^{(\pm)}_F = (\lambda_{\beta^-_b})^{(\pm)}_F/\tilde{\lambda}_{\beta^-_b}$ and indices $(\pm)$ stand for the polarizations of the neutron.

For $g_S = g_T = 0$ these angular distributions of the decay probabilities agree well with those obtained by Song in [9]. Our polar angle ϑ is related to the polar angle θ in Song's paper as $\vartheta = \pi - \theta$.

The angular distributions, given in Eq. (24), can be used for the experimental search for the bound-state β^--decay of the polarized neutron into hydrogen in the hyperfine state with $F = 1$. Since in the directions $\cos\vartheta = \mp 1$ the angular distributions of the probabilities of the production of hydrogen in the hyperfine state with $F = 0$ vanish, so for $\cos\vartheta = \mp 1$ one can detect only the bound-state β^--decays of the neutron into hydrogen in the hyperfine state with $F = 1$.

The probabilities of decays into hydrogen in the certain hyperfine states are equal to

$$\begin{aligned}
R_{F=0} &= \frac{(\lambda_{\beta^-_b})_{F=0}}{\lambda_{\beta^-_b}} = \frac{1}{4}\frac{(1+3g_A)^2}{1+3g_A^2}\frac{1}{1+b}\left(1 + 2\frac{g_S + 3g_T}{1+3g_A}\right) \\
&= 0.987(2)\left(1 + 2\frac{g_S + 3g_T}{1+3g_A}\right),
\end{aligned}$$

$$R_{F=1} = \frac{(\lambda_{\beta_b^-})_{F=1}}{\lambda_{\beta_b^-}} = \frac{3}{4}\,\frac{(1-g_A)^2}{1+3g_A^2}\,\frac{1}{1+b}\left(1+2\,\frac{g_S-g_T}{1-g_A}\right) = 0.010(0)\left(1+2\,\frac{g_S-g_T}{1-g_A}\right), \quad (25)$$

where we have used the numerical values $g_A = 1.2750(9)$ and $b = 0.0032(23)$.

VI. ELECTRON SPECTRUM OF CONTINUUM-STATE β^--DECAY OF THE NEUTRON WITH CORRELATION COEFFICIENTS

The experimental measurement of the value of the axial coupling constant g_A can be carried out by measuring the electron energy spectrum and correlation coefficients [3]. The electron energy spectrum of the continuum-state β^--decay of the neutron is equal to

$$\frac{d^5\lambda^{(\gamma)}_{\beta_c^-}}{dE_e d\Omega_e d\Omega_{\tilde{\nu}_e}} = \left(1+3g_A^2+g_S^2+3g_T^2\right) \times \frac{G_F^2|V_{ud}|^2}{16\pi^5}(Q_{\beta_c^-}+m_e-E_e)^2E_e\sqrt{E_e^2-m_e^2} \times F(E_e,Z=1)\left(1+\frac{\alpha}{2\pi}\,g(E_e)\right) \times \left(1+a\,\frac{\vec{k}_e\cdot\vec{k}_{\tilde{\nu}_e}}{E_eE_{\tilde{\nu}_e}}+b\,\frac{m_e}{E_e}+A\,\frac{\vec{\xi}\cdot\vec{k}_e}{E_e}+B\,\frac{\vec{\xi}\cdot\vec{k}_{\tilde{\nu}_e}}{E_{\tilde{\nu}_e}}\right), \quad (26)$$

where the coefficients a, A, and B define the correlations between momenta of electron and antineutrino, neutron spin and electron momentum, and neutron spin and antineutrino momentum, respectively, $\vec{\xi}$ is the unit polarization vector of the neutron. The Fierz term b [3] describes a deviation from the $V - A$ theory of weak interactions. The correlation coefficients are equal to

$$a = \frac{1-g_A^2-g_S^2+g_T^2}{1+3g_A^2+g_S^2+3g_T^2},$$
$$b = \frac{2(g_S+3g_Ag_T)}{1+3g_A^2+g_S^2+3g_T^2},$$
$$A = -2\,\frac{g_A(g_A-1)+g_T(g_S-g_T)}{1+3g_A^2+g_S^2+3g_T^2}, \quad (27)$$
$$B = +2\,\frac{g_A(g_A+1)+g_T(g_S+g_T)}{1+3g_A^2+g_S^2+3g_T^2} + 2\,\frac{g_T+g_A(g_S+2g_T)}{1+3g_A^2+g_S^2+3g_T^2}\,\frac{m_e}{E_e}.$$

Neglecting the contribution of g_S^2, g_T^2, and g_Sg_T we get

$$a = \frac{1-g_A^2}{1+3g_A^2},\quad b = 2\,\frac{g_S+3g_Ag_T}{1+3g_A^2},$$
$$A = -2\,\frac{g_A(g_A-1)}{1+3g_A^2},\quad B = +2\,\frac{g_A(g_A+1)}{1+3g_A^2} + 2\,\frac{g_T+g_A(g_S+2g_T)}{1+3g_A^2}\,\frac{m_e}{E_e}. \quad (28)$$

The coefficients a and A agree well with the results adduced in [3], whereas the coefficient B differs from that given in [3] by the term inversely proportional to the energy of the electron and linear in scalar and tensor coupling constants. The value of the Fierz term $b = 0.0032(23)$ is given in Eq. (17).

VII. NUMERICAL VALUE OF CKM MATRIX ELEMENT $|V_{ud}|$ IN $V - A$ THEORY OF WEAK INTERACTIONS

For the calculation of the lifetime of the neutron we have used the numerical value $|V_{ud}| = 0.97419(22)$ of the CKM matrix element proposed in [4].

In this section we calculate the value of the CKM matrix element $|V_{ud}|$ in the $V - A$ theory of weak interactions using our expression for the continuum-state β^--decay rate of the neutron Eq. (8) calculated for the axial coupling constant $g_A = 1.2750(9)$ [3] and accounting for the radiative corrections and the experimental values of the lifetimes of the neutron [1,4]. From Eq. (8) with $f(Q_{\beta_c^-}, Z = 1) \to f^{(\gamma)}(Q_{\beta_c^-}, Z = 1)$ we get

$$|V_{ud}|^2 = \frac{4910.22}{\tau^{(\mathrm{exp})}_{\beta_c^-}(1+3g_A^2)}. \quad (29)$$

Using the experimental values of the lifetimes $\tau^{(\mathrm{exp})}_{\beta_c^-} = 878.5(8)\,\mathrm{s}$ and $\tau^{(\mathrm{exp})}_{\beta_c^-} = 885.7(8)\,\mathrm{s}$, given in [1] and [4], respectively, we obtain

$$|V_{ud}| = \begin{cases} 0.9752(7), & \tau^{(\mathrm{exp})}_{\beta_c^-} = 878.5(8)\,\mathrm{s} \\ 0.9713(7), & \tau^{(\mathrm{exp})}_{\beta_c^-} = 885.7(8)\,\mathrm{s}. \end{cases} \quad (30)$$

In Fig. 1 we show a dependence of the CKM matrix element on the values of the lifetime of the neutron and

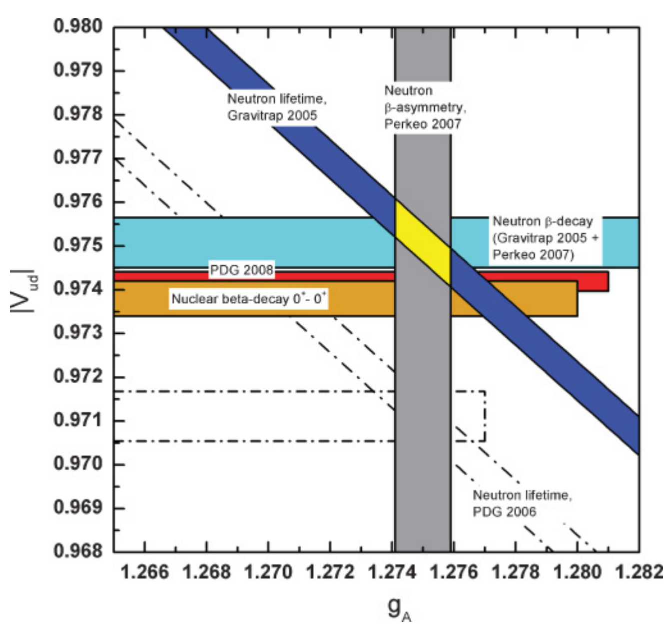

FIG. 1. (Color online) The dependence of the CKM matrix element $|V_{ud}|$ on the values of the lifetime of the neutron and the axial coupling constant g_A.

the axial coupling constant g_A. The yellow area shows that the value $|V_{ud}| = 0.9752(7)$, calculated for the lifetime $\tau^{(\mathrm{exp})}_{\beta^-_c} = 878.5(8)$ s, agrees with both $|V_{ud}| = 0.97419(22)$ and $|V_{ud}| = 0.9738(4)$.

One can see that the value $|V_{ud}| = 0.9713(7)$, calculated for the lifetime $\tau^{(\mathrm{exp})}_{\beta^-_c} = 885.7(8)$ s and $g_A = 1.2750(9)$, is ruled out by the experimental value $|V_{ud}| = 0.9738(4)$, measured from the superallowed $0^+ \to 0^+$ nuclear β^--decays, caused by pure Fermi transitions only [3,18], and the unitarity of the CKM matrix elements giving $|V_{ud}| = 0.97419(22)$ [4].

VIII. CONCLUSIVE DISCUSSION

We have recalculated the continuum-state and bound-state β^--decay rates of the neutron. Taking into account the contributions of weak and strong interactions for the lifetime of the neutron we get the value $\tau_{\beta^-_c} = 914.8(1.2)$ s, where the error ± 1.2 s is caused by the experimental error of the axial coupling constant $g_A = 1.2750(9)$ and the CKM matrix element $|V_{ud}| = 0.97419(22)$ [4]. Including the radiative corrections [13], the theoretical value of the lifetime of the neutron changes to $\tau^{(\gamma)}_{\beta^-_c} = 880.4(1.1)$ s. It agrees well with the experimental value $\tau^{(\mathrm{exp})}_{\beta^-_c} = 878.5(8)$ s [1].

We would like to accentuate that the radiative corrections are universal and make up about 3.9 %. The theoretical value of the radiative corrections, calculated in this paper as

$$R_{\mathrm{RC}} = \frac{f^{(\gamma)}(Q_{\beta^-_c}, Z = 1)}{f(Q_{\beta^-_c}, Z = 1)} = 1.03912, \qquad (31)$$

agrees well with the value $R_{\mathrm{RC}} = 1.03886(39)$ given in [3], and $R_{\mathrm{RC}} = 1.0390(8)$ calculated in [13].

Thus, the agreement of the theoretical value of the lifetime of the neutron $\tau_{\beta^-_c} = 880.4(1.1)$ s with the experimental value $\tau^{(\mathrm{exp})}_{\beta^-_c} = 878.5(8)$ s, measured in [1], is fully due to the axial coupling constant $g_A = 1.2750(9)$ and the CKM matrix element $|V_{ud}| = 0.97419(22)$ [4] (see Table III and the discussion below).

Using our expression (8) for the continuum-state β^--decay rate with the Fermi integral, accounting for the contribution of radiative corrections, the axial coupling constant $g_A = 1.2750(9)$ and the experimental lifetimes of the neutron $\tau^{(\mathrm{exp})}_{\beta^-_c} = 878.5(8)$ s [1] and $\tau^{(\mathrm{exp})}_{\beta^-_c} = 885.7(8)$ s we get the values of the CKM matrix element $|V_{ud}| = 0.9752(7)$ and $|V_{ud}| = 0.9713(7)$, respectively.

It seems that $|V_{ud}| = 0.9713(7)$ is ruled out by the values $|V_{ud}| = 0.9738(4)$ and $|V_{ud}| = 0.97419(22)$ defined from the superallowed $0^+ \to 0^+$ nuclear β^--decays [3,18] and the unitarity condition for the CKM matrix elements [4], respectively. One should emphasize that it is valid only for the axial coupling constant $g_A = 1.2750(9)$.

What lifetime time of the neutron is singled out by the nature either $\tau^{(\mathrm{exp})}_{\beta^-_c} = 878.5(8)$ s [1] or $\tau^{(\mathrm{exp})}_{\beta^-_c} = 885.7(8)$ s [4]?

Of course, the reply to this question should be obtained by further experiments in terrestrial laboratories. In regards to new measurements of the lifetime of the neutron we would like to mention the recent experimental value $\tau^{(\mathrm{exp})}_{\beta^-_c} = 878.2(1.9)$ s, reported by Ezhov *et al.* [32]. It is important to emphasize that the experimental procedure of Ezhov's experiments, based on the use of the magnetic trap for neutrons, differs from the procedure of Serebrov's experiments [1]. In addition to the experimental data of terrestrial laboratories some hints on the value of the lifetime of the neutron can be obtained far from Earth, for example, from cosmology [25,26].

TABLE II. The correlation coefficients of the energy spectrum of the continuum-state β^--decay of the neutron calculated for $g_A = 1.2750(9)$ [3]; $C = -0.27484\,(A + B)$ is the proton asymmetry measured in [29].

	Experiment	Theory: $g_A = 1.2750\,(9)$
$\tau_{\beta^-_c}$	878.5 (8) [1]	$880.4\,(1.1)_{V-A}$
a	−0.103 (4) [3]	$-\,0.1065\,(3)_{V-A}$
b	–	0.0032 (23)
A	−0.11933 (34) [3]	fit
B	0.9821 (40) [27,28]	$0.9871\,(4)_{V-A}$
C	−0.2377 (26) [29]	$-0.2385\,(1)_{V-A}$
$\lvert V_{ud}\rvert$	0.97419 (22) [4]	$0.9752\,(7)_{V-A}$
$\lvert V_{ud}\rvert$	0.9738 (4) [18]	$0.9752\,(7)_{V-A}$
g_S	–	−0.0251 (181)
g_T	–	+0.0090 (65)

The theoretical values of the correlation coefficients, calculated for $g_A = 1.2750(9)$, are given in Table II. We remind the reader that the value $g_A = 1.2750(9)$ of the axial coupling constant has been calculated from the fit of the experimental value of the coefficient of the correlation between the neutron spin and electron momentum $A^{(\mathrm{exp})} = -\,0.11933(34)$, which has been obtained in [3] as an averaged value over PERKEO II measurements [30,31].

The deviation of the theoretical value of the lifetime of the free neutron $\tau^{(\mathrm{th})}_{\beta^-_c} = 880.1(1.1)$ s from the experimental one $\tau^{\mathrm{exp}}_{\beta^-_c} = 878.5(8)$ s [1] allows to take into account the contributions of scalar and tensor weak interactions, which can be added to the standard $V - A$ baryon-lepton weak interactions with coupling constants g_S and g_T, respectively. From the fit of the experimental value of the lifetime of the neutron $\tau^{\mathrm{exp}}_{\beta^-_c} = 878.5(8)$ s [1] we have found $g_S + 3g_A g_T = 0.0094(70)$, caused by the value of the Fierz term $b = 0.0032(23)$, Eq. (17).

The standard $V - A$ weak interactions describe well the experimental data on the coefficient of the neutron spin and antineutrino momentum correlation (see Table II). Therefore, in order to estimate the values of the scalar and tensor coupling constants, we set to zero the contribution of the energy-dependent term to the coefficient B. This gives

$$g_T + g_A(g_S + 2g_T) = 0. \qquad (32)$$

We would like to emphasize that there is no other reason for the constraint (32) except a sufficiently good agreement between the experimental and theoretical values of B (see Table II). Of course, more precise experimental data on the coefficient B, showing an energy dependence, may destroy such a constraint. Solving Eq. (32) together with the Fierz term Eq. (17) we estimate the scalar and tensor coupling constants

$$g_S = +\frac{b}{2}\,\frac{(1+2g_A)\big(1+3g_A^2\big)}{1+2g_A-3g_A^2},\qquad g_T = -\frac{b}{2}\,\frac{g_A\big(1+3g_A^2\big)}{1+2g_A-3g_A^2}. \tag{33}$$

The numerical values are given in Table II.

The coupling constants g_S and g_T are related to the weak coupling constants C_V, C_S, and C_T used in [8], as $g_S = C_S/C_V$ and $g_T = C_T/C_V$. The scalar $g_S = -0.0251(181)$ and tensor $g_T = +0.0090(65)$ differ from the values $g_S = +0.0013(13)$ and $g_T = -0.0046(42)$ found in [8]. This can imply only that the energy-dependent term in the correlation coefficient B cannot be set zero. The coupling constants g_S and g_T can be obtained experimentally by measuring the energy dependence of the coefficient B of the correlation of the neutron spin and the antineutrino momentum in parallel to the measuring of the bound-state β^--decay rates of the neutron into hydrogen in certain hyperfine states. As we have shown the measurement of the angular distributions of the probabilities of the bound-state β^--decay of the polarized neutron into hydrogen in the hyperfine states with $F = 1$ can be carried out at $\vartheta = 0$ or $\vartheta = \pi$.

Our angular distributions for the probabilities of the bound-state β^--decay rates of the neutron into hydrogen in certain hyperfine states agree at $g_S = g_T = 0$ with those obtained by Song [9].

A. Comments on the use of the axial coupling constant $g_A = 1.2695(29)$

Concluding our analysis of the β^--decays of the neutron we would like to discuss the theoretical lifetime of the neutron, the correlation coefficients, the CKM matrix element $|V_{ud}|$, and the estimate of the scalar and tensor coupling constants, calculated for the axial coupling constant $g_A = 1.2695(29)$. The results are given in Table III.

The use of the axial coupling constant $g_A = 1.2695(29)$, given in [4] (see also [19]), allows to describe the world averaged value of the lifetime of the neutron well. However, due to the sufficiently large uncertainty, which is by a factor 3 larger compared with that of $g_A = 1.2750(9)$, the theoretical values for the correlation coefficients agree with the experimental data within two standard deviations, and the contribution of the scalar and tensor weak interactions is ruled out, since the numerical values of the Fierz term, g_S and g_T, are compared with zero.

TABLE III. The correlation coefficients of the energy spectrum of the continuum-state β^--decay of the neutron calculated for $g_A = 1.2695(29)$ [4]; $C = -0.27484\,(A+B)$ is the proton asymmetry, measured in [29].

	Experiment	Theory: $g_A = 1.2695\,(29)$
$\tau_{\beta_c^-}$	885.7 (8) [4]	886.7 (3.4)
a	−0.103 (4) [3]	$-0.1048\,(9)_{V-A}$
b	–	0.0017 (59)
A	−0.11933 (34) [3]	$-0.11727\,(109)_{V-A}$
B	0.9821 (40) [27,28]	$0.9876\,(23)_{V-A}$
C	−0.2377 (26) [29]	$-0.2392\,(7)_{V-A}$
$\lvert V_{ud}\rvert$	0.97419 (22) [4]	$0.9747\,(18)_{V-A}$
$\lvert V_{ud}\rvert$	0.9738 (4) [18]	$0.9747\,(18)_{V-A}$
g_S	–	−0.0135 (470)
g_T	–	+0.0049 (169)

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