Properties of cold dense nuclear matter based on a nonperturbative approach inspired by chiral perturbation theory

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(Received 23 October 2008; revised manuscript received 19 April 2009; published 29 September 2009)

The effective chiral Lagrangian for cold dense nuclear matter is constructed, in which the chemical potential corresponding to the baryon number density of QCD is introduced and included in the effective Lagrangian as an external field. A nonperturbative approach inspired by chiral perturbation theory is employed to calculate the baryon number density and its susceptibility, pressure, and quark condensate. The relevant discussions of the obtained results are given.

DOI: 10.1103/PhysRevC.80.034909 PACS number(s): 24.10.Pa, 12.39.Fe, 11.30.Rd, 11.30.Qc

I. INTRODUCTION

The phase transition of strongly interacting matter is of great interests to physicists. The expected phase diagram of neutral strongly interacting matter given by Ref. [1] in the framework of the Nambu-Jona-Lasinio (NJL) model is shown in Fig. 1.

As the temperature and the chemical potential increase, there is a transition from hadron matter to quark gluon plasma (QGP). For small chemical potential, experiments at the BNL Relativistic Heavy Ion Collider (RHIC) have found evidence for the existence of ideal-liquid-like QGP, whose properties need further investigations [2–5], and the CERN Large Hadron Collider (LHC) gives more information about QGP [6,7]. Theoretically, lattice QCD predicts that the transition of hadron phase to QGP is a crossover with a critical temperature in the range $150 \sim 190$ MeV [5,8–10]. The mechanism of crossover has been studied in many models [11–16]. For the low temperature and high chemical potential regime, our knowledge is quite limited. Experimentally, it is impossible to achieve such a condition in laboratories on earth. A natural laboratory holding such cold, highly compressed matter is a neutron star, about which more observations and theoretical work still need to be done [17,18]. Theoretically, lattice QCD has difficulties in a low temperature and high chemical potential regime. Models based on NJL or PNJL analyses predict that the phase transition of chiral restoration is of first order [1,5,19,20], as shown in Fig. 1. However, in Refs. [21,22], Glozman and Wagenbrunn indicated that according to a Dyson-Schwinger equation (DSE) method under the large Nc approximation (Nc denotes the number of colors), the transition is of second order. Nevertheless, it is certain that chiral symmetry is spontaneously broken when the chemical potential μ is below a critical value. And when μ goes beyond this value, chiral restoration takes place, and then the hadron matter transits into QGP. As the chemical potential further increases to a certain point, QGP is expected to change into a superconductivity phase [23,24], the existence of which still needs to be verified.

To understand the evolution properties of cold dense strongly interacting matter with chemical potential, one should first study how the baryon number density and the pressure change with the chemical potential and investigate the spontaneous breaking of chiral symmetry and its restoration where the key order parameter is the quark condensate. At present, an analysis of the evolution properties of cold dense strongly interacting matter from first principles of QCD is not possible, so one has to resort to various nonperturbative QCD approaches and models. Chiral perturbation theory (χPT) [25–28] (for recent overviews, see Refs. [29,30]) provides an efficient method for investigating the problem in the low energy regime where chiral symmetry is spontaneously broken. In this paper, we construct the chiral effective Lagrangian for cold dense nuclear matter. Chemical potential corresponding to the baryon number density of QCD is introduced and included in the effective Lagrangian as an external field. This method is analogous to the one used in Refs. [26,27]. Here it should be noted that χPT has the limitation that its use is constrained to the low energy regime and cannot be extrapolated to the regime where resonances or virtual particles appear in dense nuclear matter. Oller, Oset, and coworkers developed a nonperturbative approach that extends the idea of χPT beyond the region of its validity [31–34]. With this method, the scattering matrix can be calculated to the regime below 1.2 GeV, and the existence of resonances can be revealed. In this work, we use this nonperturbative idea to calculate the baryon number density and the quark condensate, and consequently we obtain the baryon number susceptibility and the pressure.

This paper is organized as follows. In Sec. II, the effective Lagrangian for cold dense nuclear matter is constructed. In Sec. III, the baryon number density, baryon number susceptibility, and pressure are calculated and the equation of state is obtained. In Sec. IV, the evolution of quark condensate with chemical potential is considered. Our results are consistent with the low temperature result in Ref. [35]. Conclusions are given in Sec. V.

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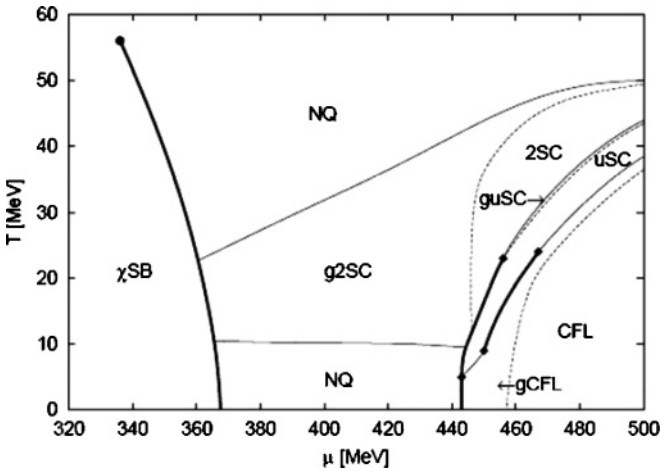

FIG. 1. Phase diagram of QCD for neutral strongly interacting matter given by Ref. [1].

II. EFFECTIVE CHIRAL LAGRANGIAN AT ZERO TEMPERATURE AND FINITE BARYON NUMBER DENSITY

In QCD, the baryon number density operator that corresponds to the conserved charge of $U(1)_B$ symmetry is $\frac{1}{3}q^+q$, where $q(x)$ denotes the quark field operator. The baryon number density of the system is given by $\langle\frac{1}{3}q^+q\rangle$ (the expectation value of the baryon number density operator on the vacuum state at definite μ). At present, it is impossible to calculate $\langle\frac{1}{3}q^+q\rangle$ directly from the QCD Lagrangian. However, based on Weinberg's idea [25] that a hadron system could be described by an effective Lagrangian including all the possible terms consistent with the assumed symmetry principles, one can calculate the vacuum expectation value of quark operators effectively in χPT.

The calculation of $\langle\bar{q}\gamma_\mu q\rangle$, $\langle\bar{q}\gamma_5\gamma_\mu q\rangle$, $\langle\bar{q}q\rangle$, and $\langle\bar{q}\gamma_5 q\rangle$ was carried out in Refs. [26,27] as follows. Introducing four external fields v_μ, a_μ, s, and p, which are coupled to the vector, axial vector, scalar, and pseudoscalar current, respectively, the presence of these external fields implies that there are four additional terms added to the usual QCD Lagrangian. In the chiral limit, it can be written as

$$\mathcal{L}_{\rm ext}^{\rm QCD} = \mathcal{L}_0^{\rm QCD} + \bar{q}\gamma^\mu(v_\mu + \gamma_5 a_\mu)q - \bar{q}(s - i\gamma_5 p)q, \quad (1)$$

where $\mathcal{L}_0^{\rm QCD}$ is the QCD Lagrangian in the chiral limit. The transformation properties of external fields are determined by the invariance of $\mathcal{L}_{\rm ext}^{\rm QCD}$ under local chiral transformation. In the end of the calculation, the scalar external field s is set to the mass matrix of quarks. According to Refs. [25–27], if in the low energy regime the effective Lagrangian consists of all the possible terms, the results calculated from this Lagrangian are equivalent to those calculated from QCD. So, the generating functional deduced from this effective Lagrangian is equivalent to the one deduced from QCD. The generating functionals are functionals of external fields which are included in the effective Lagrangian in a systematic way to ensure local chiral symmetry. The vacuum expectation value of quark operators $\bar{q}\gamma_\mu q$, $\bar{q}\gamma_5\gamma_\mu q$, $\bar{q}q$, and $\bar{q}\gamma_5 q$ can be obtained from the derivative of the generating functional of the hadron system with respect to the external fields corresponding to these operators.

To calculate the expectation value of the baryon number density $\langle\frac{1}{3}q^+q\rangle$ for cold dense nuclear matter, we use the same method and introduce the chemical potential as follows. Under local $U(1)_B$, the quark field transforms as

$$q \to e^{iB\theta(x)}q, \quad (2)$$

where B is the baryon number. To ensure local $U(1)_B$ symmetry of the system, one must introduce an external field b_μ with the transformation property

$$b_\mu \to b_\mu + \partial_\mu\theta(x) \quad (3)$$

and replace the derivative acting on the quark field by the canonical one

$$\partial_\mu q \to \partial_\mu q - iB^q b_\mu q = \partial_\mu q - \frac{i}{3}b_\mu q. \quad (4)$$

Set

$$b_\mu = \delta_{\mu 0}\mu_b \quad (5)$$

in the end of the calculation, then μ_b is the baryon chemical potential. Here and in the following calculations, we assume flavor SU(2) symmetry, which means equal chemical potentials and equal masses for u and d quarks. The density of the conserved charge corresponding to $U(1)_B$ symmetry is the baryon number density $\frac{1}{3}\bar{q}\gamma_0 q = \frac{1}{3}q^+q$. Its vacuum expectation value can be calculated from the derivative of generating functional as

$$\left\langle\frac{1}{3}q^+q\right\rangle = \left.\frac{\partial Z[b_\mu]}{\partial b_0}\right|_{b_0=\mu_b, b_i=0}. \quad (6)$$

In our approach, this is equivalent to replacing b_0 by μ_b and then doing the partial differentiation with respect to μ_b, and the result is

$$\frac{\partial Z[\mu_b]}{\partial\mu_b}. \quad (7)$$

On the other hand, under local $U(1)_B$ transformation, the nucleon field transforms as

$$\psi \to e^{iB\theta(x)}\psi. \quad (8)$$

The derivative of the nucleon field should be replaced by

$$\partial_\mu\psi \to \partial_\mu\psi - iB^N b_\mu\psi = \partial_\mu\psi - i\delta_{0\mu}\mu_b\psi. \quad (9)$$

Accordingly, the pion-nucleon effective Lagrangian $\mathcal{L}_{\pi N}$ for cold dense nuclear matter differs from the usual one in χPT [29,30] in that the canonical derivative of the nucleon field has an additional term $-i\delta_{0\mu}\mu_b\psi$. The Lagrangian for the pure pion fields $\mathcal{L}_\pi$ is the same as the one in the usual χPT, where the pions are taken as Goldstone bosons corresponding to the spontaneous breaking of chiral symmetry.

The pionic Lagrangian at leading order reads

$$\mathcal{L}_\pi^{(2)} = \frac{F_\pi^2}{4}Tr(\partial_\mu U\partial^\mu U^+) + \frac{F_\pi^2}{4}Tr(\chi U^+ + U\chi^+), \quad (10)$$

where U denotes the Goldstone boson fields

$$U = \exp\left(\frac{i\vec{\tau}\cdot\vec{\pi}}{F_\pi}\right), \quad (11)$$

with $\vec{\tau}$ being the three Pauli matrices, $\vec{\pi}$ being the pion field, and

$$\vec{\tau}\cdot\vec{\pi} = \begin{pmatrix} \pi^0 & \sqrt{2}\pi_+ \\ \sqrt{2}\pi_- & -\pi^0 \end{pmatrix}, \tag{12}$$

$\chi = 2B_0(s+ip)$ with s and p being the external fields introduced in Eq. (1), and F_π is the pion decay constant. In our calculation, $p = 0$ and s equals the mass matrix of u, d quarks. Under flavor $SU(2)_f$, χ can be expressed as

$$\chi = 2B_0 M_q = 2B_0\hat{m} = M_\pi^2. \tag{13}$$

For π-N Lagrangians, to the order of $O(p^4)$, the terms contributing to our later calculations are

$$\begin{aligned}
\mathcal{L}^{(1)}_{\pi N} &= \bar{\psi}(i\not{D} - m)\psi + \frac{1}{2}g_A\bar{\psi}\not{u}\gamma_5\psi, \\
\mathcal{L}^{(2)}_{\pi N} &= c_1\langle\chi_+\rangle\bar{\psi}\psi - \frac{c_2}{4m^2}\langle u_\mu u_\nu\rangle(\bar{\psi}D^\mu D^\nu\psi + \text{h.c.}) \\
&\quad + \frac{c_3}{2}u_\mu u^\mu\bar{\psi}\psi + \cdots, \\
\mathcal{L}^{(4)}_{\pi N} &= -\frac{e_1}{16}\langle\chi_+\rangle^2\bar{\psi}\psi + \cdots,
\end{aligned} \tag{14}$$

where

$$\begin{aligned}
D_\mu\psi &= \partial_\mu\psi - i\delta_{0\mu}\mu_b\psi + \Gamma_\mu\psi, \\
\Gamma_\mu &= \tfrac{1}{2}[u^+, \partial_\mu u], \\
u^2 &= U, \quad u_\mu = iu^+\partial_\mu U u^+.
\end{aligned} \tag{15}$$

In Eq. (14), m denotes the nucleon mass in the chiral limit, c_1, c_2, c_3, and e_1 are the low energy constants. The values for the constants used in our calculation are listed in Table I.

Here it should be noted that the effective Lagrangian contains second- and even higher order derivative terms of the nucleon field, such as the second term in $\mathcal{L}^{(2)}_{\pi N}$. Because of the presence of these terms, the density of the conserved charge of $U(1)_B$ symmetry in the nuclear system is not $\psi^+\psi$. This is why we introduce the baryon chemical potential at the quark level instead of naively adding a $\mu_b\psi^+\psi$ term to the effective Lagrangian. Besides, it should also be noted that the Feynman rules are different from those of the usual χPT owing to the additional term in the canonical derivative of the nucleon fields.

III. EVOLUTION PROPERTIES OF COLD DENSE NUCLEAR MATTER WITH CHEMICAL POTENTIAL

To study the evolution properties of cold dense nuclear matter with chemical potential, we first calculate the baryon number density and its susceptibility. According to the analysis in the above section, the baryon number density can be calculated from the effective Lagrangian of the nuclear system. The pion-nucleon effective Lagrangian can be written as

$$\mathcal{L}_{\pi N} = -\bar{\psi}\hat{K}\psi, \tag{16}$$

where according to Eq. (14), $\hat{K}$ is of the form

$$\hat{K} = -(i\gamma_\mu\partial_\mu + \gamma_0\mu_b - m + \hat{O}) \tag{17}$$

with $\hat{O}$ denoting all the remaining terms of $\hat{K}$. The baryon number density of the nucleon can be calculated to be

$$\begin{aligned}
n(\mu_b) &= \left\langle\frac{1}{3}q^+q\right\rangle = \frac{\partial}{\partial\mu_b}\int D\bar{\psi}\,D\psi\,DU\,e^{i\int d^4x\mathcal{L}_{\text{eff}}} \\
&= \frac{\partial}{\partial\mu_b}\int DU(\det K)e^{i\int d^4x\mathcal{L}_\pi} \\
&= \int DU(\det K)\text{Tr}\left(K^{-1}\frac{\partial}{\partial\mu_b}K\right)e^{i\int d^4x\mathcal{L}_\pi},
\end{aligned} \tag{18}$$

where $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi N} + \mathcal{L}_\pi$. K and O denote the results of the path integral after integrating out the nucleon field. The most important contribution to $\langle\frac{1}{3}q^+q\rangle$ from K is

$$\text{Tr}\,(K^{-1}\gamma_0). \tag{19}$$

The remaining term

$$\text{Tr}\left(K^{-1}\frac{\partial}{\partial\mu_b}O\right) \tag{20}$$

comes from higher order terms in the effective Lagrangian, which include the canonical derivative of the nucleon field, such as the second term in $\mathcal{L}^{(2)}_{\pi N}$. The contributions of such terms are far smaller than the leading term due to the smallness of c_2. Therefore, we can neglect them in our later calculation and set

$$n(\mu_b) \doteq \int DU(\det K)\text{Tr}\,(K^{-1}\gamma_0)e^{i\int d^4x\mathcal{L}_\pi}. \tag{21}$$

After doing the path integral on the pion fields U, K^{-1} becomes the nucleon propagator S. Then the baryon number density is

$$n(\mu_b) = \frac{-i}{(2\pi)^4}\int d^4p\,\text{Tr}\,(\gamma_0 S). \tag{22}$$

Remember that the chemical potential is introduced in Euclidean space, p_0 is imaginary, and the integral is of the form

$$\int d^4p = \int d^3\vec{p}\int_{-i\infty}^{i\infty} dp_0. \tag{23}$$

Besides, note that the baryon number density vanishes at zero chemical potential, so if we let $p_0' = p_0 + \mu_b$, then

$$n(\mu_b) = \frac{-i}{(2\pi)^4}\int d^3\vec{p}\left(\int_{-i\infty+\mu_b}^{i\infty+\mu_b} dp_0' - \int_{-i\infty}^{i\infty} dp_0'\right)\text{Tr}\,(\gamma_0 S). \tag{24}$$

TABLE I. Numerical values for m, M_π, F_π, g_A, c_1, c_2, c_3, and e_1 used in our calculation. c_1, c_2, and c_3 are the same as the ones used in Ref. [30]. m and e_1 are fitted through the nucleon mass.

M_π (MeV)	m (MeV)	F_π (MeV)	g_A	c_1 (MeV^{-1})	c_2 (MeV^{-1})	c_3 (MeV^{-1})	e_1 (MeV^{-2})
137	896	92.4	1.27	-0.90×10^{-3}	3.3×10^{-3}	-4.7×10^{-3}	1×10^{-9}

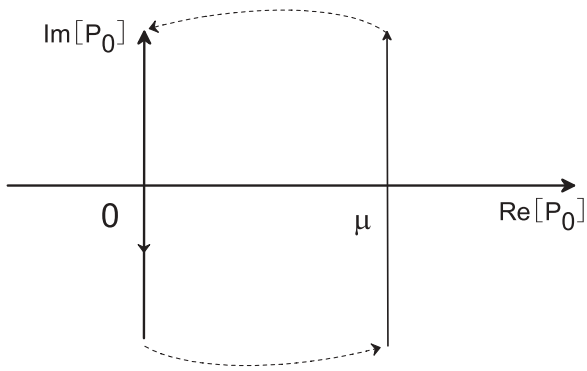

FIG. 2. Integration path of p_0 for the baryon number density of the nucleon in Eq. (24).

The integration path in the complex plane is closed as is shown in Fig. 2.

According to the Cauchy integral formula, the value of the integral is determined by the analytical properties of the integrand in the region encircled by the integration path.

Obviously, ordinary χPT requires real momentum, and an extrapolation of the results to the complex plane would be apparently illegal according to perturbation expansion when any singularity appears. As a result, a nonperturbative scheme becomes necessary in this case. Following the approach developed in Refs. [31,32], in which the T matrix was calculated nonperturbatively from the L-S equation, we calculate the nucleon propagator in the same way as follows. The full nucleon propagator satisfies the following DSE

$$S = S_0 + S_0 \Sigma S, \tag{25}$$

where S_0 is the free nucleon propagator and Σ is the self-energy. In principle, Σ include the full propagator S in the integral and the DSE should be solved by iteration. Here we make the approximation of replacing the full propagator in the self-energy with the free one. And thus

$$S = (1 - S_0\Sigma)^{-1} S_0 = \left[S_0^{-1}(1 - S_0\Sigma)\right]^{-1} = \frac{1}{S_0^{-1} - \Sigma}. \tag{26}$$

One may notice that this expression for S is just what one uses for mass renormalization. However, in ordinary χPT it holds only near the mass shell. Extending the result to the full complex plane and calculating $n(\mu)$ in terms of such an S means adopting a nonperturbative approach instead of an order-by-order perturbative expansion. Σ is calculated perturbatively from the effective chiral Lagrangian. To be consistent with ordinary χPT, counter terms are added, and the divergence of Σ is removed by mass renormalization. So, when $\mu = 0$, S has a pole at the physical value of the nucleon mass. To the order of $O(p^4)$, the diagrams that contribute to the self-energy Σ are shown in Fig. 3. The explicit expression for the self-energy is

$$\begin{aligned}
\Sigma &= -4c_1 M^2 + \Sigma_a + \Sigma_b + \Sigma_c + e_1 M^4 + O(p^5),\\
\Sigma_a &= \frac{3g_A^2}{4F_\pi^2}(m + \not{p}')\left\{M_\pi^2 I + (m - \not{p}')\ \not{p}' I^{(1)}\right\},\\
\Sigma_b &= \frac{3M_\pi^2 \Delta_\pi}{F_\pi^2}\left\{2c_1 - \frac{p'^2}{m^2 d}c_2 - c_3\right\},\\
\Sigma_c &= -4c_1 M_\pi^2 \frac{\partial \Sigma_a}{\partial m},
\end{aligned} \tag{27}$$

where $d = 4$, $\not{p}' = \not{p} + \gamma_0 \mu_b$,

$$\begin{aligned}
I = &-\frac{1}{8\pi^2}\frac{\alpha\sqrt{1-\Omega^2}}{1 + 2\alpha\Omega + \alpha^2}\arccos\left(-\frac{\Omega + \alpha}{\sqrt{1 + 2\alpha\Omega + \alpha^2}}\right)\\
&- \frac{1}{16\pi^2}\frac{\alpha(\Omega + \alpha)}{1 + 2\alpha\Omega + \alpha^2}(2\ln\alpha - 1),
\end{aligned} \tag{28}$$

$$I^{(1)} = \frac{1}{2p'^2}\left\{\left(p'^2 - m^2 + M_\pi^2\right)I + \Delta_\pi\right\},\ \ \Delta_\pi = \frac{M_\pi^2}{8\pi^2}\ln\frac{M_\pi}{m},$$

and

$$\begin{aligned}
\alpha &= \frac{M_\pi}{m},\\
\Omega &= \frac{p'^2 - m^2 - M_\pi^2}{2m M_\pi}.
\end{aligned} \tag{29}$$

It should be noted that the free nucleon propagator in cold dense nuclear matter is different from the usual one, because in the Feynman diagram the momentum of nucleon is always added with a term $\delta_{\mu 0}\mu_b$. If the calculation is restricted to order $O(p^4)$, then the self-energy is the same as that shown in Ref. [28], except that the momentum p of the nucleon field is replaced by $p' = \{p_0 + \mu_b, \vec{p}\}$, wherever it appears. The renormalization scheme used here is the one introduced in Ref. [28] based on the "infrared regularization," which we denote as IR. The baryon number density can now be calculated to be

$$n(\mu_b) = \frac{-i}{(2\pi)^4}\int d^4 p \mathrm{Tr}\left(\frac{\gamma_0}{\not{p}' - m - \Sigma(\not{p}')}\right), \tag{30}$$

However, a direct application of the IR results [28] in the full complex p_0 plane is illegal. In IR, the loop integral H is divided into two parts, the singular part I and the regular part R, and the R part is dropped out in renormalization because it does not fit the infrared singularity of the integral. Considering that the analysis of the singularity is based on an expansion of the nucleon momentum about the nucleon mass m_N, when the nucleon momentum goes far beyond m_N, the expansion is illegal and so the regular part R cannot be dropped out. In the regime where the momentum of the nucleon is much larger or

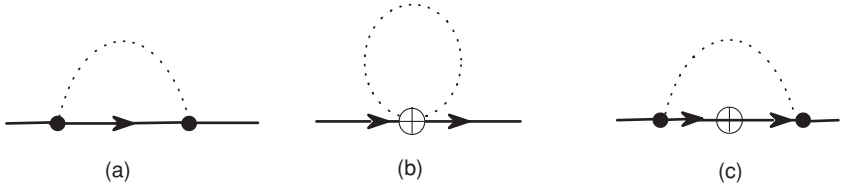

FIG. 3. One-loop diagrams contributing to the self-energy of the nucleon. The crossed vertices denote the contribution from $\mathcal{L}_{\pi N}^{(2)}$.

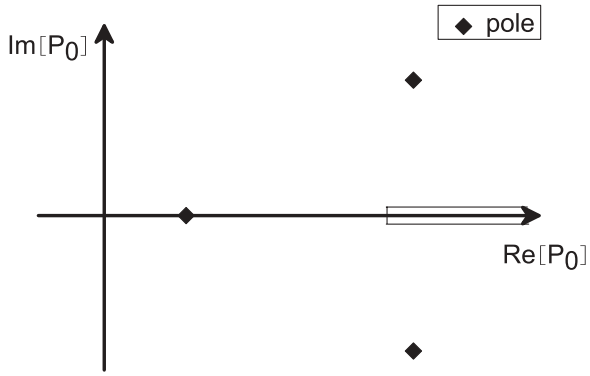

FIG. 4. Poles and branch cut of the integrand in Eq. (24) in the complex p_0 plane. The positions of the three poles are at $p_0 = \sqrt{\vec{p}^2 + p_n^2}$, where n denotes a, b, and c. p_n for each pole is $p_a = 938$, $p_b = 1152 + i337$, and $p_c = 1152 - i337$. The branch cut starts at d, where $p_0 = \sqrt{\vec{p}^2 + (m + M_\pi)^2}$.

much smaller than m_N, the result of the loop integral should be $H = I + R$, not I.

According to above consideration, the integrand in Eq. (24) has three poles and one branch cut in the complex p_0 plane with the real part of p_0 positive and not larger than 1200 MeV. The regime beyond 1200 MeV is out of the consideration because the spontaneous breaking of chiral symmetry is expected to get restored there. The poles and branch cut relevant to our calculation are shown in Fig. 4.

Obviously, the pole denoted as a on the real axis corresponds to the ground state of the nucleon (N938). The branch cut starts at $p_0 = \sqrt{\vec{p}^2 + (m + M_\pi)^2}$. The positions of b and c are far from the real axis and are on the right-hand side of the starting point of the cut line.

The results for the baryon number density are shown in Figs. 5–7. The contributions from the poles and those from the branch cut are shown in Figs. 5 and 6, respectively. The total contributions from the poles and the branch cut are shown in Fig. 7. In Fig. 7, the result for baryon number density from χPT is denoted by a thick black line, while the free-nucleon approximation result [36] is the gray line.

From Fig. 7, it can be seen that the baryon number density $n(\mu)$ is zero when the baryon chemical potential μ_b is smaller than the mass of the nucleon. That is, $\mu = 938$ MeV is a singularity. This result agrees qualitatively with the general conclusion in Ref. [36]. In that reference, based on a universal argument, it is pointed out that the existence of some singularity at the point $\mu = \mu_0$ and $T = 0$ is a robust

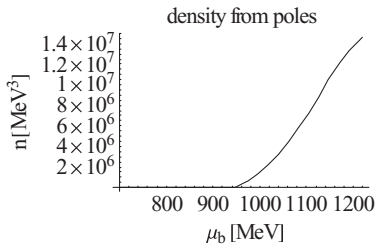

FIG. 5. Contributions to the baryon number density of cold dense nuclear matter from poles.

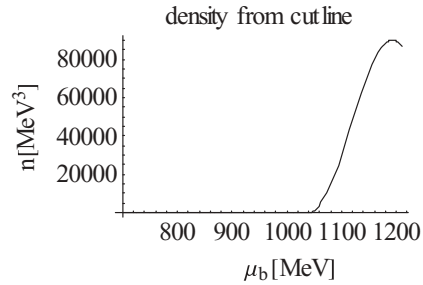

FIG. 6. Contributions to the baryon number density from the branch cut.

and model-independent prediction. A recent model calculation using the rainbow approximation of the DSE method [37] also supports this. Here one complementary point should be stressed: physically, the critical point for $n(\mu)$ to go from zero to a nonzero value should be smaller than 938 MeV because of the contribution from the binding energy. However, our calculation cannot reveal such a contribution, because the pion loops are not considered. It is expected that when calculating to higher order, the results would be better and the quenching effect would be revealed. When μ_b goes beyond that starting point, our result for $n(\mu_b)$ is larger than the free-nucleon result. And when μ_b is larger than about 1152 MeV, the rate for the increase of $n(\mu)$ is slowed down. (Here it should be noted that our calculation is based on a nonperturbative approach inspired by χPT. Since this is not a rigorous theory, keeping four digits of precision here is not very meaningful. Nevertheless, for convenience of discussion below, we shall still keep the four digits of precision in our results.)

The baryon number susceptibility $\chi(\mu_b)$ is defined as

$$\chi(\mu_b) = \frac{\partial}{\partial \mu_b} n(\mu_b), \tag{31}$$

which comes into existence when the temperature of the system stays the same. The results of $\chi(\mu_b)$ is shown in Fig. 8.

In Fig. 8, a peak around 1152 MeV can be clearly seen. Here it is interesting to note that the chemical potential at which the rate for the increase of $n(\mu)$ is slowed down coincides with the one at which the peak in the curve for baryon number susceptibility appears.

Once $n(\mu_b)$ is obtained, one can calculate the pressure $P(\mu)$ of the nuclear matter. The relation of $n(\mu_b)$ and $P(\mu)$ is given

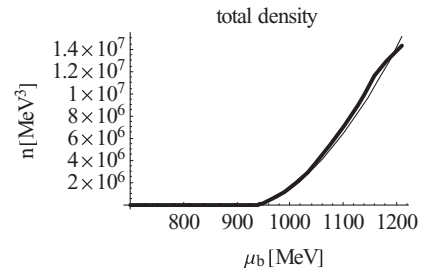

FIG. 7. Total contributions to the baryon number density from both poles and the branch cut, where thick black line is our result from χPT and the gray line is the free-nucleon approximation result.

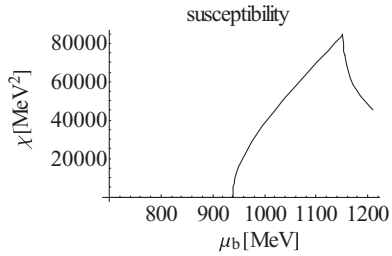

FIG. 8. Curve for the susceptibility of cold dense nuclear matter.

by (for more details, please see Ref. [37])

$$n(\mu_b) = \frac{\partial P(\mu)}{\partial \mu}. \tag{32}$$

The $P(\mu)$-$n(\mu)$ curve for the equation of state of cold dense nuclear matter is shown in Fig. 9. Our results for baryon number density and equation of state are quite similar to those calculated from the free-nucleon approximation. This could explain why one can adopt the almost independent particle model in the study of nuclear matter. In the following, we will explore more details about the phase transition by studying the evolution of quark condensate with the chemical potential.

IV. EVOLUTION OF THE QUARK CONDENSATE

According to $\mathcal{L}_{\rm ext}^{\rm QCD}$ in Eq. (1), the quark condensate $\langle\bar{q}q\rangle$ can be calculated from the derivative of the generating functional with respect to the corresponding external fields, i.e.,

$$\langle\bar{q}q\rangle = \frac{\partial}{\partial s}\int D\bar{\psi}\,D\psi\,DU e^{i\int d^4x\mathcal{L}_{\rm eff}}|_{s=M_q}. \tag{33}$$

By analogy with the deduction in the last section, the result comes out to be

$$\frac{-i}{(2\pi)^4}\int d^4p\,{\rm Tr}\left(S\frac{\partial\Sigma}{\partial\hat{m}}\right), \tag{34}$$

where $\hat{m}$ denotes the mass of u and d quarks under flavor SU(2) symmetry. The integral is performed in the same way

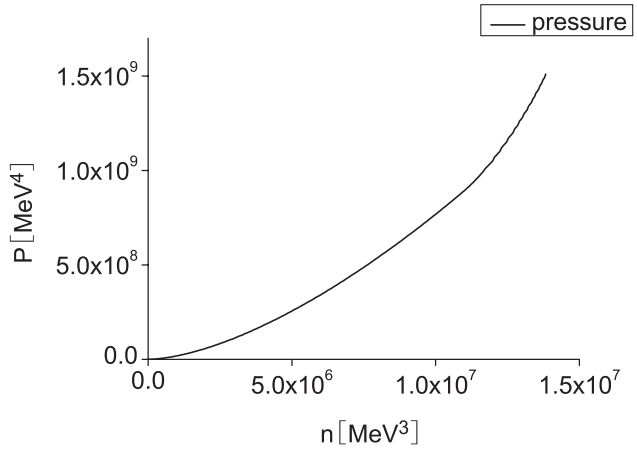

FIG. 9. Curve for the equation of state of cold dense nuclear matter.

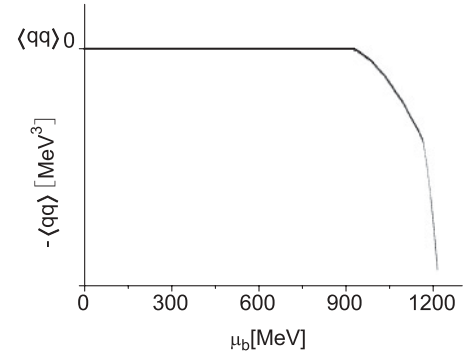

FIG. 10. Curve for the evolution of $-\langle\bar{q}q\rangle$.

as in Eq. (24), that is,

$$\int d^4p = \int d^3\vec{p}\left(\int_{-i\infty+\mu_b}^{i\infty+\mu_b} dp_0' - \int_{-i\infty}^{i\infty} dp_0'\right), \tag{35}$$

with the integration path shown in Fig. 2. However, since the vacuum quark condensate does not vanish, the result of the integral in Eq. (34) is not $\langle\bar{q}q\rangle$ but instead $\langle\bar{q}q\rangle - \langle\bar{q}q\rangle_0$, where $\langle\bar{q}q\rangle_0$ denotes the vacuum quark condensate.

The curve for the quark condensate $-\langle\bar{q}q\rangle$ is shown in Fig. 10. Since the value for $\langle\bar{q}q\rangle_0$ is not well determined yet, we just give a sketch map in Fig. 10.

When μ_b is below a certain value μ_c (where in our work $\mu_c = m_N$), $\langle\bar{q}q\rangle$ is kept unchanged from its vacuum value, i.e., $\langle\bar{q}q\rangle = \langle\bar{q}q\rangle_0$. For μ_b larger than μ_c, the absolute value of the quark condensate tends to decrease as μ_b increases. Recent work based on the rainbow approximation of the DSE method in Ref. [38] and the large Nc approximation in Ref. [21] give the same result. From Fig. 10, it can be seen that the decrease of quark condensate speeds up when μ_b is larger than 1152 MeV. When the chemical potential reaches 1200 MeV, the quark condensate vanishes. Therefore, our results are applicable only for μ_b smaller than 1200 MeV, where the spontaneous breaking of chiral symmetry still works. This range of application is in agreement with that in Refs. [31,32], where the upper range is obtained by fitting the experiments. In addition, our results are consistent with the HBχPT result at low temperature in Ref. [35], where the evolution of the quark condensate is investigated for different temperatures. In principle, our results are also consistent with those in Refs. [39,40].

V. CONCLUSION

The effective chiral Lagrangian for cold dense nuclear matter is constructed, in which the chemical potential corresponding to the baryon number density of QCD is introduced and included in the effective Lagrangian as an external field. Using a nonperturbative approach inspired by chiral perturbation theory, the evolution properties of cold dense nuclear matter with chemical potential are analyzed, and the evolution curve of the baryon number density, the baryon number susceptibility, and the curve of the equation of state are obtained. When the baryon chemical potential is around

1152 MeV, a peak is found in the curve of baryon number susceptibility. These results for baryon number density and equation of state are quite similar to those calculated from the free-nucleon approximation. This could explain why one can adopt the almost independent particle model in the study of nuclear matter. The evolution curve for quark condensate indicates that the quark condensate is kept unchanged from its vacuum value when $\mu_b < m_N$, and it decreases as μ_b increases when $\mu_b > m_N$. This result is also consistent with that in Refs. [21,38] obtained using the DSE method. When the chemical potential goes beyond 1152 MeV, the quark condensate decreases rapidly to zero. To understand the full phase transition properties of cold dense strongly interacting matter, one needs to investigate the region above the phase transition point using a method, such as the DSE method in Refs. [41–43], which is applicable to cold dense quark matter. As is indicated in Sec. III, the contribution of binding energy and the quench effect are not included in our calculation. To improve our results, we need to calculate self-energy to order of $O(p^6)$.

ACKNOWLEDGMENTS

We have benefited greatly from discussions with L. Chang, Y. X. Liu, and C. D. Roberts. This work is supported in part by the Key Research Plan of Theoretical Physics and Cross Science of China (Grant No. 90503011), the National Natural Science Foundation of China (Grant No. 10575050), and the Research Fund for the Doctoral Program of Higher Education (Grant No. 20060284020).

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