Higher-order QED calculation of ultrarelativistic heavy-ion production of $\mu^+\mu^-$ pairs

A. J. Baltz

Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA

A higher-order QED calculation of the ultraperipheral heavy-ion cross section for $\mu^+\mu^-$ pair production at the Relativistic Heavy Ion Collider and the Large Hadron Collider is carried out. The so-called Coulomb corrections lead to an even greater percentage decrease of $\mu^+\mu^-$ production from perturbation theory than the corresponding decrease for e^+e^- pair production. Unlike the e^+e^- case, the finite charge distribution of the ions (form factor) and the necessary subtraction of impact parameters with matter overlap are significant effects in calculation an observable ultraperipheral $\mu^+\mu^-$ total cross section.

I. INTRODUCTION

In recent years sufficient progress has been made in evaluating higher-order QED corrections to e^+e^- pair production in ultraperipheral heavy-ion collisions to allow a meaningful comparison with data. The comparison of calculated e^+e^- pair production rates [1] with STAR data [2] provides the first evidence of higher-order QED effects at the Relativistic Heavy Ion Collider (RHIC). The analogous higher-order corrections to $\mu^+\mu^-$ pair production are now of interest with the anticipated program of ultraperipheral heavy-ion collisions at the Large Hadron Collider (LHC) [3]. In particular it has been suggested by Kharlov and Sadovsky (see Ref. [3]) that because muon pairs are easy to detect (in the ALICE detector) and simple to calculate (in perturbation theory) they can be used as a luminosity monitor at the LHC. And a recent article by Hencken, Kuraev, and Serbo has presented approximate analytical calculations indicating that the higher-order Coulomb corrections to $\mu^+\mu^-$ pair production are small [4]. However, in this article I will present less approximate numerical calculations showing that higher-order QED corrections to the $\mu^+\mu^-$ rates are not small (and thus would require inclusion, e.g., in calculations to obtain a meaningful luminosity measurement). I will further argue that an additional reduction in $\mu^+\mu^-$ rates, relatively insignificant in the analogous e^+e^- process, arises from unitarity corrections necessitated by the exclusion of ion trajectories with nuclear overlap.

In Sec. II I discuss the method used here to calculate higher-order QED lepton pair production with heavy ions. Section III considers the differences between rates for e^+e^- and $\mu^+\mu^-$ pair production in the usual impact-parameter-independent calculation: how they scale and the relatively larger effect of the heavy-ion form factor for the $\mu^+\mu^-$ pairs. Section IV introduces the impact-parameter-dependent representation necessary for consideration of unitarity effects, especially the exclusion of ion overlap trajectories. A summary of the results for computed total $\mu^+\mu^-$ pair production cross sections at RHIC and LHC is presented in Sec. V.

II. HIGHER-ORDER QED: METHOD OF CALCULATION

The method employed in this article for higher-order $\mu^+\mu^-$ pair production is an extension of previous work on higher-order e^+e^- pairs. A broad review of issues involved in the e^+e^- pair calculations has been presented in a recent review [5], and a discussion of the factorization of the different electromagnetic processes and the applicability of the semiclassical description is found in Ref. [6]. Here I review the history relevant to the present calculations.

The possibility of accurate higher-order e^+e^- pair calculation originated with the realization that in an appropriate gauge [7], the electromagnetic potential of a relativistic heavy ion is to a very good approximation a delta function in the direction of motion of the heavy ion times the two-dimensional solution of Maxwell's equations in the transverse direction [8,9]. This led to the closed form solution of the time-dependent Dirac equation for lepton pair production [10–12]. However, this original solution needed to be corrected for the following reason. One apparent consequence of the original solution was that rates for pair production in the exact solution agreed with the corresponding perturbation theory result. It was subsequently pointed out by Ivanov, Schiller, and Serbo [13] that this heavy-ion conclusion was contrary to the well-known fact that photoproduction of e^+e^- pairs on a heavy target shows a negative (Coulomb) correction proportional to Z^2 that is well described by the Bethe-Maximon theory [14]. These authors went on to compute large Coulomb corrections to the pair total cross section by considering higher-order Feynman diagrams in a leading logarithm approximation. Lee and Milstein [15,16] came to essentially the same result for the Coulomb corrections. They pointed out that the original Dirac equation solution involved an integral over the transverse spatial coordinates that was not well regularized. Lee and Milstein constructed an appropriate regularized transverse integral in the low-transverse-momentum (k) approximation that could be solved analytically to obtain the Coulomb corrections. They also noted that replacing the original transverse potential $-2iZ\alpha \ln(\rho)$ with $2iZ\alpha K_0(\rho\omega/\gamma)$ gives a properly regularized expression for the original transverse integral

$$F(\mathbf{k}) = 2\pi \int d\rho\rho J_0(k\rho)\{\exp[2iZ\alpha K_0(\rho\omega/\gamma)] - 1\}, \quad (1)$$

that goes over into the correct lowest-order expression

$$F_0(\mathbf{k}) = \frac{4i\pi Z\alpha}{k^2 + \omega^2/\gamma^2}. \quad (2)$$

in the perturbative limit. The modified Bessel function $K_0(\rho\omega/\gamma) = -\ln(\rho)$ plus constants for small ρ and cuts off exponentially at $\rho \sim \gamma/\omega$, where γ is the relativistic boost of the ion producing the photon and ω is the energy of the photon. I previously carried out numerical calculations using these expressions and obtained results identical to those of Lee and Milstein in their small k limit [17]. In my previous cross-section calculations [18,19] and in what follows, Eq. (1) is used for the higher-order calculations and Eq. (2) for lowest order.

For impact-parameter-dependent cross sections the calculations presented here make use of the methods of calculating e^+e^- pair probabilities previously described [19]. The impact-parameter- (b) dependent amplitude presents a particular numerical challenge because it involves a rapidly oscillating phase $\exp(i\mathbf{k}\cdot\mathbf{b})$ in the integral over the transverse momentum $\mathbf{k}$ transferred from the ion to the lepton pair. The usual method of evaluating the perturbative impact-parameter-dependent probability is to first square the amplitude and then integrate over the sum and difference of $\mathbf{k}$ and $\mathbf{k}'$. Here I have integrated before squaring, and I deal with the rapid oscillations with the piecewise analytical integration method previously described [19]. In that previous b-dependent calculation of the total cross section for e^+e^- production, half of the contribution comes from $b > 5000$ fm and contributions up to $b = 10^6$ fm are considered. Due to the large values of b contributing, that calculation was somewhat crude. However, integration over b reproduced the known cross sections calculated with the b-independent method or calculated from the very accurate analytical Racah formula [20] to about 3%. It can also be noted that the computed perturbative b-dependent probabilities in that article were in relatively good agreement with calculations in the literature [21] available for $b < 7000$ fm.

III. SCALING OF $\mu^+\mu^-$ WITH e^+e^- CROSS SECTIONS

Let us begin by reviewing the scaling of $\mu^+\mu^-$ cross sections from the corresponding e^+e^- cross sections. For point charge heavy ions (no form factor) if length is expressed in terms of $1/m_l$ and energy in terms of m_l then the total lepton pair cross section $\sigma(\mu^+\mu^-)$ is identical to $\sigma(e^+e^-)$.

A form factor $g(k)$ may be defined that modifies the expressions for $F(k)$ in Eqs. (1) and (2). If one assumes a simple form factor

$$g(k) = \frac{1}{1 + k^2/\Lambda^2} \qquad (3)$$

where for Au or Pb

$$\Lambda \simeq 80 \text{ MeV} = 160\, m_e = 0.75\, m_\mu, \qquad (4)$$

then Eq. (2) for the perturbative limit becomes

$$F_0^f(\mathbf{k}) = \frac{4i\pi Z\alpha}{(k^2 + \omega^2/\gamma^2)(1 + k^2/\Lambda^2)}. \qquad (5)$$

k is cut off at the low end when $k^2 \ll (\omega/\gamma)^2$. In the perturbative case for e^+e^- pairs it has been shown that the effect of the form factor seems to be present only where the impact parameter is of the same size as the nuclear radius [22].

However, the situation is different for μ pairs. At the high end the form factor cuts off when $k^2 \gg \Lambda^2$. The form factor contributes if Λ^2 is comparable to or less than $(\omega/\gamma)^2$, the cutoff of k without the form factor. Assume that at this high end cutoff without the form factor $k \simeq 100\omega/\gamma \simeq \omega$ for RHIC. Clearly for μ pair production the sum of the ωs for the two virtual photons must be greater than twice the mass of the muon. Thus for even the lowest-energy μ pairs (corresponding to large impact parameters) at least one $\omega > m_\mu$ and the form factor is important. However, the form factor is relatively insignificant for the total $\sigma(e^+e^-)$ and contributes only at electron energies some two orders of magnitude above the electron mass, comparable to the value of Λ. Without a form factor

$$\frac{\sigma(\mu^+\mu^-)}{\sigma(e^+e^-)} = \left(\frac{m_e}{m_\mu}\right)^2 = 2.34 \times 10^{-5}. \qquad (6)$$

But with a form factor the perturbation theory result calculations give

$$\frac{\sigma(\mu^+\mu^-)}{\sigma(e^+e^-)} = 0.61 \times 10^{-5} = 0.26 \times \left(\frac{m_e}{m_\mu}\right)^2 \qquad (7)$$

for RHIC and

$$\frac{\sigma(\mu^+\mu^-)}{\sigma(e^+e^-)} = 1.16 \times 10^{-5} = 0.50 \times \left(\frac{m_e}{m_\mu}\right)^2 \qquad (8)$$

for LHC.

To include a form factor in the eikonalized expression with Coulomb corrections Eq. (1) then the most obvious prescription is to apply the form factor to the transverse potential:

$$F^f(\mathbf{k}) = 2\pi \int d\rho\rho\, J_0(k\rho)\, \{\exp[2iZ\alpha g(k)K_0(\rho\omega/\gamma)] - 1\}. \qquad (9)$$

This expression obviously goes into the correct perturbative limit Eq. (5). A simpler expression is to take the form factor only to first order but the Coulomb corrections to higher order

$$F^{f0}(\mathbf{k}) = 2\pi g(k) \int d\rho\rho\, J_0(k\rho)\, \{\exp[2iZ\alpha K_0(\rho\omega/\gamma)] - 1\}. \qquad (10)$$

Again, this expression obviously goes to the correct perturbative limit Eq. (5). This is the expression that will be used in this article. A discussion of the validity of this approximation is given in Appendix A.

For simplicity in calculation and simplicity in comparing with Ref. [4], the form factor $g(k)$ Eq. (3) has also neglected any dependence on longitudinal momentum. Including a longitudinal-momentum dependence would make a small reduction in cross section values, about 5% for RHIC and 1% for LHC, as discussed in Appendix B.

I have previously calculated [18] that there is a 17% reduction at RHIC and a 11% reduction at LHC in the exact total $\sigma(e^+e^-)$ from the perturbation theory result. For the $\sigma(\mu^+\mu^-)$ here the corresponding reduction from perturbation theory is even greater, 22% at RHIC and 14% LHC. The present perturbative $\sigma(\mu^+\mu^-)$ calculations are in fairly good

agreement with the calculations of Hencken, Kuraev, and Serbo [4], but the present exact cross-section calculations are in disagreement with their argument that Coulomb corrections are relatively insignificant for μ pairs.

So far the calculations presented have been performed in the impact-parameter-independent representation. There is an additional reduction that comes into play for an observable $\sigma(\mu^+\mu^-)$ that arises from unitarity considerations, and one must make use of the impact-parameter representation discussed in the following section.

IV. IMPACT PARAMETER AND UNITARITY

The perturbative (Born) cross section and corresponding cross sections with higher-order Coulomb corrections discussed in the previous section correspond to an inclusive cross section, constructed from a probability corresponding to the number operator for a given process. If one considers an exclusive cross section, e.g., exciting a μ pair and nothing else in a heavy-ion reaction, then one must consider unitarity corrections for competing processes in an impact-parameter representation as will be seen below. For μ pair production the main unitarity corrections arise in principle from competing e^+e^- pair production, Coulomb dissociation of the heavy ions, and nuclear processes at ion-ion overlap.

Hencken, Kuraev, and Serbo have observed that while unitarity corrections are small for e^+e^- cross sections they are large for corresponding $\mu^+\mu^-$ pair production [4]. The perturbative Born cross section for e^+e^- production, corresponding to an inclusive cross section, is little increased from the exclusive cross section. The perturbative Born cross section for $\mu^+\mu^-$ production also corresponds to the inclusive cross section, but the exclusive cross section is significantly reduced by unitarity corrections due to the simultaneous production of e^+e^- pairs along with the $\mu^+\mu^-$ pairs. Lee and Milstein have recently developed a quasianalytical procedure to include the higher-order Coulomb corrections in calculating the impact parameter dependence the e^+e^- pair production [23]. Based on their procedure Jenschura, Hencken, and Serbo have updated the consideration of the e^+e^- pair unitarity corrections [24].

In practice some unitarity corrections are relevant to what is actually measured and some are not. While it is an enlightening theoretical exercise to consider e^+e^- pair unitarity corrections to μ pair rates, in practice the dominant contributions of soft e^+e^- pairs are of an energy scale orders of magnitude too small to be observed in an experiment designed to observe μ pairs. However, when calculating μ pair rates in an impact-parameter representation, a correction must be made to exclude the lowest impact parameters of ion-ion overlap, where the dominant processes are nuclear.

If one assumes independence of the various heavy-ion reaction processes, then the probability of a reaction leading to a final state differing from the incoming channel is given by the usual Poisson distribution. If $P(b)$ is the sum of the probabilities for producing an excited state by a heavy-ion reaction at a given impact parameter b
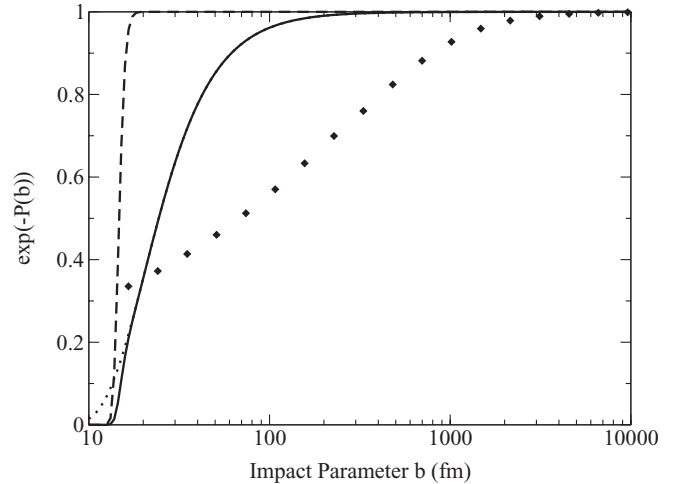
$$P(b) = \Sigma_i P_i(b), \tag{11}$$

FIG. 1. Unitarity reduction factor for relevant processes in the calculation of exclusive μ pair production at RHIC: dashed line = nuclear collisions; dotted line = Coulomb excitation of one of the heavy ions; solid line = nuclear collision or Coulomb excitation; diamonds = e^+e^- pair production.

where each $P_i(b)$ is the inclusive probability of a specific final state i, then the exclusive probability for a given final state $P_i^u(b)$ is

$$P_i^u(b) = P_i(b) \exp[-P(b)], \tag{12}$$

and the probability of remaining in the initial state is

$$P_o^u(b) = 1 - \exp[-P(b)]. \tag{13}$$

Figure 1 shows the unitarity reduction factor $\exp[-P(b)]$ in Eq. (12) evaluated for the probabilities of various processes for the case of Au+Au at RHIC. In agreement with the previously discussed above work in the literature [4,23,24] the unitarity effect of e^+e^- pair production (diamonds) is significant for low and intermediate impact parameters. Following the methods of Refs. [25,26] I have also calculated the unitarity reduction factor $\exp[-P(b)]$ for Coulomb dissociation and nuclear dissociation.

It is instructive to compare the impact-parameter dependence of the contribution to μ pair production and e^+e^- pair production at RHIC. Figure 2 shows the distribution for for μ pairs and Fig. 3 for e^+e^- pairs. In both figures dashes correspond to perturbation theory and the solid line the higher-order calculation. The shift in scale mentioned in the previous section is evident. Comparing the region of e^+e^- reduction shown in Fig. 1 with the regions of dominant cross-section contribution for μ pairs (Fig. 2) and e^+e^- pairs (Fig. 3) makes evident the reasoning of Hencken, Kuraev, and Serbo [4] that unitarity corrections are small for e^+e^- cross sections and large for $\mu^+\mu^-$. Also clearly the region of nuclear collisions (dashed line in Fig. 1) would provide no reduction of the e^+e^- pair cross section (Fig. 2) but would slightly reduce the μ pair cross section (Fig. 1). It is obvious that even with a momentum-dependent form factor there is significant contribution to the μ pair cross section here in the region of ion-ion overlap. This contribution must be eliminated for
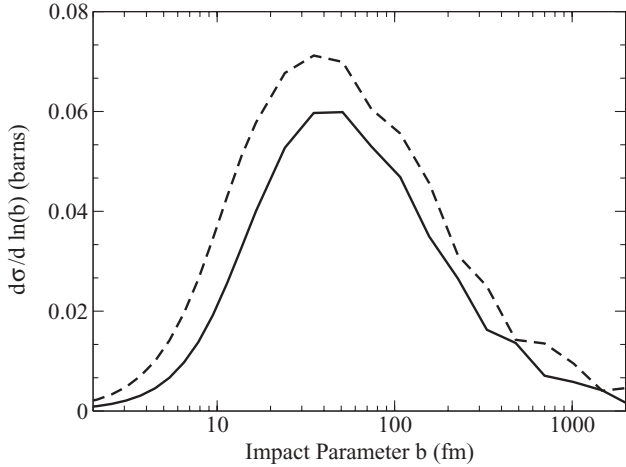
FIG. 2. Impact parameter dependence of contribution to total cross section for μ pair production at RHIC: dashed line = perturbation theory; solid line = higher-order calculation.

ultraperipheral collisions and leads into the discussion in the rest of this section.

As noted above, what states are considered in the unitarity consideration can be determined or defined by the energy scale of the detected particles. For example, when considering $\mu^+\mu^-$ pair production, one might not consider the dominant soft e^+e^- pairs as part of the excited spectrum for purposes of unitarity normalization. This might be a reasonable definition corresponding to the experimental detection conditions. However, to construct a calculated cross section corresponding to the observed pair production events without any other final-state particles arising from ion-ion overlap, one should include only ultraperipheral impact parameters.

In terms of a **b**- (impact parameter) dependent amplitude $M(\mathbf{k}, \mathbf{b}) \exp(i\mathbf{k} \cdot \mathbf{b})$ an appropriate nonunitarized probability
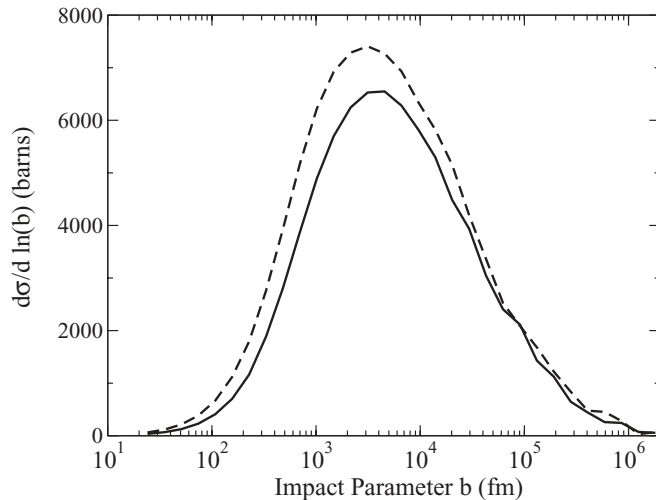

FIG. 3. Impact-parameter dependence of contribution to total cross section for e^+e^- pair production at RHIC: dashed line = perturbation theory; solid line = higher-order calculation.

can be written

$$P_i(b) = \left| \int d^2k M_i(\mathbf{k}, \mathbf{b}) \exp(i\mathbf{k} \cdot \mathbf{b}) \right|^2. \quad (14)$$

Let $P_i(b)$ be a nonunitarized probability for exciting a $\mu^+\mu^-$ pair and $P_j(b)$ the corresponding probability for a nuclear reaction. Then define a partially exclusive cross section as one that excludes nuclear interaction only

$$\sigma_i = \int_0^\infty d^2b P_i^u(b) = \int_0^\infty d^2b P_i(b) \exp[-P_j(b)]. \quad (15)$$

Because the nuclear interactions occur only below some $b_0 = R_1 + R_2$, it is convenient to express this cross section as a difference

$$\sigma_i = \int_0^\infty d^2b P_i(b) + \int_0^{b<b_0} d^2b P_i(b)\{\exp[P_j(b) - 1]\}. \quad (16)$$

The first term can be evaluated as was done before without a specific impact parameter representation [18]. The second term can then be evaluated using the method of Ref. [19]: because **b** is limited to the lowest impact parameters the $\exp(i\mathbf{k} \cdot \mathbf{b})$ factor is still numerically tractable even though **k** scales up by a factor of m_μ/m_e as compared to the e^+e^- case. A similar trick to subtract the nuclear interaction at small impact parameters has previously been used [27].

V. NUMERICAL RESULTS

Table I summarizes the total cross section results for $\mu^+\mu^-$ pair production at RHIC. In the first row the **b**-independent perturbative cross section for $\sigma(\mu^+\mu^-)$ is 211 mb and the cross section with higher-order effects is 164 mb. As previously noted in Sec. III, this reduction of 22% from perturbation theory is greater than the 17% reduction in the exact $\sigma(e^+e^-)$ from the perturbation theory seen in Ref. [18]. The second row shows the results from integrating the **b**-dependent computation of the cross section shown in Fig. 2. The numbers in parentheses correspond to the negative of the second right-hand term of Eq. (16). A rough check can be done by comparing the results of the **b**-dependent and **b**-independent calculations of the total $\mu^+\mu^-$ production cross section. Even though the **b**-dependent becomes more inaccurate beyond the low impact parameters it still reproduces the **b**-independent results to about 10%. The lowest impact parameters have the greatest accuracy and we calculate the higher-order cross section from the overlap impact parameters to be subtracted off as 20 mb. Thus the final computed best cross section (third

TABLE I. RHIC: Au + Au, $\gamma = 100$, $\mu^+\mu^-$ total cross section.

Results (mb)	Perturb.	Exact
b-independent formulation	211	164
b integration (ion overlap)	232 (36)	181 (20)
b-independent minus ion overlap	175	144
Hencken *et al.* ($\gamma = 108$)	230	230

TABLE II. LHC: Pb + Pb, $\gamma = 2760$, $\mu^+\mu^-$ total cross section.

Results (b)	Perturb.	Exact
b-independent formulation	2.43	2.09
b integration ion overlap	0.13	0.06
b-independent minus ion overlap	2.29	2.03
Hencken *et al.* ($\gamma = 3000$)	2.60	2.60

row) is 144 mb, a 32% reduction from the perturbation theory calculation.

Table II shows calculations of colliding Pb + Pb ions at the LHC. The perturbative $\mu^+\mu^-$ production cross section shown in the first row is 2.43 b and higher-order effects reduced it by 14% to 2.09 b. Again this reduction is greater than the 11% reduction in the exact $\sigma(e^+e^-)$ from the perturbation theory result [18]. Due to the higher values of transverse momentum transferred from the virtual photons in this LHC case it was not feasible to compute the **b**-dependent cross-section contributions throughout the entire impact parameter range. However, in the region of ion overlap the impact parameter was small enough that the rapidly oscillating phase $\exp(i\mathbf{k} \cdot \mathbf{b})$ in the integral over the transverse momentum **k** transferred from the ion to the lepton pair remained tractable and Eq. (16) could be used. The additional reduction from exclusion of overlap impact parameters was 0.06 b for a best value of 2.03 b, an overall 16% reduction from perturbation theory.

The present perturbative $\sigma(\mu^+\mu^-)$ calculations are in fair agreement with the calculations of Hencken, Kuraev, and Serbo [4], but the present exact cross-section calculations are in disagreement with their argument that Coulomb corrections are relatively insignificant for μ pairs. In this work I have shown that unlike the case for e^+e^- pair production, the finite size of the colliding nuclei provides an important modification for both the perturbative and higher-order calculated total cross sections $\sigma(\mu^+\mu^-)$. The form factor reduces the higher-order calculation by an even greater percentage than it does for perturbation theory. Furthermore, making the necessary elimination of interactions where the ions overlap further reduces the higher-order cross section from perturbation theory.

APPENDIX A: HIGHER-ORDER FORM FACTOR EFFECTS

As noted in Sec. III, to include a form factor in the eikonalized expression for the transverse integral with Coulomb corrections, the most obvious prescription is to apply the form factor to the transverse potential, leading to Eq. (9):

$$F^f(\mathbf{k}) = 2\pi \int d\rho\rho J_0(k\rho)\{\exp[2iZ\alpha g(k)K_0(\rho\omega/\gamma)] - 1\}.$$
$$\text{(A1)}$$

But including the form factor in the transverse potential is equivalent to letting the coupling constant $Z\alpha$ run as a function of k, analogous to the situation in QCD. To do this makes the numerical integration of Eq. (1) more complicated and has not been done in this article. However, by a relatively simple modification of the b independent expression for the higher-order cross section one can put an upper limit on the modification to higher-order effect of using the more proper Eq. (9) rather than the expression Eq. (10) used throughout this this article.

The numerical integration of Eq. (1) is most conveniently carried out after a change of variables to $\xi = k\rho$ and

$$F(\mathbf{k}) = \frac{2\pi}{k^2} \int d\xi\xi J_0(\xi)\{\exp[2iZ\alpha K_0(\xi\omega/\gamma k)] - 1\}. \quad \text{(A2)}$$

This integral is carried out for various values of the parameter $k\gamma/\omega$. F actually has a two-dimensional parametrization in $k\gamma/\omega$ and k, but the $1/k^2$ dependence trivially factors out of the integral. Likewise the additional k dependence in expression Eq. (10) used in this article factors out trivially. However, the additional k dependence in the more exact expression Eq. (9) does not factor out trivially, leading to the additional complication.

To put a limit on the error incurred by using the expression Eq. (9) rather than Eq. (10), I begin by recalling that the organization of the b-independent computer code used in Ref. [18] involves a difference

$$|\Delta F(\mathbf{k})^2| = |F(\mathbf{k})|^2 - |F_0(\mathbf{k})|^2 \quad \text{(A3)}$$

between the squared value of the higher-order expression Eq. (1) and that of the perturbative expression, Eq. (2). Because the effect of the form factor $g(k)$ is the same as reducing the value of Z as a function of k at large k (like running coupling in QCD) then evaluation of Eq. (A2) for various values of Z may be used as a proxy for the higher-order dependence on $g(k)$. That is, for a given Z if the form factor is reduced from unity by some percentage then it is equivalent to no form factor and just reducing Z by the same percentage. Numerical calculations of $|\Delta F(\mathbf{k})^2|$ show that it scales as Z^4 for low values of Z and a little less than Z^4 as Z is increased. This scaling is consistent with the integral over (A3)

$$G = \int \frac{d^2k}{(2\pi)^2} k^2 [|F(\mathbf{k})|^2 - |F_0(\mathbf{k})|^2] \quad \text{(A4)}$$

in Lee and Milstein's analysis of higher-order Coulomb corrections [15,16], which takes the analytical form

$$G = -8\pi(Z\alpha)^2[\text{Re}\psi(1 + iZ\alpha) + \gamma_{\text{Euler}}], \quad \text{(A5)}$$

where $\psi(1 + iZ\alpha)$ is the digamma function and γ_{Euler} is Euler's constant. This expression may be alternatively expressed as

$$G = -8\pi(Z\alpha)^2 f(Z\alpha), \quad \text{(A6)}$$

where $f(Z\alpha)$ is the same function that was presented by Bethe, Maximon, and Davies [14] for Coulomb corrections to e^+e^- photoproduction on heavy nuclei and takes the form

$$f(Z\alpha) = (Z\alpha)^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + (Z\alpha)^2)}. \quad \text{(A7)}$$

G obviously scales as Z^4 for low values of Z and a little less than Z^4 as Z is increased.

The expression Eq. (A3) modified with the lowest-order implementation of the form factor utilized in this article takes the form

$$\begin{aligned}|\Delta F^{f0}(\mathbf{k})^2| &= |F^{f0}(\mathbf{k})|^2 - \left|F_0^{f0}(\mathbf{k})\right|^2 \\ &= g(k)^2(|F(\mathbf{k})|^2 - |F_0(\mathbf{k})|^2).\end{aligned} \quad \text{(A8)}$$

If one consider the analogous expression with the form factor to higher order, then replacing the $g(k)^2$ dependence of Eq. (A7) with $g(k)^4$ suggested by the Z^4 scaling seen in the difference without a form factor,

$$\begin{aligned}|\Delta F^{f}(\mathbf{k})^2| &= |F^{f}(\mathbf{k})|^2 - \left|F_0^{f}(\mathbf{k})\right|^2 \\ &= g(k)^4(|F(\mathbf{k})|^2 - |F_0(\mathbf{k})|^2),\end{aligned} \quad \text{(A9)}$$

should slightly overstate the higher-order effect of the form factor in Coulomb corrections.

Recalculation of the exact b-independent RHIC Au + Au cross section of Table I with the $g(k)^4$ form factor scaling of Eq. (A9) gives 171 mb, a 19% reduction from perturbation theory in comparison with the 164-mb 22% reduction using the more approximate $g(k)^2$ of Eq. (A8). Likewise for Pb + Pb at LHC the exact calculation with $g(k)^4$ gives 2.12 b, a 13% reduction from perturbation theory in comparison with the 2.09 b 14% reduction with $g(k)^2$.

Both recalculations make only a small change from the lowest-order treatment of the form factor in this article. And because it is far from trivial to implement the more proper higher-order treatment of the form factor of Eq. (9), especially in b-dependent calculations, I have not done so in this article. It seems that once the k-dependent cutoff of the form factor is put in, sharpening the cutoff by an additional squaring has a relatively small effect. Even with the higher-order $g(k)^4$ scaling calculations, the reduction from perturbation theory in $\sigma(\mu^+\mu^-)$ are still larger than the reductions in the analogous exact $\sigma(e^+e^-)$ without a form factor from perturbation theory.

APPENDIX B: LONGITUDINAL FORM FACTOR EFFECTS

One might include longitudinal form factor effects by modifying Eq. (3) to make $g(k)$ a function of $k^2 + \omega^2/\gamma^2$ rather than simply a function of k^2:

$$g(k) = \frac{1}{1 + (k^2 + \omega^2/\gamma^2)/\Lambda^2}. \quad \text{(B1)}$$

I have recalculated b independent cross sections using Eqs. (5), (10), and (B1) in place of (3), and I find a 5% reduction for both the perturbative and higher-order computations for RHIC but only a corresponding 1% reduction for LHC. The 5% reduction for RHIC is equivalent to calculations without a longitudinal factor, but with the value of Λ reduced from 80 to 75.5 MeV.

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