Angular momentum effects and barrier modification in sub-barrier fusion reactions using the proximity potential in the Wong formula

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Using the capture cross-section data from ^{48}Ca + ^{238}U, ^{48}Ca + ^{244}Pu, and ^{48}Ca + ^{248}Cm reactions in the superheavy mass region, and fusion-evaporation cross sections from ^{58}Ni + ^{58}Ni, ^{64}Ni + ^{64}Ni, and ^{64}Ni + ^{100}Mo reactions known for fusion hindrance phenomenon in coupled-channels calculations, the Wong formula is assessed for its angular momentum and barrier-modification effects at sub-barrier energies. The simple, $\ell = 0$ barrier-based Wong formula is shown to ignore the modifications of the barrier due to its inbuilt ℓ dependence via ℓ summation, which is found to be adequate enough to explain the capture cross sections for all the three above-mentioned ^{48}Ca-based reactions forming superheavy systems. For the capture (equivalently, quasifission) reactions, the complete ℓ-summed Wong formula is shown to be the same as the dynamical cluster-decay model expression, of one of us (R.K.G.) and collaborators, with the condition of fragment preformation probability $P_0^\ell = 1$ for all the angular momentum ℓ values. In the case of fusion-evaporation cross sections, however, a further modification of barriers is required for below-barrier energies, affected in terms of either the barrier "lowering" or barrier "narrowing" via the curvature constant. Calculations are made for use of nuclear proximity potential, with effects of multipole deformations included up to hexadecapole, and orientation degrees of freedom integrated for both the coplanar and noncoplanar configurations.

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I. INTRODUCTION

The Wong formula [1] has been used quite extensively for a variety of fusion reactions, from a dominantly fusion-evaporation cross section [2–4] to fission cross section [5,6] and the capture cross section [7]. The unexpected behavior of fusion cross sections, in particular the fusion-evaporation cross sections at energies far below the Coulomb barrier, has challenged the theoretical models to explain the so-called fusion hindrance phenomenon in coupled-channels calculations (ccc) [8]. Misicu and Greiner [7] were the first to have shown that the M3Y potential, *with an additional repulsive core*, describes the capture cross sections, for at least the ^{48}Ca + ^{238}U reaction, using equally well either the Wong formula or the ccc with deformations included up to hexadecapole ($\beta_2 > 0$, $\beta_4 > 0$) and the orientation degree of freedom integrated over all allowed values in the same plane. The same prescription, for both the Wong formula and ccc, however, failed for other two reactions, ^{48}Ca + ^{244}Pu and ^{48}Ca + ^{248}Cm, in the superheavy mass region. In a further application of the M3Y + repulsive-core potential in ccc, Misicu and Esbensen [4] succeeded in describing the three well-known ^{58}Ni + ^{58}Ni, ^{64}Ni + ^{64}Ni, and ^{64}Ni + ^{100}Mo reactions whose measured fusion-evaporation cross sections show clear signatures of hindrance phenomenon in their true ccc. The repulsive core is shown [4] to modify the shape of the inner part of the potential in terms of a thicker barrier and shallower pocket. Thus, the only acceptable explanation for hindrance phenomenon in ccc so far is the "modified shape of potential inside the barrier" at sub-barrier energies, also supported by the dynamical cluster-decay model (DCM) of preformed clusters [9,10] in a compound system where the property of "lowering of barriers" at sub-barrier energies arises in a simple way [11] in its fitting of the neck-length parameter.

In this article, we re-examine Wong formula for different types of dominant cross sections for fusion reactions, in particular the approximations introduced by Wong to obtain a simplified form of expression for cross sections in terms of s-wave ($\ell = 0$) barrier characteristics. We find that, at least for ^{48}Ca-based reactions forming suprheavy systems, a modified interaction potential (such as through an additional repulsive core in M3Y [7] or through the neck-length parameter in the DCM [9–11]) is required simply because of Wong's approximations for the centrifugal potential. In other words, Wong's specific ℓ-summation procedure, leading to the use of only the $\ell = 0$ barrier, seems to exclude the modifications entering the potential due to its ℓ dependence. In fact, ignoring Wong's specific ℓ-summation procedure leads us to an alternative model, the DCM with its fragment preformation factor $P_0^\ell = 1$ (see below) where the ℓ summation is carried out explicitly. However, the same method of explicit ℓ summation in the Wong formula, when applied to reactions known for hindrance effects in ccc (^{58}Ni + ^{58}Ni, ^{64}Ni + ^{64}Ni, and ^{64}Ni + ^{100}Mo), still require *further* "barrier modifications" at sub-barrier energies. Note that the dominant cross sections involved in these later reactions are the fusion-evaporation cross sections where the condition of the incoming nuclei keeping their identity (satisfied for capture or quasifission reactions) does not apply. Also, in the DCM the condition of $P_0^\ell = 1$ is satisfied only for capture (equivalently, quasifission) data [9], which means to suggest that the "barrier modification" for fusion-evaporation cross sections in case of Wong formula is required for compensating the effects of $P_0^\ell \neq 1$ in the DCM (see below).

The article is organized as follows. Section II gives the details of the Wong formula, and its possible extention to complete ℓ-summation effects. The relation between the Wong

formula and the DCM is also worked out in this section. The nuclear proximity potential due to Blocki *et al.* [12] is used, with effects of deformations included (up to β_4) and orientation degrees of freedom integrated for both the coplanar [13] and noncoplanar [14] configurations of nuclei. The calculations are presented in Sec. III, and our discussion of results and conclusions are given in Sec. IV.

II. THE THEORY

A. Wong formula and its extension to include explicit summation of angular momentum effects

1. Wong formula

According to Wong [1], the fusion cross section, in terms of angular-momentum ℓ partial waves, for two deformed and oriented nuclei (with orientation angles θ_i), lying in two different planes (with azimuthal angle Φ between the planes), and colliding with center-of-mass (c.m.) energy $E_{\rm c.m.}$, is

$$\sigma(E_{\rm c.m.},\theta_i,\Phi)=\frac{\pi}{k^2}\sum_{\ell=0}^{\ell_{\max}}(2\ell+1)P_\ell(E_{\rm c.m.},\theta_i,\Phi), \quad (1)$$

with $k=\sqrt{\frac{2\mu E_{\rm c.m.}}{\hbar^2}}$ and μ as the reduced mass. Here, P_ℓ is the transmission coefficient for each ℓ that describes the penetration of barrier $V_\ell(R,E_{\rm c.m.},\theta_i,\Phi)$ and $\ell_{\max}$ is the maximum angular momentum, defined later. Using the Hill-Wheeler [15] approximation of assimilating the shape of the interaction barrier $V_\ell(R,E_{\rm c.m.},\theta_i,\Phi)$ through an inverted harmonic oscillator [$V_\ell(R,E_{\rm c.m.},\theta_i,\Phi)=V_B^\ell(E_{\rm c.m.},\theta_i,\Phi)-\frac{1}{2}\mu\omega^2(R-R_B^\ell)^2$], the penetrability P_ℓ, in terms of its barrier height $V_B^\ell(E_{\rm c.m.},\theta_i,\Phi)$ and curvature $\hbar\omega_\ell(E_{\rm c.m.},\theta_i,\Phi)$, is

$$P_\ell=\left[1+\exp\left(\frac{2\pi(V_B^\ell(E_{\rm c.m.},\theta_i,\Phi)-E_{\rm c.m.})}{\hbar\omega_\ell(E_{\rm c.m.},\theta_i,\Phi)}\right)\right]^{-1}, \quad (2)$$

with $\hbar\omega_\ell(E_{\rm c.m.},\theta_i,\Phi)$, evaluated at the barrier position $R=R_B^\ell$ corresponding to the maximum barrier height $V_B^\ell(E_{\rm c.m.},\theta_i,\Phi)$, given as

$$\hbar\omega_\ell(E_{\rm c.m.},\theta_i,\Phi)=\hbar\left[|d^2V_\ell(R)/dR^2|_{R=R_B^\ell}/\mu\right]^{1/2}, \quad (3)$$

and, the R_B^ℓ obtained from the condition

$$|dV_\ell(R)/dR|_{R=R_B^\ell}=0.$$

Instead of solving Eq. (1) explicitly, which requires the complete ℓ-dependent potentials $V_\ell(R,E_{\rm c.m.},\theta_i,\Phi)$, Wong [1] carried out the ℓ summation in Eq. (1) approximately under the conditions:

(i) $\hbar\omega_\ell\approx\hbar\omega_0$, and
(ii) $V_B^\ell\approx V_B^0+\frac{\hbar^2\ell(\ell+1)}{2\mu R_B^{0\,2}}$,

which means to assume $R_B^\ell\approx R_B^0$ also. In other words, both V_B^ℓ and $\hbar\omega_\ell$ are obtained in terms of its $\ell=0$ values, with V_B^0 given as the sum of nuclear proximity potential V_P and Coulomb potential V_C at $R=R_B^0$,

$$V_B^0=V_P(R=R_B^0,A_i,\beta_{\lambda i},E_{\rm c.m.},\theta_i,\Phi)+V_C(R=R_B^0,Z_i,\beta_{\lambda i},E_{\rm c.m.},\theta_i,\Phi), \quad (4)$$

where $\beta_{\lambda i}$, $\lambda=2,3,4$ are the static quadrupole, octupole, and hexadecapole deformations.

Using the above two approximations, and replacing the ℓ summation in Eq. (1) by an integral, gives on integration the Wong formula [1]

$$\sigma(E_{\rm c.m.},\theta_i,\Phi)=\frac{R_B^{0\,2}\hbar\omega_0}{2E_{\rm c.m.}}\ln\left[1+\exp\left(\frac{2\pi}{\hbar\omega_0}\left(E_{\rm c.m.}-V_B^0\right)\right)\right], \quad (5)$$

which on integration over the orientation angles θ_i and azimuthal angle Φ gives the fusion cross section

$$\sigma(E_{\rm c.m.})=\int_{\theta_i,\Phi=0}^{\pi/2}\sigma(E_{\rm c.m.},\theta_i,\Phi)\sin\theta_1 d\theta_1\sin\theta_2 d\theta_2 d\Phi. \quad (6)$$

Note that noncoplanar configurations ($\Phi\neq0$) are included here for the first time. Also, the $E_{\rm c.m.}$ dependence (equivalently, temperature T dependence) of interaction potential $V_\ell(R)$ is introduced in the Wong formula for the first time (see below), related to the incoming center-of-mass energy $E_{\rm c.m.}$ or the compound nucleus excitation energy E^* as

$$E^*=E_{\rm c.m.}+Q_{\rm in}=\frac{1}{a}AT^2-T \qquad (\text{T in MeV}), \quad (7)$$

with $a=9$ or 10, respectively, for intermediate mass or superheavy systems. $Q_{\rm in}$ is the entrance channel Q value. It is important to note that the characteristics of only the $\ell=0$ barrier play role in Wong formula (5).

2. Extension of the Wong formula to include the ℓ summation explicitly

For an explicit summation over ℓ in Eq. (1), the ℓ-dependent interaction potential $V_\ell(R)$, entering Eq. (2) via V_B^ℓ, R_B^ℓ, and $\hbar\omega_\ell$ (also calculated in Hill-Wheeler method of inverted harmonic oscillator for each ℓ value), is given by

$$V_\ell(R)=V_P(R,A_i,\beta_{\lambda i},T,\theta_i,\Phi)+V_C(R,Z_i,\beta_{\lambda i},T,\theta_i,\Phi)+\frac{\hbar^2\ell(\ell+1)}{2\mu R^2}, \quad (8)$$

with details of Coloumb interaction and nuclear proximity potential for deformed, oriented nuclei in the same plane (coplanar, $\Phi=0$) as well as in different planes (noncoplanar, $\Phi\neq0$), given in Ref. [16] and explicity for $\Phi=0$ and $\Phi\neq0$, respectively, in Ref. [13] and [14]. The ℓ summation in Eq. (1) is then carried out for the $\ell_{\max}$ determined empirically for a best fit to measured cross section. This procedure of explicit ℓ summation works very well for capture (equivalently, quasifission) reactions, such as ^{48}Ca + ^{238}U, ^{244}Pu, and ^{248}Cm, forming the superheavy systems.

The temperature effects in both the V_C and V_P are introduced via the radius vectors of two nuclei, as follows:

$$R_i(\alpha_i,T)=R_{0i}(T)\left[1+\sum_\lambda\beta_{\lambda i}Y_\lambda^{(0)}(\alpha_i)\right] \quad (9)$$

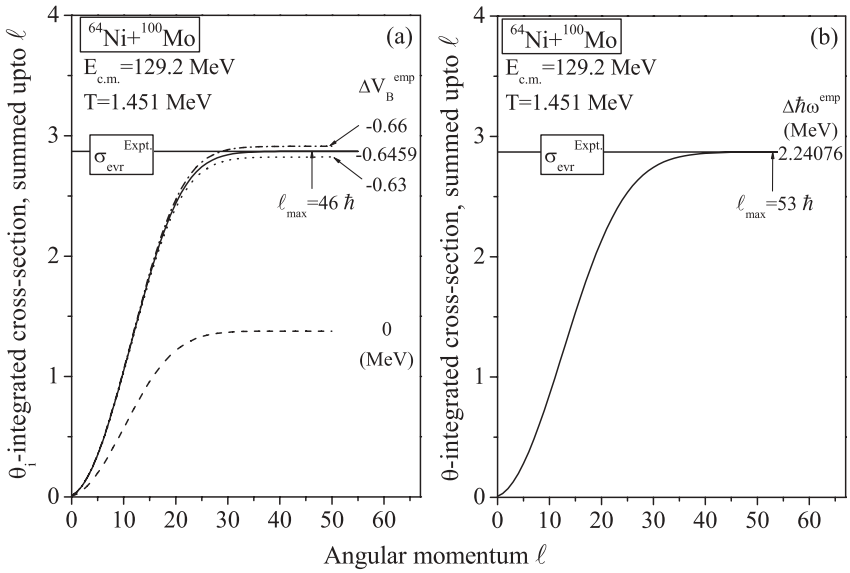

FIG. 1. Variation of the θ_i-integrated ($\Phi = 0$) cross section summed up to the angular momentum ℓ as a function of ℓ itself for different (a) $\Delta V_B^{\rm emp}$ values, and (b) $\Delta\hbar\omega^{\rm emp}$, for the ^{64}Ni + ^{100}Mo reaction at $E_{\rm c.m.} =$ 129.2 MeV, showing the minimum value of $\Delta V_B^{\rm emp}$ (or $\Delta\hbar\omega^{\rm emp}$) (solid line) required to fit the data at an $\ell_{\rm max}$ value (a saturation condition is reached for the best fitted $\Delta V_B^{\rm emp}$ or $\Delta\hbar\omega^{\rm emp}$). Panel (a) also shows the failure of the $\Delta V_B^{\rm emp} = 0$ value (dashed line) to reach the experimental cross section, and its uniqueness by its failure again at a slightly above (dot-dashed line) and slightly below (dotted line) the exact $\Delta V_B^{\rm emp}$ value fitting the data.

with the temperature dependence of R_{0i}, as in Ref. [17],

$$R_{0i}(T) = \left[1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}\right](1 + 0.0007T^2). \tag{10}$$

Here the orientation angle θ_i is the angle between the nuclear symmetry axis and the collision Z axis, measured in the counterclockwise direction, and angle α_i is the angle between the symmetry axis and the radius vector of the colliding nucleus, measured in the clockwise direction from the symmetry axis (see, e.g., Fig. 1 of Ref. [13]).

However, for reactions such as ^{58}Ni + ^{58}Ni, ^{64}Ni + ^{64}Ni, or ^{64}Ni + ^{100}Mo, where the fusion-evaporation cross section $\sigma_{\rm evr}$ is measured, the above procedure of explicit ℓ summation fails to reproduce the data at sub-barrier energies and possibly demands modification of the barrier, which we carry out here empirically by either (i) keeping the curvature $\hbar\omega_\ell$ same and modifying the barrier height V_B^ℓ, obtained from Eq. (8), by $\Delta V_B^{\rm emp}$, i.e., define

$$V_B^\ell(\text{modified}) = V_B^\ell + \Delta V_B^{\rm emp}$$

or (ii) keep the barrier height V_B^ℓ same and modify the curvature $\hbar\omega_\ell$ as

$$\hbar\omega_\ell(\text{modified}) = \hbar\omega_\ell + \Delta\hbar\omega^{\rm emp}.$$

This is illustrated in Fig. 1, first for the failure of ℓ-summation method (dashed line in Fig. 1(a) for $\Delta V_B^{\rm emp} = 0$), and then for both of cases of modifying V_B^ℓ or $\hbar\omega_\ell$, where $\Delta V_B^{\rm emp}$ (or $\Delta\hbar\omega_\ell^{\rm emp}$) is determined empirically to be minimum for the best fit to $\sigma_{\rm evr}$ at $\ell = \ell_{\rm max}$.

Alternatively, one could tend to think that the modification of the barrier could also be carried out at the level of $\ell = 0$ barrier-based Wong formula (5) itself. Of course, such a fitting procedure is possible but, as we shall show later, it leads to a completely undesirable result (see below, Fig. 9).

B. Relation between the Wong formula and the dynamical cluster-decay model

The DCM defines for each process the compound nucleus decay, or fragment formation, cross section, in terms of ℓ partial waves, as [9–11]

$$\sigma = \sum_{\ell=0}^{\ell_{\rm max}} \sigma_\ell = \frac{\pi}{k^2} \sum_{\ell=0}^{\ell_{\rm max}} (2\ell + 1) P_0^\ell P_\ell \tag{11}$$

with the additional P_0^ℓ as the fragment preformation probability, referring to $\eta[= (A_1 - A_2)/(A_1 + A_2)]$ motion. For the capture (or quasifission) process, because the two incoming nuclei do not loose their identity, $P_0^\ell = 1$ in the DCM [9], which reduces Eq. (11) to Eq. (1) of Wong. Thus, whereas the capture process is treated on similar footings (see below) in both the Wong model and the DCM ($P_0^\ell = 1$), the fusion-evaporation cross sections in Wong need the barrier modification and in the DCM the preformation factor $P_0^\ell > 0$.

A point of difference in the two models (Wong and the DCM) is that the penetrability P_ℓ in the Wong formula is calculated in the Hill-Wheeler [15] approximation of inverted harmonic oscillator for the interaction potential $V_\ell(R)$ of the *incoming channel*, whereas the same in the DCM is the WKB integral, whose first turning point $R_a = R_1(\alpha_1, T) + R_2(\alpha_2, T) + \Delta R(T)$ is defined through a neck-length parameter ΔR for the best fit to, say, the data on the fusion-evaporation cross section, which also contains the "barrier lowering" effects in it *for each decay channel* [11].

III. CALCULATIONS AND RESULTS

First, we assess Wong's approximations with a view to look for barrier modifications due to its ℓ, θ, and Φ dependences. Figure 2 shows the interaction potential for ^{48}Ca + ^{238}U reaction at $E_{\rm c.m.} = 193.57$ MeV, using an illustrative $\theta = 90^\circ$ and a few ℓ values (^{48}Ca being a spherical nucleus; only the

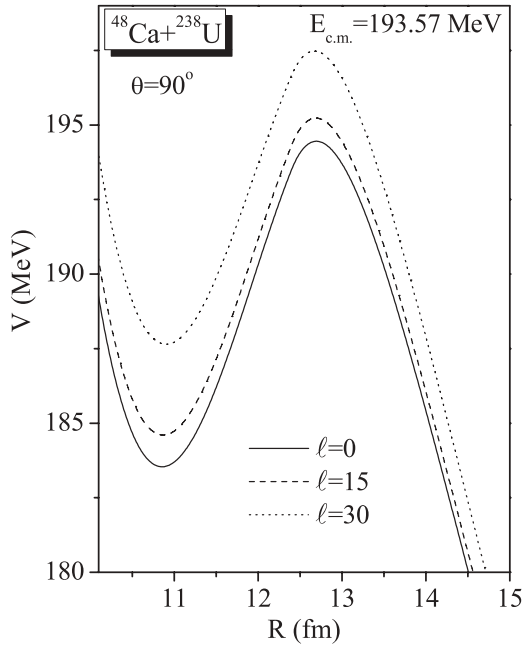

FIG. 2. Interaction potential for ^{48}Ca + ^{238}U system at $E_{\text{c.m.}} =$ 193.57 MeV (equivalently, $T = 1.091$ MeV), taking $\theta = 90^\circ$ and $\ell = 0$, 15, and 30 $\hbar$.

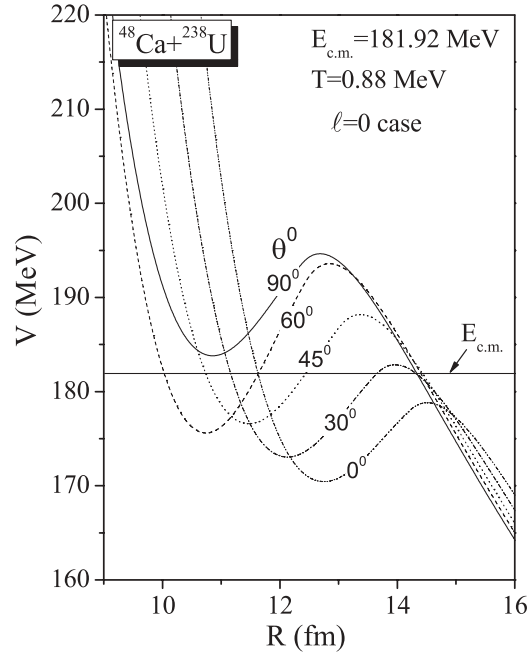

FIG. 3. Interaction potentials for the ^{48}Ca + ^{238}U system at various θ values of ^{238}U for $\ell = 0$ case of $E_{\text{c.m.}} = 181.92$ MeV.

orientation of deformed nucleus in the same plane comes into play). It is clear from the figure that as ℓ value increases the barrier characteristics (the barrier height, its position, as well as the oscillator frequency) change appreciably, i.e., barrier thickness increases and pocket gets shallower. This result is further evident from Table I that all barrier properties change due to the ℓ value, and this is more so for $\hbar\omega_\ell$ and V_B^ℓ. Hence, Wong's approximation of using only the $\ell = 0$ quantities neglect the in-built property of Eq. (1) to include the barrier-modification property through an explicit use of ℓ-dependent barriers. Similarly, the orientation θ degree of freedom modifies the barrier strongly, as is illustrated in Fig. 3 for the $\ell = 0$ case of a below-barrier center-of-mass energy. Thus, it is also important to consider the integration over θ coordinate(s). Furthermore, Fig. 4 illustrates the case of noncoplanar nuclei in ^{64}Ni + ^{64}Ni reactions where the θ_i-integrated ($\Phi = 0^\circ$) cross section is compared with Φ-integrated cross section, together with the experimental data [18]. The role of Φ is clearly shown to improve the comparison with experiments at near and below-barrier energies, thereby stressing the importance of integration over the Φ coordinate to include also the noncoplanar configurations.

TABLE I. Calculated oscillator frequencies $\hbar\omega_\ell$, barrier heights V_B^ℓ, and its position R_B^ℓ for different ℓ values for the interaction potential illustrated in Fig. 2.

ℓ ($\hbar$)	$\hbar\omega_\ell$ (MeV)	V_B^ℓ (MeV)	R_B^ℓ (fm)
0	4.4104	194.333	12.715
15	4.4295	195.112	12.708
30	4.4780	197.355	12.6895

Next, as a first application of the explicit ℓ-summed Wong formula, we consider the capture cross sections for the reactions ^{48}Ca + ^{238}U, ^{48}Ca + ^{244}Pu, and ^{48}Ca + ^{248}Cm. Figure 5 shows a comparison of the experimental data [19] for capture cross sections with explicit ℓ-summed Wong formula

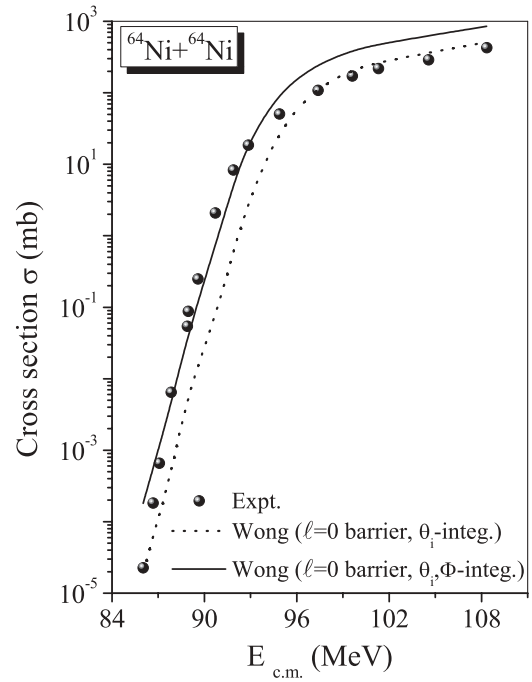

FIG. 4. Fusion-evaporation cross section for the ^{64}Ni + ^{64}Ni system, using the simple Wong formula (5), integrated over θ_i ($\Phi = 0$) alone and integrated over Φ, together with the experimental data [18].

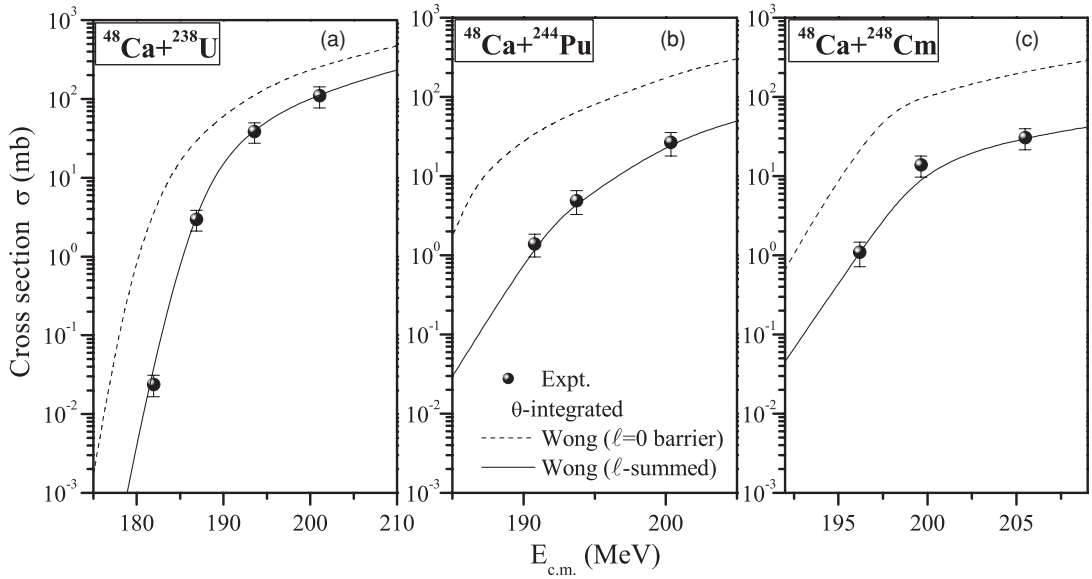

FIG. 5. The capture cross sections for the systems ^{48}Ca + ^{238}U, ^{244}Pu, and ^{248}Cm calculated by using the explicit ℓ-summed Wong formula (solid lines) compared with the simple $\ell = 0$ barrier-based Wong formula (dashed lines) and the experimental data [19] (solid circles with error bars).

(solid line) and the $\ell = 0$ barrier based Wong formula (5) (dotted line), both integrated over the θ coordinate (because ^{48}Ca is a spherical nucleus, these reactions are the cases of $\Phi = 0^\circ$, coplanar nuclei). Apparently, the simple $\ell = 0$ barrier-based Wong formula does not fit the data, but the explicit ℓ-summed Wong formula gives a nice fitting of data with $\ell_{\max}(E_{\text{c.m.}})$ deduced as shown in Fig. 6. An interesting property of the variation of $\ell_{\max}$ with $E_{\text{c.m.}}$ is its near saturation at maximum $E_{\text{c.m.}}$ and tendency toward a zero value at energies far below the barrier energies, with its average value matching the global ℓ value predicted on finite-range liquid-drop model (FRLDM) [20].

The complete ℓ-summed Wong formula, however, also fails for the lighter systems ^{58}Ni + ^{58}Ni, ^{64}Ni + ^{64}Ni, and ^{64}Ni + ^{100}Mo at below-barrier energies, as shown in Fig. 7 (dashed line, θ_i-integrated, for $\Delta V_B^{\text{emp}} = 0$). The simple $\ell = 0$ barrier-based Wong is already shown in Fig. 4 not to fit the data for both the coplanar ($\Phi = 0^\circ$) and noncoplanar ($\Phi \neq 0^\circ$) cases. As already noted above, in these reactions, the measured data are the fusion-evaporation cross sections and, in the language of the DCM, we are considering here $P_0^\ell = 1$ more suitable for capture cross sections. To include such effects, we modify here all the ℓ-dependent barriers empirically via a barrier-lowering constant ΔV_B^{emp} or barrier curvature constant $\Delta\hbar\omega^{\text{emp}}$, as

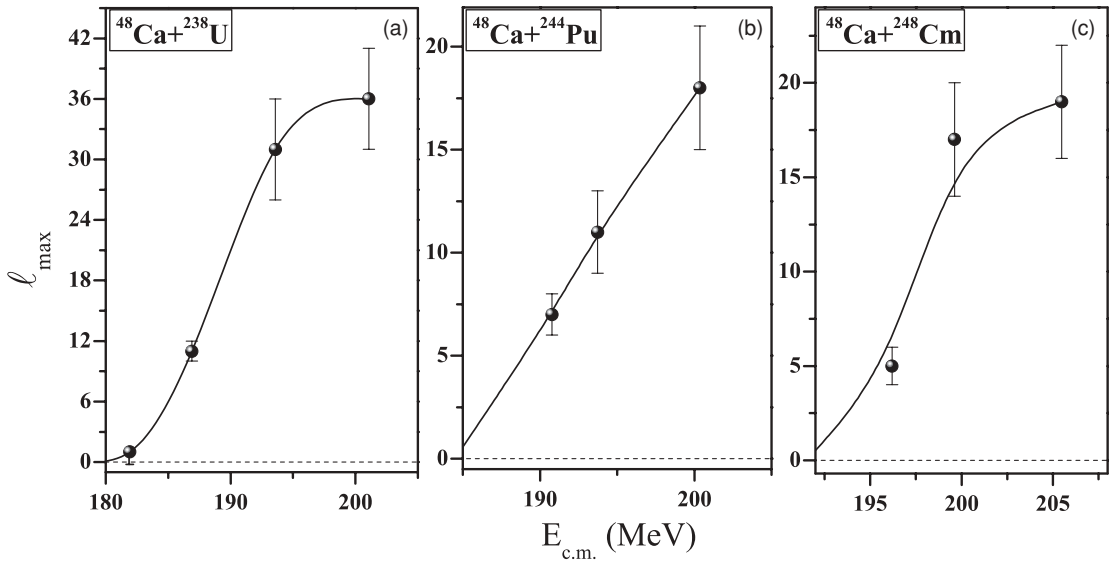

FIG. 6. Variation of deduced maximum angular momentum $\ell_{\max}$ with $E_{\text{c.m.}}$ (solid circles with calculated error bars) for the systems considered in Fig. 5. The solid lines are only a guide for the eye.

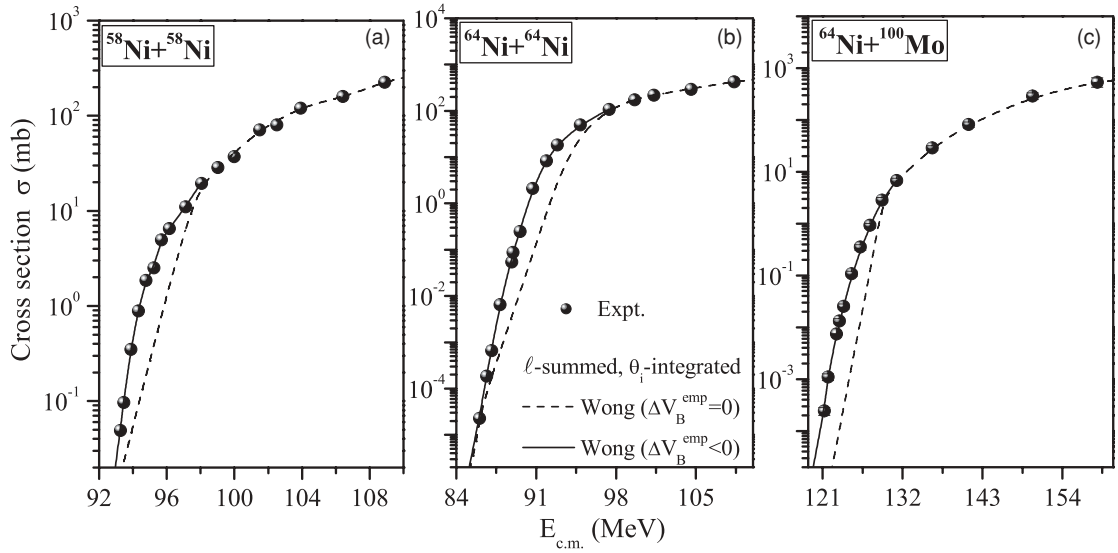

FIG. 7. The experimental data on the fusion-evaporation cross section (solid circles) compared with complete ℓ-summed Wong, integrated over θ_i coordinates ($\Phi = 0^\circ$), with $\Delta V_B^{\rm emp} = 0$ (dashed line), and best fitted $\Delta V_B^{\rm emp}$ (solid line), for the systems (a) ^{58}Ni + ^{58}Ni [21], (b) ^{64}Ni + ^{64}Ni [18], and (c) ^{64}Ni + ^{100}Mo [8].

already explained in Fig. 1. In the following, we do this for the $\Phi = 0^\circ$ case and for $\Delta V_B^{\rm emp}$. A similar fitting of data can also be carried out for $\Delta\hbar\omega^{\rm emp}$ and for the $\Phi \neq 0^\circ$ case.

Figure 8 shows the resultant $\Delta V_B^{\rm emp}$ for the best fit to data in Fig. 7 (solid line), with $\ell_{\rm max}$ determined as in Fig. 1. Evidently, no modification of the barrier is needed ($\Delta V_B^{\rm emp} = 0$) at the above-barrier energies, but then, as $E_{\rm c.m.}$ decreases, $\Delta V_B^{\rm emp}$ as a function of $E_{\rm c.m.}$ falls off steeply and then increases again depending on how good or how bad the fit is to data in Fig. 7 for the case of $\Delta V_B^{\rm emp} = 0$ (dashed line). In other words, no particular significance can be attached to the nature of curves in Fig. 8, i.e., its double valuedness, etc., except that it gives the required modification of barrier to fit the fusion-evaporation data.

Finally, we have checked if the process of barrier modification could be carrier out by using the $\ell = 0$ barrier-based Wong formula (5) itself. The result of this calculation is illustrated in Fig. 9 for both the ^{48}Ca- and Ni-based reactions. We notice in this figure that, for a fit of the cross-section data similar to that for the ℓ-summed Wong formula (solid lines in Figs. 5 and 7), the barrier-modification parameter $\Delta V_B^{\rm emp}$ is positive for the ^{48}Ca + ^{238}U reaction (same for the other two ^{48}Ca-based reactions), which means that the barrier has to be "raised," and for the ^{64}Ni + ^{64}Ni (and other two Ni-based reactions) it is negative ("lowering" the barrier) for below-barrier energies but again positive ("raising" the barrier), and ever increasing, for above-barrier energies. Note that in case of the ℓ-summed Wong formula, $\Delta V_B^{\rm emp} = 0$ for all above-barrier energies. Thus, for the $\ell = 0$ barrier-based Wong formula (5), we find that at sub-barrier energies (for Ni-based reactions) the procedure of modifying the $\ell = 0$ barrier is perhaps not very different from that for the ℓ-summed case,

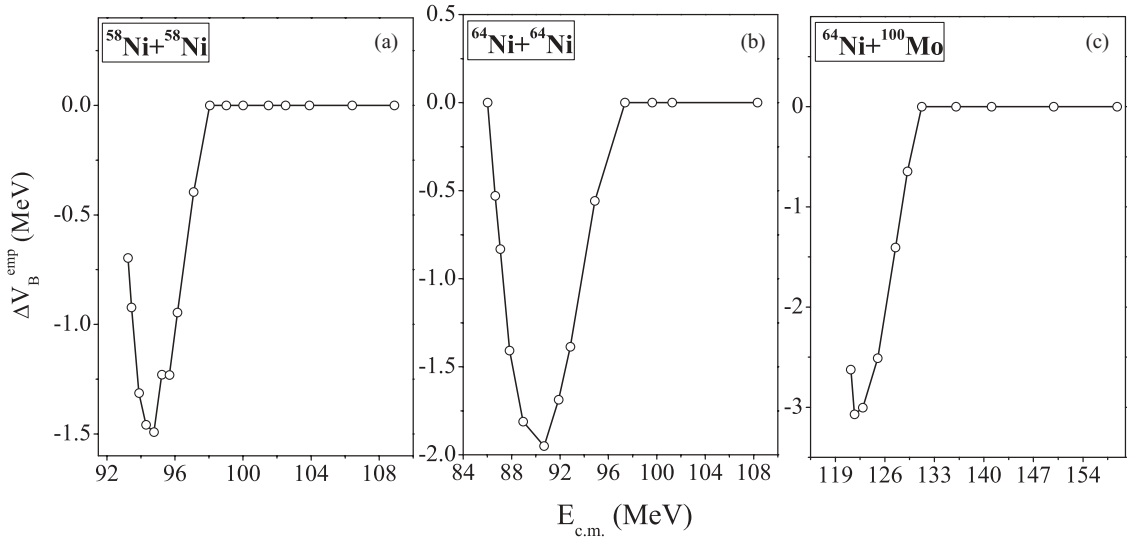

FIG. 8. The variation of $\Delta V_B^{\rm emp}$, fitting the data in Fig. 7, with center-of-mass energy $E_{\rm c.m.}$.

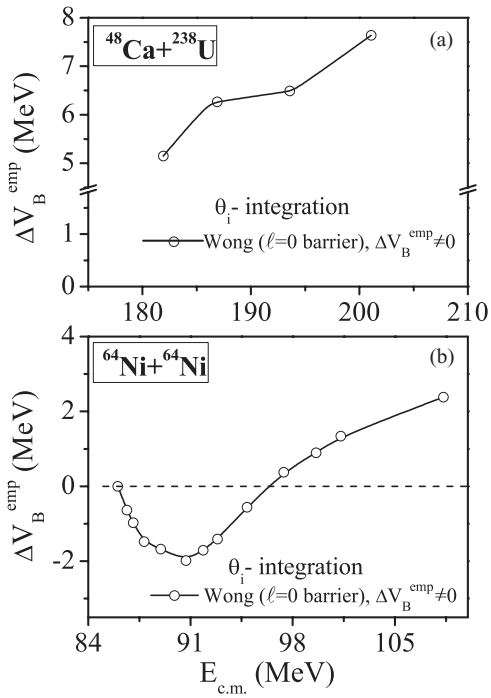

FIG. 9. Same as for Fig. 8 but for the $\ell = 0$ barrier-based Wong formula (5) fitting the data in Fig. 5(a) and 7(b) as well as for the ℓ-summed Wong formula.

but at above-barrier energies (for both the ^{48}Ca- and Ni-based reactions), instead of a zero modification of the barrier for the ℓ-summed Wong formula, it gives a completely undesirable result of requiring the "raising" of barrier to fit the data on both the capture and fusion-evaporation cross sections.

IV. CONCLUSIONS

The Wong formula is re-evaluated for its angular-momentum and barrier-modification effects at sub-barrier energies. This is done for use of nuclear proximity potential, and with multipole deformations included up to hexadecapole (β_4) and orientation degrees of freedom integrated for both the coplanar ($\Phi = 0^\circ$) and noncoplanar ($\Phi \neq 0^\circ$) nuclei. Calculations are made for the measured capture cross sections in ^{48}Ca + ^{238}U, ^{48}Ca + ^{244}Pu, and ^{48}Ca + ^{248}Cm reactions forming superheavy nuclei, and the fusion-evaporation cross sections in medium mass ^{58}Ni + ^{58}Ni, ^{64}Ni + ^{64}Ni, and ^{64}Ni + ^{100}Mo reactions where fusion hindrance phenomenon in coupled-channels calculations is known to exist.

The simple Wong formula, based on the $\ell = 0$ barrier, is first shown to ignore the modifications of the barriers entering the calculations due to their ℓ dependences. The ℓ-dependent barriers introduced via ℓ summation are found to be sufficient to explain the capture cross sections for all the three above-mentiond ^{48}Ca-based reactions. The complete ℓ-summed Wong formula is in fact the same as the DCM model expression with the condition of fragment preformation probability $P_0^\ell = 1$ for all angular-momentum ℓ values, applicable to capture (equivalently, quasifission) reactions where the incoming nuclei keep their identity. However, for the ^{58,64}Ni-based fusion-evaporation cross sections mentioned above, in agreement with Misicu and Esbensen [4] and the DCM calculations of Gupta and collaborators [11], a further modification of barriers is found essential for incident energies below the barrier, which is shown possible to be included in terms of either the "lowering of barrier" or "increasing of barrier curvature parameter." Apparently, the depth of the potential pocket plays no role, at least in the context of Wong formula. Similarly, with barrier-modification effects introduced in the simple $\ell = 0$ barrier-based Wong formula, it does not give realistic results at both the above- and below-barrier energies.

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