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Barrier distributions for weakly bound systems

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We discuss the use of reduced fusion cross sections in the derivation of fusion barrier distributions. We show that the elimination of static effects associated with system sizes and optical potentials obtained by the recently introduced fusion functions can be extended to barrier distributions. This can be a useful tool for systematic studies of breakup coupling effects in fusion processes.

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I. INTRODUCTION

Nuclear reactions induced by weakly bound projectiles have attracted considerable interest for the last two decades [1–3]. These reactions are influenced by two factors. The first is that the potential barrier for weakly bound systems tends to be lower and somewhat broader. The second factor is the strong coupling with the breakup channel. While the lower barrier leads to larger fusion cross sections, the role of breakup coupling is more complicated. The breakup process gives rise to different kinds of fusion. Besides the usual complete fusion (CF), where the whole projectile merges with the target to form the compound nucleus, the breakup process may take place, populating different reaction channels: (i) incomplete fusion (ICF), when some—but not all—projectile fragments are absorbed by the target while at least one escapes the interaction region, (ii) noncapture breakup (NCBU), when none of the fragments is absorbed by the target, and (iii) sequential complete fusion, when all fragments are absorbed sequentially. From the experimental point of view, sequential complete fusion cannot be distinguished from the usual complete fusion. Furthermore, most experiments can only measure the total fusion (TF) cross section, which corresponds to the sum $\sigma_{\rm CF} + \sigma_{\rm ICF}$.

To understand the reaction mechanisms in collisions of weakly bound systems, several experiments have been performed and various theoretical models have been developed [1]. In addition to the analysis of fusion excitation functions, useful information can be extracted from fusion barrier distributions (FBDs). They have been introduced by Rowley, Satchler, and Stelson [4,5] and are defined as

$$D(E) = \frac{d^2}{dE^2}\left[E\sigma_F(E)\right]. \qquad (1)$$

Above, E is the collision energy in the c.m. frame and $\sigma_F(E)$ is the fusion cross section. Although FBDs are hard to measure in weakly bound systems, there are results available in collisions of a few stable weakly bound projectiles with heavy targets.

To perform systematic studies of the effects of the breakup channel on fusion, it is necessary to compare the cross sections for weakly bound systems with those for tightly bound ones, as well as the results for different weakly bound systems (stable and/or radioactive) among themselves. For this purpose, it is necessary to isolate the effects of breakup coupling, eliminating the influence of other factors such as system size and the height and shape of the potential barrier. This can be achieved through an appropriate reduction of the fusion data. Although several reduction methods have been proposed [6–11], in most cases the influence of the breakup channel is entangled with some influence of other factors. Recently, we introduced a new reduction method that eliminates all static effects of weak binding and also the effects of coupling to bound channels [12]. In this method, the fusion cross section is transformed into a dimensionless cross section, which is called the fusion function. The method was then employed to analyze CF and TF data for several heavy [13] and light [14] systems. More recently, this method was also extended to total reaction cross sections [15].

In the present work, we extend the reduction method of Ref. [12] to fusion barrier distributions. In Sec. II, we review the basic concepts of fusion functions. In Sec. III, we define fusion function barrier distributions and derive an analytical expression for a universal function to which they should be compared. In Sec. IV, we discuss the numerical evaluation of these distributions and give as examples the fusion function barrier distributions for some strongly bound systems. In Sec. V, we use these concepts to study the fusion of weakly bound systems. Finally, in Sec. VI, we present our conclusions.

II. FUSION FUNCTIONS

In recent papers, Canto *et al.* [12,13] proposed a new method to reduce fusion data. In this method, the collision energy in the c.m. frame, E, and the fusion cross section, σ_F, are respectively transformed into the dimensionless quantities x and $F(x)$, defined as

$$x = \frac{E - V_B}{\hbar\omega}, \quad F(x) = \frac{2E}{\hbar\omega R_B^2}\,\sigma_F. \tag{2}$$

Above, R_B, V_B, and $\hbar\omega$ stand for the barrier radius, height, and curvature parameters, which should be obtained from a realistic model for the bare projectile-target interaction. We point out that the reduction method of Eq. (2) is model dependent. To avoid problems arising from unreasonable choices of the bare potential, one should adopt theoretical models that can be used over a wide mass range and take into account the static effects of low breakup thresholds in collisions of weakly bound nuclei. The natural candidates are double-folding models based on realistic nuclear densities and standard nucleon-nucleon interactions. We use here the Sao Paulo potential (SPP) [16–18], which has these features. This potential has no free parameter and has been used to study elastic scattering and other reaction mechanisms in collisions of several tightly and weakly bound systems [18–21], including fusion barrier distributions of weakly bound systems [22]. Barrier parameters obtained from the SPP for a large number of systems are given in Table 2 of Ref. [12]. In the same reference, one can also find a detailed description of the SPP and a study of the sensitivity of fusion functions to changes in the parameters of the bare potential.

The transformation of Eq. (2) is inspired by Wong's approximation [23] for the fusion cross section. In Wong's approximation, the barrier at the s wave is approximated by the parabola,

$$V(r) \sim V_B + \tfrac{1}{2}\,\mu\omega^2\,(r - R_B)^2, \tag{3}$$

where μ is the projectile-target reduced mass. Using the Hill-Wheeler expression for the tunneling probabilities [24] and approximating the partial-wave summation by an integral, Wong got the analytical expression

$$\sigma_F^W(x) = R_B^2\,\frac{\hbar\omega}{2E}\,\ln\left[1 + \exp\left(\frac{2\pi\,(E - V_B)}{\hbar\omega}\right)\right]. \tag{4}$$

If we now insert this approximation into Eq. (2), we get the fusion function

$$F_0(x) = \ln[1 + \exp(2\pi x)]. \tag{5}$$

The fusion function of Eq. (5) has the important property of being independent of the system. For this reason, it is called the universal fusion function (UFF) in Ref. [12].

When fusion data are available, one can evaluate experimental fusion functions $F_{\rm exp}(x)$ using the experimental cross sections in Eq. (2). In situations where Wong's approximation is valid and the fusion cross section is not influenced by channel coupling, $F_{\rm exp}(x) \simeq F_0(x)$. However, these conditions are hardly satisfied. In Refs. [12,13], it is pointed out that Wong's approximation breaks down in collisions of light systems at subbarrier energies and that the fusion cross section is frequently affected by channel coupling. The present work is not concerned with very light systems or with energies well below the barrier, where Wong's approximation breaks down. Thus, the differences between $F_{\rm exp}$ and F_0 are mainly due to couplings with bound channels and breakup. To study the influence of the breakup channel on fusion, it is then necessary to reduce the influence of bound channels on the data. For this purpose, we renormalize the experimental fusion function as

$$\bar{F}_{\rm exp}(x) = \frac{F_{\rm exp}}{\mathcal{R}(x)}, \quad \mathcal{R}(x) = \frac{\sigma_F^{\rm CC}}{\sigma_F^W}. \tag{6}$$

Above, $\sigma_F^{\rm CC}$ is the fusion cross section obtained from a coupled-channel (CC) calculation including all relevant bound channels, and σ_F^W is the one-dimensional barrier penetration cross section (with all couplings switched off) within Wong's approximation. Since $\mathcal{R}(x)$ is an estimate of the influence of couplings to bound channels on the fusion cross section, dividing the experimental fusion function by this factor minimizes this influence. The use of Wong's cross section instead of the one obtained from the direct solution of the single-channel Schrödinger equation has the advantage of eliminating deviations from the universal function that are not related to channel-coupling effects. However, we point out that this difference is not relevant for the system and energy range considered in the present work. Renormalized fusion functions were shown to be a very useful tool for studying the influence of the breakup channel in the fusion of heavy [12,13] and light [14] systems. It should be remarked that the renormalization factor $\mathcal{R}(x)$ is also model dependent. The effects of the breakup channel on the fusion cross sections will only be correctly indicated if all couplings with bound channels are accurately accounted for. If a strongly coupled bound channel is left out or its coupling strength is wrong, the influence of this channel will be entangled with that of the breakup channel and the comparison with the data may be misleading. However, the same difficulties are encountered in the usual analysis, in which the data are directly compared with predictions of CC calculations including bound channels. The advantage of the procedure of Eq. (6) is that it allows comparisons of fusion data for different systems in a single plot, which is not possible with the conventional approach. The coupled-channel calculations of the present work were performed with the code FRESCO [25].

III. FUSION FUNCTION BARRIER DISTRIBUTIONS

The same procedures used in the construction of fusion functions can be extended to fusion barrier distributions. We define the fusion function barrier distribution (FFBD) as

$$\mathcal{D}(x) = \frac{1}{2\pi}\left[\frac{d^2F(x)}{dx^2}\right]. \tag{7}$$

Note that this definition is analogous to the usual one [4,5,26], except for an overall normalization factor.

The different fusion functions considered in the previous sections give rise to different FFBDs. Of particular interest is the universal barrier distribution (UBD), which is obtained from the universal fusion function of Eq. (5). In this case, the second derivative of Eq. (7) can be evaluated analytically, and we get

$$\mathcal{D}_0(x) = \frac{\pi}{1 + \cosh(2\pi x)}. \tag{8}$$

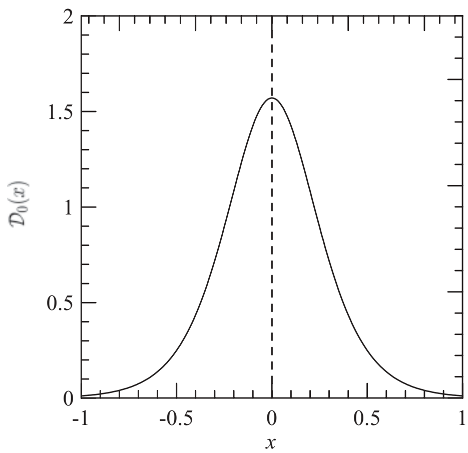

FIG. 1. Fusion barrier distribution derived from the universal fusion function.

The function $\mathcal{D}_0(x)$ is represented in Fig. 1. It is symmetric around the maximum at $x = 0$ and is normalized as

$$\int_{-\infty}^{\infty} \mathcal{D}_0(x)\, dx = 1. \tag{9}$$

Experimental fusion function barrier distributions can be obtained from $F_{\rm exp}(x)$ and $\bar{F}_{\rm exp}(x)$. That is,

$$\mathcal{D}_{\rm exp}(x) = \frac{1}{2\pi}\left[\frac{d^2F_{\rm exp}(x)}{dx^2}\right], \tag{10}$$

and

$$\bar{\mathcal{D}}_{\rm exp}(x) = \frac{1}{2\pi}\left[\frac{d^2\bar{F}_{\rm exp}(x)}{dx^2}\right]. \tag{11}$$

If Wong's approximation holds and CC effects on the fusion cross section are negligible, one should have $\mathcal{D}_{\rm exp}(x) \simeq \mathcal{D}_0(x)$. In most cases, the validity of Wong's approximation should not be a problem. Barrier distributions are usually evaluated around the Coulomb barrier, and in this region Wong's approximation is expected to be reasonable, except for very light systems [13]. The differences between $\mathcal{D}_{\rm exp}(x)$ and $\mathcal{D}_0(x)$ should then be attributed to CC effects. These effects can be accounted for in $\bar{\mathcal{D}}_{\rm exp}(x)$, if all relevant CC effects are contained in the factor $\sigma_F^{\rm CC}(x)$, used in the renormalization procedure of Eq. (6). Therefore, the differences between $\bar{\mathcal{D}}_{\rm exp}(x)$ and $\mathcal{D}_0(x)$ measure the importance of the channel coupling effects missing in $\sigma_F^{\rm CC}(x)$. In this way, the differences between $\bar{\mathcal{D}}_{\rm exp}(x)$ and $\mathcal{D}_0(x)$, in a situation where $\sigma_F^{\rm CC}(x)$ contains the effects of coupling to all relevant bound channels, represent the influence of the breakup process on the experimental FFBDs.

IV. NUMERICAL EVALUATION OF THE FFBD

As fusion functions are measured at a discrete set of x values, corresponding to a finite number of collision energies, exact evaluations of experimental FFBDs cannot be performed. Thus, one has to resort to numerical procedures, which approximate derivatives by finite differences. Typically, one starts with sets of data points $\{x_i, \bar{F}_{\rm exp}(x_i), \delta_i(\bar{F}_{\rm exp}); i = 1,\ldots,N\}$, where $\delta_i(\bar{F}_{\rm exp})$ is the statistical error associated with the measurement of $\bar{F}(x_i)$. We neglect errors arising from the factor $\sigma_F^W(x)/\sigma_F^{\rm CC}(x)$. The usual method to derive fusion barrier distributions is to use the three-point formula. To evaluate the FFBD at one value x_i of the set, one selects two other points, x_j and x_k, one above x_i and one below. In most cases, one uses an equally spaced grid. That is, $x_k - x_i = x_i - x_j = \Delta x$. The three-point formula is obtained from a linear combination of Taylor expansions of the fusion function at the points $x - \Delta x$ and $x + \Delta x$. For an equally spaced x grid, one gets

$$\mathcal{D}(x_i) \approx \frac{F(x_+) + F(x_-) - 2F(x_i)}{\Delta x^2}, \tag{12}$$

where $x_\pm = x_i \pm \Delta x$. Above, F and $\mathcal{D}$ are short-hand notations for the experimental fusion function and FFBD, respectively. The statistical error of the barrier distribution at the point x_i is

$$\delta_i^{\rm stat} \approx \frac{\sqrt{[\delta F(x_+)]^2 + [\delta F(x_-)]^2 + 4\,[\delta F(x_i)]^2}}{(\Delta x)^2}. \tag{13}$$

Equation (13) indicates that when the grid step decreases, the statistical error grows quadratically with its inverse. In principle, one can use any Δx, provided that it is compatible with the available data points. In typical evaluations of barrier distributions, the grid spacing is $\Delta E \sim 2$ MeV. This corresponds to $\Delta x \sim 0.4$–0.5, since $\hbar\omega \sim 4$ MeV. Larger grid spacings tend to wash out structures of the barrier distribution around the Coulomb barrier [5]. On the other hand, small values of ΔE are avoided, because they lead to large statistical errors, as indicated in Eq. (13).

Besides the above-discussed statistical error, there is a systematic error associated with the three-point formula. It arises from the truncation of the Taylor series at second order. It is given by

$$\delta_i^{\rm syst} \approx \frac{1}{12}\left[\frac{d^4F(x_i)}{dx_i^4}\right](\Delta x)^2. \tag{14}$$

This problem was considered in detail by Canto and Donangelo [27]. Using Wong's approximation to estimate the fourth-order derivative of the cross section, they showed that the systematic error can be quite large, specially at $E \approx V_B$. For the grid spacing $\Delta E = 2$ MeV, the systematic error in the barrier region is much larger than the statistical one. In general, the systematic error is disregarded when experimental fusion barrier distributions are compared with predictions of theoretical models. The justification for this procedure is that both distributions are evaluated using the same energy step. Therefore, the systematic error is not expected to influence the comparison. The situation is different in the present work. We have an analytical expression for the standard FFBD, $\mathcal{D}_0(x)$, to which experimental distributions should be compared. To handle the situation, we adopt two procedures. The first is to use an x-dependent step Δx, chosen to minimize the total error,

$$\delta_i^{\rm tot} = \sqrt{\left[\delta_i^{\rm stat}\right]^2 + \left[\delta_i^{\rm syst}\right]^2}. \tag{15}$$

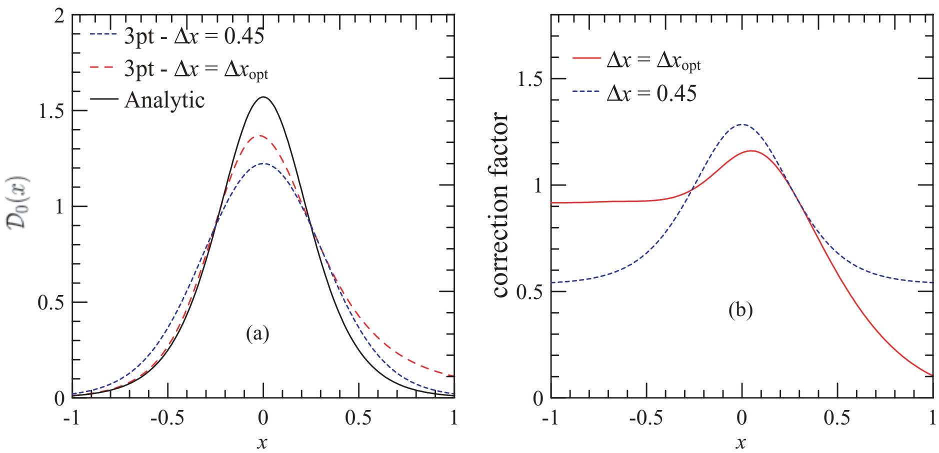

FIG. 2. (Color online) Inaccuracy of the three-point formula. (a) Comparison of the exact values of the universal distribution (solid line) with results of the three-point formula using the optimal grid (long-dashed line) and the constant step size $\Delta x = 0.45$ (short-dashed line). (b) Correction factors for the two options of the grid.

The optimal grid step is given by the parametric expression [27]

$$\Delta x_{\rm opt} = 0.5\,\epsilon^{1/4} \quad \text{for} \quad x \leqslant -0.8, \tag{16}$$

$$\Delta x_{\rm opt} = 0.5\,\epsilon^{1/4} \times f(x) \quad \text{for} \quad x > -0.8, \tag{17}$$

where $f(x)$ is the second-degree polynomial

$$f(x) = 1 + 1.8\,(x + 0.8)^2, \tag{18}$$

and ϵ is the average relative error for the three values of the fusion function appearing in Eq. (12). The second procedure is to multiply the experimental barrier distribution by a factor $C(x)$ to correct the systematic error. This factor is estimated by the ratio

$$C(x) = \frac{\mathcal{D}_0(x)}{\mathcal{D}_0^{\rm 3pt}(x)}. \tag{19}$$

Above, $\mathcal{D}_0^{\rm 3pt}(x)$ is the UBD, evaluated by the three-point formula. It can also be evaluated analytically, and the result is

$$\mathcal{D}_0^{\rm 3pt}(x) = \frac{1}{2\pi(\Delta x)^2}\ln\left\{\frac{[1+e^{2\pi(x+\Delta x)}][1+e^{2\pi(x-\Delta x)}]}{[1+e^{2\pi x}]^2}\right\}. \tag{20}$$

In Fig. 2, we illustrate the systematic error in the three-point formula. We use it to evaluate the universal distribution $\mathcal{D}_0(x)$, which is known analytically [Eq. (8)]. In Fig. 2(a), the exact results are compared with results of the three-point formula for two choices of the grid. The first is the constant step $\Delta x = 0.45$, which corresponds to $\Delta E \simeq 2$ MeV. The second is the optimized variable step given by the parametrization of Eqs. (16)–(18). In this case, the average relative statistical error was assumed to be 1%. In Fig. 2(b), we show the corresponding correction factor for each option of the grid. It is clear that important systematic errors arise from the numerical evaluation of the second derivative.

According to the discussion in this section, one should expect to get $\bar{\mathcal{D}}_{\rm exp}(x) \simeq \mathcal{D}_0(x)$ when couplings with all relevant channels are included in the calculation of $\sigma_F^{\rm CC}(x)$. We check this point in simple applications, where the relevant channels as well as their couplings are well established: the collisions $^{16}O + {}^{144,148,154}Sm$. The results are shown in Fig. 3. The data are from Refs. [26,28], and the CC calculations used to renormalize the data [see Eq. (6)] are from Ref. [29]. In Fig. 3(a), the experimental points are very different from the universal FFBDs. In Fig. 3(b), where the experimental FFBD is renormalized according to Eq. (6), the results for the three systems are similar, and they agree very well with the universal distribution.

V. APPLICATION: STUDY OF WEAKLY BOUND SYSTEMS

We now investigate the effects of the breakup channel on the experimental FFBD for collisions of stable weakly bound projectile on heavy targets. We consider the $^9Be + {}^{208}Pb$ and $^{6,7}Li + {}^{209}Bi$ systems, for which high-precision data are available [30–32]. One should have in mind that numerical evaluations of second derivatives require data with great accuracy. The numerical evaluation of the experimental FFBDs for these systems was based on the data points in the tables of Ref. [32]. Typically, these tables give CF or TF cross sections $\sigma_F(E_i)$ and the corresponding statistical error δ_i at N collision energies in the center-of-mass frame, E_i, $i = 1, \ldots, N$. Using Eqs. (2) and (6), we build a new table, each line containing $\{x_i,\ \bar{F}_{\rm exp}(x_i),\ \delta\bar{F}_i\}$. We then evaluate the barrier distribution at each of the x values appearing in the table. For this purpose, we use $\bar{F}_{\rm exp}(x_i)$ and also the fusion functions at two other x values, x_j and x_k, being $x_j < x_i$ and $x_k > x_i$. These points are chosen by the criterion of minimizing the total (statistical+systematic) error, as discussed in Ref. [27]. That is, $x_i - x_j$ and $x_k - x_i$ should be as close as possible to the optimal x-dependent energy step $\Delta x_{\rm opt}$ given in Eqs. (16)–(18). Using this procedure, we have evaluated the FFBDs associated with CF for the $^{6,7}Li + {}^{209}Bi$ and $^9Be + {}^{208}Pb$ systems. For $^9Be + {}^{208}Pb$, we study also TF, since there are experimental data available. The TF and CF results are shown respectively in Figs. 4(a) and 4(b). In the case of TF, we observe that the experimental FFBDs are quite close to the UBDs around and above the barrier. The only exception is one point at the maximum, which is slightly below. On the other hand, the experimental points at lower energies are above the universal curve. This is not surprising, since there are important contributions from ICF at subbarrier energies, which were not included in the renormalization procedure.

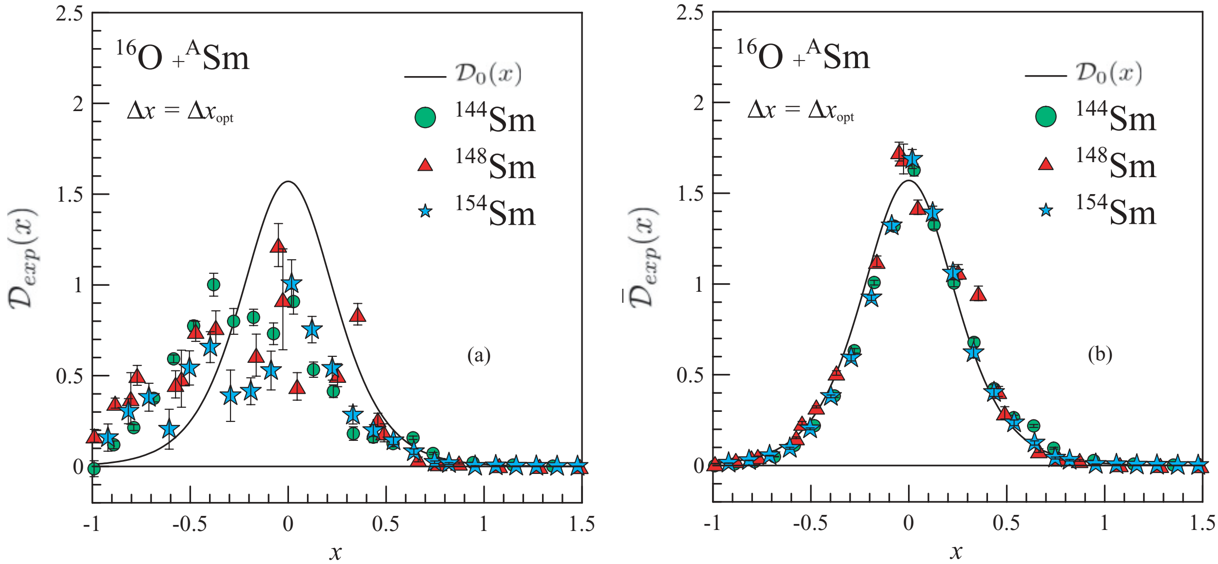

FIG. 3. (Color online) Experimental FFBDs for the $^{16}O + {}^{144,148,154}Sm$ systems in comparison with the universal distribution $\mathcal{D}_0(x)$ (solid lines). (a) $\mathcal{D}_{\rm exp}(x)$. (b) Renormalized distribution $\bar{\mathcal{D}}_{\rm exp}(x)$. The data are from Ref. [26].

We now consider the results for CF, appearing in Fig. 4(b). Examining the figure, one reaches several interesting conclusions. The first is that the results for the three systems are similar, specially in the cases of $^9Be + {}^{208}Pb$ and $^7Li + {}^{209}Bi$. This is consistent with the analysis of renormalized fusion functions of Refs. [12,13]. The second interesting point is that the experimental FFBDs for CF are close to the UBDs at energies well above and well below the Coulomb barrier. However, their maxima at $x \simeq 0$ are about 40% lower. The experimental FFBDs for the $^7Li + {}^{208}Pb$ and $^9Be + {}^{209}Bi$ are very close to the universal curve multiplied by a suppression factor of 0.6 (dashed line). This conclusion is consistent with the results of Refs. [30–32]. The FFBD for $^6Li + {}^{208}Pb$ is also similar to the universal curve multiplied by 0.6. However, in this case, where the breakup threshold has the lowest value ($B_{^6{\rm Li}} = 1.47$ MeV), the suppression is slightly stronger, and the coupling with the breakup channel produces a slight shift of the experimental curve toward positive x values. An effect of this kind was found in the CDCC calculations of Ref. [33]. The fact that breakup coupling leads to a uniform (energy-independent) reduction of the fusion function barrier distribution is not trivial. It is clear that a suppression of the fusion function propagates to the barrier distribution. However, one would expect that some subtle energy dependence would emerge from the process of taking second derivatives.

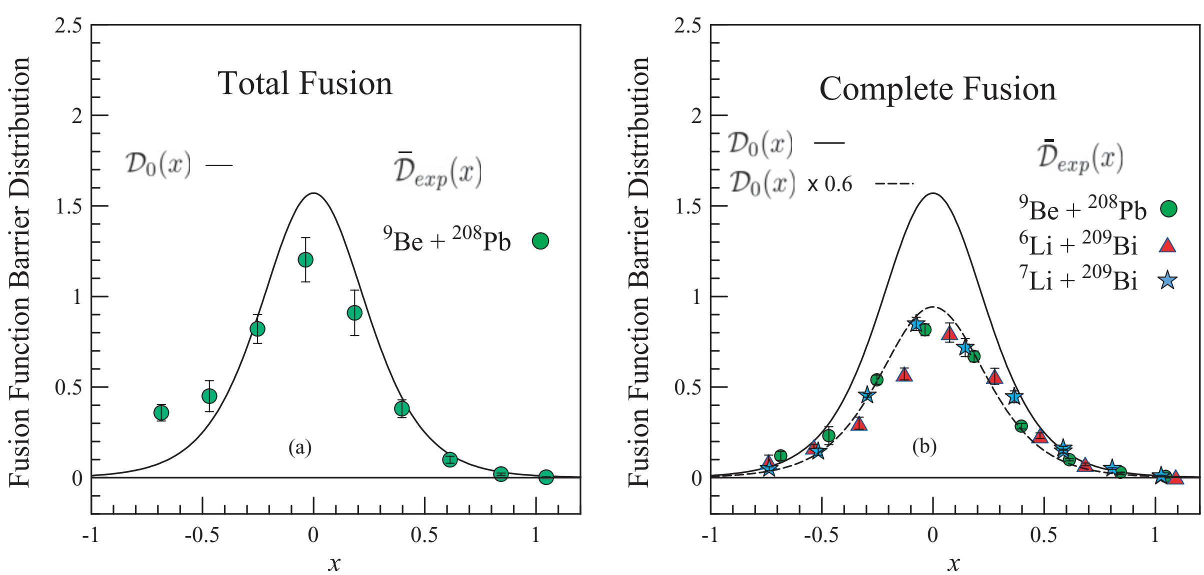

FIG. 4. (Color online) Experimental FFBDs for the $^9Be + {}^{208}Pb$ and $^{6,7}Li + {}^{209}Bi$ system, in comparison with the UBDs for total and complete fusion. The dashed line in (b) corresponds to the UFFBD multiplied by the factor 0.6. For details, see the text.

It should be emphasized that similar conclusions could be reached separately for each system, through a comparison of experimental cross sections or barrier distributions with theoretical predictions of CC calculations. However, by using reduced data, one can put together results for a variety of systems. In this way, systematic properties of weakly bound system emerge more clearly.

VI. CONCLUSIONS

We have investigated the role of the breakup channel in fusion barrier distributions for weakly bound systems. For this purpose, we obtain fusion barrier distributions from experimental fusion data reduced by a recently developed method. This method has the advantage of removing all static effects associated with system size and details of the potential barrier. It can eliminate also the influence of the couplings to bound excited channels. In this way, the effects of breakup coupling can be better investigated.

The method was used to study total fusion and complete fusion barrier distributions in the case of collisions of stable weakly bound projectiles with heavy targets, for which there are data available. The results were compared with a universal curve representing barrier distributions in the absence of breakup, and the deviations from this curve were attributed mainly to breakup coupling. We found that the distribution for total fusion is close to the universal curve except at subbarrier energies. We pointed out that this difference should arise from incomplete fusion. In the case of complete fusion barrier distributions, the maxima of the experimental distributions are about 40% lower than that of the universal curve.

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