^{26}Mg observables for the USDA and USDB Hamiltonians

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(Received 12 June 2009; published 1 September 2009)

Assignments are made between theory and experiment of corresponding levels in ^{26}Mg levels based on energies, lifetimes, branching ratios, electron scattering form factors, and reduced electromagnetic transition strengths. Results based on the new *sd*-shell interactions USDA (universal *sd*-shell interaction A) and USDB (universal *sd*-shell interaction B), as well as the older USD interaction, are compared.

DOI: 10.1103/PhysRevC.80.034301 PACS number(s): 21.10.Ky, 21.10.Jx, 21.60.Cs

I. INTRODUCTION

Two new interactions, universal *sd*-shell interaction A (USDA) and universal *sd*-shell interaction B (USDB) [1], have recently been obtained from fits of 63 two-body matrix elements and 3 single-particle energies to more than 608 binding energies and energy levels for the *sd*-shell nuclei from $A = 16$ to $A = 40$. The energy data set used for USDA and USDB was updated from the one used 25 years ago to obtain the USD interaction based on 47 linear combinations of parameters fitted to 447 energy data with a rms deviation of 150 keV [2]. The energy data have been improved and extended, in particular with more recent data for the neutron-rich *sd*-shell nuclei. As a consequence the main change from USD to USDA/B in terms of energies of low-lying states involved the most neutron-rich nuclei, and in particular features related to the position of the neutron $0d_{3/2}$ single-particle state around ^{24}O. The new interactions are used for configuration-interaction calculations involving the $0d_{5/2}$, $0d_{3/2}$, and $1s_{1/2}$ active orbitals for protons and neutrons. For USDA 30 linear combinations of one- and two-body matrix elements were varied, with the remaining 36 linear combinations fixed at values of a renormalized *G* matrix, with a resulting rms deviation between experimental and theoretical energies of 170 keV. For USDB, 56 linear combinations were varied with 10 fixed at the *G*-matrix values and with an improved rms deviation of 130 keV. Binding energies and energy levels for all *sd*-shell nuclei are shown in Ref. [3] and compared with experiment where available.

In recent work [4] extensive comparisons of observables for the low-lying states of many *sd*-shell nuclei have been made with the results based on the interactions USDA, USDB, and USD. Our goal in this article is to test these Hamiltonians for observables that go to high excitation energy, and for this purpose we choose ^{26}Mg, a nucleus near the middle of the *sd*-shell. The object is to see to what extent and to what excitation energy experimental states can be associated with states calculated in the *sd*-shell basis. Rates for the astrophysical rapid-proton-capture process depend upon calculations of γ widths for levels near the proton decay thresholds [5,6]. This study provides an example of the applicability and the accuracy of calculations limited to the *sd* shell.

Previous studies of ^{26}Mg have made extensive comparisons of predictions for the older USD interaction to data on ^{24}Mg$(p,t)^{26}$Mg [7] and high-spin states [8]. Our results are consistent with the level associations made in these works, but add much more in terms of comparison with the more recently derived interactions and in terms of more recent experimental work especially in regard to data on inelastic excitation.

For the calculation of electromagnetic transition strengths and electron scattering form factors, harmonic oscillator radial wave functions with $b = 1.769$ fm and $\hbar\omega = 13.260$ MeV have been used, and effective charges and *g* factors have been obtained from fits to large numbers of data (moments as well as transitions) in Ref. [4], unless stated otherwise.

II. COMPARISON OF ENERGIES AND LIFETIMES OF LEVELS

In Tables I and II measured energies and half-lives of ^{26}Mg levels from Ref. [9] are compared with theoretical values. Only the energies for USDB are included, whereas the half-lives for the interactions USDA and USD are also included. The experimental energies in Tables I and II are compared with the USDB energies in Fig. 1. A significant parameter to consider when assigning theoretical levels to experimental ones, in addition to energy, is the level lifetime. Where a definite association has not been made, or the spin/parity of the level is uncertain, the experimental energy is left blank. Spin/parity indications are only given in the experimental column where there is some uncertainty about the assignment. The first negative parity level (not included in Table I) is the 3^- state at 6.876 MeV.

All experimental positive parity states up to 8 MeV are included in Table I. Up to this energy, levels have a definite spin-parity assignment with the exceptions of the 6.634 MeV $(0\text{ to }4)^+$ state, the 7.200 MeV $(0,1)^+$ state, and the 7.816 MeV $(2,3)^+$ state. For the observed 6.634 MeV state, theory predicts a 1^+ state in this energy region, and the half-lives lie within the observed upper limits. The 7.200 MeV $(0,1)^+$ states does not appear to have a theoretical counterpart and thus may be the first "intruder" positive parity state, related to two nucleons excited from the $0p$ shell or

TABLE I. Levels in ^{26}Mg. Experimental energies and lifetimes are from Ref. [9], except where a new energy is suggested by the present analysis. Theoretical energies are based on USDB.

n	Energy USDB (MeV)	Energy exp (MeV)	n_J	J^π USDB	J^π exp	$T_{1/2}$ USDB (psec)	$T_{1/2}$ exp (psec)	$T_{1/2}$ USDA (psec)	$T_{1/2}$ USD (psec)
2	1.897	1.809	1	2^+		0.333	0.476(12)	0.299	0.299
3	3.007	2.938	2	2^+		0.137	0.141(8)	0.121	0.0993
4	3.635	3.589	2	0^+		28.689	6.44(14)	11.499	40.002
5	3.883	3.942	1	3^+		1.434	0.85(12)	1.566	2.503
6	4.317	4.350	2	3^+		0.0743	0.105(20)	0.0983	0.0631
7	4.365	4.319	1	4^+		0.316	0.272(16)	0.308	0.176
8	4.450	4.333	3	2^+		0.0437	0.020(3)	0.0326	0.0576
9	4.882	4.835	4	2^+		0.0355	0.028(6)	0.0320	0.0626
10	4.939	4.901	2	4^+		0.0328	0.029(6)	0.0398	0.0411
11	5.034	4.972	3	0^+		0.464	0.440(60)	0.767	0.423
12	5.386	5.292	5	2^+		0.007257	<0.010	0.00643	0.00870
13	5.523	5.476	3	4^+		0.0218	0.021(6)	0.0261	0.0278
14	5.716	5.691	1	1^+		0.003370	<0.008	0.00314	0.00352
15	5.893	5.716	4	4^+		0.0474	0.070(35)	0.0511	0.0430
16	6.133	6.256	4	0^+		0.0580	0.052(24)	0.0623	0.0914
17	6.180	6.125	3	3^+		0.003657	0.014(6)	0.00410	0.00500
18	6.620	6.634	2	1^+	$(0–4)^+$	0.004029	<0.007	0.00595	0.00467
19	6.677	6.746	6	2^+		0.005201	0.016(8)	0.00426	0.00326
20	6.730	6.622	5	4^+		0.0191	0.019(5)	0.0184	0.0265
21	6.910	7.100	7	2^+		0.003299	<0.014	0.00357	0.00305
22	7.068	6.978	1	5^+		0.0180	0.014(5)	0.0193	0.0185
		7.200			$(0,1)^+$				
23	7.149	7.371	8	2^+		0.005182		0.00626	0.00437
24	7.296	7.242	4	3^+		0.001975	<0.007	0.00267	0.00189
25	7.388	7.395	2	5^+		0.0122	<0.014	0.0123	0.0122
26	7.434	7.677	6	4^+		0.002424	<0.010	0.00337	0.00355
27	7.573		9	2^+		0.000943		0.00098	0.00222
		7.428			$(0,1)^+$				
28	7.699	7.726	5	3^+		0.003207		0.00204	0.00318
29	7.856	7.773	7	4^+		0.002492	<0.070	0.00198	0.00292
		7.816			$(2,3)^+$				
30	7.926		3	1^+		0.001291		0.001724	0.00112
31	8.040		5	0^+		0.006620		0.00463	0.00432
32	8.126	8.201	1	6^+		0.0202	<0.014	0.0182	0.0172
33	8.222	8.251	6	3^+		0.000388		0.00318	0.000652
34	8.340		10	2^+		0.000995		0.000795	0.000572
35	8.396		4	1^+		0.000273		0.000401	0.000357
36	8.418	8.459	7	3^+		0.001072		0.000292	0.001277
37	8.444		3	5^+		0.003161		0.00310	0.002869
38	8.462	8.472	2	6^+		0.0175	<0.014	0.0020	0.0248
39	8.584	8.706	8	4^+		0.000430		0.000487	0.000538
30	8.809	8.930	9	4^+		0.003415		0.00238	0.003173
41	8.820		6	0^+		0.004467		0.00518	0.007146

to the $1p0f$ shell. Above 8 MeV there are many experimental states with uncertain spin and/or parity assignments, so the association of experimental and theoretical states above 8 MeV is made on the basis of those selectively populated in electron scattering or β decay as discussed in the following sections.

The half-lives are based on calculations of all possible decays from a given level ($M1$ and $E2$) and were calculated with the programs NuShell and DENS [3]. Effective g factors and charges determined from least-square fits to a large number of data (moments as well as transitions) have been used [4]. The calculated half-lives are based on the theoretical energy levels for the electromagnetic phase-space factors. In most cases the differences obtained if experimental energies were used would be 10% or less. Up to about 7 MeV there is a good correspondence generally between experimental and calculated half-lives, and where there are only upper limits for experiment, the theory values lie within the limits. The

TABLE II. Levels in ^{26}Mg—continuation of Table I.

n	Energy USDB (MeV)	Energy exp (MeV)	n_J	J^π USDB	J^π exp	$T_{1/2}$ USDB (psec)	$T_{1/2}$ exp (psec)	$T_{1/2}$ USDA (psec)	$T_{1/2}$ USD (psec)
42	8.846	8.864	11	2^+		0.000563		0.000764	0.00876
43	9.048	9.064	4	5^+		0.000735		0.000832	0.00129
44	9.098	9.111	3	6^+		0.009448	<0.010	0.0100	0.00897
45	9.139	9.261	10	4^+		0.000530		0.00238	0.000553
46	9.160		8	3^+		0.000385		0.000467	0.000440
47	9.241		5	1^+		0.000394		0.000595	0.000418
48	9.272		12	2^+		0.000404		0.000474	0.000422
49	9.275		11	4^+		0.000765		0.000531	0.000789
50	9.327	9.24	6	1^+		0.000176		0.000129	0.000308
51	9.390		9	3^+		0.000917		0.000743	0.00167
52	9.402		12	4^+		0.000880		0.000562	0.000945
53	9.483		13	2^+		0.001731		0.000366	0.00307
54	9.511		7	1^+		0.000344		0.0001898	0.000288
55	9.523	9.541	5	5^+		0.004885		0.00429	0.00572
56	9.541		14	2^+		0.000371		0.000981	0.000250
57	9.584		10	3^+		0.000459		0.000502	0.000484
58	9.614		6	5^+		0.003004		0.002601	0.00257
59	9.615	9.383	4	6^+		0.0108	<0.007	0.0108	0.011
60	9.626	9.829	1	7^+	$(5,7)^+$	0.0722	0.037(10)	0.0695	0.0533
61	9.789		15	2^+		0.000296		0.000327	0.000909
62	9.877		13	4^+		0.000451		0.000392	0.000893
63	9.939		7	0^+		0.004347		0.00276	0.00401
64	9.941	9.989	5	6^+		0.005650	<0.007	0.00635	0.00790
65	9.963		11	3^+		0.000860		0.000455	0.00105
66	10.058	10.2	8	1^+		0.000083		0.000061	0.000060
76	10.515	10.65	9	1^+		0.000059		0.000072	0.000485
102	11.460	11.2	13	1^+		0.000053		0.000291	0.000088
110	11.712		1	8^+		0.032		0.017	0.034
139	13.31	13.33	$T = 2$	1^+					

ratios of experimental to theoretical half-lives are plotted in Fig. 2 for the first 21 states of USDB (up to 7.4 MeV experimental energy). Where there are only experimental limits the range of possible ratios are shown as vertical lines. With a few exceptions, particularly where the experimental errors are very large, the ratios of USDA and USDB lie fairly close to the value of 1, indicating good overall agreement between theory and experiment. There is a larger scatter for USD.

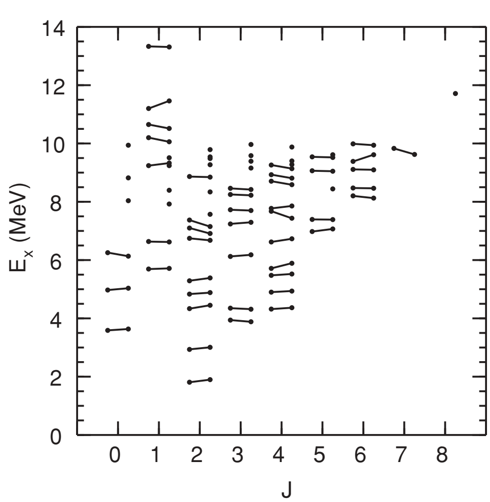

FIG. 1. Comparison of experimental energies (left) vs the USDB theoretical energies (right) for positive parity states with angular momentum J, based on the associations made in Tables I and II. All theoretical levels up to 10 MeV are shown. Levels above 10 MeV correspond to 1^+ levels shown in Fig. 6 and the lowest 8^+ level.

The largest deviation is for the 0_2^+ state (the subscript indicates the order of the level for a given J^π value); level number four in Table I. For all of the interactions the de-excitation from the second 0^+ state occurs almost exclusively via decay to the lowest 2^+ state. Compared to the experimental $B(E2)$ 0_2^+ to 2_1^+ value of 4.92(11) e^2 fm^4, the calculated $B(E2)$ values for these transitions vary considerably; 3.21 e^2 fm^4 for USDA, 1.24 e^2 fm^4 for USDB, and 0.86 e^2 fm^4 for USD, leading to corresponding variations in the lifetimes with USDA being in best agreement. These $B(E2)$ values are very small compared with $B(E2)$ 0_1^+ to 2_1^+ with an experimental value of 307(9) e^2 fm^4. The large variation of the $B(E2)$ 0_2^+ to 2_1^+ values means that the experimental value for this transition might be used as a constraint in a global fit to the effective Hamiltonian. This together with the constraints from low-lying GT and $M1$ matrix elements as pointed out in Ref. [4] might in the future be used to obtain a more precise empirical sd-shell Hamiltonian.

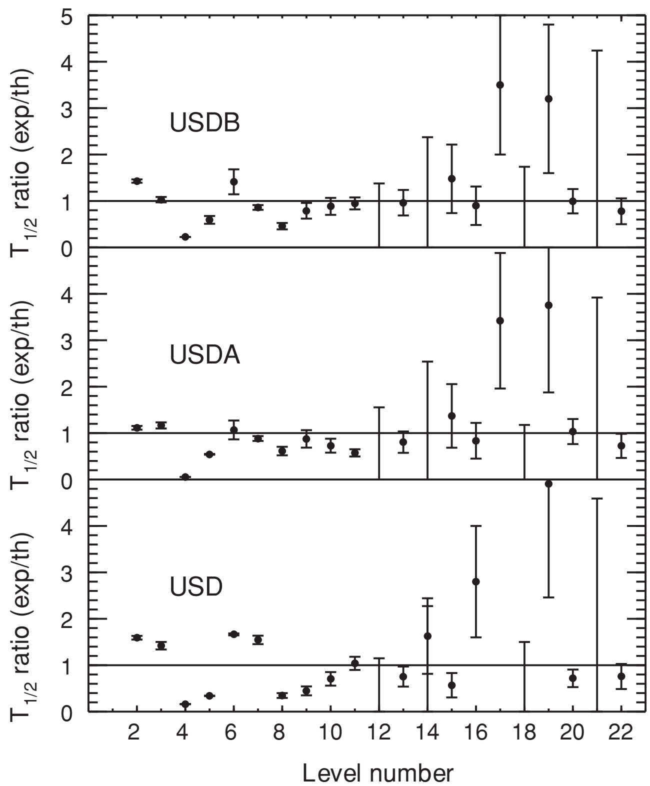

FIG. 2. Ratios of half-lives versus level number n (as given in Table I).

A comparison of calculated branching ratios with values from Ref. [9] reveals that in general there is a reasonable correspondence in terms of which branches are dominant. The branching ratios for the different interactions mostly differ by a few percent. For the predicted 1_2^+ state [experimentally the spin possibilities are $(0–4)^+$] the branching ratio agreement is quite good, which further supports our assignment of the observed state at 6.634 MeV as 1^+. For the third, fourth, and fifth 1^+ states there are no experimental counterparts. For the experimental state indicated as $1^{(+)}$ at 9.239 MeV the branching ratio agreement with our 1_6^+ is also quite reasonable. Above 10 MeV there are several suggested 1^+ states at the end of Table II. Based on a comparison of $B(M1)$ strengths from (e,e') and (γ,γ') scattering in Sec. IV the 1^+ state at an experimental energy of 10.2 MeV could be associated with either of the observed states at 10.148 or 10.319 MeV. However, a comparison of branching ratios favors the 10.148 MeV assignment. The observed state at 10.646 MeV has a measured branching ratio that agrees well with theory. The assignment of the next two 1^+ states are based on scattering data. The 13.33 MeV state is a $T = 2$ state.

For the first eight 2^+ states theory produces quite reasonable branching ratios, with 2_7^+ being the least satisfactory. The state 2_9^+ has no evident experimental counterpart. However, $B(E2)$ data in Sec. IV suggest a 2^+ state at 7.428 MeV—at this energy there is an observed state indicated as $(0,1)^+$. The next predicted 2_{10}^+ state (8.34 MeV for USDB) has branching ratios that cannot be associated with any of the observed 2^+ states in the same energy region—these observed states may well be intruder states. The state 2_{11}^+ has a good branching ratio agreement with the observed 2^+ state at 8.864 MeV, which substantiates this assignment. For higher energies there are no evident experimental counterparts.

For the first five 3^+ states there is good to reasonable agreement for branching ratios. In Table I we have made an association between two observed states at 8.251 and 8.459 MeV and the theoretical states 3_6^+ and 3_7^+, respectively. For USDB there is fair agreement with the measured branching ratios, but for USDA and USD the results for the two states are switched. Because the two states are fairly close in energy this shift in mixing can readily occur.

For the first ten 4^+ states that we have associated with experimental counterparts the agreement for the branching ratios is fair to good. For higher energies there are no experimental counterparts as yet. For the first two 5^+ states in Table I the branching ratios agree very well. The third state at 8.444 MeV (USDB) has not been matched. For the next two states at experimental energies of 9.064 and 9.541 MeV, the branching ratio agreement is excellent in the case of the former state, but only reasonable for the latter. Higher states are not matched. For the first three 6^+ states in Table I the agreement for the branching ratios is very good. For the fourth state at an experimental energy of 9.383 MeV the agreement is fair, and for the fifth state at 9.989 MeV the agreement is good. For the experimental assignment at 9.829 MeV of $(5,7)^+$ we suggest a 7^+ state—the branching ratio agreement is very good for the different interactions, which supports our assignment.

III. COMPARISON WITH FORM FACTORS FROM ELECTRON SCATTERING

Inelastic electron scattering to excited states in ^{26}Mg is one of the key methods in making associations between experimental and theoretical energy levels. In the following sections we consider available electron scattering data for states of various spins. Details of the form factor calculations are given in Ref. [10] for $E2$ and $E4$ and in Ref. [11] for $M1$, $M3$, and $M5$. For $E2$ and $E4$ we use the Tassie-model form factor for the core-polarization contribution [10]. For these multipoles the shape and the experimental and theoretical form factors are similar up through the first minimum. Thus for making comparisons between experimental and theoretical strengths is it practical to compare the magnitude of the first maximum in the respective form factors. The error bars are generally small and only indicated if significant.

For $E0$ transitions to excited 0^+ states the calculated form factors have a completely different shape compared to that of the experimental form factors [12]. The matrix elements of the first term in the expansion of the operator $\Sigma_i\, j_0(qr_i)$ [proportional to $\Sigma_i(q\, r_i)^0 = 1$] is zero because the wave functions are orthogonal. The second term in the expansion [proportional to $\Sigma_i(q\, r_i)^2$] has matrix elements that are zero with oscillator radial wave functions and are small with Woods-Saxon or Hartree-Fock radial wave functions [12]. The shape of the theoretical form factor at low q up to the peak of the first maximum is completely determined from transitions between orbitals outside the *sd* shell (such as the 0*d* to 1*d* orbitals discussed in Ref. [12]) arising from core polarization. We do not consider *E*0 form factors in this work.

A. Scattering to 3^+ states

Electron scattering from the 0^+ ground state of ^{26}Mg to 3^+ states has provided assignments of several new 3^+ states. Form factors and a set of $B(M3)$ values are given in Refs. [13] and [14]. (An eighth 3^+ state was assigned to an observed level at 9.042 MeV in Ref. [13], but an assignment of 2^- was made for this state in Ref. [14].)

The magnitudes of the first maxima of the transverse form factors are compared in Fig. 3 with values from the new interactions USDA and USDB, as well as with the USD interaction, using free-nucleon *g* factors. The vertical bins serve as a guide to the eye. For the first seven states, up to 9 MeV, it is evident that there is an unambiguous correspondence between experiment and theory. These states have been included in Table II. Above 9 MeV the calculated *M*3 strength is small and it is evident that some of the states predicted are too weak to be observed experimentally.

It was claimed in Ref. [13] that no overall quenching in the $B(M3)$ strength had been found. However, because we have used free-nucleon *g* factors it is evident from Fig. 3 that there is some quenching at the peak of the first maximum in the *M*3 form factors. The quenching factors for the summed strength are given, respectively, by 0.73 (USDA), 0.69 (USDB), and 0.76 (USD).

B. Scattering to 5^+ states

Electron scattering from the 0^+ ground state of ^{26}Mg to 5^+ states is considered next. The magnitudes of the first maxima of the transverse form factors from Ref. [14] are compared in Fig. 4 with values calculated from the three interactions. Free-nucleon *g* factors were used. The experimental peaks at 6.978, 9.064, and 9.541 MeV have counterparts in the shell-model calculations. The experimental strength for the 9.541 MeV has a large error and is not in disagreement with the small theoretical value. The two states predicted by theory around 7.4 and 8.4 MeV are presumably too weak to be observed experimentally. The first might be associated with the observed 5^+ state at 7.395 [9] on the grounds of similar energies.

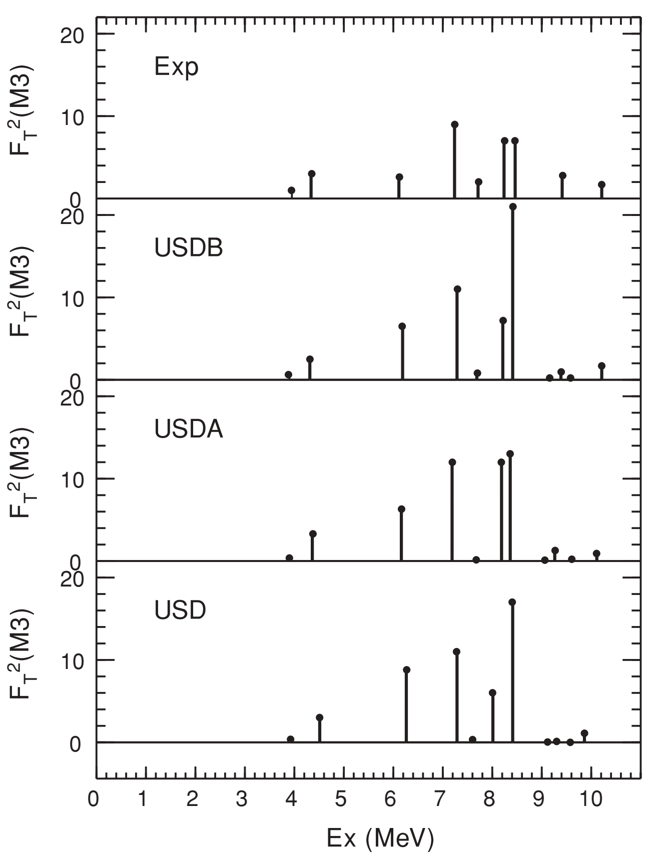

FIG. 3. Comparison of the first maxima of the experimental transverse form factors $F_T^2(M3)$ (multiplied by 10^6) with values from the interactions USDA, USDB, and USD. Free-nucleon *g*-factors were used.

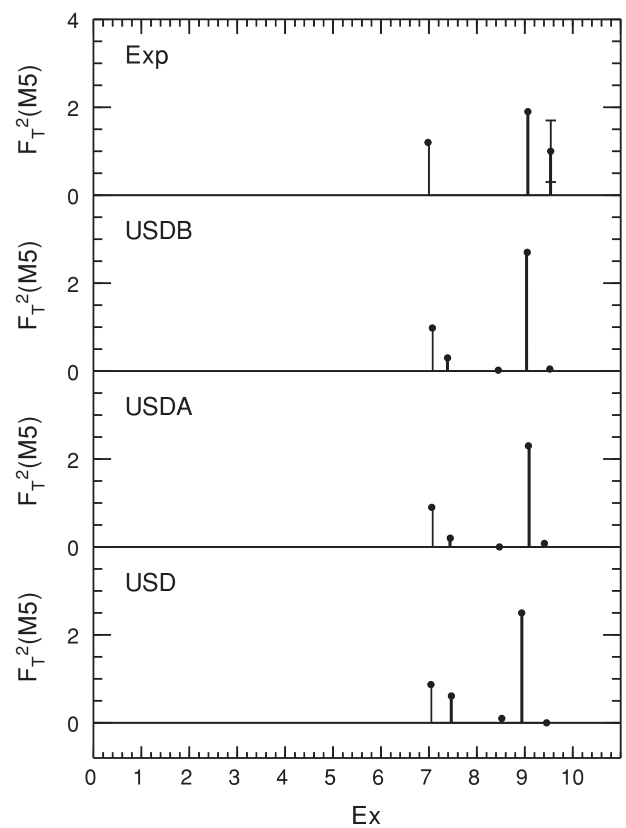

FIG. 4. Comparison of the first maxima of the experimental transverse form factors $F_T^2(M5)$ (multiplied by 10^6) with values from the interactions USDA, USDB, and USD.

IV. COMPARISON OF REDUCED TRANSITION STRENGTHS

A. Results for $B(M1)$ strengths for excitation of 1^+ states

$B(\sigma)$ values have been extracted for (p, p') scattering cross sections from the ground state of ^{26}Mg to 1^+ states up to about 15 MeV excitation [15]. $B(\sigma)$ corresponds to the $B(M1)$ reduced transition strength with the orbital contributions neglected. The qualitative proportionality between $B(\sigma)$ and (p, p') cross sections is due to the dominance of the $\sigma\tau$ part of the nucleon-nucleon interaction for proton energies around 200 MeV [16]. This proportionality is not better than about 20% because the cross sections also depend on the smaller central σ, spin-orbit, and tensor parts of the interaction, because of exchange contributions [16] and because of distortion (e.g., the interaction is dominated by the surface part of the transition density). Thus the comparison between experimental and theoretical $B(\sigma)$ values may be influenced by these factors and must be made on a qualitative level.

The $B(\sigma)$ values are shown in Fig. 5. For the 13.33 MeV excitation there is an obvious correspondence with the shell-model counterparts. However, there are numerous small theoretical values above about 11.5 MeV, and also several below about 8.5 MeV, that can easily escape detection experimentally. Some of the stronger theoretical excitations differ in energy and magnitude, making it difficult to make associations with experiment.

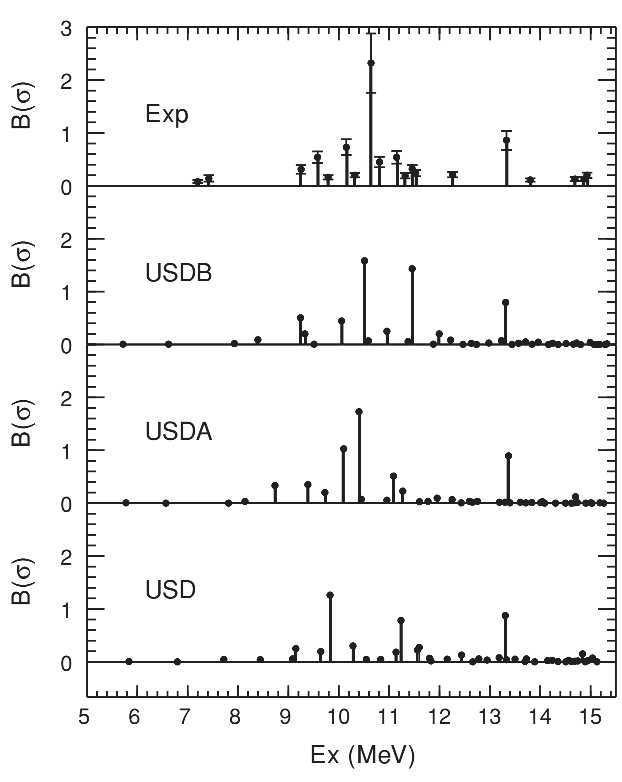

FIG. 5. Comparison of $B(\sigma)$ values (in units of μ_N^2) from (p, p') cross sections with values from the interactions USDA, USDB, and USD.

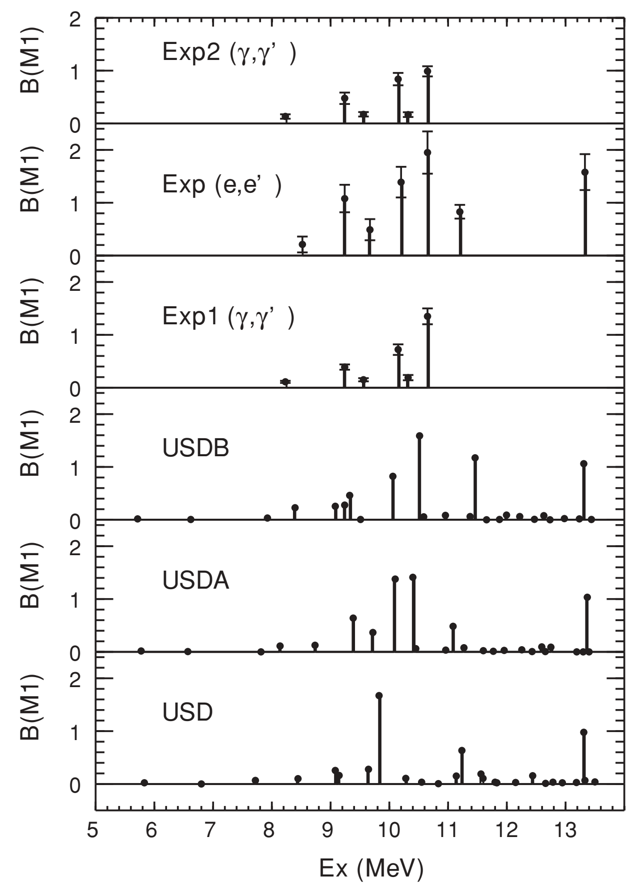

FIG. 6. Comparison of $B(M1)$ values (in units of μ_N^2) from (γ, γ') and (e, e') reactions with values from the interactions USDA, USDB, and USD.

Comparisons with $B(M2)$ strengths extracted from (γ, γ') [Ref. [17], (Exp1); Ref. [18], (Exp2)] and (e, e') reactions [19] are given in Fig. 6. It is noticeable that there are some large discrepancies in magnitude between the (e, e') and (γ, γ') values, with the (γ, γ') values consistently reduced with respect to the (e, e') values. The two (γ, γ') sets agree well except that the $B(M1)$ value at 10.65 MeV for the Exp1 set is slightly larger than that for the Exp2 set. The 13.33 MeV state excitation for (e, e') corresponds to the 13.31 $T = 2$ state of USDB, with very similar energies for USDA and USD. Again there are many small theory values above about 11 MeV that would be difficult to see experimentally. The 11.2 MeV state for (e, e') appears to correspond to the 11.46 MeV state for USDB, with slightly different energies for USDA and USD. The strong excitation at 10.65 MeV for both (e, e') and (γ, γ'), but with rather different magnitudes, would correspond to the 10.52 MeV state in USDB, and with similar energies for USDA and USD. The next strong excitation at 10.2 MeV for both (e, e') and (γ, γ') evidently corresponds to the 10.06 MeV state for USDB, with a similar energy for USDA and a slightly higher energy for USD (10.28 MeV). There are two confirmed 1^+ states in this energy region listed in Ref. [9], at 10.148 MeV and at 10.319 MeV, and this excitation could be associated with either of these. The excitation of a state observed at 9.67 MeV for (e,e') has no counterpart for (γ,γ'), and vice versa for a state observed at 9.56 MeV for (γ,γ'); therefore no conclusions can be drawn in these cases. The state observed at 9.24 MeV probably corresponds to the 9.33 MeV state for USDB, with a similar energy for USDA. The small excitations for (e,e') at 8.52 MeV and for (γ,γ') at 8.23 MeV do not seem to have obvious counterparts with theory. The suggested correspondences are included in Table II.

The comparison of $B(\sigma)$ and $B(M1)$ can provide information on the orbital part of the transitions and mesonic-exchange currents [20–22]. Because the extraction of $B(\sigma)$ from proton scattering cross sections is not exact, and the $B(M1)$ values depend upon mesonic-exchange currents that are contained in the effective g factors [20], this comparison is beyond the scope of the present work.

B. Results for $B(E2)$ strengths for excitation of 2^+ states

Comparisons of $B(E2)$ values from Ref. [14] are given in Fig. 7 with values from the three interactions. Optimal effective charges based on the fits in Ref. [4] have been used ($\Delta e_p = 0.36$, $\Delta e_n = 0.45$). Note that the $B(E2)$ value of the first excitation to the 2^+ state at 1.809 MeV should be multiplied by a factor of 20. A good correspondence can be seen between theory and experiment for the first 9 states; for the ninth state, which is assigned $(0,1)^+$ at 7.428 MeV in the compilation of Ref. [9], we suggest a spin-parity of 2^+. For higher energies the theoretical $B(E2)$ values are small and assignments between theory and experiment are less certain, except for the excitation at 8.86 MeV, which corresponds well with the predictions of the three interactions. The large $B(E2)$ seen at 9.88 MeV is not reproduced by any of the three interactions. This state might be associated with the low-energy part of the $2\hbar\omega$ $E2$ giant resonance excitation that is not explicitly included in the sd-shell calculations (it is implicitly included in terms of the effective charges for low-lying states).

C. Results for $B(E4)$ strengths for excitation of 4^+ states

Comparisons of $B(E4)$ values from Ref. [14] with values from the three interactions are given in Fig. 8. Effective charges of $\Delta e_p = 0.35$ and $\Delta e_n = 0.35$ have been used. For the first five 4^+ states there is a clear correspondence between experiment and theory which corroborates the assignments in Table I. The next two experimental states at 7.68 and 7.77 MeV appear to be associated with the two USDB states at 7.434 and 7.856 MeV, respectively. Starting at 8.7 MeV there is a group of three excitations at measured energies of 8.706, 8.930, and 9.261 MeV, of which the pattern is reproduced in all three interactions: for USDB the corresponding energies

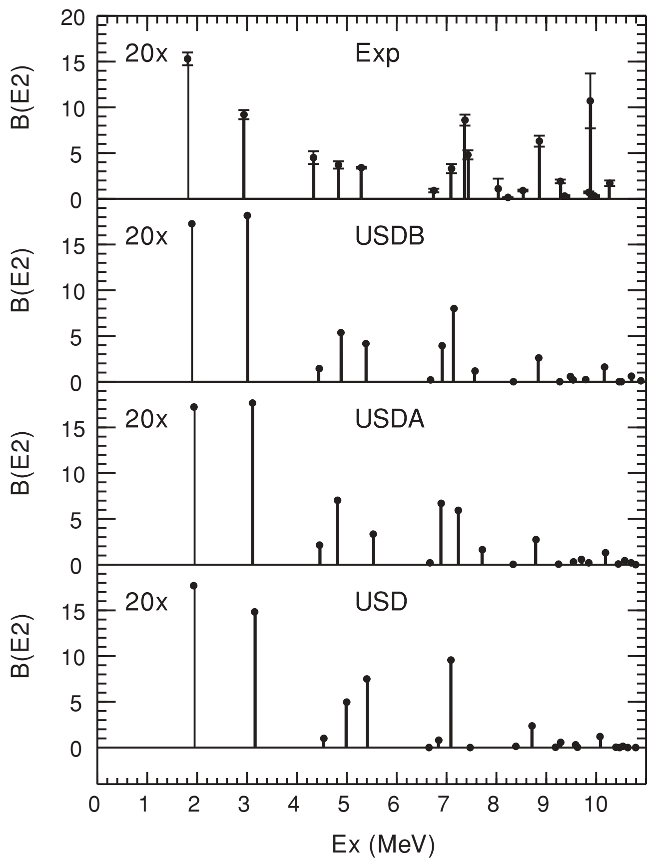

FIG. 7. Comparison of $B(E2)$ values (in units of e^2 fm^4) from Ref. [14] with values from the interactions USDA, USDB, and USD.

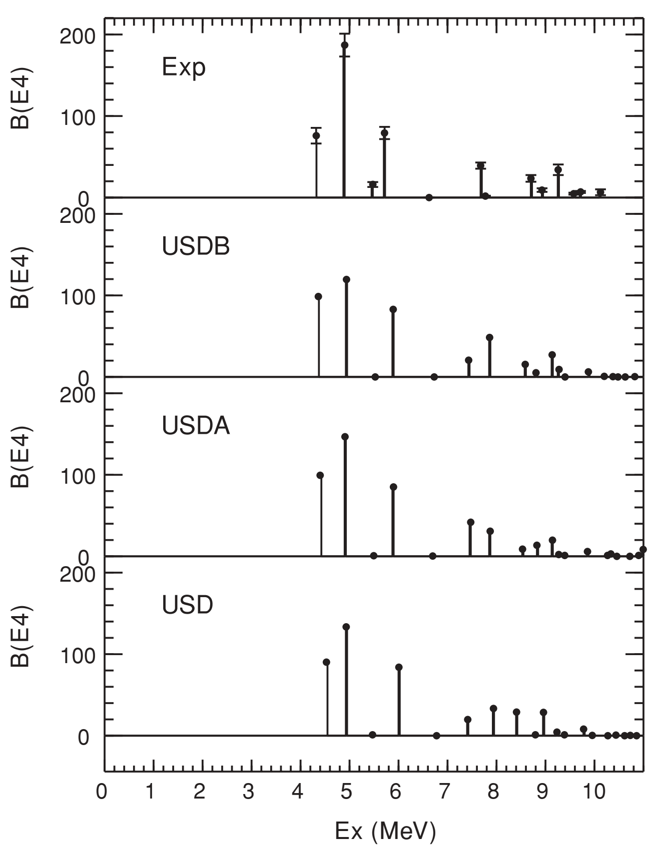

FIG. 8. Comparison of $B(E4)$ values (in units of 10^2 e^2 fm^8) from Ref. [14] with values from the interactions USDA, USDB, and USD.

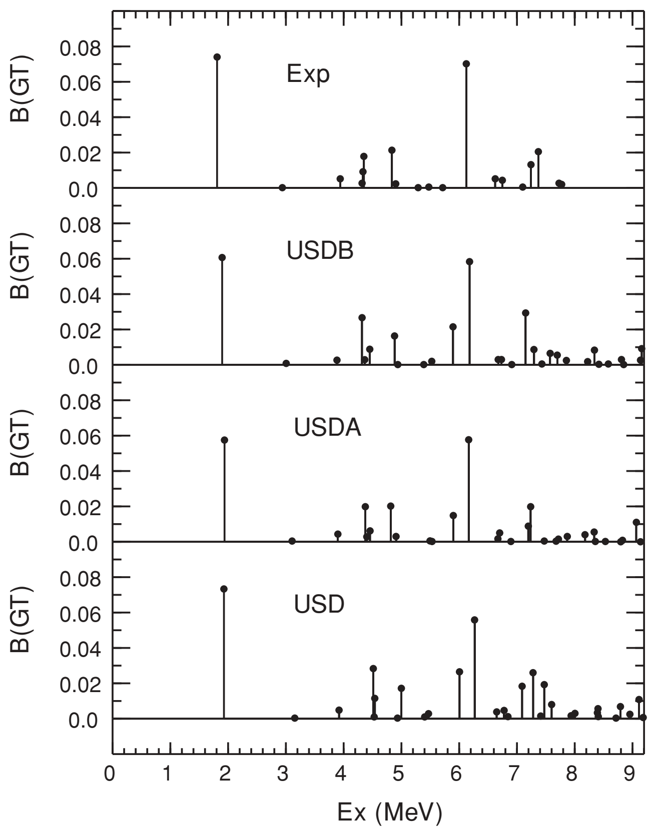

FIG. 9. Comparison of experimental $B(\mathrm{GT})$ values from Ref. [23] with values from the interactions USDA, USDB, and USD.

are 8.584, 8.809, and 9.139 MeV, respectively. These are included in Tables I and II. For higher energies both the theory and experiment give small $B(E4)$ values, which makes associations uncertain.

V. RESULTS FOR GAMOW-TELLER STRENGTHS OF β DECAY OF ^{26}Na TO STATES IN ^{26}Mg

High-precision measurements of the half-life and β-branching ratios for the β^- decay of ^{26}Na to ^{26}Mg have been made in β-counting and γ-decay experiments, respectively [23]. $B(\mathrm{GT})$ values for decays from the 3^+ ground state of ^{26}Na to 2^+, 3^+, and 4^+ states were extracted from the *ft* values determined and are compared with the different interactions in Fig. 9. We used effective operators for the calculated Gamow-Teller matrix elements with q_{GT} given in Ref. [4]. There is a good general correspondence between theory and experiment, with USDA giving the best overall agreement.

VI. CONCLUSIONS

Using the new sd-shell interactions USDA and USDB, as well as the older USD interaction, assignments between theory and experiment of corresponding levels in ^{26}Mg levels have been confirmed and new ones suggested. A comprehensive summary of corresponding levels is given in tables and graphs. Excitation energies up to about 10 MeV have been considered, and in some cases even higher energies, based on electron scattering data and electromagnetic transition strengths. Level lifetimes based on the detailed γ-decay transition schemes have also been provided. We have been able to make the association of theoretical levels to about 50 experimental levels. All (21) experimental positive parity states up to 7.1 MeV have a good match with theory. The first level not described by the sd-shell appears to be the 7.200 MeV $(0,1)^+$ state. Above 7 MeV many other experimental states can be associated with the sd-shell theory on the basis of comparison to inelastic scattering experiments. The complete set of theoretical states is given up to 10 MeV but many above 8 MeV cannot be associated with experiment due to the uncertainties in the experimental spin-parity assignments as well as the fact that many of the observed states may not be described by sd-shell configurations. Overall the new interactions USDA and USDB are better than USD with regard to detailed comparison with data. The differences between USDA and USDB appear to provide a reasonable estimate of the theoretical error in predicting the observables and we recommend that they both be used for future comparison to experiment and astrophysical applications.

ACKNOWLEDGMENTS

This work is partly supported by NSF Grant PHY-0758099 and the National Research Foundation of South Africa under Grant 2073007.

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