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Chiral asymmetry of the Fermi surface in dense relativistic matter in a magnetic field

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It is revealed that in the normal phase of dense relativistic matter in a magnetic field, there exists a contribution to the axial current associated with a relative shift of the longitudinal momenta in the dispersion relations of opposite chirality fermions. Unlike the topological contribution in the axial current at the lowest Landau level, recently discussed in the literature, the dynamical one appears only in interacting matter and affects the fermions in *all* Landau levels, including those around the Fermi surface. The induced axial current and the shift of the Fermi surfaces of the left-handed and right-handed fermions are expected to play an important role in transport and emission properties of matter in various types of compact stars as well as in heavy ion collisions.

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Introduction. At zero density and temperature, the structure of the ground state in relativistic chiral invariant theories in a magnetic field is dictated by the magnetic catalysis phenomenon: the magnetic field is a strong catalyst of spontaneous chiral symmetry breaking [1,2]. The situation becomes much more complicated in the case of dense relativistic matter in a magnetic field [3]. The main goal of this Rapid Communication is to reveal and describe some universal properties of such dynamics.

The recent studies [4] of similar dynamics in graphene in $2+1$ dimensions have revealed several types of order parameters whose analogs have not been discussed in the context of relativistic models in $3+1$ dimensions. (For earlier applications of the magnetic catalysis phenomenon in graphene, see Ref. [5].) This motivated us to reexamine the properties of dense relativistic matter in an external magnetic field in $3+1$ dimensions. As we show, an external magnetic field induces two qualitatively different contributions to the net axial current in the normal phase of dense relativistic matter. The first one is a topological contribution due to the lowest Landau level (LLL) that was previously discussed in the literature [6–9]. It exists even in free theories. (For related studies in hot quark-gluon plasma, see Ref. [10].) In this Rapid Communication we find that there is also an additional dynamical contribution to the axial current that appears only in interacting matter. The crucial new point is that *all* Landau levels, including those around the Fermi surface, contribute to this interaction-driven contribution. Its origin is related to a relative shift of the longitudinal momenta in the dispersion relations of opposite chirality fermions. The amount of the shift is proportional to a coupling constant, the magnetic field, and the fermion chemical potential. Notably, it is almost independent of temperature. As we discuss below, this effect may have profound implications for the physics of compact stars as well as heavy ion collisions.

Model. In this Rapid Communication, in order to illustrate the effect in the clearest way, a simple model will be utilized. We perform a study of a Nambu-Jona-Lasinio model with the Lagrangian density

$$\mathcal{L} = \bar{\psi}\left(iD_\nu + \mu_0\delta_\nu^0\right)\gamma^\nu\psi + \frac{G_{\text{int}}}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2], \quad (1)$$

where $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$ and μ_0 is the chemical potential. This model possesses the chiral $U(1)_L \times U(1)_R$ symmetry. The covariant derivative $D_\nu = \partial_\nu - ieA_\nu$ is taken in the Landau gauge, i.e., $A_\nu = xB\delta_\nu^2$ where B is the strength of the external magnetic field pointing in the z direction.

The structure of the full fermion propagator is given by a $(3+1)$-dimensional generalization of the ansatz used for the description of the quantum Hall effect dynamics in graphene [4], namely,

$$iG^{-1}(u,u') = [(i\partial_t + \mu)\gamma^0 - (\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) - \pi^3\gamma^3 + i\tilde{\mu}\gamma^1\gamma^2 + \Delta\gamma^3\gamma^5 - m]\delta^4(u-u'), \quad (2)$$

where $\pi_k = i(\partial_k - ieA_k)$ is the canonical momentum and $u = (t, \boldsymbol{r})$. The above propagator contains several dynamical parameters that are absent in the tree level propagator,

$$iS^{-1}(u,u') = [(i\partial_t + \mu_0)\gamma^0 - (\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) - \pi^3\gamma^3]\delta^4(u-u'). \quad (3)$$

The physical meaning of the parameters m and μ is straightforward: m is the Dirac mass and μ is the full chemical potential in interacting theory. From the structure of the inverse propagator in Eq. (2), it is clear that $\tilde{\mu}$ plays the role of the anomalous magnetic moment [in graphene, it also can be interpreted as a chemical potential related to a conserved (pseudospin) current [4]].

The meaning of the last parameter, Δ, is more subtle. In the context of graphene, Δ is a mass parameter that is odd both under time-reversal and parity transformations. In $2+1$ dimensional models, this mass is responsible for inducing the Chern-Simons term in the effective action for gauge fields [11].

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In $3+1$ dimensions, as suggested by Eq. (2), it is related to an induced axial current along the direction of the magnetic field, $\bar{\psi}\gamma^3\gamma^5\psi$. As will be shown below, Δ determines a shift of the longitudinal momenta in the dispersion relations of opposite chirality fermions in the chiral limit. We will call it a chiral shift parameter. Let us emphasize that, in the presence of an external magnetic field, the time-reversal and parity symmetries are broken and a state with the vanishing Δ is not protected by any symmetry.

The parameters m, μ, Δ, and $\tilde{\mu}$ are self-consistently determined from the gap equation, which takes the following form in the mean field approximation:

$$G^{-1}(u,u') = S^{-1}(u,u') - iG_{\text{int}}\{G(u,u) - \gamma^5 G(u,u)\gamma^5 - \text{tr}[G(u,u)] + \gamma^5\,\text{tr}[\gamma^5 G(u,u)]\}\delta^4(u-u'). \tag{4}$$

While the parameters Δ and μ do not break the chiral $U(1)_L \times U(1)_R$ symmetry, nonzero values of m and $\tilde{\mu}$ break it down to $U(1)_{L+R}$. In the mean field approximation used here, we find that $\tilde{\mu}=0$ in a self-consistent solution to the gap equation. While this fact simplifies the analysis, we note that $\tilde{\mu}$ may be nonvanishing in more refined approximations and in models with other types of interactions [4,12]. On the other hand, as suggested by the analysis in graphene, a nonzero $\tilde{\mu}$ should not change the main qualitative features of the phase with an induced Δ [4].

The spectrum of fermionic quasiparticles is determined by the poles in the full propagator (2) with $\tilde{\mu}=0$. As in Refs. [2,4], expanding the propagator over the Landau levels, we arrive at the following dispersion relation:

$$\omega_{n,\sigma} = -\mu \pm \sqrt{\left[\sqrt{m^2+k_3^2} + \sigma\Delta\,\text{sign}(eB)\right]^2 + 2n|eB|}, \tag{5}$$

where n labels the Landau levels, $\sigma=\pm 1$, and k_3 is the momentum along the direction of the magnetic field. In the chiral limit with $m=0$, the states with the quantum number $\sigma=\pm 1$ have a shifted longitudinal momentum, $k^3 \to k^3 \pm \Delta\,\text{sign}(eB)$. As will be shown below, in this limit, the two different values of σ correspond to fermions with opposite chiralities. As a result, the Fermi surfaces for left-handed and right-handed fermions become shifted.

Results. In accordance with the magnetic catalysis scenario [1,2], the ground state in the NJL model at vanishing μ_0 is characterized by a nonzero Dirac mass m_0 that breaks the chiral $U(1)_L \times U(1)_R$ symmetry. Such a vacuum state can withstand a finite stress due to a nonzero chemical potential. However, when μ_0 exceeds a certain critical value μ_c, the chiral symmetry restoration and a new ground state are expected. As we show here, the new state is characterized by a nonvanishing chiral shift parameter Δ and a nonzero axial current in the direction of the magnetic field. Since no symmetry of the theory is broken, this state describes the *normal* phase of matter that happens to have a rather rich chiral structure.

The value of the Dirac mass m_0 in the vacuum state was calculated in Ref. [2]. In the weakly coupled regime, $g \equiv G_{\text{int}}\Lambda^2/(4\pi^2) \ll 1$, the solution reads

$$m_0^2 = \frac{1}{\pi l^2}\exp\left(-\frac{\Lambda^2 l^2}{g}\right), \tag{6}$$

where $l = 1/\sqrt{|eB|}$ is the magnetic length and Λ is the ultraviolet cutoff in the model at hand. This zero temperature solution exists for $\mu_0 < m_0$. In this solution, the full chemical potential $\mu=\mu_0$.

Our analysis shows that, in addition to the solution with a nonzero Dirac mass m, the gap equation also allows a solution with $m=0$ and a nonzero chiral shift parameter Δ,

$$\Delta = g\mu\frac{eB}{\Lambda^2[1+2ag]+g|eB|}, \tag{7}$$

where a is a dimensionless constant of order 1, which is determined by the regularization scheme used in the analysis (see below). Interestingly, the temperature dependence of Δ comes only through the chemical potential, which has a weak temperature dependence when $T \ll \mu$ [13]. At $T=0$, the chemical potential satisfies the following equation:

$$\mu = \mu_0 - \frac{g}{(\Lambda l)^2}[\mu - \Delta\,\text{sign}(eB)] - \frac{2g\,\text{sign}(\mu)}{(\Lambda l)^2}\sum_{n=1}^{\infty}\sqrt{\mu^2 - 2n|eB|}\,\theta(\mu^2 - 2n|eB|). \tag{8}$$

The approximate solution to this equation is $\mu \simeq \mu_0$ up to power corrections in small g.

The free energies of the two states are given by [13]

$$\Omega_m \simeq -\frac{m_0^2}{2(2\pi l)^2}(1+(m_0 l)^2\ln|\Lambda l|) \tag{9}$$

and

$$\Omega_\Delta \simeq -\frac{\mu_0^2}{(2\pi l)^2}\left(1 - g\frac{|eB|}{\Lambda^2}\right), \tag{10}$$

respectively. In deriving the last expression, we used the approximate relations $\mu \simeq \mu_0$ and $\Delta \simeq g\mu_0 eB/\Lambda^2$. By comparing the free energies in Eqs. (9) and (10), we see that the ground state with a nonzero Δ becomes favorable when $\mu_0 \gtrsim m_0/\sqrt{2}$. This is analogous to the Clogston relation in superconductivity [14].

The properties of the two types of quasiparticles corresponding to $\sigma=\pm 1$ in the dispersion relation in Eq. (5) with $m=0$ are further clarified by the structure of the full propagator:

$$G(u,u) = G_0^-\mathcal{P}_- + \sum_{n=1}^{\infty}(G_n^-\mathcal{P}_- + G_n^+\mathcal{P}_+), \tag{11}$$

where $\mathcal{P}_\pm \equiv \frac{1}{2}[1 \pm i\gamma^1\gamma^2\,\text{sign}(eB)]$ are the spin projectors. (For $u' \neq u$, the propagator will be presented elsewhere [13].)

The functions $G_n^{\pm}$ with $n \geqslant 0$ are given by

$$G_n^{\pm} = \frac{i|eB|\gamma^0}{2\pi}\int\frac{d\omega dk^3}{(2\pi)^2} \times\left[\frac{\omega+\mu\pm[k^3-\Delta\,\text{sign}(eB)]}{(\omega+\mu)^2-2n|eB|-[k^3-\Delta\,\text{sign}(eB)]^2}\mathcal{P}_5^- + \frac{\omega+\mu\mp[k^3+\Delta\,\text{sign}(eB)]}{(\omega+\mu)^2-2n|eB|-[k^3+\Delta\,\text{sign}(eB)]^2}\mathcal{P}_5^+\right], \quad (12)$$

where $\mathcal{P}_5^{\pm} \equiv \frac{1}{2}[1 \pm \gamma^5\,\text{sign}(eB)]$ are the chirality projectors for a fixed sign of eB. As follows from this equation, the quasiparticles of opposite chiralities have dispersion relations that differ from those in the free theory by the shift of their longitudinal momentum $k^3 \to k^3 \pm \Delta\,\text{sign}(eB)$. This has profound implications for the physical properties of matter.

The ground state with $\Delta \neq 0$ is characterized by a nonvanishing expectation value of the axial current density,

$$\langle j_5^3(u)\rangle = -\text{tr}[\gamma^3\gamma^5 G(u,u)] = \frac{eB}{2\pi^2}\mu - \frac{|eB|}{2\pi^2}\Delta - \frac{|eB|}{\pi^2}\Delta\sum_{n=1}^{\infty}\kappa(\sqrt{2n|eB|},\Lambda), \quad (13)$$

where $\kappa(x,\Lambda)$ is a smooth cutoff function defined by the value of the cut-off energy Λ and a certain width of the region in which the value of the function drops from 1 down to 0 [i.e., $\kappa(x,\Lambda) \simeq 1$ for $x \ll \Lambda$ and $\kappa(x,\Lambda) \simeq 0$ for $x \gg \Lambda$]. Taking this into account, we find that $\sum_{n=1}^{\infty}\kappa(\sqrt{2n|eB|},\Lambda) = a\Lambda^2/|eB|$ and $a = O(1)$.

While the first two terms in the current density (13) come from the LLL, the last term is due to the higher Landau levels, $n \geqslant 1$. Notably, all higher Landau levels below the (smeared) cut-off energy give (nearly) identical contributions to the induced axial current. The contribution proportional to μ is topological in nature and appears even in the free theory [7]. All other terms, which are proportional to Δ, are the result of interactions and have not been revealed in the literature before.

We expect that the interaction-driven contributions can strongly modify transport and emission properties of dense relativistic matter in a magnetic field. Indeed, the corresponding induced axial currents are the consequence of a relative longitudinal flow of opposite chirality quasiparticles, including those in higher Landau levels around the Fermi energy. This is in contrast to the role of the topological contribution that is exclusively due to the LLL, which is to a large degree quenched from the low-energy dynamics by the Pauli exclusion principle in many realistic cases.

Discussion and Summary. In this Rapid Communication we found that, in accordance with the magnetic catalysis scenario, the vacuum state of relativistic matter in a magnetic field is characterized by a nonzero Dirac mass given by Eq. (6) at weak coupling. However, when the chemical potential exceeds a certain critical value, $\mu_c \simeq m_0/\sqrt{2}$, such a state is replaced by the normal ground state that is characterized by the following two properties: (i) the presence of an induced axial current along the magnetic field and (ii) the presence of the dynamically generated chiral shift parameter Δ, which is a $3+1$ dimensional analog of the parity odd mass term in $2+1$ dimensions leading to the Chern-Simons term. We find that, in addition to the previously known topological term in the induced current, there are also interaction-driven contributions from the lowest as well as from the higher Landau levels. In fact, the newly found contributions are directly related to a dynamically generated value of the chiral shift parameter $\Delta \simeq g\mu_0 eB/\Lambda^2$. This parameter quantifies the relative shift of the longitudinal momenta in the dispersion relations of opposite chirality quasiparticles.

It might be appropriate to mention that, for instructional purposes, our study here is simplified: we used an NJL-type local interaction and utilized the mean-field approximation. These limitations may lead to the results that are not always quantitatively reliable, e.g., in the context of stellar matter. Nevertheless, it is expected that our results should remain qualitatively the same even when more realistic models are used. Indeed, the fact that the expression for Δ in Eq. (7) is linear in g in the lowest order indicates that the corresponding dynamics is essentially perturbative. Apparently this is a general feature that should not depend on whether the interactions are short range, as in the NJL model, or long range, as in QCD or QED. In either case, a vanishing Δ is not protected by any symmetry.

Another limiting assumption of this study is the exact chiral symmetry. However, most of the results are not modified much when the symmetry is at least approximate, i.e., when the fermions have nonzero bare Dirac masses, but such masses are small compared to the value of the chemical potential [13]. For the applications in protoneutron stars suggested below, this approximation is justified, but the general study will be of interest, e.g., in relation to the electron plasma in white dwarfs.

In the future, it will be also of interest to address a possible interference of the dynamics responsible for the generation of the chiral shift parameter with color superconductivity in quark matter [15]. Here we just mention that the normal ground state with a nonzero Δ seems to be the only possibility at higher temperatures, which are of main interest for us here.

We expect that the generation of the chiral shift parameter may affect physical properties of the quark matter in quark and/or hybrid stars, the electron gas in neutron stars, and possibly even the electron gas in white dwarfs. The corresponding fermionic systems are degenerate ($T \ll \mu$) and the results of this Rapid Communication apply directly. It is appropriate to mention, however, that the chiral shift parameter does not vanish even in the nondegenerate limit ($T \gg \mu$), although the analysis of the dynamics becomes more involved [13]. Therefore, the generation of a nonzero Δ can also affect the chiral magnetic effect in heavy ion collisions [10].

One of the consequences of the phenomenon discussed in this Rapid Communication is the possibility of a qualitatively new mechanism for the pulsar kicks [16]. In the presence of a magnetic field, almost any type of relativistic matter inside a protoneutron star should develop axial currents as in Eq. (13). The main carriers of such currents are the electrons in the nuclear matter, and the quarks together with the electrons in the deconfined quark matter. Since the induced currents and the chiral shift parameter have only a weak temperature dependence (assuming $T \ll \mu$), this phenomenon may provide a robust anisotropic medium even at the earliest stages of the protoneutron star. This is important because moderately hot matter with 10 MeV $\lesssim T \lesssim$ 50 MeV, present during the first few tens of seconds of the protoneutron star evolution [17], may have a large enough amount of the thermal energy to power the strongest (with $v \gtrsim 1000$ km/s) pulsar kicks observed [16]. In contrast, the constraints of the energy conservation make it hard, if not impossible, to explain such kicks if the interior matter is cold ($T \lesssim 1$ MeV). The common difficulty of using a hot matter, however, is the very efficient thermal isotropization that washes out a nonisotropic distribution of neutrinos produced by almost any mechanism [18,19]. In the mechanism proposed in this Rapid Communication, however, the asymmetric distribution of the neutrinos *develops* as a result of their multiple elastic scattering on the left-handed electrons or quarks, flowing in the whole bulk of the stellar matter in one direction along the magnetic field.

In passing, let us mention that the robustness of the axial currents in hot magnetized matter may also provide an additional neutrino push to facilitate successful supernova explosions as suggested in Ref. [20]. Further details of this mechanism will be discussed elsewhere [13].

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