# On-shell consistency of the Rarita-Schwinger field formulation

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We prove that any bilinear coupling of a massive spin-3/2 field can be brought into a gauge-invariant form suggested by Pascalutsa by means of a nonlinear field redefinition. The corresponding field transformation is given explicitly in a closed form and the implications for chiral effective field theory with explicit $\Delta(1232)$ isobar degrees of freedom are discussed.

A consistent formulation of a field theory involving interacting spin-3/2 degrees of freedom is an old and much studied problem (see, e.g., Refs. [1–3]). Recently, it has attracted renewed interest in the context of chiral effective field theory of QCD with explicit $\Delta(1232)$ isobar degrees of freedom and its applications to, for example, pion-nucleon scattering or photonuclear processes (see Refs. [4,5] for recent review articles). In the commonly used Rarita-Schwinger formalism, the spin-3/2 field is represented by a vector-spinor $\psi_\mu$ (where here and in what follows, we omit the spinor indices). It is well known how to write the most general free Lagrangian for $\psi_\mu$ that describes the proper number of degrees of freedom [6]. The unphysical spin-1/2 degrees of freedom are projected out in the resulting free equations of motion. It is much more difficult to ensure the decoupling of the unphysical degrees of freedom in the case of interacting spin-3/2 fields. An elegant way to achieve this goal is to require that all interactions have the same type of gauge invariance as the kinetic term of the spin-3/2 field [7]. This requirement of gauge invariance is, however, not compatible with the nonlinear realization of the chiral symmetry [8,9], which is commonly adopted in chiral effective field theory and ensures the chiral invariance of the effective Lagrangian on a term-by-term basis. In this context, an important observation was made in Ref. [10], where it was shown that any gauge-noninvariant linear coupling of the spin-3/2 fields can be brought into the gauge-invariant form via a suitably chosen field redefinition. In that work, a conjecture was made that such a field redefinition should also be possible for bilinear couplings, but no explicit form for this transformation was given. In this work we fill this gap and prove that such a transformation indeed exists for arbitrary bilinear couplings. Moreover, we are able to give the transformation explicitly in a closed form.

The most general Lagrangian for spin-3/2 fields can be written in the following form (after setting the point-transformation parameter $A = -1$) [6,11]:

$$\mathcal{L} = \bar{\psi}^\mu((i\partial)_{\mu\nu} - m_{\mu\nu} + V_{\mu\nu})\psi^\nu = \bar{\psi}(i\partial\!\!\!/ - m + V)\psi. \quad (1)$$

Here and in what follows, we use the short-hand notation

$$(i\partial\!\!\!/)^{\mu\nu} = \gamma^{\mu\nu\alpha}i\partial_\alpha, \quad m^{\mu\nu} = \gamma^{\mu\nu}m, \quad (2)$$

with $m$ being the mass of the spin-3/2 field and $\gamma^\nu$ the Dirac matrices. The quantities $\gamma^{\mu\nu}$, $\gamma^{\mu\nu\alpha}$ are defined according to $\gamma^{\mu\nu} = [\gamma^\mu, \gamma^\nu]/2$, $\gamma^{\mu\nu\alpha} = \{\gamma^{\mu\nu}, \gamma^\alpha\}/2$ and are completely antisymmetric with respect to the Dirac indices. We will also need the inverse of the mass operator $m^{\mu\nu}$, which in $d$ dimensions takes the form

$$\left[\frac{1}{m}\right]^{\mu\nu} = -\frac{1}{m}\left(g^{\mu\nu} + \frac{1}{1-d}\gamma^\mu\gamma^\nu\right). \quad (3)$$

Notice that because of the completely antisymmetric nature of $\gamma^{\mu\nu\alpha}$, the free massless Lagrangian is invariant under the gauge transformation $\psi_\mu \to \psi_\mu + \partial_\mu\epsilon$, where $\epsilon$ is an arbitrary spinor. We call the interacting field theory gauge invariant if all *interaction* terms are invariant under this gauge transformation. A gauge-invariant theory can be easily quantized using the standard path-integral technique. Indeed the path-integral formulation leads, by using the Faddeev-Popov trick, very naturally to the constraints that have the same form as in the free field theory. Let us denote the action of a gauge-invariant theory by

$$S = S[\bar{\psi}_\mu, \psi_\mu, \phi] = S_0[\bar{\psi}_\mu, \psi_\mu, \phi] - \int d^4x \bar{\psi} m \psi. \quad (4)$$

Here we denote by $S_0$ the gauge invariant part of the theory,

$$S_0[\bar{\psi}_\mu + \partial_\mu\bar{\epsilon}, \psi_\mu + \partial_\mu\epsilon, \phi] = S_0[\bar{\psi}_\mu, \psi_\mu, \phi], \quad (5)$$

and $\phi$ represents all other fields that interact with the spin-3/2 particle. The partition function in the path-integral formulation is given by

$$Z = \int D\psi_\mu D\psi_\mu^\dagger D\phi \, e^{iS}. \quad (6)$$

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Inserting the Faddeev-Popov one-operator we get

$$Z = \int D\psi_\mu D\bar{\psi}_\mu D\phi \, D\epsilon D\bar{\epsilon} \delta[(\bar{\psi}_\mu + \partial_\mu\bar{\epsilon})\gamma^\mu]$$
$$\times \delta[\gamma_\mu(\psi^\mu - \partial^\mu\epsilon)] \frac{1}{\text{Det}[-\gamma_\mu\partial^\mu]^2} \, e^{iS}. \tag{7}$$

Using gauge invariance we can rewrite the action as

$$S[\bar{\psi}_\mu, \psi_\mu, \phi] = S[\bar{\psi}_\mu + \partial_\mu\bar{\epsilon}, \psi_\mu - \partial_\mu\epsilon, \phi]$$
$$+ \bar{\psi}_\mu\gamma^{\mu\nu}m\partial_\nu\epsilon + \bar{\epsilon}\,\partial_\mu\gamma^{\mu\nu}m\,\psi_\nu. \tag{8}$$

Substituting additionally the spin-3/2 fields by $\bar{\psi}_\mu \to \bar{\psi}_\mu - \partial_\mu\bar{\epsilon}$, $\psi_\mu \to \psi_\mu + \partial_\mu\epsilon$, we obtain for the partition function

$$Z = \frac{1}{\text{Det}[-\gamma_\mu\partial^\mu]^2} \int D\psi_\mu D\bar{\psi}_\mu D\phi \, D\epsilon D\bar{\epsilon}\delta(\bar{\psi}_\mu\gamma^\mu)$$
$$\times \delta(\gamma_\mu\psi^\mu) \, e^{\{iS\}} e^{-im\int d^4x[\bar{\psi}_\mu\partial^\mu\epsilon + \bar{\epsilon}\,\partial^\mu\psi_\mu]}. \tag{9}$$

Finally, integrating over the spinor fields $\epsilon$ and $\bar{\epsilon}$ yields

$$Z = \text{const} \int \text{D}\psi_\mu \text{D}\bar{\psi}_\mu \text{D}\phi$$
$$\times \delta(\gamma_\mu\psi^\mu)\,\delta(\bar{\psi}_\mu\gamma^\mu)\delta(\partial_\mu\psi^\mu)\,\delta(\partial^\mu\bar{\psi}_\mu)\, e^{iS}. \tag{10}$$

This is exactly the same expression as given in Eq. (46) of Ref. [7], which was derived for a specific gauge-invariant model using the Dirac-Faddeev quantization framework [12–14] (see also Ref. [15] for a related work). Notice that the interacting theory has the same constraints as the free one. Clearly, this is the consequence of the fact that the gauge invariance is only broken by the mass term in both theories (see Ref. [7] for an extended discussion).

We now approach our main goal and demonstrate the on-shell equivalence of gauge-invariant and gauge-noninvariant formulations of the interacting theories with the massive spin-3/2 fields extending the earlier work [10] to the case of bilinear couplings. Consider the Lagrangian for interacting massive spin-3/2 particles given in its general form

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m + V)\psi, \tag{11}$$

where $V_{\mu\nu}$ represents an arbitrary local interaction. We claim that such a theory[1] is on-shell equivalent to the gauge-invariant one described by the Lagrangian

$$\mathcal{L}' = \bar{\psi}\left\{i\partial\!\!\!/ - m + i\partial\!\!\!/\frac{1}{m}V\frac{1}{m}\left[1 - (i\partial\!\!\!/ + m)\frac{1}{m}V\frac{1}{m}\right]^{-1}i\partial\!\!\!/\right\}\psi. \tag{12}$$

To prove this statement we rely on the equivalence theorem [8,16], which states that two field theories are equivalent on mass shell provided there exists a field transformation $\psi \to \psi + P[\psi, \phi]$ that transforms one Lagrangian into another. Here, $P[\psi, \phi] = \mathcal{O}(\psi^2, \psi\phi)$ denotes a local polynomial of the fields $\psi, \phi$ and their derivatives. In the following, we will

show that the original Lagrangian in Eq. (11) can be brought into the gauge-invariant form given in Eq. (12) by the following field transformation:

$$\psi \to S^{-1}[X - 1]^{-1}S\,\psi, \quad \bar{\psi} \to \bar{\psi}\,\bar{S}[\bar{X} - 1]^{-1}\bar{S}^{-1}, \tag{13}$$

where the operators $X$ and $\bar{X}$ are defined via

$$X = 1 + \left[1 - \frac{1}{m}V\frac{1}{m}(i\partial\!\!\!/ + m)\right]^{1/2},$$
$$\bar{X} = 1 + \left[1 - (i\partial\!\!\!/ + m)\frac{1}{m}V\frac{1}{m}\right]^{1/2}. \tag{14}$$

Further, the quantities $S$ and $\bar{S}$ have the form

$$S = \frac{1}{2}X\left[1 - X^{-1}\frac{1}{m}V\right], \quad \bar{S} = \frac{1}{2}\left[1 - V\frac{1}{m}\bar{X}^{-1}\right]\bar{X}. \tag{15}$$

To prove the equivalence of the two Lagrangians we have to show that

$$\bar{S}[\bar{X} - 1]^{-1}\bar{S}^{-1}(i\partial\!\!\!/ - m + V)S^{-1}[X - 1]^{-1}S$$
$$\overset{!}{=} i\partial\!\!\!/ - m + i\partial\!\!\!/\frac{1}{m}V\frac{1}{m}\left[1 - (i\partial\!\!\!/ + m)\frac{1}{m}V\frac{1}{m}\right]^{-1}i\partial\!\!\!/. \tag{16}$$

We now use the following useful relation:

$$i\partial\!\!\!/\frac{1}{m}V\frac{1}{m}[\bar{X} - 1]^{-2}i\partial\!\!\!/ = [2\,\bar{S}[\bar{X} - 1]^{-1} - 1](i\partial\!\!\!/ + m)$$
$$\times [[X - 1]^{-1}2\,S - 1] - i\partial\!\!\!/ - m + V \tag{17}$$

to rewrite the right-hand side of Eq. (16), so that the relation to be verified becomes

$$\bar{S}^{-1}(i\partial\!\!\!/ - m + V)S^{-1}$$
$$\overset{!}{=} [\bar{X} - 1]\bar{S}^{-1}(i\partial\!\!\!/ - m + V)S^{-1}[X - 1] + 4(i\partial\!\!\!/ + m)$$
$$- 2[\bar{X} - 1]\bar{S}^{-1}(i\partial\!\!\!/ + m) - 2(i\partial\!\!\!/ + m)S^{-1}[X - 1]. \tag{18}$$

The left-hand side of this relation can be rewritten in the following way:

$$\bar{S}^{-1}(i\partial\!\!\!/ - m + V)S^{-1} = 2[\bar{X}^{-1}(i\partial\!\!\!/ + m)S^{-1} - \bar{S}^{-1}m]. \tag{19}$$

Here we used $i\partial\!\!\!/ - m + V = 2\bar{S}\bar{X}^{-1}(i\partial\!\!\!/ + m) - 2mS$. The relation to prove then becomes

$$4(i\partial\!\!\!/ + m) - 2[\bar{X} - 1]\bar{S}^{-1}(i\partial\!\!\!/ + m) - 2\bar{X}^{-1}(i\partial\!\!\!/ + m)$$
$$\times S^{-1}X + 2\bar{S}^{-1}mX - 2\bar{X}\bar{S}^{-1}m[X - 1] \overset{!}{=} 0. \tag{20}$$

Using the commutation-like relation

$$m\left[1 - X^{-1}\frac{1}{m}V\right]^{-1} = \left[1 - V\frac{1}{m}\bar{X}^{-1}\right]^{-1}m \tag{21}$$

we can express $S^{-1}$ as follows:

$$S^{-1} = \frac{1}{m}V\frac{1}{m}\bar{S}^{-1}mX^{-1} + 2X^{-1}. \tag{22}$$

Let us denote the left-hand side of Eq. (20), which we want to show to be equal to zero, by $Y$. Inserting this expression

for $S^{-1}$ from Eq. (22) into the third term in Eq. (20) we obtain

$$Y = 4(i\partial\!\!\!/ + m) - 2[\bar{X} - 1]\bar{S}^{-1}(i\partial\!\!\!/ + m)$$
$$- 2\bar{X}^{-1}(i\partial\!\!\!/ + m)\frac{1}{m}V\frac{1}{m}\bar{S}^{-1}m - 4\bar{X}^{-1}(i\partial\!\!\!/ + m)$$
$$+ 2\bar{S}^{-1}mX - 2\bar{X}\bar{S}^{-1}m[X - 1]. \quad (23)$$

Applying the relations

$$\bar{X}^{-1}(i\partial\!\!\!/ + m)\frac{1}{m}V\frac{1}{m} = 2 - \bar{X},$$
$$X^{-1}\frac{1}{m}V\frac{1}{m} = \frac{1}{m}V\frac{1}{m}\bar{X}^{-1} \quad (24)$$

allows us to rewrite Eq. (23) in the form

$$Y = 4(i\partial\!\!\!/ + m) - 2[\bar{X} - 1]\bar{S}^{-1}(i\partial\!\!\!/ + m) - 4\bar{X}^{-1}(i\partial\!\!\!/ + m)$$
$$+ 2[\bar{X} - 1]\bar{S}^{-1}V\frac{1}{m}\bar{X}^{-1}(i\partial\!\!\!/ + m). \quad (25)$$

Finally, putting together the second and the last term in Eq. (25) yields the desired result:

$$Y = 4(i\partial\!\!\!/ + m) - 4[\bar{X} - 1]\bar{S}^{-1}\bar{S}\bar{X}^{-1}(i\partial\!\!\!/ + m)$$
$$- 4\bar{X}^{-1}(i\partial\!\!\!/ + m) = 0. \quad (26)$$

Thus, the redefinition of the spin-3/2 field specified in Eq. (13) in the underlying gauge-noninvariant Lagrangian $\mathcal{L}$ in Eq. (11) leads to the gauge-invariant form $\mathcal{L}'$ given in Eq. (12). Notice that even if there is only a finite number of interaction terms in the original Lagrangian, the transformed Lagrangian always contains an infinite number of terms.

Let us now briefly discuss possible linear couplings such as the couplings of the $\Delta$ to nucleon fields. Once the bilinear terms are gauge invariant the entire Lagrangian can be transformed into gauge-invariant form by much simpler transformation (which was extensively discussed by Pascalutsa [10]). We briefly repeat the arguments. We start with the Lagrangian

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m + V)\psi + \bar{\psi}\mathcal{O}_{\Delta N}N + \bar{N}\mathcal{O}_{N\Delta}\psi + \ldots, \quad (27)$$

where $N$ and $\bar{N}$ represent nucleon fields, the operators $V, \mathcal{O}_{\Delta N}$, and $\mathcal{O}_{N\Delta}$ represent interactions between nucleons and deltas and pions or other fields, and the ellipses represent all other possible interactions that do not include delta degrees of freedom. After the transformation given in Eq. (13) we obtain the Lagrangian

$$\mathcal{L}' = \bar{\psi}(V_{\Delta\Delta} - m)\psi + \bar{\psi}\tilde{V}_{\Delta N}N + \bar{N}\tilde{V}_{N\Delta}\psi + \ldots, \quad (28)$$

where the operators $V_{\Delta\Delta}, \tilde{V}_{\Delta N}$, and $\tilde{V}_{N\Delta}$ are given as follows:

$$V_{\Delta\Delta} = i\partial\!\!\!/ + i\partial\!\!\!/\frac{1}{m}V\frac{1}{m}\left[1 - (i\partial\!\!\!/ + m)\frac{1}{m}V\frac{1}{m}\right]^{-1}i\partial\!\!\!/,$$
$$\tilde{V}_{\Delta N} = \bar{S}[\bar{X} - 1]^{-1}\bar{S}^{-1}\mathcal{O}_{\Delta N}, \quad (29)$$
$$\tilde{V}_{N\Delta} = \mathcal{O}_{N\Delta}S^{-1}[X - 1]^{-1}S.$$

Note that the interaction $V_{\Delta\Delta}$ includes only gauge-invariant vertices. However the interactions $\tilde{V}_{\Delta N}$ and $\tilde{V}_{N\Delta}$ are in general

not gauge invariant. To bring them into a gauge-invariant form let us shift the delta fields like

$$\bar{\psi}_\mu \to \bar{\psi}_\mu + \bar{\xi}_\mu, \quad \psi_\mu \to \psi_\mu + \xi_\mu, \quad (30)$$

where $\bar{\xi}_\mu$ and $\xi_\mu$ are not yet specified but assumed to have the property of not depending on the delta fields. The transformed Lagrangian reads

$$\mathcal{L}'' = \bar{\psi}(V_{\Delta\Delta} - m)\psi + \bar{\xi}V_{\Delta\Delta}\psi + \bar{\psi}V_{\Delta\Delta}\xi$$
$$+ \bar{\psi}(\tilde{V}_{\Delta N}N - m\xi) + (\bar{N}\tilde{V}_{N\Delta} - \bar{\xi}m)\psi + \cdots. \quad (31)$$

To get a gauge-invariant form we now fix the fields $\xi_\mu$ and $\bar{\xi}_\mu$ to

$$\bar{\xi} = \bar{N}\tilde{V}_{N\Delta}\frac{1}{m}, \quad \xi = \frac{1}{m}\tilde{V}_{\Delta N}N. \quad (32)$$

With this choice the Lagrangian becomes gauge invariant:

$$\mathcal{L}'' = \bar{\psi}(V_{\Delta\Delta} - m)\psi + \bar{\psi}V_{\Delta N}N + \bar{N}V_{N\Delta}\psi + \ldots, \quad (33)$$

where the gauge-invariant delta-nucleon interactions are given by

$$V_{\Delta N} = V_{\Delta\Delta}\frac{1}{m}\tilde{V}_{\Delta N}, \quad V_{N\Delta} = \tilde{V}_{N\Delta}\frac{1}{m}V_{\Delta\Delta}. \quad (34)$$

The equivalence of the two formulations by means of the nonlinear field redefinition as discussed here has important consequences for calculations within chiral effective field theory. It implies that $S$-matrix elements can be calculated from the standard effective Lagrangian with the chiral symmetry being realized on a term-by-term basis using naive Feynman rules.

Let us briefly discuss what happens with the off-shell parameters, which account for the unphysical spin-1/2 degrees of freedom. The original interaction $\bar{\psi}V\psi$ can be written as a series of interactions with different off-shell parameters $Z_i$ and $Z_i'$:

$$\bar{\psi}V\psi = \sum_i \bar{\psi}\Theta(Z_i')V_i\Theta(Z_i)\psi + \text{h.c.}, \quad (35)$$

with $\Theta(Z_i)_{\mu\nu} = g_{\mu\nu} + Z_i\gamma_\mu\gamma_\nu$. Multiplication of the off-shell function $\Theta(Z_i)$ with $\frac{1}{m}i\partial\!\!\!/\psi$ or $\bar{\psi}i\partial\!\!\!/\frac{1}{m}$ from right or left, respectively, projects out the off-shell parameters $Z_i$:

$$\left[\Theta(Z_i)\frac{1}{m}i\partial\!\!\!/\psi\right]_\mu = \left[\frac{1}{m}i\partial\!\!\!/\psi\right]_\mu. \quad (36)$$

Similarly one can show that

$$\left[\bar{\psi}i\partial\!\!\!/\frac{1}{m}\Theta(Z_i')\right]_\mu = \left[\bar{\psi}i\partial\!\!\!/\frac{1}{m}\right]_\mu. \quad (37)$$

Because of these relations all the off-shell parameter contributions on the most left- and most right-hand sides of the gauge-invariant interactions $V_{\Delta\Delta}, V_{\Delta N}$, and $V_{N\Delta}$ are projected out. However, all the other off-shell parameter contributions are not zero but can be accounted for by counterterms. So for a given field theory to be independent of the off-shell parameters counterterms are needed to absorb all the off-shell dependence. The theory becomes automaticaly an effective field theory with infinitely many counterterms. In this picture, the off-shell parameters play the role of renormalization

scale: The counterterms do explicitly depend on the off-shell parameters in a way that all the calculated observables remain independent of them. In the case of the theory with the chiral Lagrangian it remains to be clarified whether the originally present counterterms are sufficient to absorb all the off-shell parameter contributions. Note that the gauge-invariant interactions $V_{\Delta\Delta}$, $V_{\Delta N}$, and $V_{N\Delta}$ involve not only covariant derivatives that appear in the chiral Lagrangian. For this reason, it is not obvious that the chiral-invariant counterterms in the original Lagrangian can completely compensate the off-shell dependence. For a related discussion we refer to the work of Ellis and Tang [11,17]. Notice further that at least in the case of pion-nucleon scattering at leading one-loop order the off-shell parameters can be absorbed in low-energy constants accompanying certain local interaction operators. This was confirmed in the explicit calculation by Fettes and Meißner [18] based on the so-called small-scale expansion [19].

The general result proved algebraically here can also be understood on the level of Feynman diagrams. The underlying mechanism is even more evident in that case. We now show that the manipulations discussed here simply correspond to the partial fraction decomposition of the free Feynman propagator. Consider, for example, a tree diagram with two insertions of $V$. The corresponding expression in operator form reads

$$i\partial\frac{1}{m}i\,V\frac{i}{i\partial-m}i\,V\frac{1}{m}i\partial=i\partial\frac{1}{m}i\,V\frac{1}{m}m\frac{i}{i\partial-m}m\frac{1}{m}i\,V\frac{1}{m}i\partial. \tag{38}$$

Using the partial fraction decomposition

$$m\frac{1}{i\partial-m}m=-(i\partial+m)+i\partial\frac{1}{i\partial-m}i\partial \tag{39}$$

we can rewrite this diagram in a gauge-invariant form:

$$i\partial\frac{1}{m}i\,V\frac{i}{i\partial-m}i\,V\frac{1}{m}i\partial=i\partial\frac{1}{m}i\,V\frac{1}{m}i\partial\frac{i}{i\partial-m}$$
$$\times i\partial\frac{1}{m}i\,V\frac{1}{m}i\partial+i\partial\frac{1}{m}i\,V\frac{1}{m}(i\partial+m)\frac{1}{m}V\frac{1}{m}i\partial. \tag{40}$$

Notice that because of the appearance of the operators $\frac{1}{m}i\partial$ and $i\partial\frac{1}{m}$ the first part includes only gauge-invariant vertices, which cannot depend on off-shell parameters. The second term is just a vertex that accounts for all the off-shell parameters. So by partial fraction decomposition one can naturally split the true spin-3/2 propagating structures, which are gauge-invariant, and all nonpropagating spin-1/2 contributions, which are collected into additional vertices.

In this paper, we have explicitly constructed a nonlinear field transformation that brings the interaction Lagrangian of spin-3/2 fields as given in Eq. (11) in a form that is invariant under gauge transformation. This proves a conjecture made in Ref. [10] where it was shown how a gauge-noninvariant Lagrangian with linear couplings in the spin-3/2 fields can be brought into a gauge-invariant form via a suitably chosen field redefinition. We have also given a diagrammatic explanation of our main result, which allows us to split the truly propagating spin-3/2 components from the nonpropagating spin-1/2 contributions that are subsumed in local operators existing in the most general Lagrangian of deltas coupled to nucleons, pions, and external fields.

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