

Nucleon-nucleon charge symmetry breaking and the $dd \rightarrow \alpha\pi^0$ reaction

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We show that using parameters consistent with the charge symmetry violating difference between the strong nn and pp scattering lengths provides significant constraints on the calculated amplitude for the $dd \rightarrow \alpha\pi^0$ reaction.

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The concepts of charge independence and charge symmetry provide powerful tools to organize the multiplet structure of hadrons and nuclei. These symmetries are not perfect; diverse, small but interesting violations have been discovered [1,2]. Our concern here is with the breaking of charge symmetry (CS). This symmetry is defined as invariance under a rotation by 180° around the second axis in isospin space. In quantum chromodynamics (QCD), CS implies that dynamics are invariant under the exchange of the up and down quarks [1]. However, because the up and down quarks do have different masses ($m_u \neq m_d$) [3], the QCD Lagrangian is not charge symmetric. This symmetry violation is called charge symmetry breaking (CSB). The different electromagnetic interactions of the up and down quarks also cause CSB as well as the breaking of charge independence. Observing the effects of CSB interactions therefore provides a probe of m_u and m_d , once the electromagnetic interactions are treated.

It has long been known that CSB is violated in the 1S_0 state of nucleon-nucleon scattering, with $a_{pp} - a_{nn} \equiv \Delta a = 1.5 \pm 0.5$ fm [1], where a denotes the scattering length. The nn interaction is more attractive than the pp interaction. There are many explanations for this, all which use meson exchange mechanisms [4,5]. Each of these mechanisms involving the strong interaction can be traced to the mass difference between the up and down quarks. Nucleon-nucleon potentials that are consistent with this scattering length difference are successful in reproducing (along with electromagnetic effects) the measured binding energy differences between mirror nuclei [5,6]. It is interesting to search for further manifestations of the up-down quark mass difference.

Two exciting observations of CSB in experiments involving the production of neutral pions have stirred interest in this subject. CSB was observed in the reaction $np \rightarrow d\pi^0$ at TRIUMF by measuring the CSB forward-backward asymmetry of the differential cross section as $A_{fb} = [17.2 \pm 8(\text{stat}) \pm 5.5(\text{sys})] \times 10^{-4}$ [7]. Furthermore, the final experiment at the IUCF cooler ring reported a relatively large $dd \rightarrow \alpha\pi^0$ cross section ($\sigma = 12.7 \pm 2.2$ pb at $T_d = 228.5$ MeV and 15.1 ± 3.1 pb at 231.8 MeV) [8]. The $dd \rightarrow \alpha\pi^0$ reaction violates CS because the deuterons and the α particles self-conjugate under the CS operator, with a positive eigenvalue, while the neutral pion wave function changes sign.

The study of CSB π^0 production reactions presents an exciting new opportunity to determine the influence of quark

masses in nuclear physics and to use effective field theory (EFT) to improve our understanding of how QCD works [2]. This is because chiral symmetry of QCD determines the form of pionic interactions. Electromagnetic CSB is typically of the same order of magnitude as the strong one and also can be handled using EFT. The EFT for the standard model at momenta comparable to the pion mass, $Q \sim m_\pi$, is chiral perturbation theory (χ PT) [9]. This EFT has been extended to pion production [10–14], where typical momenta are $Q \sim \sqrt{m_\pi M}$, with M being the nucleon mass. EFT using the operators of Ref. [15] was used to correctly predict the sign of the forward-backward asymmetry in $np \rightarrow d\pi^0$ [16].

The purpose of this brief report is to use information regarding CSB in the nucleon-nucleon system to constrain the calculations of the $dd \rightarrow \alpha\pi^0$ reaction. We begin by describing the pion production calculations and then show how the nucleon-nucleon CSB is relevant for this calculation. We constrain the parameters by the requirement that the calculated NN CSB be consistent with observation. It turns out that this constraint reduces considerably the uncertainty in the predictions for the $dd \rightarrow \alpha\pi^0$ reaction.

Attempts to understand the $dd \rightarrow \alpha\pi^0$ reaction began with a survey of various mechanisms using initial state plane wave functions and simplified final state wave functions [11]. Next, recent significant advances in four-body theory [17,18] were used to include the effects of deuteron-deuteron interactions in the initial state and to include bound-state wave functions with realistic two- and three-nucleon interactions [19]. The resulting calculations are hybrids: the CSB pion-production operators are constructed using EFT, but the strong interactions are not. The result was that a cross section of the experimentally measured size could be obtained using leading order (LO) and next-to-next-to LO (NNLO) pion production operators. However, not all NNLO diagrams were included. A complete analysis would require a careful treatment of loop diagrams.

The calculation made use of a variety of CSB mechanisms that we briefly review now. At formally leading order, there is only one contribution, represented by Fig. 1(a): pion rescattering in which the CSB occurs through the seagull pion-nucleon terms linked to the nucleon-mass splitting.

Reference [19] shows that there is no next-to-leading order (NLO) contribution. At NNLO, suppressed by $\mathcal{O}(m_\pi/M)$, there exists a recoil correction of the LO term, labeled \mathcal{M}_{rec} . This is determined by the same parameters as the LO term.

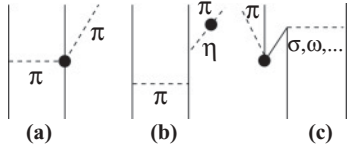


FIG. 1. Diagrams of $np \rightarrow d\pi^0$; the solid circles indicate CSB.

At the same order there are other operators. A term arises in which a one-body CSB operator ($\propto \beta_1 + \bar{\beta}_3$) is sandwiched between initial and final state wave functions, as illustrated in, e.g., Fig. 1(b). We refer to this as the one-body term (\mathcal{M}_{1b}). The terms $\beta_1 = O(\epsilon m_\pi^2/M^2)$ and $\bar{\beta}_3 = O(\alpha/\pi)$ arise from, respectively, the quark mass difference and electromagnetic contributions to the isospin-violating pion-nucleon coupling. Neither β_1 nor $\bar{\beta}_3$ can be extracted from experiment yet. These terms were estimated by modeling [20] β_1 by π - η mixing, see Fig. 1(b),

$$\beta_1 = \bar{g}_\eta \langle \pi^0 | H | \eta \rangle / m_\eta^2, \quad (1)$$

where $\langle \pi^0 | H | \eta \rangle = -4200 \text{ MeV}^2$ is the π - η mixing matrix element [21], and $\bar{g}_\eta = g_{\eta NN} f_\pi / M$ is the η -nucleon coupling constant. Nucleon-nucleon elastic scattering data show little sensitivity to η exchange and high-accuracy fits can be achieved [22] using $g_{\eta NN}^2/4\pi = 0$. Indeed, the possibility of a vanishing coupling constant had been raised earlier by the detailed analysis of NN total cross sections and $p\bar{p}$ data using dispersion relations [23] resulting in $g_{\eta NN}^2/4\pi \approx 0$. Also, in the Bonn full model [24] it is found that $g_{\eta NN}^2/4\pi \approx 0$ is consistent with the NN scattering data. A value of $g_{\eta NN}^2/4\pi = 0.51$ was used in Ref. [19].

The effects of electromagnetic interactions as well as strong CSB were included in computing the α -particle wave functions [17]. These interactions generate a small isospin $T = 1$ component of the wave function that enables a nonzero contribution of charge-symmetry conserving (CSC) production operators. The one-body operator was used to generate π^0 production [19].

A number of other CSB mechanisms enter at $N^3\text{LO}$ or higher, including loop diagrams and short-range interactions. The lowest order, where four-nucleon contact interactions start to contribute, is $N^4\text{LO}$. To estimate their strength, Ref. [11] evaluated certain tree-level contributions as indicated by Fig. 1(c). This figure represents the exchange of heavy mesons (σ , ω , ρ) via a Z -graph mechanism, with π - η mixing to generate CSB at pion emission (\mathcal{M}_σ , \mathcal{M}_ω , and \mathcal{M}_ρ). Another Z graph (labeled as $\mathcal{M}_{\rho\omega}$) arises in which the CSB occurs in the heavy-meson exchange via ρ - ω mixing along with strong pion emission at the vertex.

The inclusion of important Z graphs led to a quantitative description of the total cross section for the reaction $pp \rightarrow pp\pi^0$ near threshold [25]. The results [19] use the coupling constants and parameters of Ref. [11], see their Table I. It was found that the Z graphs give unexpectedly large contributions, especially the ρ - ω exchange operator, that add constructively and overwhelm the one-body term. This model of resonance saturation gives results in vast disagreement with the power

counting and therefore needs reassessment. Here we reassess the coupling constants used by Ref. [19]. This is only a first step, because it is also necessary to justify the use of Z graphs in resonance saturation, which would require further calculations.

The various mechanisms generate pion-production kernels that are sandwiched between final and initial state wave functions to provide a transition matrix element \mathcal{M} for $T_d = 228.5 \text{ MeV}$. These matrix elements are given in Table 1 of Ref. [19]. The transition amplitude can be written as

$$\begin{aligned} \mathcal{M} = & \mathcal{M}_{\text{PE}} + \mathcal{M}_{\text{rec}} + \mathcal{M}_{1b} + \mathcal{M}_\sigma + \mathcal{M}_\rho \\ & + \mathcal{M}_\omega + \mathcal{M}_{\rho\omega} + \mathcal{M}_{\text{WF}}, \end{aligned} \quad (2)$$

where the pion exchange term \mathcal{M}_{PE} , its recoil correction \mathcal{M}_{rec} , and the effects of CSB in the α wave function \mathcal{M}_{WF} are independent of g_η and β_1 . The one-body term and σ and ρ exchange terms involving the Z graphs, $\mathcal{M}_{1b} + \mathcal{M}_\sigma + \mathcal{M}_\rho$, are proportional to β_1 . The terms $\mathcal{M}_\omega + \mathcal{M}_{\rho\omega}$, which arise from ω and ρ - ω exchange, are proportional to β_1 and g_ω , the strong ω -nucleon coupling constant. Given the contributions to \mathcal{M} expressed in Table 1 of Ref. [19] in units of 10^{-4} fm^{-2} , the cross section can be written as

$$\sigma = 4.303 \text{ pb} |\mathcal{M} \times 10^4 \text{ fm}^2|^2. \quad (3)$$

Results were obtained [19] using either the Argonne V_{18} (AV18) [26] or CD-Bonn (CDB) [22] two-nucleon potentials combined with a properly adjusted Tucson-Melbourne (TM99) [27] three-nucleon force. The combination guarantees that the α -particle binding energy is reproduced with high accuracy.

We now turn to CSB in the nucleon-nucleon (NN) system due to meson mixing. Our CSB NN calculation is described in Ref. [5]. Because we have to use meson-nucleon form factors in our NN scattering calculations, while in Refs. [11] and [19] no such form factors are applied, we explain the definition of meson-nucleon coupling constants in conjunction with form factors. For this, we define a coupling constant as a function of the momentum-transfer t by

$$g_\alpha(t) \equiv F_\alpha(t) g_\alpha(t = m_\alpha^2), \quad (4)$$

with

$$F_\alpha(t) = \frac{\Lambda_\alpha^2 - m_\alpha^2}{\Lambda_\alpha^2 - t}, \quad (5)$$

where α stands for any meson, m_α denotes the meson mass, and Λ_α denotes the so-called cutoff mass. We use $\Lambda_{\rho,\omega} = 1400 \text{ MeV}$. In the relativistic three-dimensional theory used for CD-Bonn, the momentum transferred between the two nucleons is $t = -(\mathbf{q}' - \mathbf{q})^2$, with \mathbf{q} and \mathbf{q}' being the center-of-mass nucleon three-momenta before and after scattering. Coupling constants used in theories without form factors should be compared to coupling constants at $t = 0$, i.e., $g_\alpha(0)$, of a theory with form factors. Thus, in our CSB NN calculations, we use the coupling constants given in Table I of Ref. [11] and identify them with $g_\alpha(0)$, except for the ω -nucleon coupling where we use

$$\frac{g_\omega^2(0)}{4\pi} = 5.0 \quad (6)$$

instead of the 10.6 applied in Ref. [11]. Our lower value for the ω coupling is more consistent with a fixed- s dispersion relations analysis by Hamilton and Oades [28] in which a value of 5.7 ± 2.0 was obtained. Moreover, $SU(3)_{\text{flavor}}$ implies $g_\omega = 3g_\rho$, which, for the ρ coupling used, yields $g_\omega^2/4\pi = 3.9$. Applying the ω coupling constant stated in Eq. (6) and the other meson parameters as given in Table I of Ref. [11] and using $\langle \rho^0 | H | \omega \rangle = -4300 \text{ MeV}^2$ for the ρ - ω mixing matrix element [21], we find a CSB contribution to the 1S_0 scattering length difference from ρ - ω mixing of

$$\Delta a_{\rho\omega} = 1.45 \text{ fm.} \quad (7)$$

Obviously, this term entirely accounts for the observed CSB scattering length difference. We have also calculated η - π^0 mixing to find

$$\Delta a_{\pi\eta} = 0.33 \text{ fm} \quad (8)$$

using the same strong coupling constants and mixing matrix element as in Ref. [19]. Because the ρ - ω mixing explains all CSB, this might be considered as yet another argument that $g_\eta = 0$.

To explore a variety of possibilities, we rewrite the results of Ref. [19] as

$$\begin{aligned} \mathcal{M}_{\text{CDB}} = & \left(-2.75 + 3.1i + (-3.4 + 2.82i) \sqrt{\frac{g_\eta^2/4\pi}{.51}} \right. \\ & + (-.53 + .44i) \sqrt{\frac{g_\eta^2/4\pi}{.51}} \frac{g_\omega^2/4\pi}{10.6} \\ & + (-1.32 + 1.51i) \sqrt{\frac{g_\omega^2/4\pi}{10.6}} \\ & \left. + .51 - .13i \right) 10^{-4} \text{ fm}^{-2} \quad (9) \end{aligned}$$

or

$$\begin{aligned} \mathcal{M}_{\text{AV18}} = & \left(-1.65 + 1.58i + (-2.44 + 2.21i) \sqrt{\frac{g_\eta^2/4\pi}{.51}} \right. \\ & + (-.35 + .34i) \sqrt{\frac{g_\eta^2/4\pi}{.51}} \frac{g_\omega^2/4\pi}{10.6} \\ & + (-0.84 + 1.07i) \sqrt{\frac{g_\omega^2/4\pi}{10.6}} \\ & \left. + .41 - .14i \right) 10^{-4} \text{ fm}^{-2}, \quad (10) \end{aligned}$$

depending on the potential used. In Ref. [19] it was observed that this model dependence visibly influences the cross section result, requiring a more consistent treatment of the NN interaction and production operator in the future.

As a first step, we note that using the parameters of Ref. [19] leads to the results

$$\mathcal{M}_{\text{CDB}} = (-7.49 + 7.74i)10^{-4} \text{ fm}^{-2}, \quad (11)$$

$$\mathcal{M}_{\text{AV18}} = (-4.87 + 5.06i)10^{-4} \text{ fm}^{-2}, \quad (12)$$

with cross sections

$$\sigma_{\text{CDB}} = 499 \text{ pb}, \quad (13)$$

$$\sigma_{\text{AV18}} = 212 \text{ pb}. \quad (14)$$

Next we replace $g_\omega^2/4\pi$ by the value stated in Eq. (6), which yields

$$\mathcal{M}_{\text{CDB}} = (-6.80 + 7.03i)10^{-4} \text{ fm}^{-2}, \quad (15)$$

$$\mathcal{M}_{\text{AV18}} = (-4.42 + 4.54i)10^{-4} \text{ fm}^{-2}. \quad (16)$$

This would lead to about a 20% reduction of the cross section. However, using π - η mixing along with ρ - ω mixing overestimates Δa . Therefore, it is reasonable to explore the consequences of using $g_\eta = 0$, which results in

$$\mathcal{M}_{\text{CDB}} = (-3.15 + 4.00i)10^{-4} \text{ fm}^{-2}, \quad (17)$$

$$\mathcal{M}_{\text{AV18}} = (-1.82 + 2.17i)10^{-4} \text{ fm}^{-2}, \quad (18)$$

with cross sections

$$\sigma_{\text{CDB}} = 111.5 \text{ pb}, \quad (19)$$

$$\sigma_{\text{AV18}} = 34.5 \text{ pb}. \quad (20)$$

The cross section for AV18 is now only a factor of 2 or so bigger than the data, while CD-Bonn is off by a factor of about 7.

In summary, past theoretical work [19] on the cross section of the $dd \rightarrow \alpha\pi^0$ reaction at 228.5 MeV was plagued by the problem that the predictions were off by factors of between 15 and 30. In this brief report, we have shown that constraining the coupling constants involved by the requirement that the CSB in the 1S_0 NN scattering length is correctly reproduced reduces the overprediction to just a factor of about 2 (using the AV18 potential). In relative terms, this is substantial progress in understanding the $dd \rightarrow \alpha\pi^0$ reaction. However, significant differences between the use of the AV18 and CD-Bonn potentials remain, signaling that a deeper understanding is needed.

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