# Neutron Fermi liquids under the presence of a strong magnetic field with effective nuclear forces

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Landau's Fermi liquid parameters are calculated for non-superfluid pure neutron matter in the presence of a strong magnetic field at zero temperature. The particle-hole interactions in the system, where a net magnetization may be present, are characterized by these parameters in the framework of a multipolar formalism. We use either zero- or finite-range effective nuclear forces to describe the nuclear interaction. Using the obtained Fermi liquid parameters, the contribution of a strong magnetic field on some bulk magnitudes such as isothermal compressibility and spin susceptibility is also investigated.

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## I. INTRODUCTION

The description of low temperature homogeneous fermion systems was developed by Landau in 1957 [1] with the characterization of the low energy properties of the system in terms of a set of parameters, i.e., the Landau parameters, which determine the effective interaction of two quasiparticles near the Fermi surface. Since then, there have been many attempts to characterize nuclear and quark matter as Fermi liquids. Even if the general framework of the Fermi liquid theory (FLT) is well defined, there are some issues in the context of nuclear Fermi liquids because the microscopic derivation of the Landau parameters is not yet fully solved and their values depend on many-body effects and details of the nucleon-nucleon ($NN$) interaction that need to be treated carefully. Microscopic calculations of Landau parameters for nuclear and neutron matter for realistic interactions have been the subject of several investigations [2–4]. These calculations have also been evaluated in the framework of effective interactions, which allow for a Hartree-Fock description of the system and for a clean definition of the quasiparticle excitations. In this context they have been calculated for symmetric and asymmetric nuclear matter [5,6].

Because of the tiny value of the nuclear magneton $\mu_N = 3.1524512326(45) \times 10^{-18}\,\mathrm{MeV}\,G^{-1})$ [7] the values of magnetic field strength needed to provide some degree of polarization in a nuclear plasma are of the order $B \gtrsim 10^{16}$ G [8]. The only place where we have experimental indication of the existence of such intense magnetic fields is in astrophysical objects called *magnetars* [9]. On their surface magnetic field strengths can be of the order $B_{\mathrm{magnetar}} \approx 10^{14}$–$10^{15}$ G. According to the scalar virial theorem [10] allowed field strengths could be, in principle, as big as $B \approx 10^{18}$ G; however, a full detailed study of the gravitational stability condition of the maximum sustainable magnetic field strength in a star remains to be carried out.

From a theoretical point of view, FLT constitutes an excellent tool for the study of the properties of fermionic components of matter in the interior of neutron stars where the low temperature approximation is relevant. In the intermediate or high density regime, (1–4)$\rho_0$($\rho_0$ being the nuclear saturation density), where the nucleon picture is expected to be valid, some works have approached the contribution of magnetic fields to properties of $\beta$-equilibrated matter in a relativistic fashion, for example, analyzing some plasma properties using relativistic mean field theories like in Ref. [11] or equations of state (EOS) like in Refs. [12–14]. For higher densities, where quark matter may be relevant, there have been some attempts like those described in Refs. [15–17] where they find that only very large fields, $B \gtrsim 10^{19}$ G, seem to affect the EOS in a non-negligible way. In the opposite density limit, the solid outer crustal properties have been somewhat explored in the absence of magnetic fields, such as in the non-homogeneous *pasta* phases [18–21], or in the presence of strong quantizing magnetic fields as described in Ref. [22].

In a previous contribution we analyzed the effects of a strong magnetic field of astrophysical origin in a non-superfluid pure neutron system [8]. There, we found that using different parametrizations for the nonrelativistic effective $NN$ interaction, such as Skyrme and Gogny forces and in the context of Hartree-Fock calculations, a net magnetization is energetically allowed. One should notice that some effective nuclear interactions such as Skyrme forces predict that even in the absence of a magnetic field a spontaneous magnetization at sufficiently high densities may arise [23–26]. However, modern calculations using realistic $NN$ potentials, such as the auxiliary field diffusion Monte Carlo (AFDMC) method [27], the lowest order constrained variational (LOCV) method [28], Brueckner-Hartree-Fock calculations [29,30], and relativistic mean field [31] or relativistic Brueckner-Hartree-Fock calculations [32], which have studied the energetics of spin polarized neutron matter in the absence of magnetic fields, seem to prevent a spontaneous ferromagnetic phase transition and therefore there is the tendency to consider this fact as a pathology of the model used for the effective interaction. Notice also that the modern finite-range Gogny effective interactions do not predict this instability [25]. In any case,

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the transition into a ferromagnetic state could have important consequences for the evolution of a protoneutron star, in particular, for the spin correlations in the medium that do strongly affect the neutrino opacities inside the star [33,34].

In this work we evaluate the Landau parameters of polarized neutron matter up to dipolar order, characterized for having two Fermi spheres corresponding to the two possible spin orientations of neutrons. This study requires the use of an extension of the FLT to a two component system, which is presented in Sec. II, together with the derivation of the Landau parameters for two parametrizations of the nuclear interaction, namely, effective Skyrme and Gogny forces. Using a multipolar expansion of the particle-hole matrix elements we examine Landau's Fermi liquid parameters up to dipolar order. This allows one to see the residual contribution of energy and momentum because they do not appear in the commonly used monopolar calculation. Additionally, the consistent treatment of the induced magnetization in the plasma, already discussed in a previous publication [8], allows one to see what are the limits of validity of present non-magnetized neutron matter calculations. In this sense, this work constitutes a first step into future applications to systems with more particle flavors and will be interesting to compare to for the sake of completeness. To size up the relative contribution of Landau parameters and other in-medium observables to bulk properties we compute magnitudes such as isothermal compressibility and spin susceptibility for this type of magnetized one flavor plasma. Results are presented in Sec. III, and a summary and conclusions are given in Sec. IV.

## II. POLARIZED NEUTRON MATTER AS A FERMI LIQUID

In this section we briefly review the basic formalism of the normal FLT [35] for a homogenous neutron system in the presence of a strong magnetic field.

We consider a uniform magnetic field in the $z$ direction, $\mathbf{B} = B\mathbf{k}$, which also defines the quantization axis for the spin. In such a fermionic system particles can have spin projection on the $z$ axis, $\sigma$, which can be either $\sigma = +1$ or $\sigma = -1$ for spins aligned parallel or antiparallel to the magnetic field, respectively. Let us remind the reader here that for a neutron the magnetic moment is antiparallel to the spin. The neutron number density, $\rho$, is the sum of spin up (+) and down (−) particles,

$$\rho = \rho_+ + \rho_-. \tag{1}$$

These densities define the Fermi momenta, i.e., the Fermi surface of each fermion component with spin projection $\sigma$. At zero temperature the number density for each component is given by (we set $\hbar = c = 1$)

$$\rho_\sigma = \frac{k_{F,\sigma}^3}{6\pi^2}. \tag{2}$$

In the neutron system the net magnetization density is defined as

$$m = \mu_n \Delta \rho, \tag{3}$$

where $\mu_n = -1.9130427(5)\mu_N$ is the neutron magnetic moment in units of the nuclear magneton [7] and $\Delta$ is the spin excess or polarization of the system.

$$\Delta = \frac{\rho_+ - \rho_-}{\rho}. \tag{4}$$

The total magnetization of a given volume is then $M = \int m dV$. The relevant thermodynamical potential to study neutron matter at zero temperature under an external magnetic field, $H$, is the Helmholtz free energy, $F$, defined as [36]

$$F = E - HM, \tag{5}$$

where $E$ is the energy of the system. Note that previously we have used $B$ to designate the magnetic field strength; however, the total magnetic field is the sum of the external magnetic field and the induced magnetization, $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$. In this work we will assume that the ratio $|H/B|$ will always be close to unity. In what follows we will keep the notation using $B$ to designate the total magnetic field strength.

In the context of FLT it is assumed that ph excitations happen around the Fermi surface; therefore, this treatment is valid, in principle, for the low temperature regime where $T \ll T_{F,\sigma}$ [35]. Then, there is a one-to-one correspondence of states in the interacting system with states in a free Fermi gas. As particles with given momentum $\mathbf{k}$ and spin projection $\sigma$ are added addiabatically to the system, an eigenstate of the real gas is obtained and, thus, a distribution of quasiparticles, $n_{\mathbf{k},\sigma}$.

Because the lifetime of the quasiparticle varies inversely to the square of the departure of its energy from the Fermi energy [35] we consider that the ph perturbation mechanism takes place around the available polarized Fermi seas. Assuming small deviations of the distribution function of the $\sigma$-polarized quasiparticles in the plasma, $n_{\mathbf{k},\sigma}$, with respect to the ground state distribution, $n^0_{\mathbf{k},\sigma}$ we have $\delta n_{\mathbf{k},\sigma} = n_{\mathbf{k},\sigma} - n^0_{\mathbf{k},\sigma}$.

In the FLT the variation of the free energy, $\delta F$, can be written up to second order in $\delta n_{\mathbf{k},\sigma}$ as

$$\delta F = \sum_{\mathbf{k},\sigma} (\epsilon_{\mathbf{k},\sigma} - \mu_n \sigma B)\delta n_{\mathbf{k},\sigma} + \frac{1}{2} \sum_{\mathbf{k},\mathbf{k}',\sigma,\sigma'} f_{\mathbf{k},\sigma,\mathbf{k}',\sigma'} \delta n_{\mathbf{k},\sigma} \delta n_{\mathbf{k}',\sigma'} + O(\delta n^3). \tag{6}$$

As seen in Ref. [8] the change in free energy under the presence of a magnetic field is due to the energy of (anti)alignment of spins in the field and to the change in the quasiparticle distribution function. The single particle energy can be obtained as a functional derivative,

$$\epsilon_{\mathbf{k},\sigma} - \mu_n \sigma B = \frac{\partial F}{\partial n_{\mathbf{k},\sigma}}, \tag{7}$$

as well as the quasiparticle interaction coefficients,

$$f_{\mathbf{k},\sigma,\mathbf{k}',\sigma'} = \frac{\partial^2 F}{\partial n_{\mathbf{k},\sigma} \partial n_{\mathbf{k}',\sigma'}}. \tag{8}$$

In the general case of polarized neutron matter, the ph matrix element can be written as [37,38]

$$V_{\rm ph}(\mathbf{q_1}, \mathbf{q_2}, \mathbf{q}) = \langle \mathbf{q} + \mathbf{q_1}\sigma_1, \mathbf{q_1}\sigma_3 | V | \mathbf{q} + \mathbf{q_2}\sigma_4, \mathbf{q_2}\sigma_2 \rangle, \tag{9}$$

where we keep the same notation as defined in Refs. [5,6,37,38] to designate $\mathbf{k_1} = \mathbf{q} + \mathbf{q_1}$, $\mathbf{k_2} = \mathbf{q_2}$, $\mathbf{k_3} = \mathbf{q_1}$, and $\mathbf{k_4} = \mathbf{q} + \mathbf{q_2}$ as the participating momenta of the ph interaction with initial momenta $\mathbf{q_1}$, $\mathbf{q_2}$ and $\mathbf{q}$ is the transferred three-momentum. Assuming that the ph perturbation mechanism takes place around each Fermi surface, each quasiparticle incoming momentum, $\mathbf{q_i}$, must be replaced by the corresponding polarized Fermi momentum $\mathbf{k}_{\mathbf{F},\sigma}$ and the momentum transfer is supposed to be small $\mathbf{q} \approx 0$.

When computing the quasiparticle interaction matrix elements we must note that they only depend on the Fermi momentum of each polarized component and the relative angle, $\theta$, of interacting quasiparticles' three-momenta $\mathbf{q_1}$, $\mathbf{q_2}$. For a nonpolarized system it is usually written as an expansion in Legendre polynomials [35],

$$V_{\mathrm{ph}} = \sum_{l=0}^{\infty}[f_l + g_l \sigma_\mathbf{1} \cdot \sigma_\mathbf{2}] P_l(\cos\theta). \tag{10}$$

Previous works [5,34,38,39] have studied a nonpolarized pure neutron system using a ph interaction at the monopolar and dipolar ($l = 0, 1$) level. In that case, it is convenient to define dimensionless parameters $F_l = f_l N_0$ and $G_l = g_l N_0$, where $N_0$ is the quasiparticle level density at the Fermi surface at $T = 0$. For non polarized neutron matter density is given by $\rho = g k_F^3/(6\pi^2)$ where $N_0 = \frac{g m^* k_F}{2\pi^2}$, $g = 2$ is the spin degeneracy and $m^*$ is the effective mass at the Fermi surface.

Considering the two possible spin orientations $\sigma = \pm 1$ the polarized interaction matrix elements can be written using coefficients depending on the polarizations involved, $f_l^{(\sigma,\sigma')}$. On a nonpolarized system they fulfill the following relations,

$$f_l = \frac{f_l^{(\sigma,\sigma)} + f_l^{(\sigma,-\sigma)}}{2}, \tag{11}$$

$$g_l = \frac{f_l^{(\sigma,\sigma)} - f_l^{(\sigma,-\sigma)}}{2}. \tag{12}$$

In the same way, as mentioned for the nonpolarized case, one can define dimensionless coefficients in the form $F_l^{(\sigma,\sigma')} = f_l^{(\sigma,\sigma')}\sqrt{N_{0\sigma}N_{0\sigma'}}$, where $N_{0\sigma}$ is the quasiparticle level density at the Fermi surface of each component at $T = 0$, $N_{0\sigma} = \frac{m^*_\sigma k_{F,\sigma}}{2\pi^2}$. The Landau coefficients for the nonpolarized case can be recovered from the general polarized coefficients as $\Delta \to 0$,

$$F_l^{(\sigma)} \equiv \frac{F_l^{(\sigma,\sigma)} + F_l^{(\sigma,-\sigma)}}{2} \to \frac{F_l}{2}\,(B = 0), \tag{13}$$

$$G_l^{(\sigma)} \equiv \frac{F_l^{(\sigma,\sigma)} - F_l^{(\sigma,-\sigma)}}{2} \to \frac{G_l}{2}\,(B = 0), \tag{14}$$

where we have defined $F_l^{(\sigma)}$ and $G_l^{(\sigma)}$. Note that some works define combinations of coefficients in a different way [40]. Contrary to the nonpolarized case, now the $F_l^{(\sigma,\sigma')}$ interaction displays a $2 \times 2$ matrix structure depending on the spin projections considered. To size up the importance of the polarization in neutron matter it is useful to consider a ratio of these coefficients $(F_l^{(\sigma)}, G_l^{(\sigma)})$ with respect to the $B = 0$ case. We define

$$R^\sigma_{Fl} = \frac{2F_l^{(\sigma)} - F_l(B = 0)}{|F_l(B = 0)|} \tag{15}$$

and

$$R^\sigma_{Gl} = \frac{2G_l^{(\sigma)} - G_l(B = 0)}{|G_l(B = 0)|}. \tag{16}$$

In this work we concentrate on $R^-_{Fl}$, $R^-_{Gl}$, which correspond to the energetically favorable dominant population fraction with magnetic moments (spins) aligned parallel (antiparallel) to the magnetic field.

In this work we are interested in retaining just the terms with $l \leqslant 1$. Notice that constraining results just to dipolar multipolarity is exact for the zero-range Skyrme forces but for the finite-range Gogny forces the coefficients at higher multipolarities are not zero, although they decrease rather rapidly.

Within the context of the Landau FLT, additional static physical quantities of interest [41,42] can be obtained. We have derived the expressions for these quantities for the polarized neutron matter case. The quasiparticle effective mass is related via Galilean invariance with the dipolar matrix elements [35],

$$m^*_\sigma/m = 1 + \frac{1}{3}N_{0\sigma}\left[f_1^{(\sigma,\sigma)} + \left(\frac{k^2_{F,-\sigma}}{k^2_{F,\sigma}}\right) f_1^{(\sigma,-\sigma)}\right]. \tag{17}$$

The isothermal compressibility, $K = 9\frac{\partial P}{\partial \rho}$, is related to the study of the variation of the pressure and density profiles for a neutron star. At zero temperature it can be written as

$$K = \frac{9}{\rho}\sum_\sigma \frac{\rho^2_\sigma}{N_{0\sigma}}\left(1 + N_{0\sigma}\left[f_0^{(\sigma,\sigma)} + \left(\frac{k^2_{F,-\sigma}}{k^2_{F,\sigma}}\right) f_0^{(\sigma,-\sigma)}\right]\right). \tag{18}$$

The thermodynamical magnitude that characterizes the change in the magnetization of the medium when an external magnetic field exists is the spin susceptibility $\chi$. It is obtained by considering the change of the quasiparticle distribution functions for the up and down components [43],

$$m = \chi B = \mu_n(\delta\rho_+ - \delta\rho_-). \tag{19}$$

For a polarized system we have obtained

$$\chi = \sum_\sigma \frac{\mu_n^2 N_{0\sigma}}{1 + N_{0\sigma}\left[f_0^{(\sigma,\sigma)} - \left(\frac{k^2_{F,-\sigma}}{k^2_{F,\sigma}}\right) f_0^{(\sigma,-\sigma)}\right]}. \tag{20}$$

The bulk magnitudes $K$ and $\chi$ for polarized neutron matter are affected by a factor involving the quasiparticle interaction matrix elements and the level densities with respect to the polarized free Fermi gas value for each population component.

A necessary ingredient in the calculation of the free energy functional in the neutron system, $F$, is the nuclear interaction. In the next subsections of the present article we consider either zero-range Skyrme or finite-range Gogny forces as illustrative examples of effective nuclear interactions. Using the nonrelativistic Hartree-Fock approximation, the free energy per particle in neutron matter under the presence of a magnetic field $B$ can be calculated minimizing $F$ as a function of $B$ for given thermodynamical conditions to obtain $\Delta_{\min}$ (polarization at the minimum). As a rule of thumb, for the maximum magnetic field strength considered in this work, $B = 10^{18}$ G, a $\Delta_{\min} < 40\%$ is allowed for densities $0.5 < \rho/\rho_0 < 3.3$ at zero temperature [8].

### A. Skyrme force

We have considered, in the first place, the phenomenological Skyrme interaction that appears in the literature under the rather general form [44]

$$\begin{aligned} V_{NN}^{\text{Skyrme}}(\mathbf{r}_1,\mathbf{r}_2) &= t_0(1+x_0P^\sigma)\delta(\mathbf{r}) \\ &\quad + \tfrac{1}{2}t_1(1+x_1P^\sigma)[\mathbf{k}'^2\delta(\mathbf{r})+\delta(\mathbf{r})\mathbf{k}^2] \\ &\quad + t_2(1+x_2P^\sigma)\mathbf{k}'\cdot\delta(\mathbf{r})\mathbf{k} \\ &\quad + \tfrac{1}{6}t_3(1+x_3P^\sigma)\rho^\alpha(\mathbf{R})\delta(\mathbf{r}), \end{aligned} \quad (21)$$

where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$, and $\mathbf{k} = (\nabla_1 - \nabla_2)/2i$ is the relative momentum acting on the right and $\mathbf{k}'$ its conjugate acting on the left. $P^\sigma$ is the spin exchange operator. Note that we have omitted the spin-orbit term not relevant for the total energy of homogeneous systems. Its effect on the RPA response function has been shown to be sizable only at values of the momentum transfer much higher than the Fermi momentum [45].

As is widely known, this effective interaction allows for a good reproduction of finite nuclei observables and bulk matter EOS relevant to neutron stars [44]. In the Skyrme model the ph interaction matrix elements retain the density and polarization dependence and their multipolar decomposition gives contribution up to $l = 1$. We have derived analytical expressions for both parallel and antiparallel coefficients for an arbitrary polarization in neutron matter.

The monopolar terms can be written as

$$\begin{aligned} f_0^{(\sigma,\sigma)} &= \tfrac{1}{6}t_3(1-x_3)[\alpha(\alpha-1)\rho^{\alpha-2}\rho_+\rho_+ \\ &\quad + 2\alpha\rho^{\alpha-1}(\rho-\rho_\sigma)] + t_2(1+x_2)k_{F,\sigma}^2, \end{aligned} \quad (22)$$

$$\begin{aligned} f_0^{(\sigma,-\sigma)} &= t_0(1-x_0) + \tfrac{1}{6}t_3(1-x_3)[\alpha(\alpha-1)\rho^{\alpha-2}\rho_+\rho_- \\ &\quad + (\alpha+1)\rho^\alpha] + \tfrac{1}{4}[t_1(1-x_1) \\ &\quad + t_2(1+x_2)]\left(k_{F,\sigma}^2 + k_{F,-\sigma}^2\right), \end{aligned} \quad (23)$$

and the dipolar terms can be written as

$$f_1^{(\sigma,\sigma)} = -t_2(1+x_2)k_{F,\sigma}^2, \quad (24)$$

$$f_1^{(\sigma,-\sigma)} = -\tfrac{1}{2}[t_1(1-x_1)+t_2(1+x_2)]k_{F,\sigma}\mathrm{k}_{\mathrm{F},-\sigma}. \quad (25)$$

In this article we use the SLy7 parametrization [44] of the Skyrme force, which not only provides good values for binding of nuclei but also a neutron matter EOS in agreement with microscopic calculations obtained using realistic interaction and giving values of maximum neutron star masses around $1.5M_\odot$. In Table I we summarize the values of some observables for symmetric nuclear matter (SNM) for the effective interaction models used in this work: saturation density, $\rho_0$; binding energy for symmetric nuclear matter, $a_v$; symmetry energy, $a_s$; and incompressibility modulus, $K_\infty$.

TABLE I. Values of some observables for symmetric nuclear matter in the absence of magnetic field with the Skyrme and Gogny forces considered in this work [44,46].

| Model | $\rho_0$ (fm$^{-3}$) | $K_\infty$ (MeV) | $a_v$ (MeV) | $a_s$ (MeV) |
|---|---|---|---|---|
| SLy7 | 0.158 | 229.7 | −15.89 | 31.99 |
| D1P | 0.1737 | 266 | −16.19 | 34.09 |

### B. Gogny force

The Gogny force has also been extensively used in the literature. In this article we use the D1P parametrization [46], which allows for a good description of both finite nuclei and EOS of pure neutron matter [25]. It includes a sum of two Gaussian-shaped terms that mimic the finite-range effects of a realistic interaction in the medium. Usually, it also contains a density-dependent zero-range term. The interaction potential is written as

$$\begin{aligned} V_{NN}^{\text{Gogny}}(\mathbf{r}_1,\mathbf{r}_2) &= \sum_{i=1}^{2}\{[W_i + B_iP^\sigma - H_iP^\tau - M_iP^\sigma P^\tau] \\ &\quad \times e^{-|\mathbf{r}_1-\mathbf{r}_2|^2/\mu_i^2} + t_{3i}(1+x_{3i}P^\sigma)\rho^{\alpha_i}\delta(\mathbf{r})\}. \end{aligned} \quad (26)$$

The two first finite-range terms contain the usual mixing of exchange operators for spin, $P^\sigma$, isospin, $P^\tau$, and spin-isospin, $P^\sigma P^\tau$, and the second contact term is a functional of the nuclear density, $\rho$. The values of the parameters can be found in Ref. [46]. Some observables for SNM with the Gogny D1P force appear in Table I. We have derived the expressions of the monopolar and dipolar Landau parameters with the Gogny interaction,

$$\begin{aligned} f_0^{(\sigma,\sigma)} &= \sum_{i=1,2}(W_i - H_i + B_i - M_i)\pi^{3/2}\mu_i^3 \\ &\quad \times \left[1 - \frac{1}{k_{F,\sigma}^2\mu_i^2}\left(1 - e^{-k_{F,\sigma}^2\mu_i^2}\right)\right] \\ &\quad + t_{3i}(1-x_{3i})[2\alpha_i\rho^{\alpha_i-1}(\rho-\rho_\sigma) \\ &\quad + \alpha_i(\alpha_i-1)\rho^{\alpha_i-2}\rho_+\rho_-], \end{aligned} \quad (27)$$

$$\begin{aligned} f_0^{(\sigma,-\sigma)} &= \sum_{i=1,2}(W_i - H_i)\pi^{3/2}\mu_i^3 - (B_i - M_i)\frac{\pi^{3/2}\mu_i}{k_F(\sigma)k_F(-\sigma)} \\ &\quad \times \left[e^{-\frac{1}{4}(k_{F,\sigma}-k_{F,-\sigma})^2\mu_i^2} - e^{-\frac{1}{4}(k_{F,\sigma}+k_{F,-\sigma})^2\mu_i^2}\right] \\ &\quad + t_{3i}(1-x_{3i})\left[(\alpha_i+1)\rho^{\alpha_i} + \alpha_i(\alpha_i-1)\rho^{\alpha_i-2}\rho_+\rho_-\right], \end{aligned} \quad (28)$$

and for the $f_1$ terms,

$$\begin{aligned} f_1^{(\sigma,\sigma)} &= \sum_{i=1,2} -(W_i - H_i + B_i - M_i)\frac{3\pi^{3/2}\mu_i}{k_{F,\sigma}^2}\left[1 - \frac{2}{k_{F,\sigma}^2\mu_i^2}\right. \\ &\quad \left. + \left(1 + \frac{2}{k_{F,\sigma}^2\mu_i^2}\right)e^{-k_{F,\sigma}^2\mu_i^2}\right], \end{aligned} \quad (29)$$

$$f_1^{(\sigma,-\sigma)} = \sum_{i=1,2} -(B_i - M_i)\frac{3\pi^{3/2}\mu_i}{k_{F,\sigma}k_{F,-\sigma}} \times \left[\left(1 - \frac{2}{k_{F,\sigma}k_{F,-\sigma}\mu_i^2}\right)e^{-\frac{1}{4}(k_{F,\sigma}-k_{F,-\sigma})^2\mu_i^2} + \left(1 + \frac{2}{k_{F,\sigma}k_{F,-\sigma}\mu_i^2}\right)e^{-\frac{1}{4}(k_{F,\sigma}+k_{F,-\sigma})^2\mu_i^2}\right]. \quad (30)$$

It is worth mentioning at this point that, once the ph interaction matrix elements have been calculated, the response functions of neutron matter can be obtained along the lines described in Ref. [45].

## III. RESULTS

In this section we discuss the properties of polarized neutron matter at zero temperature under the presence of a strong magnetic field. Such properties are described in terms of ($l = 0, 1$) Landau parameters, which are calculated from Skyrme and Gogny effective interactions. Under each thermodynamic condition ($\rho, T = 0, B$) the corresponding induced magnetization of the neutron system has been obtained by minimizing the free energy within a Hartree-Fock calculation, as described in Ref. [8]. To be physically meaningful we have considered that the nucleon picture holds up to a maximum limiting density $\rho = 4\rho_0(\rho_0 = 0.1737\text{fm}^{-1})$. Similarly, more detailed calculations are needed when considering densities below $\rho = 0.5\rho_0$, where nuclear pasta [18–21] may be present.

To measure the effects of the magnetic field on the Landau parameters we calculate the variation ratios $R^{\sigma}_{Fl}$ and $R^{\sigma}_{Gl}$ as defined in Eqs. (15) and (16) for the dominant neutron component with magnetic moment aligned parallel to the magnetic field. This corresponds to the spin anti-aligned component due to the fact that $\mu_n < 0$.

In Fig. 1 we show the ratios $R^-_{F0}$, $R^-_{G0}$, $R^-_{F1}$, and $R^-_{G1}$ (we omit the superscript) computed with the Skyrme SLy7 (a) and with the Gogny D1P (b) interactions for a characteristic low density of $\rho = 0.5\rho_0$ as a function of the logarithm of the magnetic field. At this low density the change due to the magnetic field strength in the monopolar and dipolar coefficients is very mild. For Skyrme the ph interactions are mostly slightly more repulsive in the density and spin channel but for Gogny this behavior is reversed. The variation of the dipolar terms, as computed with the Gogny interaction, is larger than that for the monopolar case. However, dipolar terms are about one order of magnitude smaller [38] so the overall effect is mostly unchanged with respect to the monopolar description. For magnetic field strengths below $B \approx 10^{16}$ G, the change in the ratios is negligible because of the tiny value of the neutron magnetic moment, which results in a vanishing value of the induced magnetization, i.e., of the polarization of the neutron matter.

In Fig. 2 we show the same ratios shown in Fig. 1 as computed with the Skyrme (a) and Gogny D1P (b) interactions for a high density case, $\rho = 3.3\rho_0$ as a function of the logarithm of the magnetic field. We can see that for the Skyrme SLy7 interaction the presence of a ferromagnetic transition at density close to the one selected [8] induces a change larger than that at lower densities in the monopolar and dipolar terms as the magnetic field grows. The attraction in the ph density excitation channel increases while in the spin channel it becomes more repulsive as the magnetic field strength grows. For the dipolar terms this tendency is reversed. For the Gogny interaction the variation of the ratios is very small except for the $G_1$ term. However, the large variation of $G_1$ does not modify the tendency shown by the monopolar terms because in this range of densities it is also an order of magnitude smaller than $G_0$.

We now analyze some other quantities related to the Landau parameters in polarized neutron matter. In Fig. 3 we show the effective neutron mass for the Skyrme SLy7 (a) and Gogny D1P (b) interactions as a function of the logarithm of the magnetic field strength. In each plot the spin down (upper curve) and spin up (lower curve) polarized components at saturation density $\rho_0$ are shown. Skyrme interaction predicts smaller effective masses and a larger relative variation for the spin up and down components than in the Gogny case. These two effects will, in turn, largely affect the level density at the Fermi surfaces of the spin components in magnetized neutron matter.

In Fig. 4 the two spin contributions to the isothermal compressibility for neutron matter are shown as a function of density for a magnetic field strength $B = 5 \times 10^{17}$ G and zero temperature for the Skyrme SLy7 (solid line) and Gogny D1P (dashed line) interactions. Contributions from the spin down

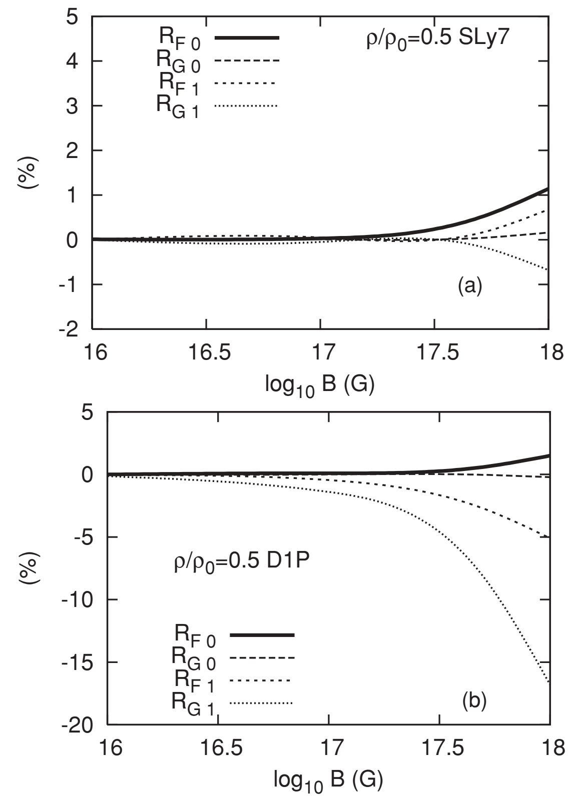


FIG. 1. Variation ratio of Landau coefficients with the SLy7 (a) and D1P (b) parametrizations at density $\rho = 0.5\rho_0$ and zero temperature as a function of the logarithm of the magnetic field strength.

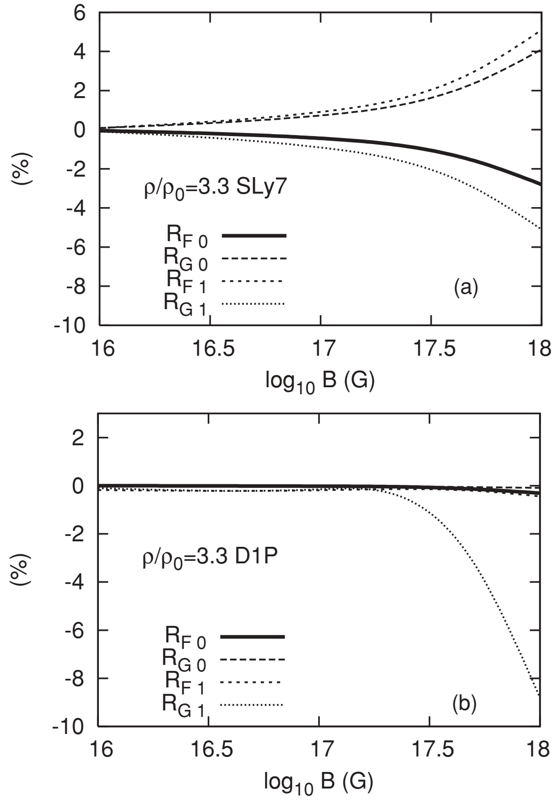


FIG. 2. Same as Fig. 1 computed at density $\rho = 3.3\rho_0$.

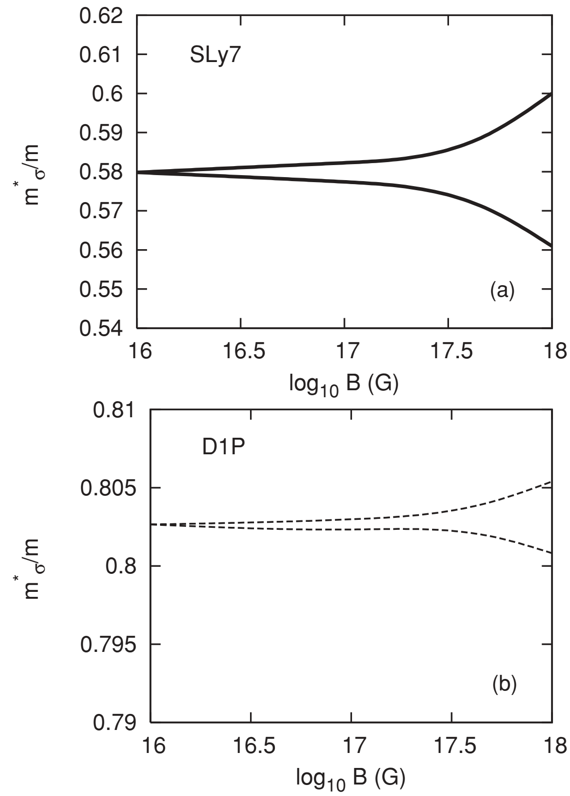


FIG. 3. Effective neutron mass at density $\rho_0$ as a function of the logarithm of the magnetic field strength for the Skyrme SLy7 (a) and Gogny D1P (b) interactions. For each model, upper (lower) curves correspond to spin down (up) polarized particles.

(up) polarized particles correspond to the upper (lower) curves for each interaction model. In some works compressibility values are plot related to the Fermi gas value; in our case, because at each density the relative populations of the up and down spin polarized particles change, we have chosen to plot absolute values. Because of the mild variation of the Landau coefficients with the magnetic field (lower than 20%) we can see that the main effect of the magnetization in the system is the change in the level densities. As the density increases the dominant spin down fraction of the polarized plasma becomes stiffer. However, there is a dramatic decrease in the global compressibility in the proximity of the ferromagnetic transition in the Skyrme case. This transition density decreases as the magnetic field grows [8]. In fact, the results beyond this density are not physically meaningful but we show them for the sake of completeness in the density range considered in this work.

In Fig. 5 contributions to the isothermal compressibility for both spin components in neutron matter are shown as a function of the logarithm of the magnetic field strength at saturation density for the Skyrme SLy7 (solid line) and Gogny D1P (dashed line) interactions. For each model, upper (lower) curves refer to spin down (up) polarized particles. As the magnetic field strength increases there is a splitting of the spin up and down component behavior with respect to the unpolarized case ($\Delta = 0$). Global compressibility is obtained by adding the contributions of both spin fractions and changes very mildly with the magnetic field. The relative variation of both components at the maximum field strength is of 20% (28%) for the Skyrme SLy7 (Gogny D1P) interaction at this density.

We now consider the effect of a strong magnetic field on the static magnetic susceptibility. In Fig. 6 we plot the magnetic susceptibility for neutron matter as a function of density in units of the nuclear magnetic moment squared, $\mu_n^2$, for a magnetic field $B = 5 \times 10^{17}$ G. The results obtained with the Skyrme SLy7 (Gogny D1P) interaction are shown with solid

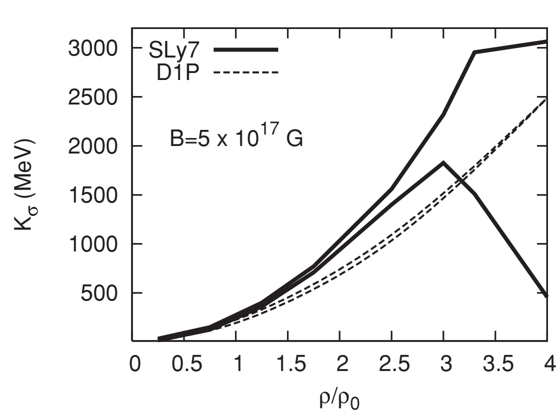


FIG. 4. Isothermal compressibility components obtained for the Skyrme SLy7 (solid curve) and Gogny D1P (dashed line) models as a function of density. Spin down (upper curve) and up (lower curve) polarized components are shown for each model.

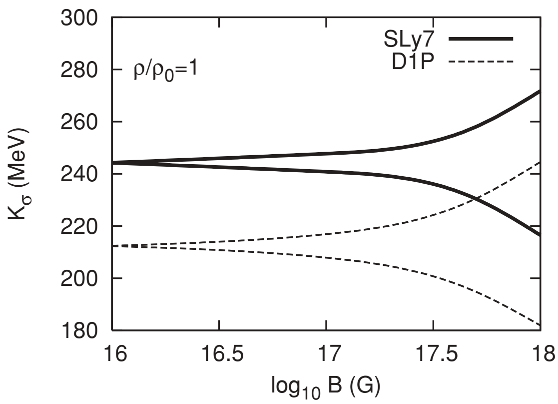


FIG. 5. Isothermal compressibility components obtained for the Skyrme SLy7 (solid line) and Gogny D1P (dashed line) models as a function of the logarithm of the magnetic field strength at density $\rho_0$. Upper (lower) curves refer to spin down (up) polarized contributions.

(dashed) lines. For Skyrme results we can see that the onset of a ferromagnetic transition (that for this specific parametrization takes place at a density around $\rho = 3.3\rho_0$ and is signaled with a vertical dotted line on the plot) drives a divergence of the susceptibility and the subsequent second-order phase transition. In this model the upper curve corresponds to the $\chi^-/\mu_n^2$ value and the lower curve to the $\chi^+/\mu_n^2$ value. The onset of the ferromagnetic transition is due to the vanishing values approached by the quantities in the denominator of Eq. (20). For the Skyrme SLy7 interaction the meaningful regime, though, is limited for values before the transition takes place. For the Gogny D1P interaction (dashed line) the splitting between the contributions from the two spin orientations is less than 1%, therefore, indistinguishable on the plot. There is no ferromagnetic transition for this parametrization and the susceptibility keeps finite values for the whole density range.

In Fig. 7 we show the magnetic susceptibility in units of the nuclear magnetic moment squared, $\mu_n^2$, for neutron matter as a function of the logarithm of the magnetic field strength at saturation density. We plot results obtained with the Skyrme SLy7 (Gogny D1P) interaction with solid (dashed) lines. Upper (lower) curves for each model refer to spin down (up) polarized

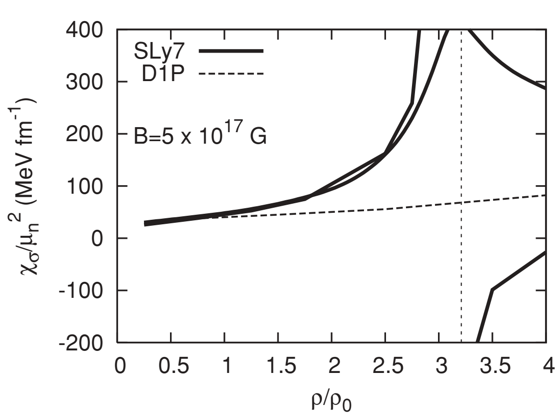


FIG. 6. Magnetic susceptibility as a function of density for the Skyrme SLy7 (solid line) and Gogny D1P (dashed line) models. See text for details.

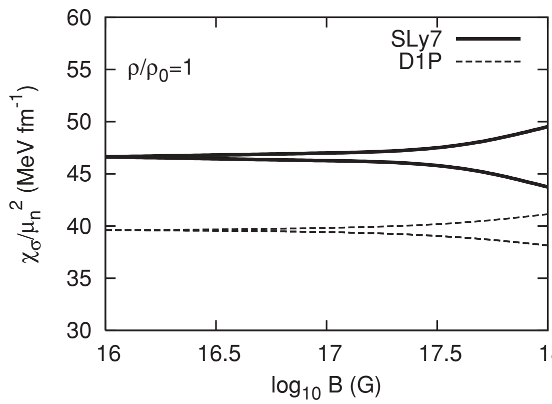


FIG. 7. Magnetic susceptibility for a pure neutron system as a function of the logarithm of the magnetic field strength for the Skyrme SLy7 (solid line) and Gogny D1P (dashed line) at saturation density. For each model upper (lower) curves refer to down (up) spin polarized components. See text for details.

population fractions. We see that contributions from the down (up) polarized components give larger (smaller) contributions to the total susceptibility that can be obtained as the sum of both. Notice that the global susceptibility remains almost unchanged as the magnetic field grows. As $B$ increases the Skyrme model predicts a higher tendency of the system to achieve a net magnetization than in the Gogny case at this density. No transition is near at this density for the Skyrme model and a smooth behavior is observed in the full range of variation of the magnetic field.

## IV. SUMMARY AND CONCLUSIONS

In this work we have investigated, in the context of the Landau theory of normal Fermi liquids, the variation of monopolar ($l = 0$) and dipolar ($l = 1$) Landau parameters that describes the particle-hole interaction matrix elements for pure neutron matter in the presence of a strong magnetic field at zero temperature. We have used effective nuclear interactions such as the zero-range Skryme SLy7 and finite-range Gogny D1P and obtained their analytical expressions valid in the case of a polarized Fermi sea. We have computed the ratios of variation of these coefficients with respect to the $B = 0$ case for the dominant spin component in the system. For the Skyrme and Gogny interactions the variation is very mild, keeping below $|R| \approx 20\%$ for the maximum magnetic field strength studied in this work, $B \approx 10^{18}$ G. This is a direct consequence of the small polarization induced by such strong fields in the system in the meaningful range of densities considered in this work.

We have also analyzed the effect of the presence of a strong magnetic field in some other static properties in the neutron plasma as deduced from the Landau theory of Fermi liquids. Effective neutron masses at the Fermi surface for each spin polarized component are calculated for both interactions from the dipolar $F_1$ coefficients. The magnetization in the plasma causes a splitting in the values of the up and down spin polarized components. Spin down (up) polarized particles show an increase (decrease) in their effective masses with respect to the nonpolarized case as the magnetic field strength grows. This will affect the level densities in polarized neutron matter. In addition, Skyrme interactions predict a variation in the splitting larger than that of the Gogny forces with increasing magnetic field $B$. Other magnitudes, such as the isothermal compressibility $K$, relate to the dipolar Landau coefficients. In the presence of a strong magnetic field the compressibility is a growing function of density, stiffer in the Skyrme case than in the Gogny case. When a ferromagnetic transition is near, as it happens for the Skyrme interaction used in this work (as a representative case of almost all Skyrme parametrizations), there is a dramatic decrease of the compressibility around the density of the onset of the ferromagnetic instability. The magnetic susceptibility, $\chi$, shows a divergent behavior at densities close to that of the onset of the phase transition for the Skyrme case. The contribution to the total susceptibility from the polarized populations in the system is related to the Landau coefficients in the spin channel that approach values driving the system to an instability. For the Gogny case there is no divergence because magnetization of the system stays very mild. For densities in the intermediate range, not close to the transition, spin down (up) contributions give a higher (lower) susceptibility as it is more (less) easy to polarize the system as the external field strengthens. Although a lot of work has been devoted to the study of polarized nuclear systems, additional effort should be made to explore properties of nuclear asymmetric matter at finite temperature. The presence of magnetic fields should be carefully studied in the future with direct simulation techniques for the low density case. This will allow exploration of the possibilities that exotic matter shapes in the crust of neutron stars may give as an additional source of opacity to the neutrino cooling in the early stages of supernova cooling.

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