

Dissociation of quarkonium in a hot QCD medium: Modification of the interquark potentialVineet Agotiya,^{1,*} Vinod Chandra,^{2,†} and Binoy K. Patra^{1,‡}¹*Department of Physics, Indian Institute of Technology Roorkee, Roorkee 247 667, India*²*Department of Physics, Indian Institute of Technology Kanpur, Kanpur 208 016, India*

(Received 15 August 2008; published 31 August 2009)

We have studied the dissociation of heavy quarkonium states in a hot QCD medium by investigating the medium modifications to a heavy quark potential. Our model shows that in-medium modification causes the screening of the charge in contrast to the screening of the range of the potential. We have then employed the medium-modified potential to estimate the dissociation pattern of the charmonium and bottomonium states and also explore how the pattern changes as we go from the perturbative to nonperturbative domain in the Debye mass. The results are in good agreement with the other current theoretical works both from the spectral function analysis and the potential model study.

DOI: [10.1103/PhysRevC.80.025210](https://doi.org/10.1103/PhysRevC.80.025210)

PACS number(s): 12.39.Hg, 12.38.Gc, 12.38.Mh

I. INTRODUCTION

The study of the fundamental forces among quarks and gluons is an essential key to the understanding of QCD and the occurrence of different phases which are expected to show up when going from low to high temperatures and/or baryon number densities. For instance, at small or vanishing temperatures quarks and gluons get confined inside a hadron by the strong force while at high temperatures a quite different medium consisting of quarks and gluons known as quark-gluon plasma (QGP) is expected. One of the most important features of the QGP formation is the color screening of static chromo-electric fields [1]. The suppression of heavy quarkonia (J/ψ , χ_c , ψ' , Υ), due to the color screening analogous to Debye screening in QED plasma, has long been proposed as a probe of deconfinement in a dense partonic medium. In the deconfined state, the interaction between heavy quarks and antiquarks gets reduced due to color screening leading to a suppression in J/ψ yields [2,3].

Thus quarkonia at finite temperature are an important tool to know the status of the matter (confined/deconfined) formed in heavy ion collisions (see, e.g., Ref. [4]). Many efforts have been devoted to determine the dissociation temperatures of $Q\bar{Q}$ states in the deconfined medium, using either lattice calculations of quarkonium spectral functions [5–8] or non-relativistic calculations based upon some effective (screened) potentials [9–14]. However, the properties of the heavy quarkonia states determined from the screened potentials do have a poor matching with the results obtained from the lattice spectral functions. None of the potential model studies and spectral functions in lattice to study quarkonia give a complete framework to study the properties of quarkonia at finite temperature. It is not yet clear at the moment up to what extent one may understand the modifications of quarkonium spectral functions in terms of the Debye screening picture. One should not expect a precise quantitative agreement with

the lattice correlators because of the uncertainties coming from a variety of sources [14]. Not only is the determination of the effective potential still an open question but also there are other related issues such as relativistic effects, thermal width of the states, and contribution from quantum corrections that need to be taken care of. On the other hand, lattice correlators are also affected by their own uncertainties. These may be due to the use of different lattices (isotropic or anisotropic). Additionally, the finite lattice-spacing might significantly alter the continuum part of the spectrum. However, some degree of qualitative agreement had been found for the S -wave correlators. This finding was somehow ambiguous for the P -wave correlators and the temperature dependence of the potential model was even qualitatively different from the lattice one.

In a recent work, Umeda [15] found that lattice calculations of meson correlators at finite temperature contain a constant contribution due to the presence of zero modes in the spectral functions. The presence of a zero mode in the vector channel had already been discussed in the literature while in the P -wave channels it had generally been overlooked. Recently Alberico *et al.* [16] updated their previous calculation [14] of quarkonium Euclidean correlators at finite temperatures in a potential model by including the effect of zero modes in the lattice spectral functions. These contributions cure most of the previously observed discrepancies with lattice calculations. This observation supports the use of potential models at finite temperature as an important tool to complement lattice studies.

The short and intermediate distance properties of the heavy quark interaction are important for the understanding of in-medium modifications of the heavy quark bound states. On the other hand, the large distance behavior of the heavy quark interaction plays a crucial role in understanding the bulk properties of the QCD plasma phase, viz. the screening property of the quark gluon plasma, the equation of state [17,18], and the order parameter (Polyakov loop) [19,20].

In all of these studies, deviations from perturbative calculations and the ideal gas behavior are expected and are indeed found at temperatures which are only moderately larger than the deconfinement temperature. This calls for quantitative nonperturbative calculations. The phase transition in full QCD appears as a crossover rather than a ‘true’ phase transition

* vinetdph@iiitr.ernet.in† vinodc@iitk.ac.in‡ binoyfph@iiitr.ernet.in

with related singularities in thermodynamic observables (in the high-temperature and low density regime) [21]. Therefore, it is reasonable to assume that the string tension does not vanish abruptly above T_c . So one should study its effects on the behavior of quarkonia in a hot QCD medium. This issue, usually overlooked in the literature, is certainly worth investigation. In the present paper we considered this potentially interesting issue by correcting the full Cornell potential with a dielectric function embodying the effects of the deconfined medium and not only its Coulomb part as usually done in the literature. We have found that this leads to a long-range Coulomb potential with a reduced effective charge (inversely proportional to the square of the Debye mass) of the heavy quark in addition to the usual Debye-screened form employed in most of the literature. With such an effective potential, we investigate the effects of different possible choices of the Debye mass on the dissociation temperatures of different quarkonium states. Since a Coulomb interaction always admits bound states, a criterion has to be adopted to define a dissociation temperature: a state is then considered to be melted when its binding energy becomes of the same order as the temperature. For this purpose, we consider a gauge-invariant, nonperturbative form of the Debye mass by Kajantie *et al.* [22] and study systematically the effects of perturbative and nonperturbative terms in Debye mass on the dissociation pattern of quarkonia in pure gauge, two-flavor, and three-flavor QCD, respectively. Additionally, we consider the lattice parametrized form of the Debye mass [23].

The paper is organized as follows. In-medium modifications to heavy quark potential is discussed in Sec. II. In Secs. II A and II B, we study the medium dependence of quarkonia binding energy and then determine their dissociation temperatures in a hot QCD medium. Finally, we conclude in Sec. III.

II. IN-MEDIUM MODIFICATIONS TO HEAVY-QUARK POTENTIAL

Because of the large quark mass $m = m_{c,b} \gg \Lambda_{\text{QCD}}$, the velocity of heavy quarks in the bound state is small and the binding effects in quarkonia at zero temperature can be understood in terms of nonrelativistic potential models [24]. More recently, the potential has been derived from QCD using a sequence of effective field theories (for a review see [25]). The present analysis also employs this idea to study the quarkonia with the nonrelativistic potential model.

Let us now turn our attention to study the medium modifications to a heavy quark potential which is considered as the Cornell potential

$$V(r) = -\frac{\alpha}{r} + \sigma r, \quad (1)$$

where α and σ are the phenomenological parameters. The former accounts for the effective coupling between a heavy quark and its antiquark and the latter gives the string coupling.

The medium modification enters in the Fourier transform of the heavy quark potential as

$$\tilde{V}(k) = \frac{V(k)}{\epsilon(k)}, \quad (2)$$

where $\epsilon(k)$ is the dielectric permittivity given in terms of the static limit of the longitudinal part of gluon self-energy [26,27]

$$\epsilon(k) = \left(1 + \frac{\Pi_L(0,k,T)}{k^2}\right) \equiv \left(1 + \frac{m_D^2}{k^2}\right). \quad (3)$$

Note that the result for the static limit of the dielectric permittivity is the perturbative one. If one assumes nonperturbative effects such as the string tension survive even above the deconfinement point then the dependence of the dielectric function on the Debye mass may get modified. So there is a *caveat* about the validity of the linear dependence of the dielectric function (ϵ) on the square of the Debye mass M_D^2 . For the sake of simplicity we put in all the nonperturbative effects together in the effective charge ($2\sigma/m_D^2$) of the medium modified potential. The quantity $V(k)$ in (2) is the Fourier transform (FT) of the Cornell potential. The evaluation of the FT of the Cornell potential is not so straightforward and can be done by assuming r as distribution ($r \rightarrow r \exp(-\gamma r)$). After the evaluation of FT we let γ tend to zero. Note that the FT of Coulomb part is straightforward to compute. The Fourier transform of the linear part $\sigma r \exp(-\gamma r)$ is

$$= -\frac{i}{k\sqrt{2\pi}} \left\{ \frac{2}{(\gamma - ik)^3} - \frac{2}{(\gamma + ik)^3} \right\}.$$

If we put $\gamma = 0$, we obtain the Fourier transform of σr denoted as

$$(\tilde{\sigma}r) = -\frac{4\sigma}{k^4\sqrt{2\pi}}. \quad (4)$$

Now the FT of the full Cornell potential can be written as

$$\mathbf{V}(k) = -\sqrt{(2/\pi)} \frac{\alpha}{k^2} - \frac{4\sigma}{\sqrt{2\pi}k^4}. \quad (5)$$

Substituting Eqs. (3) and (5) into Eq. (2) and then evaluating its inverse Fourier transform one obtains the r -dependence of the medium modified potential [28] as

$$\mathbf{V}(r) = \left(\frac{2\sigma}{m_D^2} - \alpha\right) \frac{\exp(-m_D r)}{r} - \frac{2\sigma}{m_D^2 r} + \frac{2\sigma}{m_D} - \alpha m_D. \quad (6)$$

This potential has a long-range Coulombic tail in addition to the standard Yukawa term. In the limit $r \gg 1/m_D$, we can neglect the Yukawa term and for large values of temperature the product αm_D will be much greater than $2\sigma/m_D$. So, finally the potential (6) becomes

$$V(r) \sim -\frac{2\sigma}{m_D^2 r} - \alpha m_D. \quad (7)$$

The above form (apart from a constant term) is a Coulombic type as encountered in a hydrogen atom problem with identifying the fine structure constant e^2 with the effective charge $2\sigma/m_D^2$. Since m_D is an increasing function of temperature, the effective charge $2\sigma/m_D^2$ gets waned as the temperature is increased and finally results in a screening of the charge. The constant terms in the full potential (6) are introduced by hand in order to remove short-distance medium effects. However,

such terms could arise naturally from the basic computations of a real time static potential in hot QCD [29] and from the real and imaginary time correlators in a thermal QCD medium [30]. These terms in the potential are needed in computing the masses of the quarkonium states and to compare the results with the lattice studies. It is equally important while comparing our effective potential with the free energy in lattice studies. However, these terms are not needed to compare the values of the dissociation temperatures obtained in our calculation with the values in lattice spectral studies because we have used different criteria to evaluate the dissociation temperatures.

It may not be out of context to mention that the expression for the potential in a hot QCD medium is not the same as the lattice parametrized heavy quark free-energy in the deconfined phase (which is basically a screened Coulomb, for the exact form we refer the reader to Refs. [31,32]). As emphasized by Dixit [33] that one-dimensional Fourier transform of the Cornell potential in the medium yields the similar form as used in the lattice QCD to study the quarkonium properties which assumes the one-dimensional color flux tube structure. However, at finite temperature that may not be the case since the flux tube structure may expand in more dimensions [31]. Therefore, it is better to consider the three-dimensional form of the medium modified Cornell potential which has been done exactly in the present work. The medium modified potential thus obtained has a Coulomb tail in addition to the screened Coulomb part. The strength of the Coulombic part decreases with the increase in temperature and at a certain temperature one may ignore it.

To compare our in-medium effective potential with the color-singlet free-energy [34] extracted from the lattice data which do not display at all any long-range term, we have plotted a full effective potential from Eq. (6) as a function of rT in Fig. 1. The lattice free energy goes to zero much faster than our effective potential due to the presence of the Coulomb tail. However, our potential employing the nonperturbative form of the Debye mass deviates largely from the lattice results [34].

Let us now proceed to study the charmonium and bottomonium spectrum and their binding energy with three possible choices of the Debye mass. Additionally, we take advantage of all the available lattice data, obtained not only in quenched QCD ($N_f = 0$) but also including two and, more recently, three light flavors. This enables us to study the flavor dependence of

the dissociation process, a perspective not yet achieved by the parallel studies of the spectral functions.

A. Binding energy of heavy quarkonia

Spectral function method defines binding energy of a quarkonium state as the distance between the peak position and the continuum threshold, $E_{\text{bin}} = 2m_{c,b} + V_{\infty}(T) - M$ with M being the resonance mass. In our case, it is defined as the ‘ionization potential’ because of the similarity of our approximated effective potential (7) with the hydrogen atom problem. The Schrödinger equation gives the energy eigenvalues for the ground states and the first excited states for charmonium (J/ψ , ψ' , etc.) and bottomonium (Υ , Υ' , etc.) spectra. Invoking the translational invariance, we can ignore the constant term in Eq. (7) and the energy of the n th eigenstate is given by Bohr’s theory:

$$E_n = -\frac{E_I}{n^2}; \quad E_I = \frac{m_Q \sigma^2}{m_D^4}, \quad (8)$$

where m_Q is the mass of the heavy quark and E_I is the energy of the $Q\bar{Q}$ state in the first Bohr state. The allowed energies for $Q\bar{Q}$ states are $E_n = -E_I, -\frac{E_I}{4}, -\frac{E_I}{9}, \dots$. These energies are known as the ionization potentials/binding energies for the n th bound states. It becomes a temperature-dependent quantity through the temperature dependence in the Debye mass and it decreases with the increase in temperature.

There are other states in the charmonium and bottomonium spectroscopy, viz. χ_c ’s and χ_b ’s for which the determination of the medium-dependent binding energy is beyond the scope of our present calculation. For χ_c ’s and χ_b ’s, one should take into account the spin dependence of the quark-antiquark potential [38]. Figures 2 and 3 show the variation of the binding energy (in GeV) with the temperature (in units of critical temperatures) for J/ψ and Υ , respectively. Similar variations for other quarkonia (ψ' , Υ') can also be shown.

In the present analysis, we consider three possible forms of the Debye masses, viz. the leading-order term in QCD coupling (m_D^{LO}), nonperturbative corrections to it (m_D^{NP}), and the lattice parametrized form (m_D^{L}) to study the dissociation phenomena of quarkonium in a hot QCD medium. The Debye mass at high

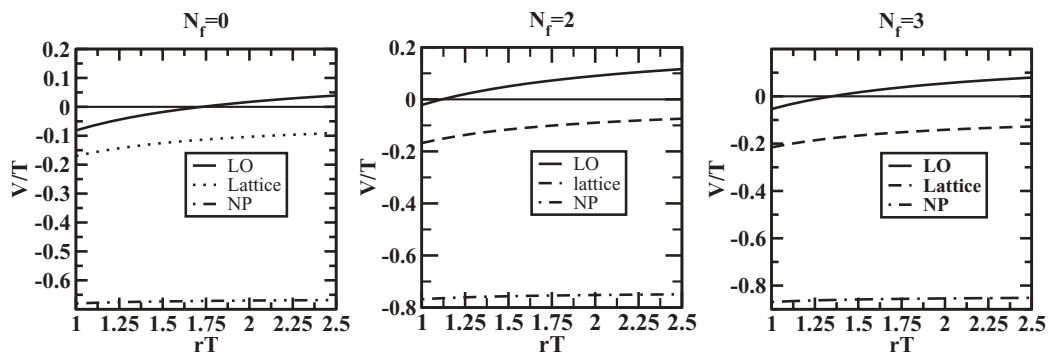


FIG. 1. The behavior of $V(r,T)/T$ as a function of rT for a fixed $T/T_c = 3.32$ [34]. The different curves denote the choice of the Debye masses.

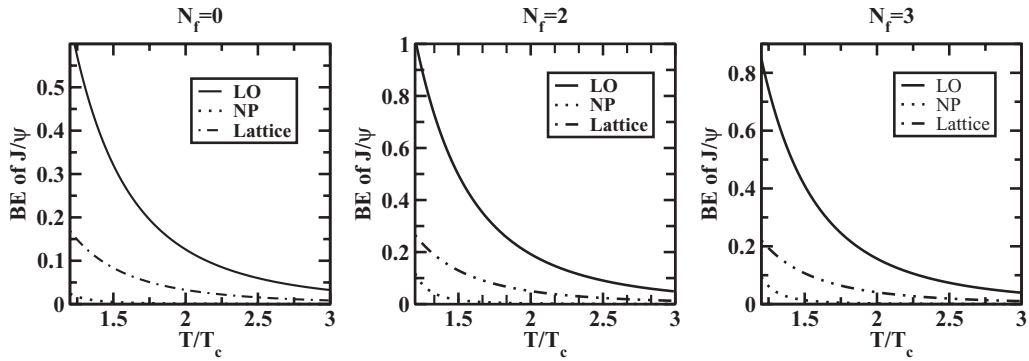


FIG. 2. The temperature dependence of the J/ψ binding energy (in GeV). The different curves denote the choice of the Debye masses.

temperature in the leading order is known from a long time and is perturbative [35] in nature. Recently Kajantie *et al.* [22] computed the nonperturbative contributions of $O(g^2T)$ and $O(g^3T)$ from a three-dimensional effective field theory which we consider in the present work. However we also consider the Debye mass obtained by fitting the (color-singlet) free energy in lattice QCD [23]. Their forms are given below:

$$\begin{aligned}
 m_D^{\text{LO}} &= g(T)T \sqrt{\frac{N}{3} + \frac{N_f}{6}}, \\
 m_D^{\text{NP}} &= m_D^{\text{LO}} + \frac{N g^2 T}{4\pi} \ln \frac{m_D^{\text{LO}}}{g^2 T} + c_N g^2 T + d_{N,N_f} g^3 T, \quad (9) \\
 m_D^{\text{L}} &= 1.4 m_D^{\text{LO}},
 \end{aligned}$$

where the coefficient c_N captures the nonperturbative effects and d_{N,N_f} is related to the choice of the scale in m_D^{LO} . We employ the two-loop expression for the QCD coupling constant at finite temperature [36] and choose the renormalization scale determined in [37].

Note that different curves in each figure denote the choice of the Debye masses (10) used to calculate the binding energy from Eq. (8). There is a common observation in Figs. 2 and 3 that there is a strong decrease in binding energy with the increase in temperature. In particular, binding energies obtained from m_D^{LO} and m_D^{L} give a realistic variation with the temperature. The temperature dependence of the binding energies shows a qualitative agreement with the similar variations shown in [23].

However, when we employ the nonperturbative form of the Debye mass m_D^{NP} the binding energies become unrealistically small compared to the binding energy at $T = 0$ and also compared to the binding energies employing m_D^{LO} and m_D^{L} . This can be understood by the fact that the value of m_D^{NP} is significantly larger than both m_D^{LO} and m_D^{L} . This observation hints that the present form of the nonperturbative corrections to the Debye mass may not be the complete one, the situation may change when nonperturbative corrections of higher order $O(g^4T)$ are added to the Debye mass and then use it to calculate the binding energy.

Thus a study of the temperature dependence of the binding energy is poised to provide a wealth of information about the dissociation pattern of quarkonium states in a thermal medium which will now be used to determine the dissociation temperatures of different states.

B. The dissociation temperatures for heavy quarkonia

Dissociation of a two-body bound state in a thermal medium can be understood as when the binding energy of a resonance state drops below the mean thermal energy of a parton, the state becomes feebly bound. The thermal fluctuations then can destroy it by transferring energy and exciting the quark-antiquark pair into its continuum. The spectral function technique in potential models defines the dissociation temperature as the temperature above which the quarkonium spectral function shows no resonance-like

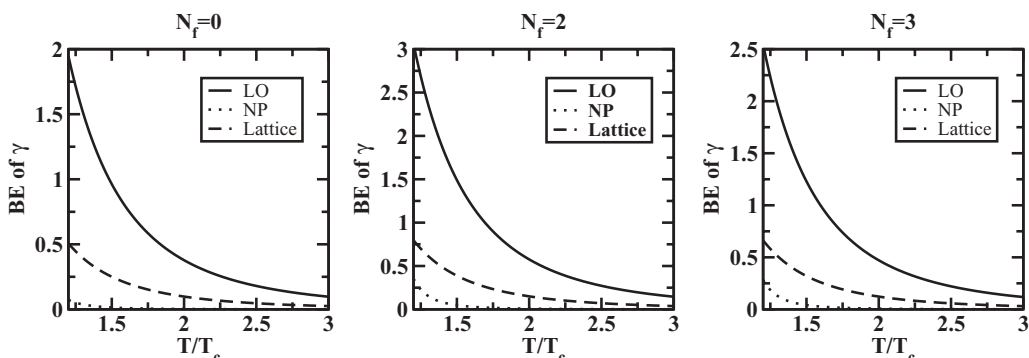


FIG. 3. Dependence of Υ binding energy (in GeV) on temperature T/T_c .

TABLE I. Lower (upper) bound on the dissociation temperature (T_D) for the quarkonia states (in units of T_c) using the leading-order term in the Debye mass m_D^{LO} .

State	Pure QCD	$N_f = 2$	$N_f = 3$
J/ψ	1.1 (1.4)	1.3 (1.7)	1.2 (1.6)
ψ'	0.8 (1.0)	0.9 (1.2)	0.9 (1.1)
Υ	1.4 (1.8)	1.7 (2.1)	1.6 (2.0)
Υ'	1.0 (1.3)	1.2 (1.6)	1.2 (1.5)

structures but the widths shown in spectral functions from current potential model calculations are not physical. The broadening of states with the increase in temperature is not included in any of these models. In Ref. [23], the authors argued that one need not have the binding energy (E_{bin}) to be zero for the dissociation. Rather a weaker condition $E_{\text{bin}} < T$ causes a state weakly bound and the thermal fluctuations can then destroy it. Since the (relativistic) thermal energy of the partons is $3T$, the lower bound on the dissociation temperature (T_D) is obtained from the relation

$$\frac{1}{n^2} \frac{m_Q \sigma^2}{m_D^4(T_D)} = 3T_D, \quad (10)$$

where the string tension (σ) is taken as 0.184 GeV^2 , and critical temperatures (T_c) are taken as 270 MeV, 203 MeV, and 197 MeV for pure, two-flavor, and three-flavor QCD medium, respectively [39]. The dissociation temperatures for the ground states and the first excited states of $c\bar{c}$ and $b\bar{b}$ are listed in Table I with the Debye mass in the leading-order m_D^{LO} . It is seen from Table I that ψ' is dissociated in the vicinity of critical temperature while J/ψ and Υ' are dissociated around $1.2T_c$. Υ is dissociated at a relatively higher temperature $1.6T_c$. These values agree quantitatively with the recent values reported by Mocsy and Petreczky [23]. On the other hand, when we use the Debye mass from lattice parametrized free energy (m_D^L), the values become much lower than the leading-order results (see Table II). However, nonperturbative corrections to the Debye mass (m_D^{NP}) make the values unrealistically small. These observations can be understood from the hierarchy in their numerical values: $m_D^{\text{LO}} < m_D^L < m_D^{\text{NP}}$.

The fact that m_D^{NP} leads to unrealistic smaller values of dissociation temperatures does not imply that one should ignore the nonperturbative terms in the Debye mass. In fact, nonperturbative terms cannot be ignored in the regime where coupling is strong which is indeed the case dealt with. It

TABLE II. Lower (upper) bound on the dissociation temperatures using the lattice parametrized form of the Debye mass m_D^L .

State	Pure QCD	$N_f = 2$	$N_f = 3$
J/ψ	0.8 (1.0)	0.9 (1.2)	0.9 (1.1)
ψ'	0.5 (0.7)	0.7 (0.8)	0.6 (0.8)
Υ	1.0 (1.3)	1.2 (1.6)	1.2 (1.5)
Υ'	0.7 (0.9)	0.9 (1.1)	0.8 (1.0)

TABLE III. Same as Table I but using the full potential (6).

State	Pure QCD	$N_f = 2$	$N_f = 3$
J/ψ	1.2 (1.5)	1.3 (1.72)	1.3 (1.7)
ψ'	0.8 (1.2)	1.0 (1.4)	1.1 (1.2)
Υ	1.4 (1.8)	1.7 (2.3)	1.6 (2.1)
Υ'	1.0 (1.4)	1.2 (1.7)	1.2 (1.5)

would rather be of interest to raise the question why this nonperturbative result obtained with a dimensional reduction is not in agreement with the Debye mass arising from Polyakov-loop correlators. This could be partially due to the arbitrariness in the definition of dissociation temperature, since strictly speaking a Coulomb potential always admits bound states in its spectrum. Indeed the choice of the average thermal energy $3T$ is not rigid because even at low temperatures $T < T_c$ (say) the Bose/Fermi distributions of partons will have a high energy tail with partons of mechanical energy greater than the binding energy. So, we calculate the upper bound of the dissociation temperatures by replacing the average thermal energy $\sim T$ which is listed within the first bracket in the tables where the values are increased by 30% approximately.

The results for the dissociation temperatures of various quarkonia listed above in Table I and Table II are obtained by dropping all the finite-range terms in the full effective potential (6). As mentioned earlier, for the s -wave states this leads to an analytically solvable Coulomb potential. To see the effects of the finite-range terms in Eq. (6), we solve the Schrödinger equation numerically with the full effective potential (6) and determine the energy spectrum of the ground and the first excited states of the charmonium and bottomonium spectrum with the Debye mass in the leading order. We find that the dissociation temperatures change by ~ 10 – 20% shown in Table III. The dissociation temperature for J/Ψ in the pure gauge case becomes $1.2T_c$ which was earlier $1.1T_c$ (Table I) and for ψ' , it now becomes $1.0T_c$ for $N_f = 2$ which was earlier $0.9T_c$. The same trend follows for other charmonium and bottomonium states. This slight increase in the dissociation temperatures is caused by the increase in the binding energies due to the finite-range terms in the full potential (6).

Finally, to compare our results with a recent calculation [23] having the same input based on a potential study for a three-flavor QCD with $T_c = 192 \text{ MeV}$, we calculated the upper bound of dissociation temperatures in Tables IV and V with the same form of Debye mass used in Ref. [23]. It shows a good agreement with their results [23]. However, the agreement holds well even with the full effective potential.

TABLE IV. Upper bound on the dissociation temperatures (T_D) with $T_c = 192 \text{ MeV}$ [23] using the lattice parametrized form of the screening mass (m_D^L).

State	ψ'	J/ψ	Υ'	Υ
T_D	$\leq 0.9T_c$	$1.2T_c$	$1.1T_c$	$1.6T_c$

TABLE V. Same as Table IV but using the full potential (6).

State	ψ'	J/ψ	Υ'	Υ
T_D	$\leq 1.0T_c$	$1.3T_c$	$1.2T_c$	$1.6T_c$

III. CONCLUSIONS AND OUTLOOK

In conclusion, we have studied the dissociation phenomena of quarkonia in a hot QCD medium by investigating the in-medium modifications to a heavy quark potential. We have found that medium modification causes a dynamical screening of color charge which, in turn, leads to a temperature dependent binding energy. We have systematically studied the temperature dependence of binding energy for the ground and first excited states of charmonium and bottomonium spectra in pure and realistic QCD medium. We have then determined the dissociation temperatures employing the perturbative result of the Debye mass (m_D^{LO}) and the lattice parametrized form m_D^L . Our estimates are consistent with the finding of recent theoretical works based on potential models [23]. However, these values are significantly smaller than the predictions of Refs. [4, 6, 14, 40] based on the first principle lattice calculations which are however plagued by its inherent uncertainties. In contrast, the inclusion of nonperturbative contributions to the Debye mass lowers the dissociation temperatures substantially which looks unfeasible. Thus, this study provides us a handle to decipher the extent up to which and how much nonperturbative effects should be incorporated into the Debye mass.

In brief, J/ψ is found to be dissociated at temperature above the critical temperature (around $1.2T_c$) when the leading-order term in the Debye mass has been employed. However, it is dissociated just below the T_c when the lattice parametrized form of the Debye mass (nonperturbative) has been employed. This finding ensues a basic question about the nature of the dissociation of quarkonium in a hot QCD medium.

Finally, our approach based on the in-medium modifications provides charmonium and bottomonium dissociation temperatures which agree nicely with recent quarkonium spectral function studies using a potential model [23]. This is true only for the perturbative result for the Debye mass but nonperturbative corrections to it make the melting temperatures too low to compare to the spectral analysis of the lattice temporal correlator of the mesonic current. This leaves an open problem of the agreement between these two kinds of approaches. This could partially be due to the arbitrariness in the definition of dissociation temperature. To examine this point we have estimated the upper bound of the dissociation temperatures from the condition: $E_{\text{bin}} = T$. We found that these estimates obtained by employing the lattice parametrized Debye mass show good agreement with the predictions in [23] which was not true for the earlier definition (10): $E_{\text{bin}} = 3T$. However, a numerical solution of the Schrödinger equation with the full effective potential (6) gives in general a slightly higher value of the dissociation temperatures. For $\chi_c(\chi_b)$ melting temperatures, one would start with the spin-dependent heavy quark potential and follow the same procedure. We will look into it in the future. It would be of interest to study the corresponding quarkonium spectral function (and temporal correlator) after giving these states a thermal width. Finally, it would be of interest to extend the present study to the medium modified potential obtained in the recent work of Brambilla *et al.* [41] within the framework of quarkonia effective field theory which also predicts the long range terms in the potential.

ACKNOWLEDGMENTS

We acknowledge V. J. Menon, A. Ranjan, and V. Ravishankar for fruitful discussions and CSIR, New Delhi for the financial support.

-
- [1] L. D. McLerran and B. Svetitsky, Phys. Rev. D **24**, 450 (1981).
 - [2] T. Matsui and H. Satz, Phys. Lett. **B178**, 416 (1986).
 - [3] B. K. Patra and D. K. Srivastava, Phys. Lett. **B505**, 113 (2001); D. Pal, B. K. Patra, and D. K. Srivastava, Eur. J. Phys. C **17**, 179 (2000).
 - [4] H. Satz, Nucl. Phys. **A783**, 249 (2007).
 - [5] M. Asakawa and T. Hatsuda, Phys. Rev. Lett. **92**, 012001 (2004).
 - [6] S. Datta, F. Karsch, P. Petreczky, and I. Wetzorke, Phys. Rev. D **69**, 094507 (2004).
 - [7] T. Umeda, K. Nomura, and H. Matsufuru, Eur. Phys. J. C **39**, 9 (2005).
 - [8] H. Iida, T. Doi, N. Ishii, H. Sukanuma, and K. Tsumura, Phys. Rev. D **74**, 074502 (2006).
 - [9] Agnes Mocsy and Peter Petreczky, Phys. Rev. D **77**, 014501 (2008).
 - [10] S. Digal, P. Petreczky, and H. Satz, Phys. Lett. **B514**, 57 (2001).
 - [11] E. V. Shuryak and I. Zahed, Phys. Rev. D **70**, 054507 (2004).
 - [12] W. M. Alberico, A. Beraudo, A. De Pace, and A. Molinari, Phys. Rev. D **72**, 114011 (2005).
 - [13] C. Y. Wong and H. W. Crater, Phys. Rev. D **75**, 034505 (2007).
 - [14] W. M. Alberico, A. Beraudo, A. De Pace, and A. Molinari, Phys. Rev. D **75**, 074009 (2007).
 - [15] T. Umeda, Phys. Rev. D **75**, 094502 (2007).
 - [16] W. M. Alberico, A. Beraudo, A. De Pace, and A. Molinari, Phys. Rev. D **77**, 017502 (2008).
 - [17] B. Beinlich, F. Karsch, E. Laermann, and A. Peikert, Eur. Phys. J. C **6**, 133 (1999).
 - [18] F. Karsch, E. Laermann, and A. Peikert, Phys. Lett. **B478**, 447 (2000).
 - [19] O. Kaczmarek, S. Ejiri, F. Karsch, E. Laermann, and F. Zantow, Prog. Theor. Phys. Suppl. **153**, 287 (2004).
 - [20] A. Dumitru, J. Lenaghan, and R. D. Pisarski, Phys. Rev. D **71**, 074004 (2005).
 - [21] F. Karsch, hep-lat/0601013; J. Phys.: Conf. Series **46**, 122 (2006).
 - [22] K. Kajantie, M. Laine, J. Peisa, A. Rajantie, K. Rummukainen, and M. E. Shaposhnikov, Phys. Rev. Lett. **79**, 3130 (1997).
 - [23] Agnes Mocsy and Peter Petreczky, Phys. Rev. Lett. **99**, 211602 (2007).
 - [24] W. Lucha *et al.*, Phys. Rep. **200**, 127 (1991).
 - [25] N. Brambilla *et al.*, Rev. Mod. Phys. **77**, 1423 (2005).

- [26] R. A. Schneider, Phys. Rev. D **66**, 036003 (2002).
- [27] H. A. Weldon, Phys. Rev. D **26**, 1394 (1982).
- [28] Akhilesh Ranjan and V. Ravishankar, arXiv:0707.3697; Vinod Chandra, Akhilesh Ranjan, and V. Ravishankar, arXiv:0801.1286.
- [29] Mikko Laine, O. Philipsen, Marcus Tassler, and Paul Romatschke, JHEP 03 (2007) 054.
- [30] A. Beraudo, J. P. Blaizot, and C. Ratti, Nucl. Phys. **A806**, 312 (2008).
- [31] Helmut Satz, J. Phys. G: Nucl. Part. Phys. **32**, R25 (2006).
- [32] C. Young and Edward Shuryak, Phys. Rev. C **79**, 034907 (2009).
- [33] Vijai V. Dixit, Mod. Phys. Lett. A **5**, 227 (1990).
- [34] Atsushi Nakamura and Takuya Saito, Prog. Theor. Phys. **111**, 733 (2004).
- [35] E. Shuryak, Sov. Phys. JETP **47**, 212 (1978) [Zh. Eksp. Teor. Fiz. **74**, 408 (1978)].
- [36] M. Laine and Y. Schroder, JHEP 03 (2005) 067.
- [37] Suzhou Huang and Marcello Lissia, Nucl. Phys. **B438**, 54 (1995).
- [38] E. Eichten and F. Feinberg, Phys. Rev. D **23**, 2724 (1981).
- [39] O. Kazmarek and F. Zantow, PoS (LAT2005) 177.
- [40] Gert Aarts, Chris Allton, Mehmet Bugrahan Oktay, Mike Peardon, and Jon Ivar Skullerud, Phys. Rev. D **76**, 094513 (2007).
- [41] Nora Brambilla, Jacopo Ghiglieri, Antonio Vairo, and Peter Petreczky, Phys. Rev. D **78**, 014017 (2008).