Estimate of the two-photon exchange effect on deuteron electromagnetic form factors

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The corrections of two-photon exchange on deuteron electromagnetic form factors are estimated based on an effective Lagrangian approach. Numerical results for the form factors $G_{C,M,Q}$ of the deuteron with the corrections are compared to its empirical ones. Moreover, the two new form factors, due to the two-photon exchange, are analyzed. A possible way to test the two-photon exchange corrections to the deuteron form factors is discussed.

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I. INTRODUCTION

We know that the electromagnetic (EM) form factors of the proton and deuteron are usually extracted from the measurements of the differential cross sections of ep and eD elastic scatterings and from the Rosenbluth separation method [1], which is based on the one-photon-exchange (OPE) approximation. For a long time, the extracted Q^2 dependencies of the nucleon EM form factors are believed to behave like a simple dipole form. For the proton electric and magnetic form factors, $G^p_{E,M}$, one conventionally assumes

$$G_E^p(Q^2) = G_M^p(Q^2)/\mu_p \simeq 1/[1 + Q^2(\mathrm{GeV}^2)/0.71]^2, \quad (1)$$

where $\mu_p = 2.79$ is the proton magneton. Recently, the new experiments of the polarized ep elastic scattering were precisely carried out at Jefferson Laboratory [2]. The polarization transfer scattering experiments of $\vec{e} + p \to e + \vec{p}$ show that the ratio $R^p = \mu_p G_E^p(Q^2)/G_M^p(Q^2)$ behaves like $R^p(Q^2) \sim 1 - 0.158Q^2$. It means that R^p is no longer a simple constant as implied in Eq. (1). It monotonously decreases with the increasing of Q^2.

One way to resolve this discrepancy, at least partially, is to take the effect of the two-photon exchange (TPE) into account [3–8]. Usually, it is believed that TPE is strongly suppressed by EM coupling constant α_{EM} ($\sim 1/137$). However, it was argued [8] that due to a very steep decreasing of the nucleon EM form factors, the TPE process, where the Q^2 is equally shared by the two exchanging photons, may be compatible to the OPE one. Some calculations of the TPE corrections to the ep elastic scattering have been done recently [3–7,9], where only the nucleon state is considered as an intermediate state. The calculations were extended further with other nucleon resonances, like Δ, P_{11}, and D_{13} states, being considered as the intermediate states [10]. There were also several other works about the TPE effect on the proton charge radius and on the parity violating [11,12] in the ep scattering. The effect on the EM form factors of the nucleon in the timelike region was estimated in Refs. [13,14]. According to the analyses for the TPE effect on the nucleon EM form factors in the literature, it is known that the TPE corrections not only modify the conventional nucleon electric and magnetic form factors but also provide a new form factor, $Y_{2\gamma}$, to the nucleon.

The TPE corrections to the deuteron (spin 1 particle) EM form factors and to the $e^+ + e^- \to D + \bar{D}$ process have been also discussed in Refs. [15–17] qualitatively. In analogy to the TPE effect on the proton EM form factors, TPE not only modifies the conventional three EM form factors of the deuteron but also provides new form factors with new structures. The general discussion of the structures of the three new form factors can be seen [15,16]. We know that the deuteron is usually regarded as a weekly bound system of a proton and a neutron (see Fig. 1). Many calculations for the EM form factors of the deuteron, with the OPE approximation, have been performed in different approaches in the literature (see, for example, Refs. [18–21]). Recent calculations based on an effective Lagrangian approach [22,23] have shown that this approach can reasonably explain the deuteron EM form factors with phenomenological, including two-body, operators.

To study the TPE effect on the deuteron system in our effective Lagrangian approach, we note that the deuteron EM form factors receive the TPE corrections from three different sources. The first one (see Fig. 2) is that the two photons directly couple to the contact points [the contact point A (or B) of Fig. 1 is the one connects the deuteron to its composites]. The second is that one of the two photons directly couples to one of the nucleons and another to one of the contact points (see Fig. 3). The last one is that the two photons respectively couple to the two nucleons (see Fig. 4). It has been proved that gauge invariance preserves in our effective Lagrangian approach only when the three kinds of the two-photon exchange diagrams are considered simultaneously [24].

In our previous work [25], only a part of the third type of the TPE corrections to the EM form factors of deuteron is considered, where the TPE corrections to the EM form factors $G_{E,M}$ and to $Y_{2\gamma}$ of the proton and neutron are directly employed to study the deuteron properties following the formalism of Ref. [9]. In the approach, only one new form factor appears. The TPE effect considered in Ref. [25] is represented by Figs. 4(a) and 4(b) and their cross-box diagrams. In this article, to extend the work of Ref. [25] further, we'll simultaneously study the three sources of the TPE effect in Figs. 2–4. It should be stressed that although the contribution of the coupling of a photon to the contact point is expected to be smaller than the one of direct couplings of the photon to the nucleons, this type of couplings is needed to guarantee gauge invariance. This article is organized as follows. In Sec. II the above-mentioned two-photon-exchange effect in the eD

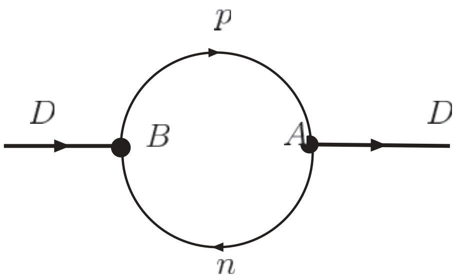

FIG. 1. Deuteron mass operator.

elastic scattering is briefly discussed. Numerical results and conclusions are given in Sec. III.

II. TWO-PHOTON-EXCHANGE IN THE *eD* ELASTIC SCATTERING

According to the OPE approximation, the electromagnetic form factors of the deuteron are defined by the matrix element of the electromagnetic current $J_\mu(x)$

$$\begin{aligned}\langle p'_D, \lambda' | J_\mu(0) | p_D, \lambda\rangle &= -e_D \left\{ \left[G_1(Q^2)\xi'^*(\lambda')\cdot\xi(\lambda) - G_3(Q^2)\frac{(\xi'^*(\lambda')\cdot q)(\xi(\lambda)\cdot q)}{2M_D^2}\right]\cdot P_\mu \right. \\ &\quad \left. + G_2(Q^2)[\xi_\mu(\lambda)(\xi'^*(\lambda')\cdot q) - \xi'^*_\mu(\lambda')(\xi(\lambda)\cdot q)]\right\}, \end{aligned} \tag{2}$$

where p'_D, ξ', λ' (or p_D, ξ, λ) denote the momentum, helicity, and polarization vector of the final (or initial) deuteron, respectively. In Eq. (2) $q = p'_D - p_D$ is the photon momentum, $P = p_D + p'_D$, $Q^2 = -q^2$ is the four-momentum transfer squared, M_D is the deuteron mass, and e_D is the charge of the deuteron. In the one-photon exchange approximation or Born approximation, the unpolarized differential cross section of the eD elastic scattering, $e(k_1, s_1) + D(p_D, \xi) \to e(k'_1, s_3) + D(p'_D, \xi')$, in the laboratory frame is [26]

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}} I_0(\text{OPE}),\\ I_0(\text{OPE}) &= A(Q^2) + B(Q^2)\tan^2\frac{\theta}{2},\end{aligned} \tag{3}$$

where θ is the scattering angle of the electron, $(d\sigma/d\Omega)_{\text{Mott}}$ is the Mott cross section for a structureless particle with recoil

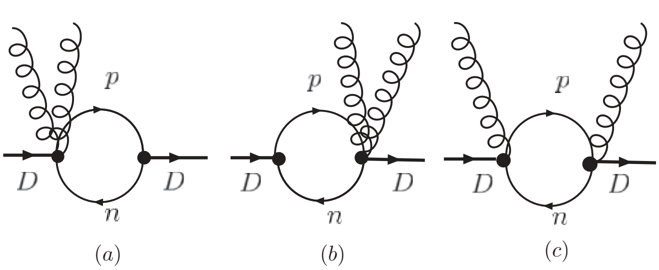

FIG. 2. Diagrams for the first type of the two-photon exchange effect. The cross-box diagrams are implied.

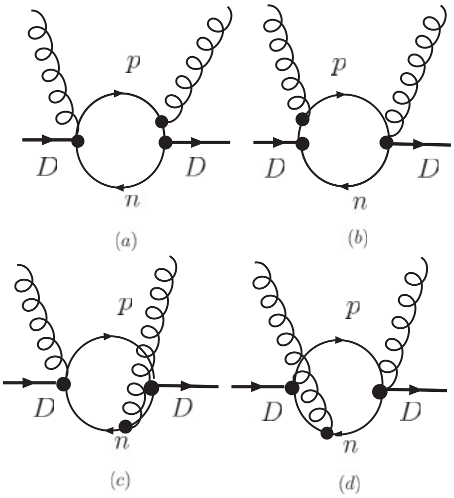

FIG. 3. Diagrams for the second type of the two-photon exchange effect. The cross-box diagrams are implied.

effect, and the two structure functions are

$$\begin{aligned}A(Q^2) &= G_C^2(Q^2) + \tfrac{2}{3}\tau_D G_M^2(Q^2) + \tfrac{8}{9}\tau_D^2 G_Q^2(Q^2),\\ B(Q^2) &= \tfrac{4}{3}\tau_D(1+\tau_D)G_M^2(Q^2).\end{aligned} \tag{4}$$

In Eq. (4), $\tau_D = Q^2/4M_D^2$ and G_M, G_C, and G_Q are the deuteron magnetic, charge, and quadrupole form factors, respectively. They can be expressed, in terms of G_1, G_2, and G_3, as

$$\begin{aligned}G_M &= G_2, \quad G_Q = G_1 - G_2 + (1+\tau_D)G_3,\\ G_C &= G_1 + \tfrac{2}{3}\tau_D G_Q.\end{aligned} \tag{5}$$

The normalizations of the three form factors are $G_C(0) = 1$, $G_M(0) = 1.714$, and $G_Q(0) = M_D^2 Q_D = 25.83$. Note that in Eqs. (3) and (4), there are two unpolarized structure functions A and B, and three independent form factors G_C, G_Q, and G_M for the deuteron. To determine the three form factors completely, one needs, at least, one polarization

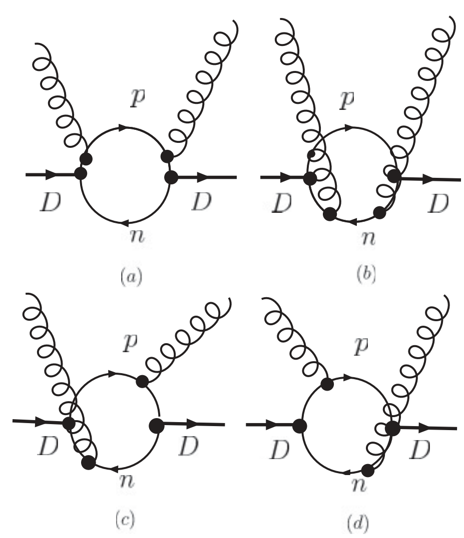

FIG. 4. Diagrams for the third type of the two-photon exchange effect. The cross-box diagrams are implied.

observable. The optimal choice is the polarization T_{20} (or P_{zz}) [27].

Considering both OPE ($\mathcal{C}=-1$) and TPE ($\mathcal{C}=+1$), and taking Lorentz, party, and charge-conjugation invariance into account, one obtains the most general form of the eD elastic scattering [15,28],

$$\mathcal{M}^{eD}=\frac{e^2}{Q^2}\bar{u}(k_1',s_3)\gamma_\mu u(k_1,s_1)\sum_{i=1}^{6}G_i'M_i^\mu, \tag{6}$$

where

$$\begin{aligned} M_1^\mu &= (\xi'^*\cdot\xi)P^\mu,\\ M_2^\mu &= [\xi^\mu(\xi'^*\cdot q)-(\xi\cdot q)\xi'^{*\mu}],\\ M_3^\mu &= -\frac{1}{2M_D^2}(\xi\cdot q)(\xi'^*\cdot q)P^\mu, \end{aligned} \tag{7}$$

and

$$\begin{aligned} M_4^\mu &= \frac{1}{2M_D^2}(\xi\cdot K)(\xi'^*\cdot K)P^\mu,\\ M_5^\mu &= [\xi^\mu(\xi'^*\cdot K)+(\xi\cdot K)\xi'^{*\mu}],\\ M_6^\mu &= \frac{1}{2M_D^2}[(\xi\cdot q)(\xi'^*\cdot K)-(\xi\cdot K)(\xi'^*\cdot q)]P^\mu, \end{aligned} \tag{8}$$

where $K=k_1+k_1'$. General speaking, the form factors G_i', with $i=1,6$, are complex functions of $s=(p_D+k_1)^2$ and $Q^2=-(k_1-k_1')^2$. They can be expressed as

$$G_i'(s,Q^2)=G_i(Q^2)+G_i^{(2)}(s,Q^2), \tag{9}$$

where G_i corresponds to the contributions arising from the one-photon exchange and $G_i^{(2)}$ stands for the rest that would come mostly from TPE. In the OPE approximation, $G_4'=G_5'=G_6'=0$. It is easy to see that G_i $(i=1,2,3)$ are of order of $(\alpha_{\rm EM})^0$ and $G_i^{(2)}$ $(i=1,\ldots 6)$ are of order $\alpha_{\rm EM}$.

To consider that a deuteron is a weakly bound state of a proton and a neutron, we take the following effective interaction between the deuteron and its composites (pn) [23]

$$\begin{aligned}\mathcal{L}_D &= g_D D^{\mu+}(x)\int dy\,\Phi_D(y^2)\bar{p}\left(x+\frac{1}{2}y\right)\\ &\quad\times C\gamma_\mu n\left(x-\frac{1}{2}y\right)+\text{H.c.},\end{aligned} \tag{10}$$

where C is the charge conjugate matrix, D^μ, p, and n are the fields of the deuteron, proton, and neutron. The correlation function Φ_D in Eq. (10) characterizes the finite size of the deuteron as a pn bound state and depends on the relative Jacobi coordinate y, in addition, x being the center-of-mass (c.m.) coordinate. The Fourier transformation of the correlation function reads

$$\Phi_D(y^2)=\int\frac{d^4p}{(2\pi)^4}\,e^{-ipy}\,\tilde{\Phi}_X(-p^2)\,. \tag{11}$$

A basic requirement for the choice of an explicit form of the correlation function is that it vanishes sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite. Here, we adopt a Gaussian form $\tilde{\Phi}_D(p_E^2)\doteq\exp(-p_E^2/\Lambda_D^2)$ for the vertex function, where p_E is the Euclidean Jacobi momentum of the deuteron and Λ_D is a size parameter. It characterizes the distribution of the constituents inside the deuteron.

We know that the low-energy theorem [29] provides a model-independent test for the reliability of different approaches [30]. For the photon deuteron (spin-1) Compton scattering, the low-energy theorem has been discussed extensively in the past [31]. A complete treatment on this issue is referred to in Ref. [32]. To the first order of the photon energy ω, the low-energy theorem tells that the forward Compton-scattering amplitude off the deuteron target is [30]

$$\begin{aligned}4\pi\mathcal{T} &= -\frac{e^2}{M_D}\vec{\epsilon}'\cdot\vec{\epsilon}\\ &\quad - i\frac{e^2}{4M_D^2}\omega(\mu_D-2)^2\vec{S}\cdot(\vec{\epsilon}'\times\vec{\epsilon})+\mathcal{O}(\omega^2),\end{aligned} \tag{12}$$

with $\vec{\epsilon}$ (or $\vec{\epsilon}'$), $\vec{S}$, and μ_D being the initial (or final) photon polarization, the deuteron spin, and its magnetic moment in unit of $e/2M_D$, respectively. In Eq. (12) the first and second terms are the Thomson and the spin-flip ones. The latter is proportional to the deuteron anomalous magnetic moment squared $\kappa_D^2=(\mu_D-2)^2$ and associates to the well-known Drell-Hearn-Gerasimov sum rule of the deuteron (see Refs. [30,33], for example). In our effective Lagrangian approach [23], the effective current of photon-deuteron has the correct structures like Eq. (2) and the numerical calculation shows that the obtained magnetic moment of the deuteron μ_D is around 1.7 (in unit of $e/2M_D$), which reasonably agrees with the experiment data. Consequentially, it is expected that the forward Compton-scattering amplitude based on our effective approach is consistent with the low-energy theorem. It should be mentioned that the above correlation function of Eq. (11), in the nonrelativistic approximation, stands for the wave function with only the S-wave of the deuteron, which contains no D-wave component. To reasonably explain the data for the deuteron quadrupole moment, we have to phenomenological include two-body operators [23]. A detailed comparison of the photon-deuteron Compton scattering amplitudes in the low-photon-energy region of our approach and of the low-energy theorem will be explicitly given in a separate article.

In our approach, the coupling g_D of $\langle p^D,\lambda|pn\rangle = g_D\xi'^*(\lambda)$ is determined by the compositeness condition [24,34–37]. It implies that the renormalization constant of the deuteron wave function is set equal to zero:

$$Z_D=1-\Sigma_D'\big(M_D^2\big)=0. \tag{13}$$

Here,

$$\Sigma_D'\big(M_D^2\big)=g_D^2\left.\frac{d\Sigma_D}{dp_D^2}\right|_{p_D^2=M_D^2} \tag{14}$$

is the derivative of the transverse part of the mass operator $\Sigma_D^{\alpha\beta}$, which conventionally splits into the transverse Σ_D and longitudinal Σ_D^L parts as:

$$\Sigma_D^{\alpha\beta}=g_\perp^{\alpha\beta}\Sigma_D\big(p_D^2\big)+\frac{p_D^\alpha p_D^\beta}{p_D^2}\Sigma_D^L\big(p_D^2\big)\,, \tag{15}$$

where $g_\perp^{\alpha\beta}=g^{\alpha\beta}-p^\alpha p^\beta/p^2$ and $g_\perp^{\alpha\beta}p_\alpha=0$. The mass operator of the deuteron in our approach is described by Fig. 1. If the size parameter Λ_D is fixed, the coupling g_D is fixed too

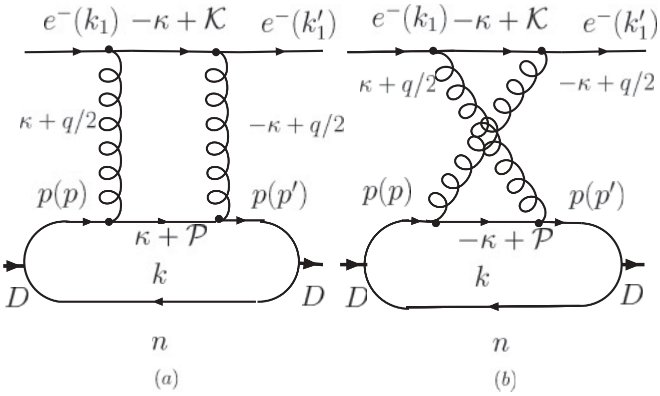

FIG. 5. Feynman diagrams for two-photon exchange: box diagram (a) and crossed box diagram (b).

according to the compositeness condition (13) (see details in Ref. [23]). Here, we reiterate that because Figs. 2, 3, and 4 are taken into account simultaneously, gauge invariance preserves in our effective Lagrangian approach.

III. NUMERICAL RESULTS AND CONCLUSIONS

To proceed a numerical calculation, we adopt the parametrization forms of the nucleon EM form factors given by Mergell, Meissner, and Drechsel [38]. Here we follow the numerical technique of Ref. [39] to simplify one of our loop integrations. The loop momentum of the box-type Feynman amplitude is parametrized in a such way that the denominators of Green function are $(\mp\kappa + q/2)^2$ for the two photons (see Fig. 5 where the cross-box diagram is explicitly shown), whereas for the electron (e) and the constituent nucleon (N), they have the forms of $(e) = (-\kappa + \mathcal{K})^2 - m_e^2$ and $(p) = (\pm\mathcal{K} + \mathcal{P})^2 - M_N^2$ with

$$\mathcal{K} = \tfrac{1}{2}(k_1 + k_1') = \tfrac{1}{2}K, \quad \mathcal{P} = \tfrac{1}{2}(p + p'). \tag{16}$$

The sign $-(+)$ is for the direct (cross-box) diagram in Fig. 5. Here, it should be mentioned that the assumption of Ref. [39] means that each of the photons carries approximately half of the transferred momentum q. It is justified on the bases of Ref. [8]. Moreover, the assumption also means that a rapid decreasing of the form factors is employed such that one can neglect the dependence on the loop momentum κ in the denominators of the photon Green function as well as in the arguments of the form factors. This results in ultraviolet divergences of the loop momentum integrals with respect to the momentum κ. Thus, a step function $\theta(M_N^2\tau_N - |\kappa^2|)$ is introduced in the loop integration. It is equivalent to apply a cut-off restriction $|\kappa^2| < M_N^2\tau$. For the photon Green function, we have

$$\frac{1}{\left|\frac{q}{2} \pm \kappa\right|^2} < \frac{1}{\mathcal{P}^2} = \frac{1}{M_N^2(1+\tau_N)} \tag{17}$$

with $\tau_N = \frac{Q^2}{4M_N^2}$. In our calculation for the effect of TPE based on the effective Lagrangian of Eq. (10), we face two loop integrations. One is the loop integration with respect to the intermediate momentum of κ, and another is the one with respect to the intermediate momentum k of the composites of the deuteron (see Fig. 5). To simplify the numerical calculation further, we also use the soft approximation for the integral variable κ in the first loop integration.

Based on the above assumption and considering the TPE effect shown in Figs. 2–4, we may estimate the TPE corrections to the deuteron EM form factors in the present effective Lagrangian approach. The effective EM interaction Lagrangians have already been given explicitly in Refs. [23,24]. With those Lagrangians, we can correctly get the normalization conditions for $G_C(0)$ and $G_M(0)$. In Figs. 6–10, we plot our numerical results for the contributions of the TPE effect to the deuteron electromagnetic form factors of G_C, G_M, G_Q and to the two additional form factors G_5 and G_6. Two different scattering angles, θ being $\pi/2$ and $\pi/10$, are selected to check the θ dependencies of the observables. In Figs. 6–8, the ratios stand for

$$R_{C,M,Q} = \frac{G^{(2)}_{C,M,Q}(s,Q^2)}{G^{\text{exp.}}_{C,M,Q}(s,Q^2)}, \tag{18}$$

where $G^{(2)}_{C,M,Q}(s,Q^2)$ represent for the TPE contributions. The individual contributions of Figs. 2, 3, and 4 and their

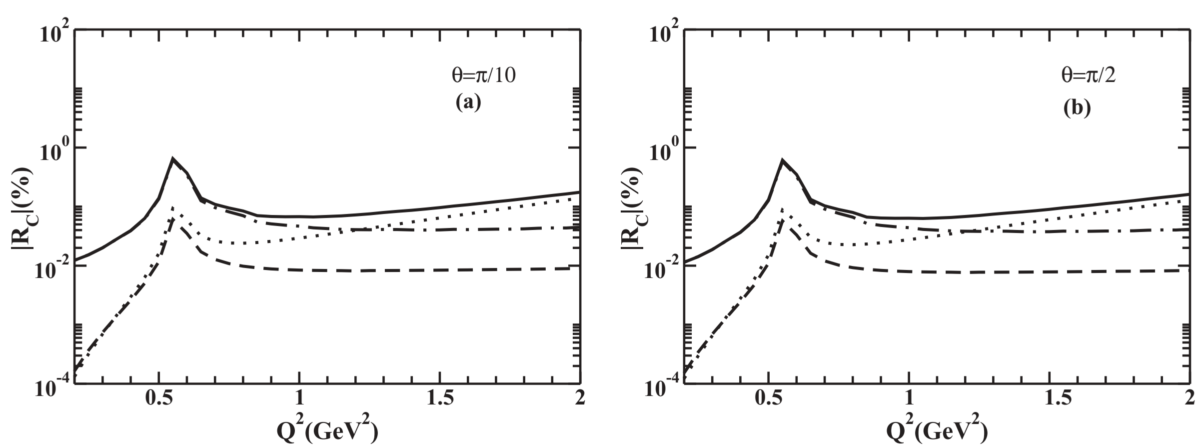

FIG. 6. $|R_C|(\%)$ for $\theta = \pi/10$ (a) and for $\theta = \pi/2$ (b). The dotted, dashed, dotted-dashed, and solid curves represent the contributions from Figs. 2, 3, and 4 and their sum.

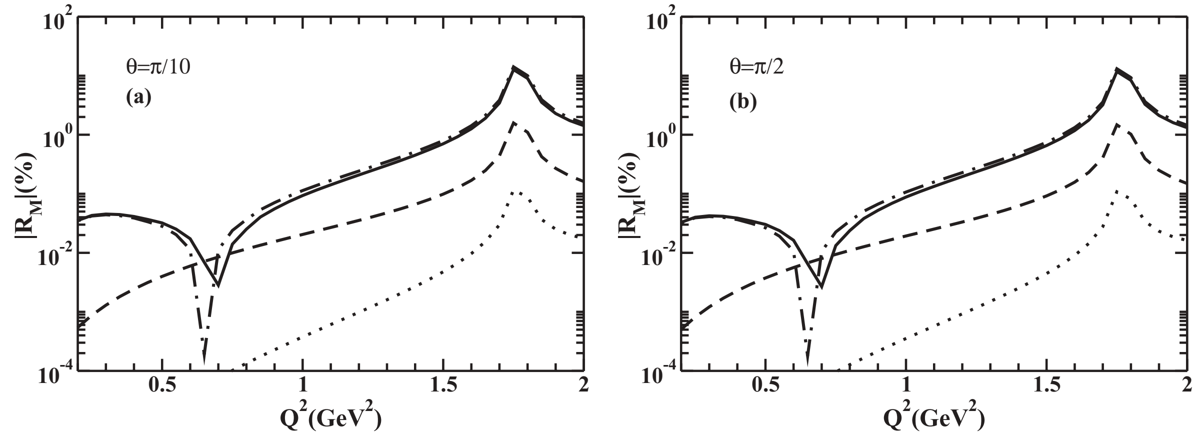

FIG. 7. $|R_M|(\%)$ for $\theta = \pi/10$ (a) and for $\theta = \pi/2$ (b). Notations as in Fig. 6.

sum to the deuteron form factors are shown explicitly. $G^{\text{exp.}}_{C,M,Q}$ in Eq. (18) are estimated by the parametrizations of Ref. [40] (θ-independent form) as the empirical data. The two maximum points in Figs. 6 and 7 are due to the two crossing points of the charge G_C and magnetic G_M form factors at about $Q^2_{\text{crossing}} \sim 0.5$ GeV^2 and $Q^2_{\text{crossing}} \sim 2.0$ GeV^2. Here, differing from the form factors of the nucleon, the form factors of the deuteron have the crossing points. From Figs. 6–8, one cannot explicitly see the θ dependence of the three ratios, because the dependencies are strongly suppressed due to the fact that the denominators of the ratios in Eq. (18) are θ independent. However, the θ dependencies can be seen explicitly in Figs. 9 and 10 for the two new form factors $G'_5 = G^{(2)}_5$ and $G'_6 = G^{(2)}_6$. It should be mentioned that we do not have the extra form factors G'_4 contributed by the TPE effect as shown in Eqs. (6) and (9) because we adopt the assumption of Ref. [39], where the κ-dependence terms in the numerator are ignored.

In our calculation, we have one parameter Λ_D in the correlation function. According to the condition that the deuteron is bound as $\langle |r^{-2}| \rangle \leqslant 0.02$ GeV^2 [18], we select a typical value for the parameter: $\Lambda_D = 0.30$ GeV that is consistent with the one used in Refs. [23,25]. Our estimates for the ratios of the deuteron electromagnetic form factors of $G'_{C,M,Q}$ tell that the TPE effect is small. To analyze the contributions of Figs. 2, 3, and 4, one sees that in the low-Q^2 region, the contributions of Figs. 2 and 3 are smaller than the one of Fig. 4. When Q^2 increases, the contributions of Figs. 2, 3, and 4 increase too. Moreover, the contributions of Figs. 2 and 3 to the form factors of $G_{C,Q}$ are always smaller than that of Fig. 4, whereas the one of Fig. 2 to G_M becomes compatible to the contribution of Fig. 4. Because the total contributions of Figs. 2 and 3 to the three conventional EM form factors of $G_{C,M,Q}$ are very small, and the θ dependencies of the ratios from the TPE corrections are suppressed, it is not easy to directly test the TPE effect from the three form factors.

However, it is expected that one may test the TPE effect from the polarizations of deuteron, because the TPE effect is θ dependent and the obtained new form factors $G_{5,6}$ are θ dependent, too. Consequently, it is reasonable to find the TPE effect in some polarizations and particularly in some angle limit. We know that if only one-photon exchange is considered, the double and single polarization observables are

$$P_{xz} = -\tau_D \frac{K_0}{M_D} \tan\frac{\theta}{2} G_M G_Q,$$
$$P_z = \frac{1}{3}\frac{K_0}{M_D}\sqrt{\tau_D(\tau_D+1)}\tan^2\frac{\theta}{2} G_M^2. \quad (19)$$

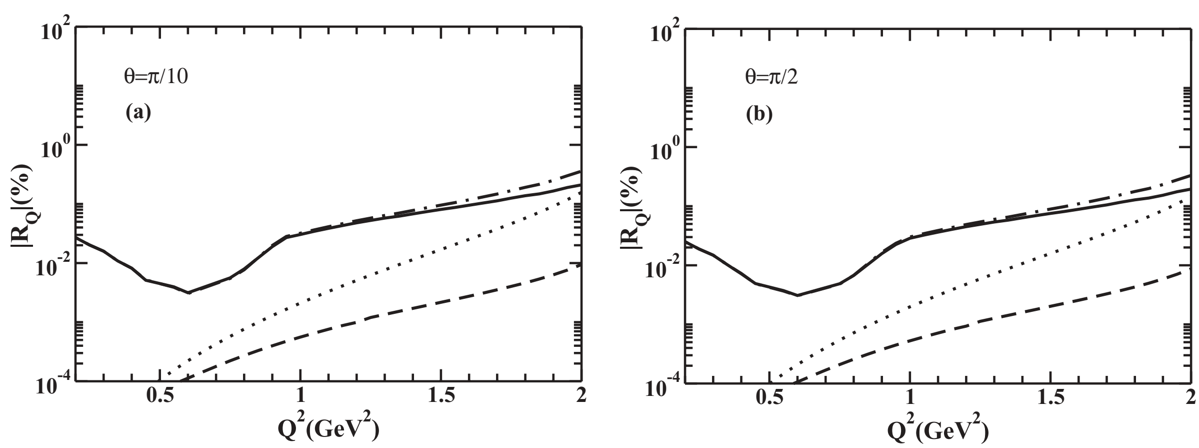

FIG. 8. $|R_Q|(\%)$ for $\theta = \pi/10$ (a) and for $\theta = \pi/2$ (b). Notations as in Fig. 6.

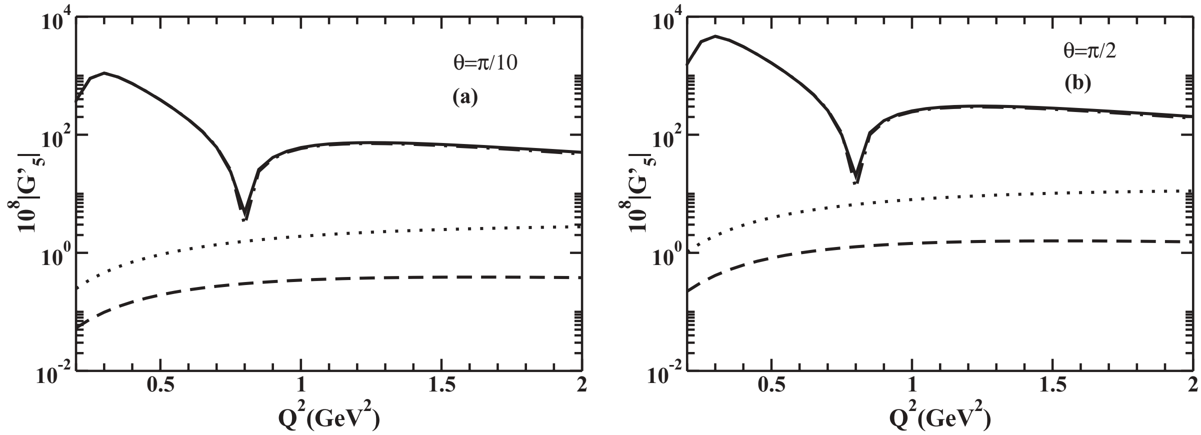

FIG. 9. $10^8|G_5'|$ for $\theta=\pi/10$ (a) and for $\theta=\pi/2$ (b). Notations as in Fig. 6.

Clearly, these two polarizations become vanishing when θ is very small because they are $\tan\frac{\theta}{2}$ and $\tan^2\frac{\theta}{2}$ dependent, respectively. However, when the TPE effect is considered in the small angle limit, its contribution is

$$\delta P_{xz}\sim 2\tau_D^2\cot\frac{\theta}{2}\left[2\left(\frac{G_1}{\tau_D+1}+G_3\right)Re(G_5') + (G_1-4G_2+2(\tau_D+1)G_3)Re(G_6')\right] \quad (20)$$

and

$$\delta P_z\sim -\frac{2\tau_D}{3}\sqrt{\frac{\tau_D}{\tau_D+1}}\left[\left(3+2(\tau_D+1)\tan^2\frac{\theta}{2}\right)G_2Re(G_5') + 2(\tau_D+1)G_2Re(G_6')\right]. \quad (21)$$

One sees that the TPE corrections to the polarizations do not vanish in the limit of $\theta\to 0$. In Fig. 11, we display the ratios $R(P_{xz})=\delta P_{xz}/P_{xz}$ for P_{xz} and $R(P_z)=\delta P_z/P_z$ for P_z calculated from Eqs. (19)–(21). The ratios should behave as $1/\tan^2(\frac{\theta}{2})$. One sees that the contributions from Figs. 2 and 3 are found to be smaller than that of Fig. 4. Moreover, one finds the sizable effect of the two new extra form factors, due to TPE, on the polarization observables P_{xz} and P_z. The remarkable θ dependencies of the ratios are also displayed in Fig. 11. Therefore, a precise measurement of the deuteron polarizations in the small angle limit is expected to test the TPE effect. Because the deuteron form factors have crossing point $Q^2_{\rm crossing}$, it is also expected to easily find the TPE effect at about $Q^2\sim Q^2_{\rm crossing}$.

Reference [25] is the first one to numerically estimate part of the TPE corrections to the deuteron form factors based on our effective Lagrangian approach. In that work, the TPE corrections, to the EM form factors of the proton and neutron following the formalism of Ref. [9], are simply employed to study the deuteron case. The corresponding TPE effect on the deuteron is shown by Figs. 4(a) and 4(b) and their cross-box diagrams. Comparing the present results to those of Ref. [25], one concludes that all the possible TPE corrections are considered in this article. Therefore, the present work gives a more systemically and sophisticated study of the TPE effect on the deuteron. Moreover, we, in this article, directly calculate the TPE exchange effect with the assumption of Ref. [39]. Clearly, the present calculation gives more information about the new deuteron form factors because we predict form factors of $G'_{5,6}$ simultaneously. The obtained results for the TPE effect are consistent with the ones of Ref. [25] qualitatively. Finally,

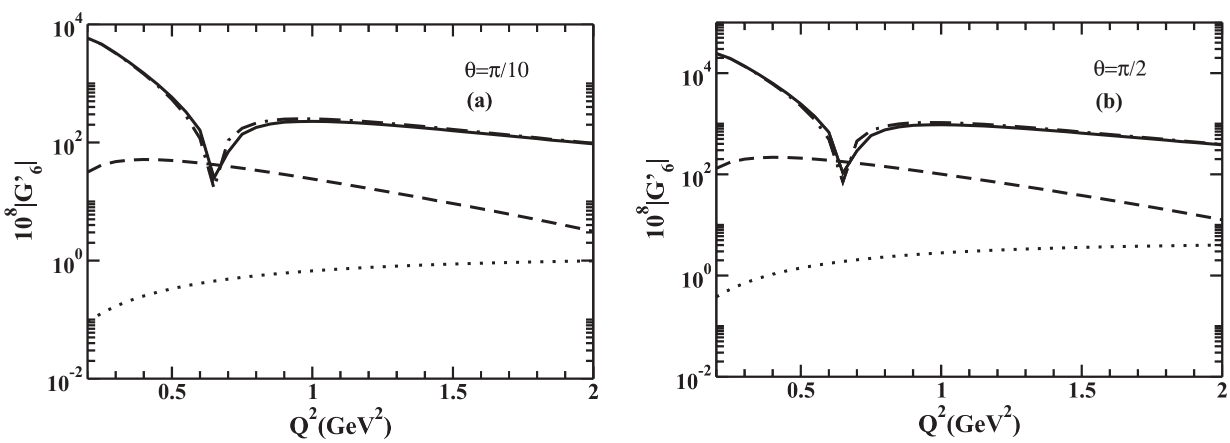

FIG. 10. $10^8|G_6'|$ for $\theta=\pi/10$ (a) and for $\theta=\pi/2$ (b). Notations as in Fig. 6.

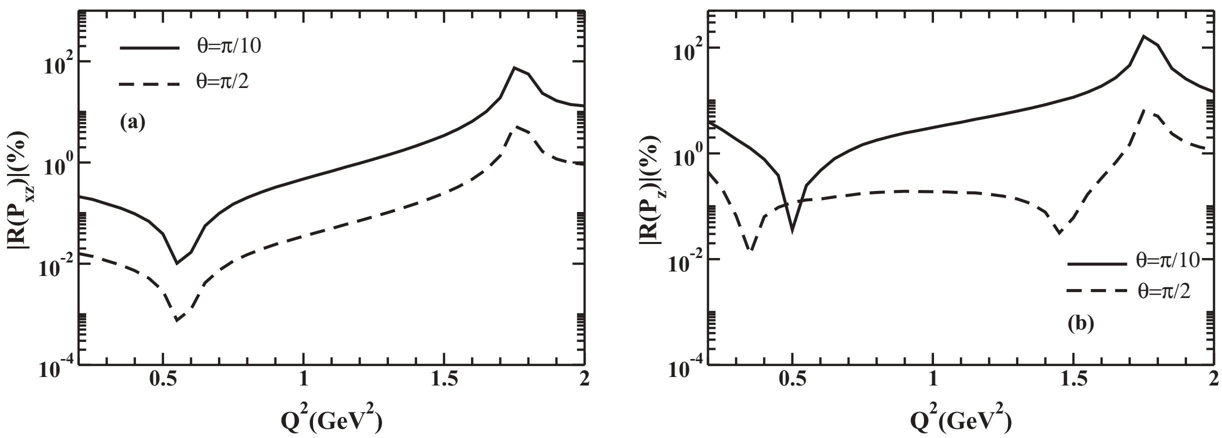

FIG. 11. Ratios for P_{xz} (a) and P_z (b).

one still cannot get any information about G'_4; this is due to the approximate methods we employed here to simplify our numerical loop integration.

To summarize, we are the first to estimate all the TPE corrections, as shown in Figs. 2–4, to the conventional form factors of the deuteron, $G_{C,M,Q}$ and of $G'_{5,6}$. Our numerical results of the TPE contributions tell that $G^{(2)}_{C,M,Q}$ are small (less than 1%). However, $G'_{5,6}$ are clearly θ dependent. The two additional form factors are expected to be tested in the future measurements of the double and single polarization observables of $P_{xz}(T_{21})$ and P_z (T_{10}) in the small angle limit and at about $Q^2 \sim Q^2_{\text{crossing}}$. Further work for an exactly full calculation of the two-photon exchange effect on the deuteron system, without using assumption of Ref. [39], is in progress.

Finally, this work is also designed to effectively treat direct electromagnetic interactions to quarks. It should be addressed that the present investigation of the two photon exchange mechanism recalls a new study of Compton scattering and it is shown that the local two-photon coupling to the same quark provides a fixed Regge singularity at $J = 0$ [41]. This subject is beyond the scope of the present work. However, it is of a great interest to see the issue for the deuteron target in our future work.

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