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Ground state energy of spin polarized quark matter with correlation

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We calculate the ground-state energy of cold and dense spin-polarized quark matter with corrections due to correlation energy ($E_{\rm corr}$). Expressions for $E_{\rm corr}$ both in the nonrelativistic and ultrarelativistic regimes have been derived and compared with the exchange and kinetic term present in the perturbation series. It is observed that the inclusion of correlation energy does not rule out the possibility of the ferromagnetic phase transition at low density within the model proposed by Tatsumi [Phys. Lett. **B489**, 280 (2000)]. We also derive the spin stiffness constant in the high-density limit of such a spin-polarized matter.

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I. INTRODUCTION

The possibility of ferromagnetic phase transition in dense quark matter was first discussed by Tatsumi [1] where it was shown that quark liquid interacting through one gluon exchange shows spontaneous magnetic instability at low densities. Such an investigation was motivated by the observation of a strong magnetic field in neutron star. Moreover, the theoretical conjectures about the possible existence of quark stars provide additional impetus to examine this issue further [2–6].

The underlying mechanism of such a phase transition for slow-moving massive quark is similar to what one observes in case of interacting electron gas [7,8] in a neutralizing positive charge background where the electrons interact only by the exchange interaction and the contribution of the direct term cancels with the background contribution. In case of interacting electron gas, the kinetic energy is minimum in unpolarized state; the exchange energy, however, favors spin alignment. These are two competing phenomena that also depend on density. It is seen that the kinetic energy dominates at higher density and as the density is lowered the exchange energy becomes larger, at some point turning the electron gas suddenly into a completely polarized state. This is the mechanism of ferromagnetism in electron gas interacting via Coulomb potential [9].

The exchange energy for quark matter interacting via one gluon exchange (OGE) is also attractive and becomes dominant at some density giving rise to ferromagnetism [1–4]. However, there are similarities and differences between quark matter and electron gas as discussed in Ref. [1]. For slow-moving massive quarks the dynamics are very similar to what happens in electron gas, while in the relativistic case a completely different mechanism works when the spin-dependent lower component of the Dirac spinor becomes important. It should also be noted that the exchange energy is negative for massive strange quark at low densities while it is always positive for massless u and d quarks, as observed in Ref. [10] and subsequently in Refs. [1,11].

The magnetic property of the quark matter was also studied in Ref. [2] by evaluating the effective potential by employing magnetic moment of a quark and treating this as an order parameter. Unlike in Ref. [1], in this model u, d, and s quarks, i.e., all of these flavors, show ferromagnetic phase transition at various densities. In Ref. [4], we revisited this problem and have evaluated Fermi liquid parameters for spin-polarized quark matter that were subsequently used to derive single-particle spectrum and total energy density as a function of the $\xi = (n_q^+ - n_q^-)/(n_q^+ + n_q^-)$. There it was shown that such a phase transition within the OGE model and parameter set of Ref. [1] is possible at very low density.

In Refs. [1,3] and [4] calculations were restricted only to the Hartree-Fock level and the higher-order terms were ignored. The computation of the ground-state energy, however, requires evaluation of the diagrams beyond the exchange loop viz. the inclusion of correlation energy as emphasized in Ref. [1]. This is rather tricky as the higher-order terms are plagued by infrared divergences due to the exchange of massless bosons like gluons (or photons), indicating the failure of naive perturbation theory. The problem can be handled by summing a class of diagrams that makes the perturbation series convergent and receives logarithmic corrections. In the case of degenerate electron matter this pioneering work was done by Gell-Mann and Brueckner (GB), commonly known as GB theory, where the correlation energy ($E_{\rm corr}$) of an electron gas at high density was calculated [7]. The correlation energy is actually the higher-order correction to the ground-state energy beyond the exchange term in the perturbation series defined by [7,8]

$$E_{\rm corr} = E - E_{\rm ex} - E_{\rm kin}. \tag{1}$$

Here, $E_{\rm corr}$, $E_{\rm ex}$, and $E_{\rm kin}$ correspond to correlation, exchange, and kinetic energy density, respectively. In general, for electron gas interacting via Coulomb force it takes the following form [7,8]

$$E_{\rm corr} = A \ln r_s + C + \mathcal{O}(r_s). \tag{2}$$

At large Fermi momentum (p_f), i.e., in the limit $r_s = 0$, the result becomes exact [12,13]. For the case of electron gas, the inverse density is set equal to $\frac{4}{3}\pi r_0^3$ and the dimensionless parameter r_s is defined as r_0 divided by Bohr radius [7]. Here

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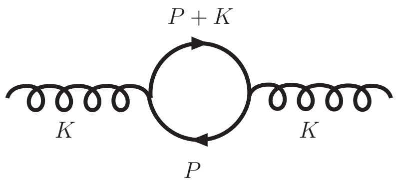

FIG. 1. Gluon self-energy.

we derive a similar expression for the dense quark matter with arbitrary spin polarization with appropriate modifications.

The model adopted in the present work is same as that of Ref. [1] except here we go beyond $\mathcal{O}(g^2)$ and include ring diagrams to evaluate the correlation energy of spin-polarized quark matter. This, together with the contribution of $E_{\rm kin}$ and $E_{\rm ex}$, as we shall see, has the small ξ expansion

$$E = E(\xi = 0) + \tfrac{1}{2}\beta_s \xi^2 + \cdots. \tag{3}$$

Here, $\beta_s = \frac{\partial^2 E}{\partial \xi^2}|_{\xi=0}$ is defined to be the spin stiffness constant in analogy with Refs. [14,15] with $r_s = g^2(\frac{3\pi}{4})^{1/3}$. It is to be noted that in Eq. (3) the first term corresponds to unpolarized matter with correlation energy having the form of Eq. (2). Clearly, this is reminiscent of what one obtains for the degenerate electron gas [14].

The derivation of $E_{\rm corr}$ here requires the evaluation of the gluon self-energy in spin asymmetric quark matter that can be used to construct the in-medium one-loop-corrected gluon propagator with explicit spin parameter dependence. Apart from the calculation of correlation energy, this might have applications in evaluation, for example, of the Fermi liquid parameter (FLP) in spin-polarized matter or spin susceptibility or quantities that can be expressed in terms of FLPs [4,11,16, 17]. In the present work, however, we restrict ourselves to the evaluation of the ring diagrams only.

The article is presented as follows. In Sec. II, we derive the expression for gluon self-energy in polarized quark matter—an essential ingredient for the calculation of correlation energy. In Sec. III, we calculate ground-state energy with correlation correction for the polarized matter. Subsequently, we also compare exchange and correlation energy density. In Sec. IV we summarize and conclude. The detailed expression of various matrix elements required to evaluate polarization tensor have been relegated to the Appendix.

II. GLUON SELF-ENERGY IN POLARIZED MATTER

To calculate the correlation energy of spin-polarized quark matter one needs to calculate the gluon self-energy in matter with arbitrary spins. This spin-dependent gluon polarization arises from the quark loop shown in Fig. 1 [18]. Mathematically [19,20],

$$\begin{aligned}\Pi_{\mu\nu} = {} & \frac{N_f g^2}{2}\int \frac{d^3p}{(2\pi)^3}\sum_{s=\pm}\frac{\theta_p^s}{2\varepsilon_p^s}\Bigg\{\frac{K^2}{K^4 - 4(P\cdot K)^2} \\ & \times \sum_{s'=\pm}\left[\mathcal{M}_{\mu\nu}^{ss'}(P+K,P) + \mathcal{M}_{\mu\nu}^{ss'}(P,P-K)\right] \\ & - \frac{2(P\cdot K)}{K^4 - 4(P\cdot K)^2}\sum_{s'=\pm}\left[\mathcal{M}_{\mu\nu}^{ss'}(P+K,P)\right. \\ & \left. - \mathcal{M}_{\mu\nu}^{ss'}(P,P-K)\right]\Bigg\}. \end{aligned} \tag{4}$$

Here, $\mathcal{M}_{\mu\nu}^{ss'}$ is related to the Compton-scattering amplitude as shown in Fig. 2. To derive Eq. (4), following Refs. [1,4] we use projection operator $\mathscr{P}(a) = \frac{1}{2}(1 + \gamma^5 \not{a})$ at each vertex. The momentum integration is performed at the Fermi surface restricted by $\theta_p^\pm = \theta(p_f^\pm - |p|)$.

Now we choose $K \equiv (k_0, 0, 0, |k|)$, $P \equiv (\varepsilon_p, |p|\sin\theta\cos\phi, |p|\sin\theta\sin\phi, |p|\cos\theta)$, $s \equiv \pm(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, and $g_{\mu\nu} = (1,-1,-1,-1)$. Note that the upper- and lowercase fonts are used to distinguish between four and three vectors.

From Fig. 2(a) the scattering amplitude becomes

$$\begin{aligned}&\mathcal{M}_{\mu\nu}^{\mathrm{dir},ss'}(P+K,P) \\ &\quad = -b_\mu P_\nu(a\cdot K) - g_{\mu\nu}(b\cdot P)(a\cdot K) + b_\mu K_\nu(a\cdot P) \\ &\qquad + a_\mu P_\nu(b\cdot K) - g_{\mu\nu}(a\cdot P)(b\cdot K) + P_\mu[-K_\nu(a\cdot b - 1) \\ &\qquad - 2P_\nu(a\cdot b - 1) + b_\nu(a\cdot K) + 2b_\nu(a\cdot P) + a_\nu(b\cdot K)] \\ &\qquad + a_\mu K_\nu(b\cdot P) + 2a_\mu P_\nu(b\cdot P) - 2g_{\mu\nu}(a\cdot P)(b\cdot P) \\ &\qquad - K_\mu[P_\nu(a\cdot b - 1) - b_\nu(a\cdot P) + a_\nu(b\cdot P)] + (K\cdot P) \\ &\qquad \times [g_{\mu\nu}(a\cdot b - 1) - b_\mu a_\nu - a_\mu b_\nu]. \end{aligned} \tag{5}$$

The components of the four-pseudovector b_μ(or a_μ) in a frame in which the particle is moving with momentum p(or $p+k$) are found by the Lorentz transformation from the rest frame as given by [1,4],

$$\begin{aligned} a_0 &= \frac{(p+k)\cdot s}{m_q}; \quad \vec{a} = s + \frac{(p+k)[(p+k)\cdot s]}{m_q(\varepsilon_{p+k} + m_q)}; \\ b_0 &= \frac{p\cdot s'}{m_q}; \quad \vec{b} = s' + \frac{p(p\cdot s')}{m_q(\varepsilon_p + m_q)}. \end{aligned} \tag{6}$$

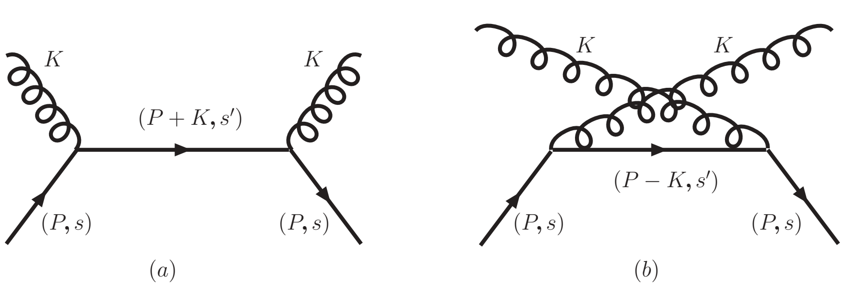

FIG. 2. Compton-scattering amplitude.

Similarly, from Fig. 2(b) we have,

$$\begin{aligned}\mathcal{M}^{\text{ex},ss'}_{\mu\nu}&(P, P-K)\\ =& -\tilde{b}_\mu P_\nu(\tilde{a}\cdot K) + g_{\mu\nu}(\tilde{b}\cdot P)(\tilde{a}\cdot K) + \tilde{b}_\mu K_\nu(\tilde{a}\cdot P)\\ &- \tilde{a}_\mu P_\nu(\tilde{b}\cdot K) + g_{\mu\nu}(\tilde{a}\cdot P)(\tilde{b}\cdot K) - P_\mu[-K_\nu(\tilde{a}\cdot\tilde{b}-1)\\ &+2P_\nu(\tilde{a}\cdot\tilde{b}-1) + \tilde{b}_\nu(\tilde{a}\cdot K) - 2\tilde{b}_\nu(\tilde{a}\cdot P) - \tilde{a}_\nu(\tilde{b}\cdot K)]\\ &- \tilde{a}_\mu K_\nu(\tilde{b}\cdot P) + 2\tilde{a}_\mu P_\nu(\tilde{b}\cdot P) - 2g_{\mu\nu}(\tilde{a}\cdot P)(\tilde{b}\cdot P)\\ &- K_\mu[-P_\nu(\tilde{a}\cdot\tilde{b}-1) + \tilde{b}_\nu(\tilde{a}\cdot P) + \tilde{a}_\nu(\tilde{b}\cdot P)]\\ &- (K\cdot P)[g_{\mu\nu}(\tilde{a}\cdot\tilde{b}-1) - \tilde{b}_\mu\tilde{a}_\nu - \tilde{a}_\mu\tilde{b}_\nu], \end{aligned} \tag{7}$$

where

$$\begin{aligned}\tilde{a}_0 &= \frac{p\cdot s}{m_q};\quad \vec{\tilde{a}} = s + \frac{p(p\cdot s)}{m_q(\epsilon_p+m_q)};\\ \tilde{b}_0 &= \frac{(p-k)\cdot s'}{m_q};\quad \vec{\tilde{b}} = s' + \frac{(p-k)[(p-k)\cdot s']}{m_q(\epsilon_{p-k}+m_q)}.\end{aligned} \tag{8}$$

Now we define matrix elements $\mathcal{M}^{ss'}_{\mu\nu}$ in terms of flip (f) and nonflip (nf) interactions where $\mathcal{M}^{\text{nf}}_{\mu\nu} = \mathcal{M}^{s=s'}_{\mu\nu}$ and $\mathcal{M}^{f}_{\mu\nu} = \mathcal{M}^{s=-s'}_{\mu\nu}$ [1,4]. Using Eq. (5) and Eq. (7) we have

$$\begin{aligned}\mathcal{M}^{f+\text{nf}}_{00}(P+K,P) + \mathcal{M}^{f+\text{nf}}_{00}(P,P-K) &= 8\epsilon_p^2\\ \mathcal{M}^{f+\text{nf}}_{00}(P+K,P) - \mathcal{M}^{f+\text{nf}}_{00}(P,P-K) &= -4(P\cdot K)\\ \mathcal{M}^{f+\text{nf}}_{33}(P+K,P) + \mathcal{M}^{f+\text{nf}}_{33}(P,P-K) &= 8p^2\cos^2\theta\\ \mathcal{M}^{f+\text{nf}}_{33}(P+K,P) - \mathcal{M}^{f+\text{nf}}_{33}(P,P-K) &= 4[2pk\cos\theta\\ &\quad+(P\cdot K)],\end{aligned} \tag{9}$$

and

$$\begin{aligned}\mathcal{M}^{f+\text{nf}}_{11}&(P+K,P) + \mathcal{M}^{f+\text{nf}}_{11}(P,P-K)\\ &= \mathcal{M}^{f+\text{nf}}_{22}(P+K,P) + \mathcal{M}^{f+\text{nf}}_{22}(P,P-K) = 4p^2\sin^2\theta,\end{aligned} \tag{10}$$

and

$$\begin{aligned}\mathcal{M}^{f+\text{nf}}_{11}&(P+K,P) - \mathcal{M}^{f+\text{nf}}_{11}(P,P-K)\\ &= \mathcal{M}^{f+\text{nf}}_{22}(P+K,P) - \mathcal{M}^{f+\text{nf}}_{22}(P,P-K) = 4(P\cdot K).\end{aligned} \tag{11}$$

The detailed expressions of the matrix element $\mathcal{M}^{ss'}_{\mu\nu}$ are given in the Appendix. In the present work we consider one flavor quark matter. Generalization for the multiflavor system is straightforward. Using Eq. (4) and Eqs. (9)–(11) we get

$$\Pi_{11} = \frac{g^2}{8\pi^3}\sum_{s=\pm}\int_0^{p_f^s}\frac{d^3p}{\epsilon_p}\left[\frac{K^2p^2\sin^2\theta - 2(P\cdot K)^2}{K^4-4(P\cdot K)^2}\right], \tag{12}$$

$$\Pi_{00} = \frac{g^2}{4\pi^3}\sum_{s=\pm}\int_0^{p_f^s}\frac{d^3p}{\epsilon_p}\left[\frac{K^2{\epsilon_p}^2 + (P\cdot K)^2}{K^4-4(P\cdot K)^2}\right], \tag{13}$$

$$\begin{aligned}\Pi_{33} =& \frac{g^2}{4\pi^3}\sum_{s=\pm}\int_0^{p_f^s}\frac{d^3p}{\epsilon_p}\\ &\times\left[\frac{K^2p^2\cos^2\theta - 2pk\cos\theta(P\cdot K) - (P\cdot K)^2}{K^4-4(P\cdot K)^2}\right].\end{aligned} \tag{14}$$

We are interested in evaluating longitudinal (Π_L) and transverse (Π_T) components of the polarization tensor. We define $\Pi_L = -\Pi_{00} + \Pi_{33}$ and $\Pi_T = \Pi_{11} = \Pi_{22}$. In the long-wavelength limit ($|p| \sim p_f$ and $|k| \ll p_f$), i.e., for low-lying excitation near the Fermi surface, K^4 can be neglected compared to $4(P\cdot K)^2$ in the denominators of Eqs. (12)–(14) [21]. The longitudinal and transverse polarization in this limit are determined to be

$$\Pi_L = \frac{g^2}{4\pi^2}(C_0^2-1)\sum_{s=\pm}p_f^s\epsilon_f^s\left[-1+\frac{C_0}{2v_f^s}\ln\left(\frac{C_0+v_f^s}{C_0-v_f^s}\right)\right], \tag{15}$$

$$\Pi_T = \frac{g^2}{16\pi^2}C_0\sum_{s=\pm}{p_f^s}^2\left[\frac{2C_0}{v_f^s}+\left(1-\frac{C_0^2}{{v_f^s}^2}\right)\ln\left(\frac{C_0+v_f^s}{C_0-v_f^s}\right)\right]. \tag{16}$$

Here, we take $C_0 = k_0/|k|$ and $v_f^\pm = p_f(1\pm\xi)^{1/3}/(p_f^2(1\pm\xi)^{2/3}+m_q^2)^{1/2}$ to cast the results in a more familiar form as presented in Ref. [21] for $\xi = 0$. It might be noted here that, although the final expressions for the longitudinal and transverse polarization look rather similar to what one obtains in the case of unpolarized matter [21] with only a difference in $v_f^\pm$ and summation over the spins, the calculation of the matrix elements with explicit spin dependencies are rather involved (see the Appendix).

Π_L and Π_T have two limiting values, corresponding to the nonrelativistic (nr) and the ultrarelativistic (ur) regime. In the nonrelativistic limit ($\epsilon_f^\pm \to m_q$)

$$\Pi_L^{\text{nr}} = -\frac{g^2}{4\pi^2}m_q\sum_{s=\pm}p_f^s\left[-1+\frac{C_0}{2v_f^s}\ln\left(\frac{C_0+v_f^s}{C_0-v_f^s}\right)\right], \tag{17}$$

$$\Pi_T^{\text{nr}} = \frac{g^2}{16\pi^2}C_0\sum_{s=\pm}{p_f^s}^2\left[\frac{2C_0}{v_f^s}+\left(1-\frac{C_0^2}{{v_f^s}^2}\right)\ln\left(\frac{C_0+v_f^s}{C_0-v_f^s}\right)\right]. \tag{18}$$

Here $v_f^s = p_f^s/m_q$. These expressions were derived in Refs. [12,22] for unpolarized electron gas. In this limit, the current-current interaction is inherently small, for which this term can be neglected compared to the Coulomb interaction to calculate correlation energy. Here, $\text{Re}\Pi_T \sim (k_0/|k|)^2$ and $\text{Im}\Pi_T \sim (k_0/|k|)$ when both $k_0 \to 0$ and $|k| \to 0$. It is apparent from this behavior of Π_T that the current-current interaction remain unscreened at zero frequency [22].

In the ultrarelativistic limit ($\epsilon_f^s \to p_f^s$) the polarization tensors take the following forms:

$$\Pi_L^{\text{ur}} = \frac{g^2}{4\pi^2}\sum_{s=\pm}{p_f^s}^2\sin^{-2}\theta_E(1-\theta_E\cot\theta_E), \tag{19}$$

$$\Pi_T^{\text{ur}} = \frac{g^2}{8\pi^2}\sum_{s=\pm}{p_f^s}^2[1-\sin^{-2}\theta_E(1-\theta_E\cot\theta_E)], \tag{20}$$

with $\theta_E = \tan^{-1}(|k|/k_0)$. For $\xi = 0$, these results are same as those of Ref. [21]. In the next section, Eqs. (15)–(20) are used to evaluate the contribution of the ring diagrams.

It might not be out of context here to mention that once we have the expressions of $\Pi_{L(T)}$, a one-loop-corrected gluon propagator in polarized quark matter can easily be constructed. This forms the basis for calculation of various physical quantities, including the FLPs, which, without such medium corrections, suffer from infrared divergences [4,11,16].

III. GROUND-STATE ENERGY WITH CORRELATION

The leading contributions to the ground-state energy are given by the three terms viz. kinetic, exchange, and correlation energy densities, i.e.,

$$E = E_{\rm kin} + E_{\rm ex} + E_{\rm corr} + \mathcal{O}(r_s). \tag{21}$$

In the high-density limit $\mathcal{O}(r_s)$ vanish, the result becomes exact [13]. $E_{\rm kin}$ is given by [1,4]

$$\begin{aligned} E_{\rm kin} = \frac{3}{16\pi^2}\Bigg\{ & p_f(1+\xi)^{1/3}\sqrt{p_f^2(1+\xi)^{2/3}+m_q^2} \\ & \times\left[2p_f^2(1+\xi)^{2/3}+m_q^2\right] \\ & -m_q^4 \ln\left(\frac{p_f(1+\xi)^{1/3}+\sqrt{p_f^2(1+\xi)^{2/3}+m_q^2}}{m_q}\right) \\ & +[\xi\to-\xi]\Bigg\}, \end{aligned} \tag{22}$$

where ξ is the polarization parameter with the condition $0 \leqslant \xi \leqslant 1$. Here n_q^+ and n_q^- represent densities of spin-up and spin-down quarks respectively and $n_q = n_q^+ + n_q^-$ denote total quark density. Then the Fermi momenta in the spin-polarized quark matter are defined as $p_f^+ = p_f(1+\xi)^{1/3}$ and $p_f^- = p_f(1-\xi)^{1/3}$, where $p_f = (\pi^2 n_q)^{1/3}$, is the Fermi momentum of the unpolarized matter ($\xi = 0$).

In the nonrelativistic (nr) and the ultrarelativistic (ur) limit kinetic energy density becomes [1,4],

$$E_{\rm kin}^{\rm nr} = \frac{3p_f^5}{20\pi^2 m_q}[(1+\xi)^{5/3}+(1-\xi)^{5/3}], \tag{23}$$

$$E_{\rm kin}^{\rm ur} = \frac{3p_f^4}{8\pi^2}[(1+\xi)^{4/3}+(1-\xi)^{4/3}]. \tag{24}$$

The first correction due to interaction to the ground-state energy is given by the exchange energy density. This arises from two quarks interchanging positions in the Fermi sea by exchanging a virtual gluon [23]. The exchange energy density was calculated in Ref. [4] within Fermi liquid theory approach. One can directly evaluate the loop diagram to calculate $E_{\rm ex}$ as shown in Fig. 3 [1].

For polarized quark matter, $E_{\rm ex}$, consists of two terms $E_{\rm ex} = E_{\rm ex}^{\rm nf} + E_{\rm ex}^{f}$. Here [1],

$$E_{\rm ex}^{\rm nf} = \frac{9}{2}\sum_{s=\pm}\iint \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}\theta\left(p_f^s-|p|\right)\theta\left(p_f^s-|p'|\right)f_{pp'}^{\rm nf}, \tag{25}$$

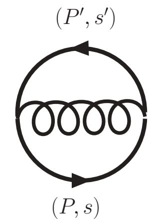

FIG. 3. Two-loop contribution to exchange energy density. Solid line represents the quark propagator and the wavy line represents gluon.

$$E_{\rm ex}^{f} = 9\iint \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}\theta(p_f^+-|p|)\theta(p_f^--|p'|)f_{pp'}^{f}, \tag{26}$$

where $f_{pp'}$ is the two-particle forward-scattering amplitude and is given by [1,4]

$$\begin{aligned} f_{pp'}^{ss'} = \frac{2g^2}{9\varepsilon_p\varepsilon'_p}\frac{1}{(P-P')^2}\Bigg\{ & 2m_q^2 - P\cdot P' - (p\cdot s)(p'\cdot s') \\ & + m_q^2(s\cdot s') + \frac{1}{(\varepsilon_p+m_q)(\varepsilon_{p'}+m_q)} \\ & \times[m_q(\varepsilon_p+m_q)(p'\cdot s)(p'\cdot s') \\ & + m_q(\varepsilon_{p'}+m_q)(p\cdot s)(p\cdot s') \\ & + (p\cdot p')(p\cdot s)(p'\cdot s')]\Bigg\}. \end{aligned} \tag{27}$$

where, $\varepsilon_p = \sqrt{p^2+m_q^2}$. In the nonrelativistic and the ultrarelativistic limit $E_{\rm ex}$ yields [1,4],

$$E_{\rm ex}^{\rm nr} = -\frac{g^2}{8\pi^4}p_f^4[(1+\xi)^{4/3}+(1-\xi)^{4/3}], \tag{28}$$

$$E_{\rm ex}^{\rm ur} = \frac{g^2}{32\pi^4}p_f^4[(1+\xi)^{4/3}+(1-\xi)^{4/3}+2(1-\xi^2)^{2/3}]. \tag{29}$$

Now we come to the central aim of the present work, i.e., the evaluation of the correlation energy of dense quark matter with arbitrary spin polarization; the leading contribution to $E_{\rm corr}$ can be obtained by adding the contributions of ring diagrams as shown in Fig. 4. It is to be noted that each of these diagrams are infrared divergent while their sum is finite [7,8,21,23,24] and are given by:

$$\begin{aligned} E_{\rm corr} = E_{\rm corr}^L + E_{\rm corr}^T = -\frac{i}{2}\int \frac{d^4K}{(2\pi)^4}\{ & [\ln(1-D^0\Pi_L) \\ & + D^0\Pi_L] + 2[\ln(1-D^0\Pi_T)+D^0\Pi_T]\}. \end{aligned} \tag{30}$$

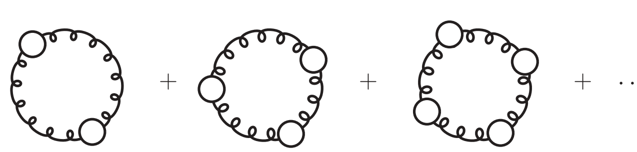

FIG. 4. The series of ring diagrams.

Here D^0 is the free gluon propagator. The spatial integral of Eq. (30) can be reduced to one for the radial variable only, because all the polarization propagators are independent of the direction of three-momentum transfer k. A Wick rotation is performed on the fourth component of the integration momentum ($k_0 \to ik_0$) so that space metric becomes Euclidean [21,25]. With $K_E^2 = k_0^2 + |k|^2 = -K^2$ and $\tan\theta_E = |k|/k_0$, Eq. (30) becomes,

$$E_{\rm corr} = \frac{1}{(2\pi)^3}\int_0^\infty K_E^2 dK_E^2 \int_0^{\pi/2} \sin^2\theta_E {\rm d}\theta_E \times \left\{\left[\ln\left(1+\frac{\Pi_L(K_E^2,\theta_E)}{K_E^2}\right) - \frac{\Pi_L(K_E^2,\theta_E)}{K_E^2}\right] + 2\left[\ln\left(1+\frac{\Pi_T(K_E^2,\theta_E)}{K_E^2}\right) - \frac{\Pi_T(K_E^2,\theta_E)}{K_E^2}\right]\right\}. \quad (31)$$

Infrared divergences would arise in Eq. (31), if we were to expand the logarithms in powers of Π_i because of the nonzero value of $\Pi_i(K_E^2,\theta_E)$ at $K_E^2 = 0$. This can be isolated by writing $K_E^2 = 0$ whenever possible in the integrand. Following Refs. [23,26], we have

$$E_{\rm corr} \simeq \frac{1}{(2\pi)^3}\int_0^\infty K_E^2 dK_E^2 \int_0^{\pi/2} \sin^2\theta_E d\theta_E \times \left\{\left[\ln\left(1+\frac{\Pi_L(0,\theta_E)}{K_E^2}\right) - \frac{\Pi_L(0,\theta_E)}{K_E^2}\right] + 2\left[\ln\left(1+\frac{\Pi_T(0,\theta_E)}{K_E^2}\right) - \frac{\Pi_T(0,\theta_E)}{K_E^2}\right] + \frac{1}{2K_E^2}\frac{1}{K_E^2+\varepsilon_f^2}\left[\Pi_L^2(0,\theta_E) + 2\Pi_T^2(0,\theta_E)\right]\right\}. \quad (32)$$

Performing K_E^2 integration the ring energy becomes [21,23,26]

$$E_{\rm corr} \simeq \frac{1}{(2\pi)^3}\frac{1}{2}\int_0^{\pi/2} \sin^2\theta_E {\rm d}\theta_E \left\{\Pi_L^2\left[\ln\left(\frac{\Pi_L}{\varepsilon_f^2}\right) - \frac{1}{2}\right] + 2\Pi_T^2\left[\ln\left(\frac{\Pi_T}{\varepsilon_f^2}\right) - \frac{1}{2}\right]\right\}. \quad (33)$$

To proceed further, we first express Π_L and Π_T in terms of polar variables. From Eq. (15) and Eq. (16) we obtain

$$\Pi_L = \frac{g^2}{4\pi^2}\sum_{s=\pm}\frac{p_f^s\varepsilon_f^s}{\sin^2\theta_E}\left[1 - \frac{\cot\theta_E}{v_f^s}\tan^{-1}\left(v_f^s\tan\theta_E\right)\right], \quad (34)$$

$$\Pi_T = \frac{g^2}{8\pi^2}\sum_{s=\pm} {p_f^s}^2\cot\theta_E\left[-\frac{\cot\theta_E}{v_f^s} + \left(1+\frac{\cot^2\theta_E}{{v_f^s}^2}\right)\tan^{-1}\left(v_f^s\tan\theta_E\right)\right]. \quad (35)$$

These are then inserted in Eq. (33) and θ_E integration is performed numerically to estimate $E_{\rm corr}$ for various ξ as shown in Fig. 5.

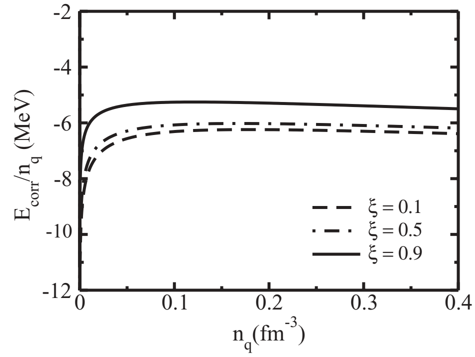

FIG. 5. Correlation energy ($E_{\rm corr}$) as a function of density for different polarization parameters.

We can also derive the analytic expression for the correlation energy in the nonrelativistic and ultrarelativistic case by using relevant Π_L and Π_T as given in Eqs. (17) and (18) and Eqs. (19) and (20) respectively.

In the nonrelativistic limit it is given by

$$E_{\rm corr}^{\rm nr} = \frac{g^4\ln g^2}{(2\pi)^6}(1-\ln 2)\frac{1}{3}m_q p_f^3. \quad (36)$$

Note that the correlation energy here is independent of spin-polarization parameter ξ. This is because it is proportional to p_f^3 when ξ-dependent terms cancel. In deriving Eq. (36) we consider exchange of longitudinal gluons only. It is to be mentioned that similar expressions for degenerate electron gas interacting via static Coulomb potential can be found in Refs. [26,27].

In the ultrarelativistic limit, the leading $g^4\ln g^2$ order contribution to $E_{\rm corr}^L$ is derived to be

$$E_{\rm corr}^{{\rm ur},L} = \frac{g^4\ln g^2}{(2\pi)^6}(1-\ln 2)\frac{1}{12}p_f^4[(1+\xi)^{4/3} + (1-\xi)^{4/3} + 2(1-\xi^2)^{2/3}]. \quad (37)$$

The term $(1-\ln 2)$ is the reminiscent of what one obtains in the nonrelativistic electron plasma as was first obtained by GB [7]. In the relativistic case, such a term does not appear in the final expression of $E_{\rm corr}$, where a similar term with opposite sign arise out of the magnetic interaction mediated by the exchange of transverse gluons as

$$E_{\rm corr}^{{\rm ur},T} = \frac{g^4\ln g^2}{(2\pi)^6}\left(\ln 2 - \frac{5}{8}\right)\frac{1}{12}p_f^4[(1+\xi)^{4/3} + (1-\xi)^{4/3} + 2(1-\xi^2)^{2/3}]. \quad (38)$$

By adding Eq. (37) and Eq. (38) one obtains

$$E_{\rm corr}^{\rm ur} = \frac{g^4\ln g^2}{2048\pi^6}p_f^4[(1+\xi)^{4/3} + (1-\xi)^{4/3} + 2(1-\xi^2)^{2/3}]. \quad (39)$$

For $\xi = 0$, the correlation energy for unpolarized matter follows [21,23,27]. That the term $\ln 2$ disappear from the relativistic ring energy is known from the work [21] where a detailed calculation of the correlation energy for the nuclear matter ground state has been performed. Furthermore, one may also note that in the nonrelativistic limit $E_{\rm ex}$ and $E_{\rm corr}$

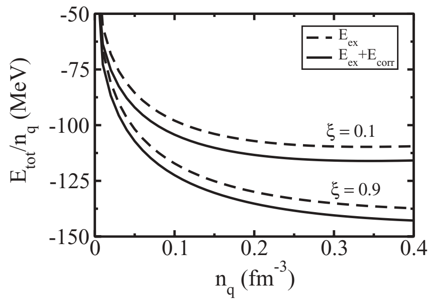

FIG. 6. Comparison of exchange and correlation energy in polarized quark matter as a function of density for different polarization parameters.

contribute with opposite sign while in the ultrarelativistic limit, both of them contribute with same sign.

Using Eq. (34) and Eq. (35) correlation energy is estimated numerically that is valid for all the kinematic regimes. For this, following Refs. [1,4], we take $\alpha_c = g^2/4\pi = 2.2$ and $m_q = 300$ MeV. In Fig. 5 we plot density dependence of correlation energy for various ξ. This shows that at a given density, with higher value of ξ, $E_{\rm corr}$ increases. In Fig. 6, we compare exchange and correlation energy density. It shows system becomes more bound when quark matter changes its phase from unpolarized to polarized matter. With increasing ξ, $E_{\rm corr}$ remains attractive; however, its value decreases as observed in both Figs. 5 and 6. In Fig. 7 we plot ground-state energy as a function of polarization parameter ξ. Hence we conclude that the quark matter interacting via OGE becomes polarized at lower density, whereas at higher density it becomes unpolarized. This clearly shows phase transition is first order and critical density is still around normal nuclear matter density $n_q^c \sim 0.16$ fm^{-3} [1,4]. In this regime, it is seen that $E_{\rm corr}$ makes the system more bound.

To derive the spin stiffness constant in the high-density limit using Eqs. (24), (29), and (39) we have

$$\beta_s = \left.\frac{\partial^2 E}{\partial \xi^2}\right|_{\xi=0} = \beta_s^{\rm kin} + \beta_s^{\rm ex} + \beta_s^{\rm corr}$$
$$= \frac{p_f^4}{3\pi^2}\left[1 - \frac{g^2}{6\pi^2} - \frac{g^4}{384\pi^4}(\ln r_s - 0.286)\right]. \quad (40)$$

Here, the logarithmic term arises from the correlation correction.

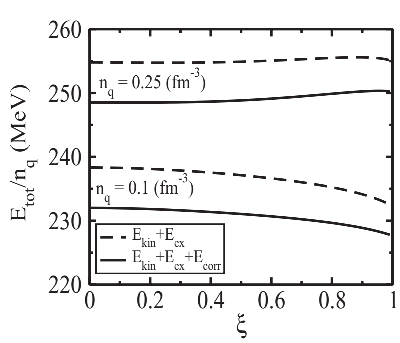

FIG. 7. Total energy of quark liquid as a function of polarization parameter at $n_q = 0.1$ fm^{-3} and $n_q = 0.25$ fm^{-3}. The critical density is around $n_q^c = 0.16$ fm^{-3} in this case.

IV. SUMMARY AND CONCLUSION

In this work we derive the expressions for the gluon self-energy in spin-polarized quark matter and calculate the ground-state energy of such a system up to term $\mathcal{O}(g^4)$ that includes corrections due to correlation effects. The analytical expressions for the correlation energy in two limiting cases (non-/ultrarelativistic) are presented and compared with $E_{\rm ex}$ and $E_{\rm kin}$. It is shown that the correlation energy for polarized quark matter is comparatively larger than the unpolarized one, although it is always attractive. We find that numerically the contribution of $E_{\rm corr}$ to the total energy is not found to be large and therefore, although qualitatively important, it is not the main factor in determining whether quark matter is ferromagnetic. With out this, however, the results remain incomplete because of the associated divergences of the terms beyond exchange diagrams [12,13]. Furthermore, this is an important first step to include the corrections due to correlations to the spin susceptibility [28–30]. In this work we present spin stiffness constant β_s of dense quark system only in the high-density limit. A detailed study of this is now underway and shall be reported elsewhere [31].

The inclusion of correlation energy, as shown here, does not rule out the possibility of ferromagnetic phase transition in quark matter at low density; rather, it makes it more probable within the model and parameter set used by Tatsumi [1] that was borrowed from the bag model and was also used in Ref. [10]. Clearly the critical density at which the spin-polarized ferromagnetic state might appear depends strongly on the quark mass and the critical density increases with increasing mass; this might change our numerical estimates.

Further uncertainty to the estimation of the critical density from the present analysis comes from the fact that we here restrict ourselves only to OGE diagrams and one flavor system. In this regime, multigluon exchange processes [5] might play an important role. More work in this direction is therefore necessary to examine this issue, especially for multiflavor systems that might appear in astrophysics. Leaving aside these questions, the evaluation of the gluon self-energy and the estimation of correlation energy in polarized matter, as mentioned in the text, nevertheless constitutes an important component for the study of the properties of dense quark system.

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APPENDIX

In the text Compton-scattering amplitudes are given as a sum of flip and nonflip terms. Here we give detail expression of $\mathcal{M}_{11}^{ss'}(P+K,P) + \mathcal{M}_{11}^{ss'}(P,P-K)$ with explicit spin indices. With the help of Eq. (5) and Eq. (7) we have

$$\mathcal{M}_{11}^{ss'}(P+K,P) + \mathcal{M}_{11}^{ss'}(P,P-K)$$
$$= T_1 + T_2 + T_3 + T_4 + T_5 + T_6, \quad (A1)$$

where

$$\begin{aligned}
T_1 &= (b\cdot P)(a\cdot K) - (\tilde{b}\cdot P)(\tilde{a}\cdot K)\\
&= \frac{\varepsilon_p k_0}{m_q^2}[(p\cdot s)(k\cdot s') + (k\cdot s)(p\cdot s')]\\
&\quad - \frac{k_0}{m_q}[(k\cdot s)(p\cdot s')] - \frac{k_0 p^2}{m_q^2(\varepsilon_p+m_q)}[(p\cdot s)(k\cdot s')\\
&\quad + (k\cdot s)(p\cdot s')] - \frac{\varepsilon_p}{m_q}[(k\cdot s)(k\cdot s')]\\
&\quad + \frac{p^2}{m_q(\varepsilon_p+m_q)}[(k\cdot s)(k\cdot s')] - \frac{\varepsilon_p(p\cdot k)}{m_q^2(\varepsilon_p+m_q)}\\
&\quad \times [(p\cdot s)(k\cdot s') + (k\cdot s)(p\cdot s')]\\
&\quad + \frac{(p\cdot k)}{m_q(\varepsilon_p+m_q)}[2(k\cdot s)(p\cdot s') - (k\cdot s)(k\cdot s')]\\
&\quad - \frac{k_0(p\cdot k)}{m_q^2(\varepsilon_p+m_q)}[(p\cdot s)(p\cdot s') - (p\cdot s)(k\cdot s')]\\
&\quad + \frac{p^2(p\cdot k)}{m_q^2(\varepsilon_p+m_q)^2}[(p\cdot s)(k\cdot s') + (k\cdot s)(p\cdot s')]\\
&\quad + \frac{p^2k^2}{m_q^2(\varepsilon_p+m_q)^2}[(p\cdot s)(p\cdot s') + (k\cdot s)(p\cdot s')]\\
&\quad + \frac{(p\cdot k)^2}{m_q^2(\varepsilon_p+m_q)^2}[(p\cdot s)(p\cdot s') - (p\cdot s)(k\cdot s')]\\
&\quad - \frac{\varepsilon_p k^2}{m_q^2(\varepsilon_p+m_q)}[(p\cdot s)(p\cdot s') + (k\cdot s)(p\cdot s')]\\
&\quad + \frac{k^2}{m_q(\varepsilon_p+m_q)}[(p\cdot s)(p\cdot s') + (k\cdot s)(p\cdot s')], \qquad \text{(A2)}
\end{aligned}$$

$$\begin{aligned}
T_2 &= (a\cdot P)(b\cdot K) - (\tilde{a}\cdot P)(\tilde{b}\cdot K)\\
&= \frac{\varepsilon_p k_0}{m_q^2}[(p\cdot s')(k\cdot s) + (p\cdot s)(k\cdot s')]\\
&\quad - \frac{\varepsilon_p}{m_q}[(k\cdot s)(k\cdot s')] - \frac{k_0}{m_q}[(p\cdot s)(k\cdot s')]\\
&\quad + \frac{\varepsilon_p(p\cdot k)}{m_q^2(\varepsilon_p+m_q)}[(p\cdot s)(k\cdot s') + (k\cdot s)(p\cdot s')]\\
&\quad + \frac{p\cdot k}{m_q(\varepsilon_p+m_q)}[2(p\cdot s)(k\cdot s') + (k\cdot s)(k\cdot s')]\\
&\quad + \frac{p^2}{m_q(\varepsilon_p+m_q)}[(k\cdot s)(k\cdot s')]\\
&\quad - \frac{p^2 k_0}{m_q^2(\varepsilon_p+m_q)}[(p\cdot s)(k\cdot s') + (k\cdot s)(p\cdot s')]\\
&\quad + \frac{p^2(p\cdot k)}{m_q^2(\varepsilon_p+m_q)^2}[(p\cdot s)(k\cdot s') + (k\cdot s)(p\cdot s')]\\
&\quad - \frac{k_0(p\cdot k)}{m_q^2(\varepsilon_p+m_q)}[(p\cdot s)(p\cdot s') + (k\cdot s)(p\cdot s')]\\
&\quad + \frac{(p\cdot k)^2}{m_q^2(\varepsilon_p+m_q)^2}[(p\cdot s)(p\cdot s') + (k\cdot s)(p\cdot s')]\\
&\quad - \frac{k^2\varepsilon_p}{m_q^2(\varepsilon_p+m_q)}[(p\cdot s)(p\cdot s') - (p\cdot s)(k\cdot s')]\\
&\quad + \frac{k^2}{m_q(\varepsilon_p+m_q)}[(p\cdot s)(p\cdot s') - (p\cdot s)(k\cdot s')]\\
&\quad + \frac{p^2k^2}{m_q^2(\varepsilon_p+m_q)^2}[(p\cdot s)(p\cdot s') - (p\cdot s)(k\cdot s')], \qquad \text{(A3)}
\end{aligned}$$

$$\begin{aligned}
T_3 &= 2{p_1}^2[(a\cdot b) + (\tilde{a}\cdot\tilde{b}) - 2]\\
&= p^2\sin^2\theta\left\{\frac{1}{{m_q}^2}[2(p\cdot s)(p\cdot s') + (p\cdot s')(k\cdot s)\right.\\
&\quad - (p\cdot s)(k\cdot s')] - 2(s\cdot s') - \frac{1}{m_q(\varepsilon_p+m_q)}\\
&\quad \times [4(p\cdot s)(p\cdot s') + 2(k\cdot s)(k\cdot s')]\\
&\quad - \frac{p^2}{m_q^2(\varepsilon_p+m_q)^2}[2(p\cdot s)(p\cdot s') - (p\cdot s)(k\cdot s')\\
&\quad + (p\cdot s')(k\cdot s)] - \frac{(p\cdot k)}{m_q^2(\varepsilon_p+m_q)^2}\\
&\quad \left.\times [(p\cdot s)(k\cdot s') + (p\cdot s')(k\cdot s)] - 2\right\}, \qquad \text{(A4)}
\end{aligned}$$

$$\begin{aligned}
T_4 &= 2[(a\cdot P)(b\cdot P) + (\tilde{a}\cdot P)(\tilde{b}\cdot P)]\\
&= 2\left\{\frac{{\varepsilon_p}^2}{m_q^2}[2(p\cdot s)(p\cdot s') + (p\cdot s')(k\cdot s) - (p\cdot s)(k\cdot s')]\right.\\
&\quad - \frac{\varepsilon_p}{m_q}[4(p\cdot s)(p\cdot s') + (p\cdot s')(k\cdot s) - (p\cdot s)(k\cdot s')]\\
&\quad + \frac{p^2}{m_q(\varepsilon_p+m_q)}[4(p\cdot s)(p\cdot s') + (p\cdot s')(k\cdot s)\\
&\quad - (p\cdot s)(k\cdot s')] - \frac{\varepsilon_p p^2}{m_q^2(\varepsilon_p+m_q)}[4(p\cdot s)(p\cdot s')\\
&\quad + 2(p\cdot s')(k\cdot s) - 2(p\cdot s)(k\cdot s')]\\
&\quad + \frac{p^2(p\cdot k)}{m_q^2(\varepsilon_p+m_q)^2}[(p\cdot s)(k\cdot s') + (p\cdot s')(k\cdot s)]\\
&\quad + 2(p\cdot s)(p\cdot s') - \frac{\varepsilon_p(p\cdot k)}{m_q^2(\varepsilon_p+m_q)}\\
&\quad \times [(p\cdot s)(k\cdot s') + (p\cdot s')(k\cdot s)]\\
&\quad + \frac{(p\cdot k)}{m_q(\varepsilon_p+m_q)}[(p\cdot s)(k\cdot s') + (p\cdot s')(k\cdot s)]\\
&\quad + \frac{p^4}{m_q^2(\varepsilon_p+m_q)^2}[2(p\cdot s)(p\cdot s') - (p\cdot s)(k\cdot s')\\
&\quad \left. + (p\cdot s')(k\cdot s)]\right\}, \qquad \text{(A5)}
\end{aligned}$$

$$\begin{aligned}
T_5 &= 2(P\cdot K)(a_1b_1 - \tilde{a}_1\tilde{b}_1)\\
&= \frac{p^2\sin^2\theta(P\cdot K)}{m_q^2(\varepsilon_p+m_q)^2}[(p\cdot s)(k\cdot s') + (p\cdot s')(k\cdot s)], \qquad \text{(A6)}
\end{aligned}$$

$$
\begin{aligned}
T_6 &= (P\cdot K)(a\cdot b-\tilde{a}\cdot\tilde{b})\\
&= \frac{(P\cdot K)}{m_q^2}\Bigg\{[(p\cdot s)(k\cdot s')+(p\cdot s')(k\cdot s)]\\
&\quad -\frac{2m_q}{(\varepsilon_p+m_q)}[(p\cdot s)(k\cdot s')+(p\cdot s')(k\cdot s)]\\
&\quad -\frac{p^2}{(\varepsilon_p+m_q)^2}[(p\cdot s)(k\cdot s')(p\cdot s')(k\cdot s)]\\
&\quad +\frac{p\cdot k}{(\varepsilon_p+m_q)^2}[2(p\cdot s)(p\cdot s')-(p\cdot s)(k\cdot s')\\
&\quad +(p\cdot s')(k\cdot s)]\Bigg\}. \qquad \text{(A7)}
\end{aligned}
$$

Similarly, one can derive terms like $[\mathcal{M}_{11}(P+K,P)-\mathcal{M}_{11}(P,P-K)]$, $[\mathcal{M}_{22}(P+K,P)\pm\mathcal{M}_{22}(P,P-K)]$, etc., with the help of Eq. (6) and Eq. (8). After explicit calculation of those terms, $\Pi_{L,T}$ can be evaluated.

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