Integer ratios of S_n/E_n in ^{40}Ca + n resonances suggesting two-oscillator excitations in the target nucleus

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In s-wave neutron resonances of ^{40}Ca at $E_n \leqslant 2.5$ MeV, S_n/E_n for many levels is found to be of the form $17(n/m)$ where n, m are small integers. Statistical tests show small probabilities for the observed dispositions of many levels at $E_n = (j/k)(1/70)G$ (j, k; small integers). To meet the requirement of time periodicity of the compound nucleus at resonance, a breathing model is developed, where the excitation energies E_x are written as a sum of inverse integers; $E_x = S_n + E_n = G\sum(1/k)$ (k: integer). In ^{40}Ca + n, the separation energy $S_n = 8362$ keV is written as $S_n = (17/70)G = (1/7 + 1/10)G$, where $G = 34.4$ MeV. G is almost equal to the Fermi energy of the nucleus. It is suggested that two oscillators of energy $(1/7)G$ and $(1/10)G$ are excited in ^{40}Ca by neutron incidence, in which the recurrence energy $(1/70)G$ is resonat with neutrons of energies at $(j/k)(1/70)G$, forming a simple compound nucleus.

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I. INTRODUCTION

For neutron-nucleus interactions, a vast amount of neutron cross-section data has been accumulated relating to reactor technology. Neutron resonance reactions are seen for every target nucleus over a wide energy region. In the compound nucleus (CN) formed by a neutron resonance reaction, the excitation energy is $E_x = S_n + E_n \sim 8$ MeV, where many degrees of freedom will be excited and coupled to form very complicated structures. Level densities are very high, and depend on target mass number; some examples are $\sim$40/MeV for ^{16}O, $\sim$400/MeV for ^{56}Fe, and $\sim 10^5$/MeV for ^{238}U. In fact, statistical properties of the observed resonances are in good agreement with the predictions of random matrix theory (RMT); good approximations are provided by the Wigner(GOE) distribution for nearest neighbor level spacings, the Porter-Thomas distribution for strengths, and Δ_3 statistics for long range correlations. Therefore the neutron resonances are surmised to be a typical example of quantum chaos [1–3].

However, several non-random properties have been reported over the past four decades, by analyzing D_{ij} (the spacing between two arbitrary levels) distributions, employing Fourier-like analyses, etc. ([4–14] and references therein). Through these analyses, special level spacings (which we call dominant spacings D_0) are found which appear more frequently than expected from RMT [5,7]. For example, for resonances in the eV region, dominant spacings are found for many target nuclei; 4.4 eV(^{177}Hf), 5.5 eV(^{123}Sb), 14.6 eV(^{238}U), 17.6 eV(^{168}Er), 142 eV(^{75}As), 213 eV(^{240}Pu), etc. In the keV or MeV resonance regions, dominant spacings are also found; 460, 515, 1515 keV(^{16}O), 575 keV(^{32}S), 184 keV(^{40}Ar), 478.4 keV(^{86}Kr), 86.2 keV(^{140}Ce), etc.

Moreover, it is very interesting that there are simple integer ratios between these dominant spacings of different nuclei [9–11,14]; in the eV region, 5.5 eV(^{123}Sb)/4.4 eV(^{177}Hf) = 5/4, 14.6 eV(^{238}U)/17.6 eV(^{168}Er) = 5/6, 142 eV(^{75}As)/213 eV(^{240}Pu) = 2/3, etc. Similarly, in the keV and MeV resonance regions, 460 keV(^{16}O)/575.6 keV(^{32}S) = 4/5, 460 keV(^{16}O)/183.8 keV(^{40}Ar) = 5/2, 575.6 keV(^{32}S)/478.4 keV(^{86}Kr) = 6/5, 517 keV(^{16}O)/86.2 keV(^{140}Ce) = 6/1, etc.

In addition, E_x/S_n is frequently found to be a ratio of integers in ^{16}O + n resonances, and $E_x/S_n = 4/3$ and 5/3 for many light nuclei [15], where calculations show a very small probability for the appearance of these integer ratios, assuming a homogeneous distribution of resonance levels. These facts suggest simple regularities in energy/spacing in the resonance region over a wide mass region, which are diametrically different from its ordinary understanding as quantum chaos. By the prevailing methods of analyses, non-statistical level structures are smeared out and lead to a plain but incorrect characterization as quantum chaos [1–3].

In this article, we report aspects of regular structures observed in the dispositions of s-wave resonances of ^{40}Ca + n, where many of the resonance energies E_n are given by $(l/m) \times$ constant, where m, l are small integers. The observations seem to suggest a dynamic mechanism of resonance reactions; excitation of two oscillators in ^{40}Ca in which the recurrence frequency is coherent with the de Broglie wave of the incident neutron. A statistical test was made on the statement $E_n = (l/m) \times$ constant. For convenience in classifying the excitation energies E_x, we use the expression $E_x = (n/m)G$, where $G \sim 34.5$ MeV and m, n are integers. In Sec. II, a brief description is given of the derivation of G. In Sec. III, integer ratios for S_n/E_n in ^{40}Ca + n are described. Probability calculations are addressed in Sec. IV, and the resonance reaction mechanisms of ^{40}Ca + n, ^{12}C + n, ^{16}O + n, and ^{140}Ce + n are described in Sec. V. A brief description of the breathing model for resonance reactions is presented in Sec. VI, and conclusions are offered in Sec. VII.

II. DERIVATION OF G

In the course of resonance level-spacing analyses by D_{ij} and Fourier-like analyses, dominant spacings are found in many nuclei. Two points become clear in such analyses:

(a) multiple integer ratios are seen among dominant spacings for a nucleus, and (b) integer ratios are found among the dominant spacings of different nuclei, which suggest the existence of a common spacing over many nuclei. In order to grasp more basic furcation properties of resonance levels, we have made level-spacing analyses on s-wave resonances of 15 even-even light target nuclei up to several hundred keV in neutron energy, where simple level furcations are expected because of the small number of degrees of freedom excited. Multiple integer ratios were found among thirty dominant spacings in the 15 nuclides, and it is clear that many of the dominant spacings D (under recoil energy correction) can be written as $D = G/mn$, where $G \sim 34.5$ MeV, and m, n : integers. Also, the excitation energies E_x themselves were found to be $E_x = G/k$ (k : *integer*) [9–11]. Our observations suggest:

(i) multi-furcation of levels characteristic to each nucleus.
(ii) a common original energy $G \sim 34.5$ MeV over many nuclei.

G is found to be almost equal to the Fermi energy, the maximum energy of a nucleon trapped in a nuclear potential.

The mechanism of CN formation by resonance reactions are reconsidered, and described in Sec. VI. Assuming an ensemble of multiple oscillators (all in normal modes) within the CN, we have developed a "recurrence model" [11,12] and a "breathing model" [15], where the time behaviors of the CN are explicitly discussed in a classical model. The recurrence times of composite oscillators are quantized by the unit time $\tau_0 = 2\pi\hbar/G = 1.20 \times 10^{-22}\text{s} = 36\,(\text{fm/C})$. Excitation energy is derived as a sum of inverse integers $E_x = G\sum_j(1/n_j)$ (n_j : integer), and the level spacings are in a form $D = G[\sum_j(1/n_j) - \sum_k(1/n_k)] = G(p/q)$ (n_j, n_k, p, q : integer), which is consistent with previous observations [9–11]. For a level composed of two oscillators, the excitation energy is $E_x = ((1/n_1) + (1/n_2))G$, with recurrence time $\tau_{\text{rec}} = LMC(n_1, n_2) \times \tau_0$, where LCM is the least common multiple of n_1, and n_2. Recurrence energy is $E_{\text{rec}} = G/\text{LCM}$.

III. S_n/E_n IN ^{40}Ca + n RESONANCES

In a previous paper, we reported the anomalously frequent appearance of integer ratios in E_x/S_n in light nuclei [12]. Here we describe the frequent appearance of integer ratios in S_n/E_n for s-wave resonances of $^{40}\text{Ca} + n$ for $E_n \leqslant 2.5$ MeV, and $S_n = 8362$ keV, where E_n is recoil-corrected energy and S_n is the neutron separation energy. Resonance data were obtained from a recent edition of the Landolt-Börnstein reference data series [13]. Forty s-wave resonances are observed in the region. Nearest neighbor level-spacing distribution ($D_{i,i+1}$) is shown in Fig. 1(a), and the spacing distribution between two arbitrary levels ($D_{i,j}$) is shown in Fig. 1(b). Nothing remarkable will be seen in these figures.

However, in 24 s-wave resonances out of 40, S_n/E_n are found to be in the form $17(l/m)$, where m, l are integers. This leads to $S_n = (17/X)G$ where X is an arbitrary integer. To maintain $G \sim 34.5$ MeV, we set $X = 70$, then $G = 34434$ keV. The excitation energies may be expressed in the form

$$E_x = S_n + E_n = ((17l + m)/70l)G$$
$$= ((17/70) + (m/l)/70)G.$$

Fourteen resonances of simple $m, l\,(m, l \leqslant 10)$ are listed in Table I, and plotted in Fig. 2. In the Table, the meaning of each column is illustrated using the first resonance as an example, where the excitation energy $E_x = S_n + E_n = 8527.2$ keV, and the neutron energy $E_n = 164.5$ keV. $g\Gamma_n$ is the neutron width multiplied by the g-factor ($g = 1$ for this case). For the first resonance, the value of S_n/E_n is 51/1, and $m = 1, l = 3$. E_x/G is $(17/70) + (1/210) = (52/210)$, and $\text{LCM} = 210$, which is proportional to the recurrence time of the CN for this resonance. Recurrence energy E_{rec} is defined as

TABLE I. S-wave resonances in $^{40}\text{Ca} + n$ which are in simple integer ratios to $E_n/492$ (keV). 492 keV = $G/70$.

j	E_x (keV)	E_n (keV)	$g\Gamma_n$ (keV)	S_n/E_n	E_x/G	LCM	E_{rec} (keV)	R	δ (keV)	$E_n/492$	G (keV)
0	8362.7	0			17/70						34434
1	8527.2	164.5	2.5	51/1	52/210	210	163.9	1	0.5	1/3	34436
2	8574.0	211.3	7.4	119/3	122/490	490	70.2	3	0.5	3/7	34436
3	8608.0	245.3	20	34/1	35/140	140	245.9	1	−0.6	1/2	34432
4	8739	376.3	0.5	68/3	71/280	280	123.0	3	7.0	3/4	34463
5	8800	437.3	10	170/9	179/700	700	49.2	9	−5.2	9/10	34413
6	8858	495.3	10	17/1	18/70	70	492.1	1	3.2	1/1	34447
7	8939.5	576.3	55	102/7	109/420	420	82.0	7	2.2	7/6	34445
8	8982.9	620.2	2	119/9	73/280	280	123.0	5	4.9	5/4	34454
9	9021.5	658.8	2.7	51/4	55/210	210	164.0	4	2.7	4/3	34445
10	9220.3	857.6	31	68/7	75/280	280	122.9	7	−3.0	7/4	34422
11	9342.5	979.8	12	17/2	19/70	70	491.7	2	−3.6	2/1	34419
12	9842.3	1479.6	2	17/3	20/70	70	492.1	3	3.2	3/1	34448
13	10339.0	1976.3	20	17/4	21/70	70	492.3	4	6.9	4/1	34463
14	10583.0	2220.3	30	34/9	43/140	140	246.1	9	5.2	9/2	34456

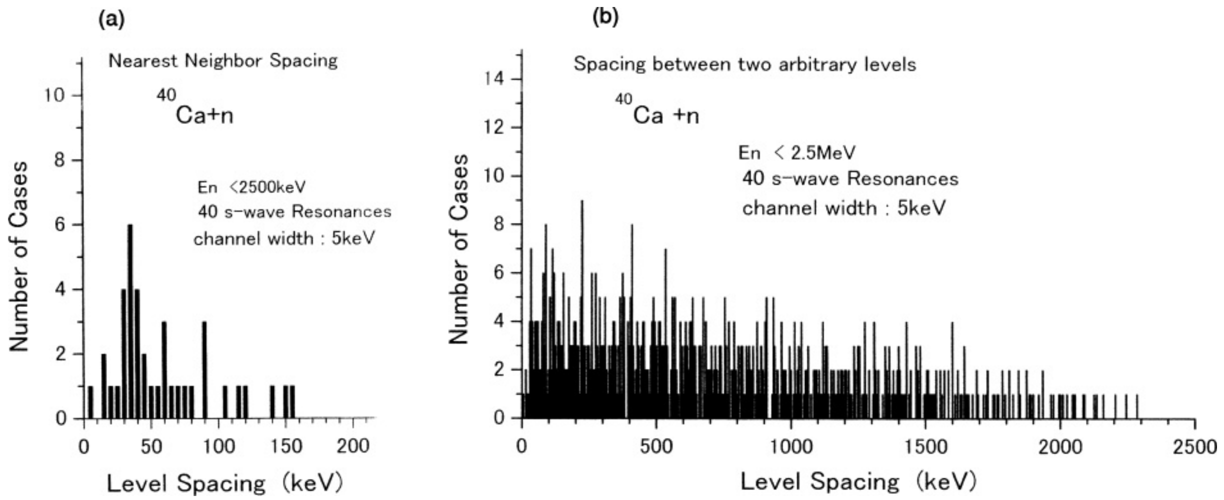

FIG. 1. (a) Nearest neighbor level spacing distribution. (b) Spacing between two arbitrary levels.

$E_{rec} = G/\text{LCM}$, and R is the integer part of E_n/E_{rec}, which is 1 for this resonance. The accuracy is $\delta = E_n - R \cdot E_{rec}$. The neutron energies are simple integer ratios to $(1/70)G =$ 492 keV, and the values of $(m/l) = E_n/492$ are shown in the second to last column. Values for G as determined by $E_x/(17/70 + (m/l)/70)$ are shown in the last column, where deviations are very small for many resonances. It is very interesting that separation energy $S_n = (17/70)G$ is a sum of the energies of two oscillators $((1/7) + (1/10))G$ of which the recurrence energy is $(1/70)G$. The mechanism of neutron resonance reactions suggested by this fact is discussed in Sec. V.

IV. STATISTICAL TEST

We now explore the statement that many of the observed values of E_n are of the form $(l/m)(1/70)G$. It is essential to determine the validity of this statement. Statistical probability calculations were made for the dispositions of many values of E_n at $(l/m)(1/70)G$, assuming a homogeneous (random) distribution of 40 levels over this energy region. Fig. 2 shows $g\Gamma_n$ of resonances vs neutron energy for $E_n \leqslant 2.5$ MeV. The average level spacing is ~60 keV. In Table I, resonances of (m/l) with $m, l \leqslant 10$ are listed. As $m, l \geqslant 8$ in only 2 cases, we restrict m and l to be to less than or equal to 7, and the number of resonances becomes 12. The possible values of (l/m) (where m, $l \leqslant 7$) are carefully counted to be 29, considering the observed energy region. Observed E_n deviated slightly from the points $(l/m)(1/70)G_{av}$ where G_{av} is the averaged value. Probability calculations were performed for two cases.

Case 1. All 12 resonances lie within a total width of deviation 13 keV. Then the candidate region, in which resonance is regarded as an integer ratio, is the sum of these widths, i.e. $29 \times 13 = 377$ keV. The ratio of the candidate

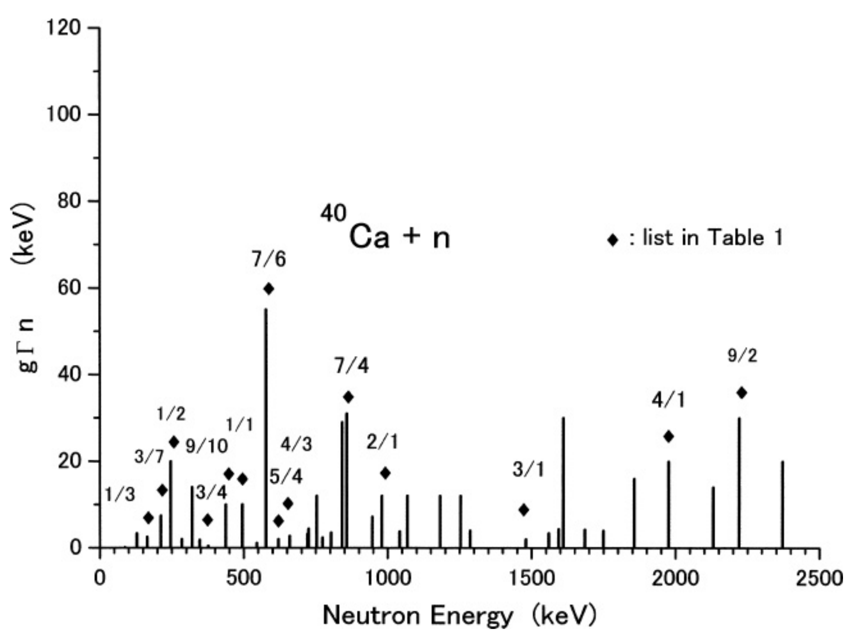

FIG. 2. Forty s-wave neutron resonances of ^{40}Ca + n. Resonances marked are $E_n = (m/n)492$ keV in Table I.

region to the total region of $(2500 - 100) = 2400$ keV is $p = 377/2400 = 0.157$, where 100 keV is an end effect. Assuming 40 resonance levels are placed homogeneously in this region, the average number of levels in the candidate region is $40p \sim 6.3$. The probability of 12 levels on the candidate region is calculated using a binomial distribution to be $P(12) = {}_{40}C_{12}\ p^{12}(1-p)^{28} = 0.011 = 1.1\%$. The summation over all levels is $\sum_{j=12}^{40} P(j) = 0.017 = 1.7\%$.

Case 2. For the 9 resonances that lie within a smaller deviation width of 5 keV, similar calculations were made: $p = 0.0604, 40p = 2.4$, and $P(9) = 0.00042 = 0.043\%$, with $\sum_{j=9}^{40} P(j) = 0.0005 = 0.05\%$.

These small probabilities are sufficient to disprove the assumption of homogeneous (random) distribution of resonance levels in the region. It is therefore reasonable to infer that the original statement is true, i.e., many of the observed values of E_n occur at $(l/m)(1/70)G$. The physical meaning relating to the resonance reactions is discussed below.

V. POSSIBLE MECHANISM OF ^{40}Ca + n RESONANCES

The separation energy of ^{40}Ca + n is $S_n = 8362.7$ keV $= (17/70)G$ which is rewritten as a sum of two oscillators $((1/7) + (1/10))G$, of which LCM = 70 and the recurrence energy is $(1/70)G$. In excitation levels of the target nucleus ^{40}Ca, two 0+ levels are seen at 3352 keV and 5211 keV [17]. It is speculated that by the incidence of a neutron, these two levels are simultaneously excited, and these energies are deformed to $(1/10)G = 3443$ keV and $(1/7)G = 4919$ keV with a recurrence energy $(1/70)G = 492$ keV. For incident neutrons of energies $(l/m)(1/70)G$, resonance reactions take place, forming a Boromian ring.

For clarity, formulations are made for the case of $n_1 = 7, n_2 = 10$, and m, l are small integers.

$$S_n = \left(\frac{1}{n_1} + \frac{1}{n_2}\right) G, \quad E_x = \left[\left(\frac{1}{n_1} + \frac{1}{n_2}\right) + \frac{1}{n_1 n_2}\frac{m}{l}\right] G,$$

$$E_n = \frac{1}{n_1 n_2}\frac{m}{l} G, \quad \frac{G}{n_1 n_2} = 492 \text{ keV}.$$

$$\frac{S_n}{E_n} = (n_1 + n_2)\frac{l}{m}, \quad \frac{E_x}{E_n} = 1 + (n_1 + n_2)\frac{l}{m}.$$

This model works fairly well for the 14 s-wave (1/2+) resonances in Table I, and for another 14 resonances with $(m, l \leqslant 10)$ of different $J\pi(1/2-, 3/2-, 3/2+, 5/2+)$.

Similar situations are found in the neutron resonances of many nuclei.

(i) In ^{12}C + n, $S_n = 4946.3$ keV $= (1/7)G = (3/21)G$. Many of the resonance energies E_n are integer ratios of $(1/21)G = 1648$ keV. For the first nine resonances of $E_n \leqslant 6$ MeV, several numerical data are shown in Table II, where simple integer ratios in $E_n/1648$ (keV) are seen.

(ii) In the case of ^{16}O + n, $S_n = 4143.3$ KeV $= (3/25)G$ is decomposed into $(3/25)G = ((1/10) + (1/50))G = ((1/15) + (1/25) + (1/75))G$, and LCM = 50 or 75. Among three 5/2+ levels of ^{17}O including S_n, the spacing $D_0 = 460$ keV $= (1/75)G$ behaves as a unit spacing [11]; 0 at the ground state(5/2+), $9D_0$ for $S_n = 4143$ keV, $16D_0$ for $E_x = 7378$ keV (resonance at $E_n = 3234$ keV), and $20D_0$ for $E_x = 9194$ keV ($E_n = 5050$ keV).

(iii) In the resonances in ^{140}Ce + n ($N = 82$, a nuclear magic number), dominant spacings of $D_0 = 86.2$ keV are found [18]. The separation energy $S_n = 5428.6$ keV $= (63/400)G$ can be decomposed into $(63/400)G = ((1/8) + (1/50) + (1/80))G$, and LCM = 400. The observed dominant spacing $D_0 = 86.2$ keV is exactly equal to $(1/400)G$. A large resonance observed at $E_n = 21.4$ keV is (1/4) of 86.2 keV.

For each reaction, several ways of decomposition are possible. However, only the minimum time recurrence (the smallest LCM) will be realized, by analogy with Fermat's principle in optics and Hamilton's principle in mechanics.

The variation of G for these four nuclides is about 1%, suggesting the existence of some simple features in nuclear excitation mechanisms. This may be related to the "Tuning effect" remarked on by S. Sukhoruchkin [8,14,18].

It is surprising that such simple arithmetic seems to be valid to describe the highly excited states of nuclei. Whereas, similar arithmetic in frequency relationship is widely observed in non-linear wave physics; examples include radio-frequency

TABLE II. First 9 resonances of ^{12}C + n which are in simple integer ratios to $E_n/1648$ (keV). 1648 keV $= G/21$.

i	J L	E_x (keV)	E_n (keV)	Γ_n (keV)	S_n/E_n	E_x/G	LCM	E_{rec} (keV)	R	δ (keV)	$E_n/1648$	G (keV)
0		4946.3	0			1/7						34624
1	5/2 2	6864	1917.7	8.6	18/7	25/126	126	274.5	7	−4.2	7/6	34594
2	5/2 2	7547	2600.7	2.0	21/11	32/147	147	235.8	11	−6.4	11/7	34669
3	3/2 2	7686	2739.7	93.9	9/5	14/63	63	549	5	−5.3	5/3	34587
4	3/2 2	8200	3253.7	1179	3/2	5/21	21	1640	2	−26.3	2/1	34440
5	1/2 1	8860	3913.7	138	15/12	9/35	35	984.4	4	−24.1	12/5	34455
6	1/2 1	9499.8	4553.5	1.7	12/11	23/84	84	413.0	11	10.1	11/4	34694
7	3/2 1	9897	4950.7	19	3/3	2/7	7	4948	1	−2.2	3/1	34639
8	1/2 0	10460	5513.7	37	9/10	19/63	63	550.5	10	8.4	10/3	34683
9	7/2 3	10753	5806.7	50	6/7	13/42	42	827.1	7	16.6	7/2	34740

waves transmitting through circuits with non-linear elements (diodes etc.), and high power laser light penetrating through a medium with non-linear polarizability. In such systems, the presence of higher and lower harmonics and the mixing of harmonics are familiar concepts [19,20].

VI. BREATHING MODEL OF NEUTRON RESONANCE REACTIONS

A brief description of the nuclear model used in the above analyses is given in this section.

By the S-matrix theory, the time response function or breathing of the CN is time-coherent with the de Broglie wave of the incident neutron. The wave packet of the incident neutron is very long ($\sim 10^{-8}$ m) compared to the nuclear size ($\sim 10^{-14}$ m), and is formally expressed as a plane wave $\exp[i(\omega t - kx)]$, where ω is frequency $= E_n/\hbar$, t is time, k the wave vector, and x the space coordinate. In crystal diffraction, an incident neutron wave is scattered from atoms at periodic lattice points, and is diffracted to directions where constructive interference of the scattered wave is maximal. The diffraction is in the (k, x) domain. In contrast, neutron resonances are in the (ω, t) domain. There will be periodic scattering centers on the time axis, which will be change size due to flare up of neutron density on the CN surface (entrance region), which is time coherent (synchronized) with the incident wave at resonances. In the absence of size change due to flare ups, only the potential scattering cross section (~ 10 barn $= 10 \times 10^{-28}$ m^2) will be observed. Therefore, a dynamic picture of the CN is needed to understand neutron resonances, and to this end we have developed the "recurrence model" [12] and the "breathing model" [15].

For a resonance, an incident neutron wave is divided into two components, a component that passes by without interaction and a penetrating component which is divided into M oscillators (normal modes) excited simultaneously in the CN. The recurrence time of each oscillator is quantized by a unit time $\tau_0 = 2\pi\hbar/G = 1.20 \times 10^{-22}$ s, and the recurrence time of the ensemble is $\tau_{\rm rec} = {\rm LCM}(n_1, n_2, \ldots, n_M)\tau_0$. At every $\tau_{\rm rec}$, the penetrating component recurs to the initial phase, gathers to form a high neutron-density flare on the CN surface, and interferes with the pass-by component. The instance of the recurrence (a few τ_0) called the coalescent phase in [12], and the gather phase in [15], is analogous to a time slit which opens to allow interference between the two components during a few τ_0 with every $\tau_{\rm rec}$. Energy spectra expected from these time structures are almost consistent with the observed facts [12], fine structure resonances, and enveloping giant resonances.

Excitation energy is in a form $E_x = G\sum_{j=1}^{M}(1/n_j)$, where the numerators will be replaced by small integers. For an oscillator of energy $(1/n)G$, we imagine a ring composed of n-elements (nucleons), on which excitation is transmitted to the next element in time τ_0, with a total excitation circulation time of $n\tau_0$. One of the elements couples to the entrance region, where interaction takes place with the pass-by component. Excitation at the entrance region excites M rings, and reappears after $\tau_{\rm rec}$ repeatedly. The incident neutron wave period $2\pi\hbar/E_n$ and its many harmonics are coherent with $\tau_{\rm rec}$ at resonances.

VII. CONCLUSION

Contrary to thier characterization as quantum chaos, neutron resonances are a regular system if we consider them from a different view point. In ^{40}Ca $+ n$ resonances, fairly simple structures are seen in resonance energies, which are revealed by the methods presented here. Similar analyses are valid for several light nuclei. In our breathing model, several new classical concepts are introduced, namely: $G \sim 34.5$ MeV, $\tau_0 = 2\pi\hbar/G = 1.20 \times 10^{-22}$ s, the recurrence time of oscillators quantized by τ_0, LCM of an oscillator ensemble, and E_x as a sum of inverse integers, equal to $E_x = G\sum_{j=1}^{M}(1/n_j)$ etc. We think that more sophisticated methods using quantum mechanical view points are needed, including the non-linear Schrödinger equation and non-linear polarizability for the incident neutron wave around nuclear surfaces. Fine structure analyses will be an interesting and fruitful area of study in nuclear physics in the 21st century.

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