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Energy of the ground and 2^+ excited states of $^{10}_{\Lambda\Lambda}$Be: A partial ten-body model

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The energies of the ground and excited 2^+ states of $^{10}_{\Lambda\Lambda}$Be have been calculated variationally in the Monte Carlo framework. The hypernucleus is treated as a partial ten-body problem in the $\Lambda\Lambda + \alpha\alpha$ model where nucleonic degrees of freedom of α's are taken into consideration ignoring the antisymmetrization between two α's. The central two-body ΛN and $\Lambda\Lambda$ and the three-body dispersive and two-pion exchange ΛNN forces, constrained by the Λp scattering data and the observed ground state energies of $^{5}_{\Lambda}$He and $^{6}_{\Lambda\Lambda}$He, are employed. The product-type trial wave function predicts binding energy for the ground state considerably less than for the event reported by Danysz *et al.*; however, it is consistent with the value deduced assuming a γ ray of 3.04 MeV must have escaped undetected in the decay of the product $^{9}_{\Lambda}\mathrm{Be}^* \longrightarrow {}^{9}_{\Lambda}\mathrm{Be} + \gamma$ of the emulsion event $^{10}_{\Lambda\Lambda}\mathrm{Be} \longrightarrow \pi^- + p + {}^{9}_{\Lambda}\mathrm{Be}^*$ and for the excited 2^+ state closer to the value measured in the Demachi-Yanagi event. The hypernucleus $^{10}_{\Lambda\Lambda}$Be has an oblate shape in the excited state. These results are consistent with the earlier four-body α cluster model approach where α's are assumed to be structureless entities.

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I. INTRODUCTION

In our earlier variational calculations [1], the hypernucleus $^{9}_{\Lambda}$Be was treated as a partial nine-body system in the $\Lambda + \alpha\alpha$ model, and the binding energy B_Λ of the hypernucleus $^{9}_{\Lambda}$Be was satisfactorily explained. Very recently [2] , we performed the detailed variational Monte Carlo (VMC) calculations for the energy spacing between the excited degenerate doublet $(3^+/2, 5^+/2)$ and the ground state of the hypernucleus $^{9}_{\Lambda}$Be following the spirit of the work in Ref. [1]. In Ref. [2], we used a number of potential parameter sets differing in C_p, the strength, and $\hat{c}$, the cutoff radius of the two-pion exchange ΛNN force, whereas the former analysis [1] used a single potential parameter set. A simple central two-body ΛN and the three-body dispersive and two-pion exchange ΛNN forces constrained by the Λp scattering data and the observed energy of $^{5}_{\Lambda}$He were employed in both analyses [1,2]. The simple trial wave functions constructed from the product of the central two-body and three-body correlations ignoring the space-exchange ΛN correlation gave good accounts of the observed energy spacing [3] of the degenerate doublet from the ground state of $^{9}_{\Lambda}$Be. The existence of nuclear degrees of freedom for the α's has been assumed from the outset in the above-mentioned analyses. The effect of NN correlations among the two α's, where each α contributes a nucleon, is simulated through gross $\alpha\alpha$ correlation, while within each α, the two-body NN correlations were explicitly incorporated. The antisymmetrization that has been ignored among the nucleons of two well separated α's is being simulated through the soft repulsive core in the $\alpha\alpha$ potential [4,5].

The three-body $\Lambda\alpha\alpha$ cluster model for $^{9}_{\Lambda}$Be had remarkable success [6–9]. The simplicity offered by the cluster model makes it seemingly a serious alternative to a partial nine-body model in explaining the observed energy of the ground and excited states of $^{9}_{\Lambda}$Be. Moreover, application of an α cluster model to the excited state is expected to give a better description than for the ground state, because the Λ particle induces less distortion in the core due to the extended $\alpha\alpha$ separation in $l = 2$ than for $l = 0$. Notwithstanding these characteristics, the success of the cluster model may be assigned to the microscopic calculation [10] which led to the introduction of the phenomenological dispersive three-body $\Lambda\alpha\alpha$ force [6–8] or to the improvement of the $\Lambda\alpha$ potential due to the modification of the odd state ΛN interaction [9]. In this context, it may be worthwhile to point out that the application of a partial nine-body model [1,2] to $^{9}_{\Lambda}$Be has revealed that the dispersive $\Lambda\alpha\alpha$ force proposed in the work of Shoeb, Mamo, and Fessahatsion [7], initially thought to be arising due to a dispersive ΛNN force alone, actually may also include the contribution of a two-pion exchange ΛNN force from the ΛNN triads where each α contributes a nucleon. The foregoing argument has been put forward, knowing full well the ambiguity in the cutoff radius dependent contributions of the two-pion exchange ΛNN force, to emphasize the importance of the partial/full nine-body problem over the cluster model, which due to its inherent limitations is highly inadequate in discerning such finer details. Therefore, an analysis using a partial/full nine-body problem will still be a preferred theoretical approach over the cluster model because of the former being more fundamental, which gives better physical insight into further improvement in the latter.

The energy of the ground and excited states of $^{10}_{\Lambda\Lambda}$Be has been extensively analyzed in the $\Lambda\Lambda\alpha\alpha$ cluster model in Refs. [6,7,9,11,12] using a variety of methods: VMC framework, Gaussian-basis coupled-rearrangement channel method, and Faddeev-Yakubovsky method. The input $\Lambda\Lambda$, $\Lambda\alpha$, and $\alpha\alpha$ potentials with soft repulsive cores and reasonable shapes, constrained by the data relevant to each interacting pair, have been used. The analyses [7,9] predict the ground state $\Lambda\Lambda$ binding of $^{10}_{\Lambda\Lambda}$Be, $B_{\Lambda\Lambda}$, to be about 15% less than the currently accepted experimental [13] value of 17.6 ± 0.4 MeV. This strengthens the speculation that in the measurement of $B_{\Lambda\Lambda}$, a γ ray [3] of 3.04 MeV must have escaped undetected from

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the decay products of ${}^{10}_{\Lambda\Lambda}$Be in the emulsion. Furthermore, the cluster model calculations of Shoeb [6] and Hiyama *et al.* [9] for the energy of the 2^+ excited state of ${}^{10}_{\Lambda\Lambda}$Be incidentally agree with the $\Lambda\Lambda$ binding energy $B_{\Lambda\Lambda} = 12.33^{+0.35}_{-0.21}$ MeV found in the E373 experiment, named the Demachi-Yanagi event [14]. Thus, the Demachi-Yanagi event, based on the calculation of the $\Lambda\Lambda\alpha\alpha$ cluster model, is identified with the 2^+ excited state of ${}^{10}_{\Lambda\Lambda}$Be.

The success of a partial nine-body model [1,2] of ${}^{9}_{\Lambda}$Be in explaining the energy of the ground and excited states motivated us to extend the model to the case of ${}^{10}_{\Lambda\Lambda}$Be and to investigate its success in explaining the energy of the ground and 2^+ excited states. The addition of a Λ to ${}^{9}_{\Lambda}$Be increases the $B_{\Lambda\Lambda}$ of the resultant system; thus on physical grounds, one expects a significant reduction in the $\alpha\alpha$ separation in comparison to what has been found in ${}^{9}_{\Lambda}$Be. The excited 2^+ state of ${}^{10}_{\Lambda\Lambda}$Be, assumed to be built on the $\alpha\alpha$ relative $l = 2$, should have an extended structure compared to the ground state because of the centrifugal barrier. The ${}^{10}_{\Lambda\Lambda}$Be is treated as a partial ten-body problem in the $\Lambda\Lambda + \alpha\alpha$ model. The expressions for the ΛN, NN, $\alpha\alpha$, and three-body ΛNN potentials are given in our earlier work [2]. Therefore, in the next section, we briefly discuss the Hamiltonians of ${}^{6}_{\Lambda\Lambda}$He as a six-body system and that of ${}^{10}_{\Lambda\Lambda}$Be as a partial ten-body problem within the α cluster model along with the construction of trial wave functions and the energy calculation. Preliminary results for the ground state energy were reported earlier [15]. The quadrupole moment of the hypernucleus ${}^{10}_{\Lambda\Lambda}$Be in the 2^+ state has also been predicted to know more about its structure. To our knowledge, this is the first time that ${}^{10}_{\Lambda\Lambda}$Be is being studied as a partial ten-body problem in the VMC approach.

In Sec. III, we present the results and discussion. The conclusion of our work and its implications are described in the last section.

II. HAMILTONIANS IN THE α CLUSTER MODEL, TRIAL WAVE FUNCTIONS, AND ENERGY CALCULATION

To analyze the energy of ${}^{10}_{\Lambda\Lambda}$Be, an investigation of the binding energies of the subsystems ${}^{5}_{\Lambda}$He and ${}^{6}_{\Lambda\Lambda}$He is needed to fix the potential parameters other than that of the two-body ΛN interaction. The ground state energy of ${}^{5}_{\Lambda}$He has already been calculated in the VMC framework for the potential parameter sets C_p, $\hat{c}$, and W_d, the strength of the dispersive ΛNN force, and therefore the potential parameter sets are taken from earlier work [2]. However, $B_{\Lambda\Lambda}$ of ${}^{6}_{\Lambda\Lambda}$He has been analyzed as a six-body problem by Shoeb [16] using VMC, but here we would like to analyze it again in detail using the same sets of potential parameters as used for ${}^{5}_{\Lambda}$He. Therefore, we are interested in making a combined study of the energies of the ground state [17] of ${}^{6}_{\Lambda\Lambda}$He and of the ground and excited states [13,14] of ${}^{10}_{\Lambda\Lambda}$Be by choosing the same sets of potential parameters. The p-shell $\Lambda\Lambda$ hypernucleus of mass number A consists of $(A-2)$ nucleons and two Λ particles. The Hamiltonian H_H^A for A baryons, in general, is written as the sum of the Hamiltonian of the $(A-2)$ nucleons of the core nucleus H_C^{A-2} and the Λ particles Hamiltonian $H_{\Lambda\Lambda}$, i.e.,

$$H_H^A = H_C^{A-2} + H_{\Lambda\Lambda}. \quad (1)$$

The nuclear Hamiltonian H_C^{A-2} for NN interacting through the two-body potential $V_{NN}(r_{ij})$ for the α cluster core nucleus is written as

$$H_C^{A-2} = \sum_{i=1}^{A-2} K_N(i) + \sum_{i<j}^{A-2} V_{NN}(r_{ij}) + V_{\alpha\alpha}^{(l)}(r_{\alpha_1\alpha_2}), \quad (2)$$

and the Λ particles Hamiltonian $H_{\Lambda\Lambda}$ is given by

$$H_{\Lambda\Lambda} = \sum_{k=1}^{2}\left(K_{\Lambda_k} + \sum_{i=1}^{A-2} V_{\Lambda N}(r_{\Lambda_k i}) + \sum_{i<j}^{A-2} V_{ij\Lambda}(r_{i\Lambda_k}, r_{j\Lambda_k})\right) + V_{\Lambda\Lambda}(r_{\Lambda_1\Lambda_2}), \quad (3)$$

where K_a is the kinetic energy operator for particle a (=N, Λ_k), and $V_{\Lambda N}$, $V_{\Lambda\Lambda}$, and $V_{ij\Lambda}$ are the two-body ΛN, $\Lambda\Lambda$, and three-body ΛNN potentials, respectively. The Hamiltonian for ${}^{6}_{\Lambda\Lambda}$He is obtained by restricting the baryon number to $A = 6$ in Eqs. (1)–(3) and suppressing the $\alpha\alpha$ potential.

The 2^+ state of ${}^{10}_{\Lambda\Lambda}$Be is assumed to be built on the first excited state $J_C = 2^+$ of the core nucleus ^{8}Be. $J_C = 2^+$ is a coupled state of $L_C = 2$ and $S_C = 0$. The two Λ particles of $s_\Lambda = 1/2$ coupled to the spin singlet function $\chi^0{}_0$ when combined with the $J_C = 2^+$ of the ^{8}Be core in ${}^{10}_{\Lambda\Lambda}$Be give rise to the $J = 2^+$ spin state. The Hamiltonian of the A(=10) baryon system ${}^{10}_{\Lambda\Lambda}$Be in the $2\alpha + 2\Lambda$ is given as

$$H_H^{10} = \sum_{i=1}^{A-2} K_N(i) + \sum_{i<j}^{4} V_{NN}(r_{ij}) + \sum_{i<j}^{A-2}{}_{i=5} V_{NN}(r_{ij}) + V_{\alpha\alpha}^{(l)}(r_{\alpha_1\alpha_2}) + \sum_{k=1}^{2}\left(K_{\Lambda_k} + \sum_{i=1}^{4} V_{\Lambda N}(r_{\Lambda_k i}) + \sum_{i=5}^{A-2} V_{\Lambda N}(r_{\Lambda_k i}) + \sum_{i<j}^{A-2} V_{ij\Lambda}(r_{i\Lambda_k}, r_{j\Lambda_k})\right) + V_{\Lambda\Lambda}(r_{\Lambda_1\Lambda_2}), \quad (4)$$

where labels i and j run over nucleons, k over Λ particles and symbols α_1 and α_2 are identified with the α particles. V_{xy} denotes the potential for a pair of particles xy(=NN,ΛN,$\Lambda\Lambda$,$\alpha\alpha$) and in the case of the $\alpha\alpha$ pair, $V_{\alpha\alpha}^{(l)}$ is the potential in the relative angular momentum l (=0 for the ground and 2 for the excited state). The three-body potential $V_{ij\Lambda}$ is the sum of the dispersive force $V^D_{\Lambda NN}$ and the two-pion exchange three-body force $V^{2\pi}_{\Lambda NN}$. The contribution of $\langle V^D_{\Lambda NN}\rangle$ to the energy, from triads ΛNN, where each α contributes a nucleon, is substantial as shown in Refs. [1,10]; neglecting it over binds not only ${}^{9}_{\Lambda}$Be but also ${}^{10}_{\Lambda\Lambda}$Be.

We have found that the results for the energy spacing of the excited degenerate doublet ($3^+/2$, $5^+/2$) from the ground state of ${}^{9}_{\Lambda}$Be for the Volkov [18] and Malfliet-Tjon (MT) [19] NN potentials are essentially the same [2]. Therefore, we have carried out calculations for the energies of ${}^{10}_{\Lambda\Lambda}$Be using the MT NN potential alone. For $\alpha\alpha$, the Ali and Bodmer [4] potential as modified by Fedorov and Jensen [5] has been used. Detailed expressions for the Urbana-type ΛN potential [10,11] and for the other potentials used here are given in Ref. [2]. The $\Lambda\Lambda$ potential in the singlet state of the three-range Gaussian form fitting the ground state binding energy of ${}^{\;6}_{\Lambda\Lambda}$He is used for analyzing the $B_{\Lambda\Lambda}$ of ${}^{10}_{\Lambda\Lambda}$Be and has the form

$$V_{\Lambda\Lambda}(r) = \sum_{i=1}^{3} v_i \exp\left(-r^2/\alpha_i^2\right), \tag{5}$$

where $v_1 = -21.49$ MeV, $\alpha_1 = 1.342$ fm; $v_2 = -379.1\gamma$ MeV, $\alpha_2 = 0.777$ fm; and $v_3 = 9324$ MeV, $\alpha_3 = 0.350$ fm; and the coefficient $\gamma = 0.6598$ from the cluster model study [7] and does not change for the six-body system, as discussed in the next section.

The expressions for the three-body forces are also given in our previous work. Here, we may remark that for the two-pion exchange ΛNN force, for ease of computation, we have restricted our calculations to two extreme values of cutoff radii $\hat{c} = 1$ and 3 fm^{-2}, because $\hat{c} = 2$ fm^{-2} is expected to give almost the same result as found earlier [2] for ${}^{9}_{\Lambda}$Be.

A. Trial wave functions

The variational wave function for ${}^{10}_{\Lambda\Lambda}$Be in the state (J,J_z) as usual is constructed from the product of central two-body correlation functions f_{xy} (for $xy = \alpha\alpha$, relative angular momentum state $l = 0$ or 2), three-body correlations $f_{\Lambda NN}$, and the ls coupled function $(y_{lm}(\Omega_{\alpha_1\alpha_2}) \otimes \chi^0{}_0)_{JJ_z}$, which is an appropriate combination of $\chi^0{}_0$, the spin singlet function of the two Λ particles, and $y_{lm}(\Omega_{\alpha_1\alpha_2})$, the spherical harmonic for the motion of two α's in the relative angular momentum state l:

$$\begin{aligned}\Psi_H^{(10)}(J,J_z) = \prod_{k=1}^{2}\Bigg(&\left[\prod_{i=1}^{4} f_{\Lambda N}(r_{\Lambda_k i})\right]\left[\prod_{i=5}^{A-2} f_{\Lambda N}(r_{\Lambda_k i})\right]\\ &\times\left[\prod_{i<j\,i=1}^{4} f_{\Lambda NN}(r_{i\Lambda_k}, r_{j\Lambda_k})\right]\\ &\times\left[\prod_{i<j\,i=5}^{A-2} f_{\Lambda NN}(r_{i\Lambda_k}, r_{j\Lambda_k})\right]\Bigg)\left[\prod_{i<j}^{4} f_{NN}(r_{ij})\right]\\ &\times\left[\prod_{i<j\,i=5}^{A-2} f_{NN}(r_{ij})\right] f_{\alpha\alpha}^{(l)}(r_{\alpha_1\alpha_2}) f_{\Lambda\Lambda}(r_{\Lambda_1\Lambda_2})\\ &\times\left(y_{lm}(\Omega_{\alpha_1\alpha_2}) \otimes \chi_0^0\right)_{JJ_z}.\end{aligned} \tag{6}$$

The central two-body spin-independent correlation functions $f_{\Lambda N}$, f_{NN}, $f_{\alpha\alpha}^{(l)}(r)$, and $f_{\Lambda\Lambda}$, similar to our earlier analyses [1,6,7,11], are obtained from the procedure developed by the Urbana group, and the three-body correlations $f_{\Lambda NN}$ have the analytical forms as used in our previous work [2]. The $f_{\Lambda NN}$ correlations arising from the triads ΛNN, where a participating nucleon comes from each α, have been ignored, as these make a negligibly small contribution owing to the large $\alpha\alpha$ separation. The wave function $\Psi_H^{(10)}$ depends on a total of 17 variational parameters $\kappa_{\Lambda N}$, $c_{\Lambda N}$, $a_{\Lambda N}$, $R_{\Lambda N}$, $s_{\Lambda N}$, κ_{NN}, c_{NN}, a_{NN}, R_{NN}, $\kappa_{\alpha\alpha}$, $c_{\alpha\alpha}$, $a_{\alpha\alpha}$, $R_{\alpha\alpha}$, $\kappa_{\Lambda\Lambda}$, $c_{\Lambda\Lambda}$, $a_{\Lambda\Lambda}$, and $R_{\Lambda\Lambda}$ for the ground state of ${}^{10}_{\Lambda\Lambda}$Be and exactly the same number of variational parameters for the excited state. The trial wave function for ${}^{\;6}_{\Lambda\Lambda}$He involving 13 variational parameters is obtained from the wave function of ${}^{10}_{\Lambda\Lambda}$Be after suppressing the $\alpha\alpha$ correlation and restricting baryon number to 6.

B. Energy calculation and moment

The energy $-B_{\Lambda\Lambda}(J,J_z)$ for a hypernucleus of baryon number A is the difference of the energy of hypernucleus in the state $\Psi_H^{(A)}(J,J_z)$ and of the nuclear core in the state $\Psi_C^{(A-2)}(J_C,M_C)$ and is written as

$$\begin{aligned}-B_{\Lambda\Lambda}&(J,J_z)\\ &= \frac{\left\langle\Psi_H^{(A)}(J,J_z)\middle|H_H^A\middle|\Psi_H^{(A)}(J,J_z)\right\rangle}{\left\langle\Psi_H^{(A)}(J,J_z)\middle|\Psi_H^{(A)}(J,J_z)\right\rangle}\\ &\quad - \frac{\left\langle\Psi_C^{(A-2)}(J_C,M_C)\middle|H_C^{A-2}\middle|\Psi_C^{(A-2)}(J_C,M_C)\right\rangle}{\left\langle\Psi_C^{(A-2)}(J_C,M_C)\middle|\Psi_C^{(A-2)}(J_C,M_C)\right\rangle}.\end{aligned} \tag{7}$$

The variational parameters entering in the wave function are varied to optimize the energy using the standard optimizing routine. The parameters $\kappa_{\Lambda N}$, κ_{NN}, $\kappa_{\alpha\alpha}$, and $\kappa_{\Lambda\Lambda}$ related to the separation energy of the pair xy are those on which the energy depends sensitively. The estimates for the energy were made for 100 000 points. The two terms in Eq. (7) were separately calculated. The second term in Eq. (7) is -31.20 MeV for ^{4}He and -62.3 MeV for ^{8}Be core for MT NN potential.

The expressions for the calculation of ΛN space-exchange energy and of quadrupole moment in the unit of e fm^2 are the same as outlined in our previous work [2].

III. RESULTS AND DISCUSSION

Before we proceed to discuss the results for the calculation of the energy of the system ${}^{10}_{\Lambda\Lambda}$Be, we need to constrain some of the potential parameters from the ground state energies of ${}^{5}_{\Lambda}$He and ${}^{\;6}_{\Lambda\Lambda}$He. For the parameters $\bar{V} = 6.15$ MeV and $\epsilon = 0.25$ of the two-body ΛN Urbana-type force, the strength W_d of dispersive force is adjusted for the sets of combinations of C_p and $\hat{c}$ from a fit to B_Λ of ${}^{5}_{\Lambda}$He for the NN potential, as described earlier [2]. The sets of ΛN and ΛNN potential parameters, so obtained, are used along with $\Lambda\Lambda$ potential Eq. (5) to analyze the ${}^{\;6}_{\Lambda\Lambda}$He. The parameter sets that give a good account of the binding energies of ${}^{5}_{\Lambda}$He and ${}^{\;6}_{\Lambda\Lambda}$He are employed to carry out the energy calculations for the ground and excited states of ${}^{10}_{\Lambda\Lambda}$Be. The experimental data for the systems ${}^{\;6}_{\Lambda\Lambda}$He and ${}^{10}_{\Lambda\Lambda}$Be are given in Refs. [13,14,17]. This is, to our knowledge, the first time that VMC approach is being applied to analyze the energy of the ground and excited 2^+ states of ${}^{10}_{\Lambda\Lambda}$Be.

A. Ground states of ${}^{\;6}_{\Lambda\Lambda}$He and ${}^{10}_{\Lambda\Lambda}$Be

The variationally calculated energy $-B_{\Lambda\Lambda}$ of ${}^{\;6}_{\Lambda\Lambda}$He for the ΛN potential [2] in conjunction with W_d of the dispersive force for a few choices of $C_p(\hat{c})$ is close to the experimental value. We note that the value of the coefficient γ remains the same as found in the cluster model analysis. The results of our calculation for ${}^{\;6}_{\Lambda\Lambda}$He are shown in Table I. From the table, we

TABLE I. VMC results ($\gamma = 0.6598$) for the total energy with statistical error for ${}^{6}_{\Lambda\Lambda}$He are given in the seventh column for combinations of three-body potentials listed in columns one and two. The third and fourth columns have the values of average kinetic $\langle T\rangle$ and two-body potential $\langle V_{BN}\rangle$ energies; the contribution of dispersive $\langle V^{D}_{\Lambda NN}\rangle$ and two-pion exchange $\langle V^{2\pi}_{\Lambda NN}\rangle$ energies are given in the fifth and sixth columns. The eighth column gives $B_{\Lambda\Lambda}$. All potential strengths and energies are in MeV, and $\hat{c}$ is in fm^{-2}. Experimental $B_{\Lambda\Lambda} = 7.25 \pm 0.19^{+0.18}_{-0.11}$ MeV, from Ref. [17].

W_d	$C_p(\hat{c})$	$\langle T\rangle$	$-\langle V_{BN}\rangle$	$\langle V^{D}_{\Lambda NN}\rangle$	$\langle V^{2\pi}_{\Lambda NN}\rangle$	$-E \pm \Delta E$	$B_{\Lambda\Lambda}$
0.012	0 (0)	93.89	136.37	3.91	0.00	38.57 ± 0.06	7.37
0.009	1 (1)	94.25	137.11	3.21	1.29	38.36 ± 0.05	7.16
0.0115	1 (3)	98.68	140.97	4.67	−0.92	38.54 ± 0.05	7.34
0.006	2 (1)	95.44	138.21	2.51	1.88	38.38 ± 0.07	7.18
0.016	2 (3)	101.54	141.33	8.07	−6.98	38.70 ± 0.07	7.50

note that as the cutoff radius changes from 1 to 3 fm^{-2}, the contribution of $V^{2\pi}_{\Lambda NN}$ changes from moderately repulsive to strongly attractive, as noticed in the earlier work [2,11], the $B_{\Lambda\Lambda}$ of ${}^{6}_{\Lambda\Lambda}$He is satisfactorily explained.

The ground state energy of ${}^{10}_{\Lambda\Lambda}$Be is optimized with respect to the variational parameters for all the sets of three-body ΛNN potential parameters that fit $B_{\Lambda\Lambda}$ of ${}^{6}_{\Lambda\Lambda}$He. For the $\alpha\alpha$ pair, the Ali and Bodmer [4,5] potential in $l = 0$ has been employed. Thus, no adjustable potential parameter is left in the calculation of ${}^{10}_{\Lambda\Lambda}$Be. In all the cases listed in Table II, the variational $B_{\Lambda\Lambda}$ values are found to lie in the range 14.64–15.21 MeV, which strongly disagree with the currently accepted experimental value [13] but are consistent with the value 14.50 MeV, deduced on the basis that a γ ray of 3.04 MeV must have escaped undetected in the old emulsion experiment [13]. Furthermore, the root mean square (rms) $\alpha\alpha$ separation $\mathbf{R}_{\alpha\alpha}$ varies between 3.46 and 3.51 fm, which is significantly smaller than found in the earlier analysis [2] for ${}^{9}_{\Lambda}$Be, showing considerable compression of core nucleus. However, we note that $\mathbf{R}_{\alpha\alpha}$ is still more than twice the rms radius of an α particle, indicating a minimal overlap of the two α's and thus justifying the internal consistency of the α cluster model.

In view of the ambiguity, arising in the experimental $B_{\Lambda\Lambda}$ value from the alternative interpretation of the event in the emulsion, in the foregoing discussion we are not in a position to say whether our potential parameter sets explain the $B_{\Lambda\Lambda}$ of ${}^{10}_{\Lambda\Lambda}$Be or not. Therefore, potential parameter sets were tested against precise and unambiguously measured experimental value for ${}^{10}_{\Lambda\Lambda}$Be corresponding to the Demachi-Yanagi event that has been theoretically assigned as a 2^+ state from the cluster model calculations [6,9].

B. Excited 2^+ of ${}^{10}_{\Lambda\Lambda}$Be

The potential parameter sets constrained by a fit to the binding energies of ${}^{5}_{\Lambda}$He, ${}^{9}_{\Lambda}$Be, and ${}^{6}_{\Lambda\Lambda}$He, were employed to calculate the energy of the excited 2^+ state of ${}^{10}_{\Lambda\Lambda}$Be. However, for the $\alpha\alpha$ pair, the Ali-Bodmer potential [4,5] is used in the relative $l = 2$ state. The results of calculation for the total and its component energies for optimum values of variational parameters are listed in Table III. The energy of the 2^+ state is found to lie between −11.63 and −11.99 MeV, which is consistent with the Demachi-Yanagi event. The calculated quadrupole moment for the 2^+ state is found to vary from −7.17 to −7.66 e fm^2 and thereby indicating that the system is highly deformed. The values of the quadrupole moment are comparable to those calculated in the four-body $\Lambda\Lambda\alpha\alpha$ cluster model by Shoeb [6]. Furthermore, $\mathbf{R}_{\alpha\alpha}$ in the 2^+ state is marginally higher than that for the ground state, as it should be because of the centrifugal barrier in the relative $l = 2$ state for the motion of two α's. The close agreement of the calculated energy of the 2^+ state in the partial ten-body problem with the Demachi-Yanagi event supports the results of

TABLE II. VMC total energy with statistical error of the ground state of ${}^{10}_{\Lambda\Lambda}$Be is listed in the seventh column. The $\alpha\alpha$ rms radii $\mathbf{R}_{\alpha\alpha}$ (in fm) for various potential parameter sets are given in the ninth column. Other quantities are the same as in Table I. Experimental value [13] $B_{\Lambda\Lambda}$ (${}^{10}_{\Lambda\Lambda}$Be) $= 17.6 \pm 0.4$ MeV [14.5 ± 0.4 MeV assuming ${}^{10}_{\Lambda\Lambda}\mathrm{Be} \longrightarrow \pi^- + p + {}^{9}_{\Lambda}\mathrm{Be}^*$; ${}^{9}_{\Lambda}\mathrm{Be}^* \longrightarrow {}^{9}_{\Lambda}\mathrm{Be} + \gamma$ (3.04 MeV) Ref. [3]].

W_d	$C_p(\hat{c})$	$\langle T\rangle$	$-\langle V_{BN}\rangle$	$\langle V^{D}_{\Lambda NN}\rangle$	$\langle V^{2\pi}_{\Lambda NN}\rangle$	$-E \pm \Delta E$	$B_{\Lambda\Lambda}$	$\mathbf{R}_{\alpha\alpha}$
0.012	0 (0)	182.10	267.07	7.57	0.00	77.40 ± 0.08	15.10	3.51
0.009	1 (1)	184.47	269.67	6.30	1.96	76.94 ± 0.08	14.64	3.50
0.0115	1 (3)	185.07	269.23	8.51	−1.51	77.16 ± 0.09	14.86	3.49
0.006	2 (1)	181.34	267.38	4.96	3.57	77.51 ± 0.11	15.21	3.46
0.016	2 (3)	186.72	266.53	13.66	−11.06	77.21 ± 0.17	14.91	3.47

TABLE III. VMC results for the 2^+ excited state of ${}^{10}_{\Lambda\Lambda}$Be. Experimental $B_{\Lambda\Lambda} = 12.33^{+0.35}_{-0.21}$ MeV, Demachi-Yanagi event [14]. The energy spacing $\Delta E_{\Lambda\Lambda}$ (in MeV) between ground and 2^+ excited states along with the statistical error is given in column nine. Experimental $\Delta E_{\Lambda\Lambda} = 5.27 \pm 0.4$ MeV from Danysz *et al.* [13] (2.17 ± 0.4 MeV deduced from the assumption of a missing γ ray). The last column has values of the quadrupole moment in the unit of $e\,\text{fm}^2$. Other quantities are the same as in the preceding table.

W_d	$C_p(\hat{c})$	$\langle T \rangle$	$-\langle V_{BN} \rangle$	$\langle V^D_{\Lambda NN} \rangle$	$\langle V^{2\pi}_{\Lambda NN} \rangle$	$-E \pm \Delta E$	$B_{\Lambda\Lambda}(2^+)$	Theoretical $\Delta E_{\Lambda\Lambda}$	$\mathbf{R}_{\alpha\alpha}$	$-\langle Q \rangle_{(2^+,2)}$
0.012	0 (0)	188.71	271.40	8.43	0.00	74.26 ± 0.08	11.96	3.14 ± 0.08	3.54	7.31
0.009	1 (1)	184.90	267.22	6.26	2.13	73.93 ± 0.08	11.63	3.01 ± 0.08	3.63	7.66
0.0115	1 (3)	185.03	266.17	8.76	−1.64	74.02 ± 0.15	11.72	3.14 ± 0.15	3.55	7.17
0.006	2 (1)	185.69	268.55	4.48	4.09	74.29 ± 0.08	11.99	3.22 ± 0.11	3.59	7.24
0.016	2 (3)	188.07	266.46	14.25	−10.04	74.18 ± 0.12	11.88	3.03 ± 0.17	3.52	7.23

cluster calculations. Assuming that the theoretically assigned 2^+ state to Demachi-Yanagi event is correct, then one can safely argue that the currently accepted $B_{\Lambda\Lambda}$ value 17.6 ± 0.4 MeV of ${}^{10}_{\Lambda\Lambda}$Be from Danysz *et al.* [13] is not correlated with Demachi-Yanagi, and therefore a remeasurement of the ground state energy of ${}^{10}_{\Lambda\Lambda}$Be is highly desirable to resolve the existing anomaly in the energy of the two states.

For the two choices of $\hat{c}$ discussed above, it should be noted from Tables II and III that for a given C_p, the contribution of $\langle V^{2\pi}_{\Lambda NN} \rangle$ is highly nonlinear for the ground and excited states of ${}^{10}_{\Lambda\Lambda}$Be, similar to the case found above for ${}^{6}_{\Lambda\Lambda}$He and for ${}^{5}_{\Lambda}$He and ${}^{9}_{\Lambda}$Be in our earlier work [2].

The differences $\Delta E_{\Lambda\Lambda}$ [$=E({}^{10}_{\Lambda\Lambda}\text{Be}^*) - E({}^{10}_{\Lambda\Lambda}\text{Be})$] between excited and ground state energies for the combination of potential parameter sets under consideration are listed in Table III. We note from the table that its value of 3.0–3.2 MeV is consistent with the cluster model calculation of Shoeb [6] but about 10% larger than the one found by Hiyama *et al.* [9]. However, theoretically calculated $\Delta E_{\Lambda\Lambda}$ lies in between the two extreme experimental values of 2.17 ± 0.4 and 5.27 ± 0.4 MeV but closer to the lower limit.

We may remark that Zhou *et al.* [20] and Žofka [21] have studied the ground state properties of hypernuclei involving core nucleus ^{8}Be apart from many other systems in the deformed Skyrme-Hartree-Fock (DSHF) framework and found that the quadrupole moment either decreases or remains practically constant with a decrease in the rms radii of the hypernuclei. However, it is not known how the deformation in the DSHF calculation will show up in the excited states for these systems. Therefore, we feel that it is premature to compare our results of the excited state with those of earlier work [20,21] in the ground state and more so in light of the remark made by Žofka [21] that “Be is too light system for the HF method to be a reliable description scheme.” We have done the VMC calculation with simplified two-body central potentials using the spherical ground state. The results of deformation for the partial ten-body ${}^{10}_{\Lambda\Lambda}$Be system in the excited state are consistent with those of the four-body cluster $\Lambda\Lambda\alpha\alpha$ model calculation [6] of ${}^{10}_{\Lambda\Lambda}$Be. The deformation in the system is built-in mainly because of the appearance of the spherical harmonic $l = 2$ for relative motion of two α's in the trial wave function. We may emphasize here that it is the cluster model calculations [6] that act as the precursor to what should be the likely outcome of the results from microscopic calculations and vice versa.

Analysis of the ground and excited states of ${}^{10}_{\Lambda\Lambda}$Be in the partial ten-body model where nuclear degrees of freedom in the α's are assumed from the outset is in essence an extension of the $\Lambda\Lambda\alpha\alpha$ cluster model [6,7,9,12] where rigid α's are assumed structureless. However, in both models, the antisymmetrization between the two α's is simulated through a soft repulsive core in the $\alpha\alpha$ potential. The cluster model calculations [6,9] of the energy for the excited 2^+ state of ${}^{10}_{\Lambda\Lambda}$Be give results in agreement with a partial ten-body system. However, for the ground state, the calculated values of the energy from the α cluster and partial ten-body models agree and are about 15% higher than the value measured by Danysz *et al.* [13] but consistent with the one from the alternative interpretation of the event. Notwithstanding the simplicity of the α cluster model and its success in describing the observed energy of ${}^{10}_{\Lambda\Lambda}$Be, the partial/full ten-body problem will still be a fundamental microscopic approach for further improving the cluster model, as discussed in our previous work [2].

IV. CONCLUSION

We conclude that we have made, to our knowledge, the first application of the VMC method in analyzing the energy of the ground and excited 2^+ states of ${}^{10}_{\Lambda\Lambda}$Be, treating it as a partial ten-body system in the $\Lambda\Lambda\alpha\alpha$ cluster model using the simple ΛN potentials and corresponding correlation functions. The Urbana-type ΛN potential consistent with the Λp scattering data along with the dispersive ΛNN or dispersive plus two-pion exchange ΛNN forces gives an energy of the 2^+ excited state of ${}^{10}_{\Lambda\Lambda}$Be close to the Demachi-Yanagi event, thus confirming the spin and parity assignment of the event as 2^+. The energy of the excited state is not correlated with the currently accepted value of the ground state; however, it agrees with the value deduced from the alternative arguments in which a γ ray is postulated to have escaped. These findings support the results of earlier cluster model analyses and thus make a stronger point for the revision of the measurement of the ground state binding energy of ${}^{10}_{\Lambda\Lambda}$Be. Furthermore, our calculation predicts that ${}^{10}_{\Lambda\Lambda}$Be in the excited state is highly deformed and has an oblate shape. Although calculations of a partial ten-body system closely agree with that of the α cluster model, the former remains a fundamental and still superior option for extracting the finer microscopic aspects of the dynamical correlations on the properties of the system and thus provide a better guidance in improving the cluster model approach. Finally, we conclude by saying that our ΛN and ΛNN potential parameter sets satisfactorily explain the energies of α cluster hypernuclei, ${}^{5}_{\Lambda}$He, ${}^{6}_{\Lambda\Lambda}$He, ${}^{9}_{\Lambda}$Be, and ${}^{10}_{\Lambda\Lambda}$Be(2^+), treating s-shell hypernuclei as A-body and p-shell as partial A-body systems, the only exception being the ground state of ${}^{10}_{\Lambda\Lambda}$Be. Therefore, our microscopic calculations strongly suggest that a fresh measurement be made of the ground state energy of ${}^{10}_{\Lambda\Lambda}$Be.

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