Nuclear response for the Skyrme effective interaction with zero-range tensor terms

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The effects of a zero-range tensor component of the effective interaction on nuclear response functions are determined in the random phase approximation approach. Explicit formulas in the case of symmetric homogeneous isotropic nuclear matter are given for each spin-isospin excitation channel. It is shown for a typical interaction with tensor couplings that the effects are quantitatively important, mainly in vector channels.

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I. INTRODUCTION

Mean-field approaches are the only ones that allow for systematic calculations of binding energy and one-body observables in the region of the nuclear chart that ranges from medium- to heavy-mass atomic nuclei from drip line to drip line [1]. These effective approaches rely on a limited number of universal parameters, usually fitted on experimental data along with properties of infinite nuclear matter derived from realistic models [2].

In the Skyrme-Hartree-Fock formulation, the energy of the system takes the form of a functional of local one-body densities derived from the effective Skyrme interaction *ansatz*. The commonly used Skyrme interaction typically depends on ten parameters and is made of contact central terms including a spin-orbit interaction. The latter, which is controlled by one single parameter, is mandatory to obtain the known sequence of magic numbers along the valley of stability. Although this is enough to reproduce the global features of nuclei, it was stressed from the beginning by Skyrme himself [3,4] that such a simple interaction would probably not be sufficient for a realistic description of nuclear spectroscopy and that a tensor interaction might be needed [5]. This last part of the effective interaction, made of two contact terms, was not considered in most of the early parametrizations of the Skyrme force possibly because of the difficulty in constraining the corresponding coupling constants.

In spite of the difficulties related to the adjustment of the parameters of the tensor terms, over the years, several attempts have been made for including them. The tensor terms as proposed by Skyrme were considered on top of the Skyrme SIII [6] effective interaction [7] or with a complete refit of the parameters [8,9]. More recently, the tensor effective interaction has regained some attention [10–15], partly because it was supposed to be the key for the reproduction of several specific spectroscopic features such as, for example, the relative shift of proton $1g_{7/2}$ and $1h_{11/2}$ levels in antimony isotopes [16]. In a recent article [17], a systematic study of the zero-range effective tensor interaction combined with a standard Skyrme functional has been made. In this work, a set of interactions was built by fixing the two parameters of the tensor terms to different values while the remaining part of the interactions was fitted using the same procedure as for the well-known SLy interaction [18–20]. It was shown that global features of spherical nuclei (masses and radii) and single-particle energies cannot be used to clearly mark the boundary of a successful domain for the tensor parameters.

In addition to looking for the parameters that lead to the "best fits" to the data, one should also worry about the values that could lead to unphysical instabilities of nuclei. Because the tensor interaction energy depends on spins and gradients of densities of the interacting nucleons, one can intuitively understand that the appearance of unphysical finite-size domains of polarized nuclear matter can be favored for some values of the coupling constants. Such situations are hard to predict and difficult to avoid during the fit of the parameters since polarized systems break time-reversal symmetry whereas the calculations entering the standard fitting protocol usually assume that nuclei are spherical and time even.

A similar kind of instability of finite systems was encountered and examined in an article devoted to the study of effective mass splitting [21]. There it was shown that the linear response formalism applied to the Skyrme energy functional can be used to predict the appearance of finite-size instabilities in nuclei. However, only the central part of the Skyrme interaction was taken into account for the building of the linear response. In the present article, we derive the full linear response for symmetric unpolarized nuclear matter from the energy calculated from a Skyrme effective interaction that contains spin-orbit and tensor terms. More specifically, we always consider that the energy is derived from an effective interaction in contrast to the spirit of the Skyrme energy density functional (Skyrme EDF) method for which this link is not required. The linear response is obtained by using the particle-hole (p-h) residual interaction. This interaction can be obtained by standard techniques of p-h configuration for the parts of the EDF that are derived from a density-independent interaction. In a more general case of an EDF not linked with a density-independent interaction, the p-h interaction is obtained as the second functional derivative of the energy.

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A significant effort is nowadays devoted to the introduction of new terms in the Skyrme EDF, the tensor term being one [10,13,17] and other popular choices being new density-dependent couplings [22–26] or higher order derivative terms [27]. As the number of parameters becomes larger, it is of particular importance to define a clever fitting strategy. Acceptable ranges of variation for the parameters can be motivated by linking the Skyrme EDF to realistic interactions using many-body perturbation theory on top of renormalized low-momentum interactions [28,29]. It is also obviously mandatory to locate the regions in parameter space that lead to instabilities. This is particularly crucial for the spin channels, since the corresponding instabilities can only develop when the parameters are plugged into calculation codes that allow for the breaking of time-reversal symmetry.

The linear response function is the tool of choice, allowing us to avoid areas of instabilities. Its use for the fit of effective interactions free of instabilities will be presented in a forthcoming article; the present one is mainly devoted to the derivation of the general formulas.

This work is organized as follows: Section II summarizes the components of the Skyrme interaction that includes a zero-range tensor. Section III recalls the standard formalism of the linear response in nuclear matter and discusses its generalization to these new tensor terms. In Sec. IV, we present some numerical calculations of responses. Finally, we discuss further possible developments in the Sec. V.

II. SKYRME INTERACTION WITH TENSOR TERMS

The usual *ansatz* for the Skyrme effective interaction [18,19] leads to an energy-density functional that can be written as the sum of a kinetic term, the Skyrme potential energy functional that models the effective strong interaction in the particle-hole channel, a pairing energy functional, the Coulomb energy functional, and correction terms to approximately remove the contribution from the center-of-mass motion. The functional discussed in this article is applied to infinite nuclear matter without pairing, so we only consider the kinetic and Skyrme potential energy terms.

Throughout this work, we will use an effective Skyrme energy functional, as written in Eq. (A1) of the Appendix, that corresponds to an antisymmetrized density-dependent two-body vertex in the particle-hole channel of the strong interaction. It can be decomposed into central, spin-orbit, and tensor contributions:

$$v_{\rm Skyrme} = v_C + v_{\rm LS} + v_T. \quad (1)$$

We will use the standard density-dependent central Skyrme force

$$\begin{aligned} v_C(\mathbf{R},\mathbf{r}) &= t_0(1+x_0\hat{P}_\sigma)\,\delta(\mathbf{r}) + \tfrac{1}{6}\,t_3\,(1+x_3\hat{P}_\sigma)\rho^\gamma(\mathbf{R})\,\delta(\mathbf{r}) \\ &\quad + \tfrac{1}{2}t_1(1+x_1\hat{P}_\sigma)[\mathbf{k}'^2\,\delta(\mathbf{r}) + \delta(\mathbf{r})\,\mathbf{k}^2] \\ &\quad + t_2(1+x_2\hat{P}_\sigma)\mathbf{k}'\cdot\delta(\mathbf{r})\,\mathbf{k}, \end{aligned} \quad (2)$$

with the usual shorthand notations for $\mathbf{r}$, $\mathbf{R}$, $\mathbf{k}$, $\mathbf{k}'$, and $\hat{P}_\sigma$ [1].

We will also use the most standard form of the spin-orbit interaction,

$$v_{\rm LS}(\mathbf{r}) = i\,W_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot[\mathbf{k}'\times\delta(\mathbf{r})\,\mathbf{k}], \quad (3)$$

which is a special case of the one proposed by Bell and Skyrme [4,30]. Finally, the tensor part of the interaction is the one proposed by Skyrme [3,5]:

$$\begin{aligned} v_T(\mathbf{r}) &= \tfrac{1}{2}t_e\,\{[3\,(\boldsymbol{\sigma}_1\cdot\mathbf{k}')(\boldsymbol{\sigma}_2\cdot\mathbf{k}') - (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\mathbf{k}'^2]\,\delta(\mathbf{r}) \\ &\quad + \delta(\mathbf{r})\,[3(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k}) - (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\mathbf{k}^2]\} \\ &\quad + t_o[3(\boldsymbol{\sigma}_1\cdot\mathbf{k}')\delta(\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{k}) - (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\mathbf{k}'\cdot\delta(\mathbf{r})\mathbf{k}], \end{aligned} \quad (4)$$

for which the derivation of the contribution to the total energy functional is discussed in detail in Refs. [31,32] as well as in Ref. [17], where the impact of such a tensor interaction on the properties of spherical nuclei is investigated.

III. LINEAR RESPONSE FORMALISM

The linear response function in nuclear matter has already been widely developed mainly in the framework of random phase approximation (RPA) based on the use of an effective interaction [33]. We adopt the presentation of the work of Margueron *et al.* [34], which was devoted to the study of the contribution from the spin-orbit term to the linear response.

We consider here the case of infinite matter as a nuclear medium at zero temperature and unpolarized both in spin and isospin spaces. At the mean-field level this system is described as an ensemble of independent nucleons moving in a self-consistent mean field generated from an effective interaction treated in the Hartree-Fock (HF) approximation. For a given density, the momentum-dependent HF mean field, or self-energy, determines the single-particle spectrum $\varepsilon(k)$ and the Fermi level $\varepsilon(k_F)$.

To calculate the response of the medium to an external field, it is convenient to introduce the Green's function, or p-h (particle-hole) propagator $G^{(\alpha)}(\mathbf{q},\omega,\mathbf{k}_1)$. As illustrated in Fig. 1, $\mathbf{k}_1$ and $\mathbf{k}_2$ are the initial and final hole momenta, respectively, and $\mathbf{q}$ is the transferred momentum. We denote by $\alpha=(S,M;I,Q)$ the spin and isospin particle-hole channels with $S=0$ ($S=1$) for the non-spin-flip (spin-flip) channel and $I=0$ ($I=1$) for the isoscalar (isovector) channel, with M and Q being the quantum numbers related with the projection of the operators $\hat{S}$ and $\hat{I}$ on the quantification axis. The latter is chosen, as usual, as the z axis along the direction of $\mathbf{q}$.

In the HF approximation, the p-h Green's function does not depend on the spin-isospin channel (α) and reads [35]

$$G_{\rm HF}(q,\omega,\mathbf{k}_1) = \frac{\theta(k_F-k_1)-\theta(k_F-|\mathbf{k}_1+\mathbf{q}|)}{\omega+\varepsilon(k_1)-\varepsilon(|\mathbf{k}_1+\mathbf{q}|)+i\eta\omega}. \quad (5)$$

To go beyond the HF approximation one takes into account long-range correlations by resumming a class of p-h diagrams to obtain the well-known RPA [35]. The interaction appearing in the RPA is the p-h residual interaction whose matrix element including the exchange part can be written as

$$V_{\rm ph}^{(\alpha,\alpha')}(q,\mathbf{k}_1,\mathbf{k}_2) \equiv \langle \mathbf{q}+\mathbf{k}_1,\mathbf{k}_1^{-1},(\alpha)|V|\mathbf{q}+\mathbf{k}_2,\mathbf{k}_2^{-1},(\alpha')\rangle. \quad (6)$$

In the general case, the residual interaction is obtained by taking the second derivative of the total energy with respect to the densities built from the Hartree-Fock solutions. In the

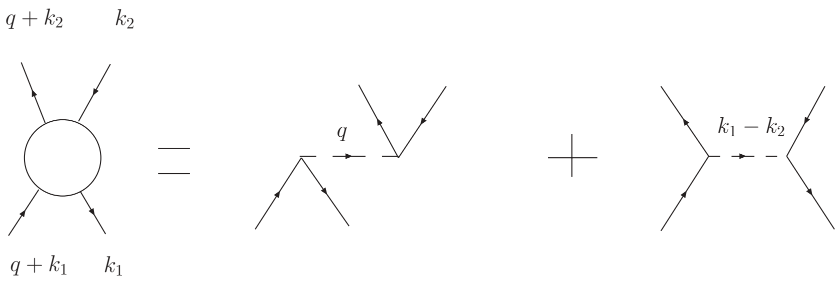

FIG. 1. Direct and exchange parts of the p-h interaction.

absence of density-dependent terms, it can also be obtained by standard techniques of particle-hole configuration [36].

The first important step is thus to determine the matrix elements for the different parts of the p-h interaction from Eqs. (2), (3), and (4).

A. The particle-hole interaction

1. Central part of the force

The central component of the p-h interaction can be written in the general form

$$V_{C,\mathrm{ph}}^{(\alpha,\alpha')}(q,\mathbf{k}_1,\mathbf{k}_2) = \delta_{\alpha\alpha'}\left\{W_1^{(\alpha)} + W_2^{(\alpha)}\left[k_1^2+k_2^2\right] - W_2^{(\alpha)}\frac{8\pi}{3}k_1k_2 \times \sum_{\mu=0,\pm1} Y_\mu^{(1)*}(\hat{k}_1)Y_\mu^{(1)}(\hat{k}_2)\right\}, \qquad (7)$$

where the $W_1^{(\alpha)}$ and $W_2^{(\alpha)}$ coefficients are functions of the Skyrme parameters (t_i, x_i) and of the transferred momentum **q** represented in Fig. 1. The convention used for the phase of the spherical harmonics is the one from Ref. [36]. The detailed expressions of the coefficients $W_1^{(\alpha)}$ and $W_2^{(\alpha)}$ have been given by Garcia-Recio *et al.* [33] and Navarro *et al.* [37] for symmetric nuclear matter and pure neutron matter, respectively; Hernandez *et al.* [38] gave them for an arbitrary neutron-proton asymmetry. The case of symmetric nuclear matter studied here is recalled in Appendix B. Because the central part of the Skyrme interaction is usually density dependent, it is not trivially related to the p-h interaction since the $W_i^{(\alpha)}$ coefficients contain rearrangement terms. One can note at this level, using only the central part of the interaction, that there is no coupling between the different spin and isospin channels.

2. Spin-orbit part of the force

To calculate the contribution of the spin-orbit term [see Eq. (3)] to the p-h interaction one has to evaluate the matrix element of the spin-orbit interaction. Since this term is density independent there is no rearrangement contribution and the result is just to add the following term to Eq. (7) (see Margueron *et al.* [34]):

$$V_{\mathrm{LS,ph}}^{(\alpha,\alpha')}(q,\mathbf{k}_1,\mathbf{k}_2) = -\delta_{II'}w(I)\sqrt{\frac{4\pi}{3}}qW_0 \times \left\{\delta_{S1}\delta_{S'0}M\left[k_1Y_{-M}^{(1)}(\hat{k}_1) - k_2Y_{-M}^{(1)}(\hat{k}_2)\right] + \delta_{S0}\delta_{S'1}M'\left[k_1Y_{M'}^{(1)}(\hat{k}_1) - k_2Y_{M'}^{(1)}(\hat{k}_2)\right]\right\}, \qquad (8)$$

where the factor $w(I)=3$ for $I=0$ and $w(I)=1$ for $I=1$ in the case of symmetric nuclear matter. It is clear from this expression that the main effect of the spin-orbit component is to couple the $S=0$ and $S=1$ channels.

3. Tensor part of the force

With the tensor force previously defined [see Eq. (4)], we have to calculate the antisymmetrized particle-hole matrix elements $\langle IQSM;\mathrm{ph}|v_T|I'Q'S'M';\mathrm{ph}\rangle$. Their analytical expressions are summarized in Table I, where we have adopted the following notation:

$$(k_{12})_M^{(1)} \equiv \sqrt{\frac{4\pi}{3}}\left[k_1Y_M^{(1)}(\hat{k}_1) - k_2Y_M^{(1)}(\hat{k}_2)\right]. \qquad (9)$$

Even if one can note from Table I that channels with different spin projection M are now coupled in a nontrivial way, these additional matrix elements are still diagonal in isospin space and act only in the vector channel. However, since we include both spin-orbit and tensor interactions in our approach, it is fundamental to note that the tensor component will impact both scalar and vector channels via the spin-orbit term.

TABLE I. Parameters of the interaction T44 [17] as defined in Eqs. (2), (3), and (4).

t_0	x_0	t_1	x_1	t_2	x_2
−2485.67	0.721557	494.477	−0.661848	−337.961	−0.803184
t_3	x_3	γ	W_0	t_e	t_o
13794.7	1.175908	1/6	161.367	173.661	7.17383

B. Response function

With the particle-hole matrix elements we are now in position to solve the RPA problem itself, that is, the Bethe-Salpeter equation satisfied by the RPA correlated Green's function $G^{(\alpha)}_{\rm RPA}(q,\omega,\mathbf{k}_1)$:

$$G^{(\alpha)}_{\rm RPA}(q,\omega,\mathbf{k}_1) = G_{\rm HF}(q,\omega,\mathbf{k}_1) + G_{\rm HF}(q,\omega,\mathbf{k}_1) \times \sum_{(\alpha')}\int \frac{d^3k_2}{(2\pi)^3} V_{\rm ph}^{(\alpha,\alpha')}(q,\mathbf{k}_1,\mathbf{k}_2) G^{(\alpha')}_{\rm RPA}(q,\omega,\mathbf{k}_2). \quad (10)$$

The response function $\chi^{(\alpha)}(q,\omega)$ in the infinite medium is related to the p-h Green's function by

$$\chi^{(\alpha)}_{\rm RPA}(q,\omega) = g\int \frac{d^3k_1}{(2\pi)^3} G^{(\alpha)}_{\rm RPA}(q,\omega,\mathbf{k}_1), \quad (11)$$

where the spin-isospin degeneracy factor g is 4 for symmetric nuclear matter. The Lindhard function $\chi_{\rm HF}$ is obtained when the free p-h propagator $G_{\rm HF}$ is used in Eq. (11).

Following the notation of Refs. [33,34], we define for any function $f(q,\omega,k_1)$

$$\langle f\rangle \equiv \int \frac{d^3k_1}{(2\pi)^3} f(k_1). \quad (12)$$

The response function can thus be written in each channel (α) as

$$\chi^{(\alpha)}_{\rm RPA} = g\langle G^{(\alpha)}_{\rm RPA}\rangle. \quad (13)$$

Finally, the quantity of interest is the dynamical structure function $S^{(\alpha)}(q,\omega)$, which is, at zero temperature, proportional to the imaginary part of the response function at positive energies:

$$S^{(\alpha)}(q,\omega) = -\frac{1}{\pi}\,{\rm Im}\,\chi^{(\alpha)}_{\rm RPA}(q,\omega). \quad (14)$$

C. Response function for the spin-orbit case

As an introduction to our full calculation we recall here some results already obtained by Margueron *et al.* [34]. When the spin-orbit force alone is included, the response function can then be written in the form (using $\hbar = c = 1$)

$$\frac{\chi_{\rm HF}}{\chi^{(\alpha)}_{\rm RPA}} = 1 - \widetilde{W}_1^{(\alpha)}\chi_0 + W_2^{(\alpha)}\left[\frac{q^2}{2}\chi_0 - 2k_F^2\chi_2\right] + \left[W_2^{(\alpha)}k_F^2\right]^2\left[\chi_2^2 - \chi_0\chi_4 + \left(\frac{m^*\omega}{k_F^2}\right)^2\chi_0^2 - \frac{m^*}{6\pi^2k_F}q^2\chi_0\right] + 2\left(\frac{m^*\omega}{q}\right)^2\frac{W_2^{(\alpha)}}{1-\frac{m^*k_F^3}{3\pi^2}W_2^{(\alpha)}}\chi_0. \quad (15)$$

In this expression k_F is the Fermi momentum and m^* denotes the effective mass of the nucleons. The functions χ_0, χ_2, and χ_4 are generalized free response functions, defined in Ref. [33] and written in Appendix D [see Eq. (D1)], and $\chi_{\rm HF} = g\,\chi_0$.

The explicit expressions of the $\widetilde{W}_1^{(\alpha)}$ coefficients are given in Appendix C, where the coupling between the $S=0$ and $S=1$ channels induced by the spin-orbit interaction can also be clearly seen. It can be noted that the coefficients $\widetilde{W}_1^{(\alpha)}$ are now complex functions of q and ω since different moments χ_{2i} enter their expressions. If we replace $\widetilde{W}_1^{(\alpha)}$ in Eq. (16) by $W_1^{(\alpha)}$ we obtain the results of Ref. [33], related to the central part of the interaction, as it should be.

D. Response function with the tensor part

As already mentioned, the tensor interaction couples vector channels with different spin projection whereas the spin-orbit interaction couples the scalar ($S=0$) and vector ($S=1$) ones. The consequence is that we obtain a nontrivial system of coupled equations for the RPA problem. As an illustration let us consider explicitly the case of isospin $I=0$ and $(S,M)=(1,1)$. In that particular channel, the Bethe-Salpeter equation for $G^{(1,1)}_{\rm RPA}(k_1)$ (where we omit the isospin index and dependence on q and ω for sake of simplicity) exhibits terms that typically read

$$G_{\rm HF}(k_1)\langle G^{(1,1)}_{\rm RPA}\rangle,\qquad G_{\rm HF}(k_1)\langle k^2G^{(1,1)}_{\rm RPA}\rangle,$$
$$k_1Y_\mu^{(1)*}G_{\rm HF}(k_1)\langle kY_\mu^{(1)}G^{(1,1)}_{\rm RPA}\rangle,\qquad k_1Y_{-1}^{(1)}G_{\rm HF}(k_1)\langle G^{(1,0)}_{\rm RPA}\rangle,$$
$$G_{\rm HF}(k_1)\langle k^2Y_{-1}^{(1)}Y_{-1}^{(1)}G^{(1,-1)}_{\rm RPA}\rangle,$$

where for example, according to Eq. (12), $\langle k^2G^{(1,1)}_{\rm RPA}\rangle$ stands for

$$\int\frac{d^3k}{(2\pi)^3}k^2G^{(1,1)}_{\rm RPA}(q,\,\omega,\,k).$$

Thus, the determination of $\langle G^{(1,1)}_{\rm RPA}\rangle$ requires knowledge of some other unknown quantities. This leads to a large system of coupled equations for

$$\langle G^{(1,M)}_{\rm RPA}\rangle,\qquad \langle k^2G^{(1,M)}_{\rm RPA}\rangle,\qquad \langle kY_0^{(1)}G^{(1,M)}_{\rm RPA}\rangle,$$
$$\langle k^2|Y_0^{(1)}|^2G^{(1,M)}_{\rm RPA}\rangle,\qquad \langle k^2|Y_1^{(1)}|^2G^{(1,M)}_{\rm RPA}\rangle,\qquad \text{for}\quad M=-1,\,0,\,1,$$

and

$$\langle k^2Y_1^{(1)}Y_1^{(1)}G^{(1,1)}_{\rm RPA}\rangle,\quad \langle k^2Y_{-1}^{(1)}Y_{-1}^{(1)}G^{(1,-1)}_{\rm RPA}\rangle,\quad \langle kY_1^{(1)}G^{(1,1)}_{\rm RPA}\rangle,$$
$$\langle kY_{-1}^{(1)}G^{(1,-1)}_{\rm RPA}\rangle,\quad \langle kY_1^{(1)}G^{(1,0)}_{\rm RPA}\rangle,\quad \langle kY_{-1}^{(1)}G^{(1,0)}_{\rm RPA}\rangle,$$

which are 21 unknown quantities for which the notation introduced in Eq. (12) was used. Fortunately, since the multipole expansion of $G_{\rm HF}$ only implies terms with $Y_0^{(L)}$, the integration over k_1 cancels all terms of the form $\langle f(k)\prod_{\{M,M',\ldots\}}Y_M^{(1)}Y_{M'}^{(1)}\cdots G_{\rm HF}\rangle$ with $M+M'+\cdots\neq 0$. Moreover, some unknown quantities can be expressed through the others and we can reduce the 21 coupled equations to three systems of 4 coupled equations for the following variables:

$$\langle G^{(1,M)}_{\rm RPA}\rangle,\quad \langle k^2G^{(1,M)}_{\rm RPA}\rangle,\quad \langle kY_0^{(1)}G^{(1,M)}_{\rm RPA}\rangle,$$
$$\langle k^2|Y_0^{(1)}|^2G^{(1,M)}_{\rm RPA}\rangle - \langle k^2|Y_1^{(1)}|^2G^{(1,M)}_{\rm RPA}\rangle,$$

for each value of M. The calculations are straightforward but tedious. The same procedure has to be repeated for isospin

$I = 1$ for $S = 1$. Indeed, only the channels with $S = 0$ are less involved.

The results are now quoted for each spin-isospin channel in a form that exhibits the symmetry properties appearing in Table I:

(i) For $S = 0$, $I = 0$:

$$\frac{\chi_{\rm HF}}{\chi_{\rm RPA}^{(0,0)}} = 1 - \widetilde{\widetilde{W}}_1^{(0,0)}\chi_0 + W_2^{(0,0)}\left(\frac{q^2}{2}\chi_0 - 2k_F^2\chi_2\right) + \left[W_2^{(0,0)}k_F^2\right]^2\left[\chi_2^2 - \chi_0\chi_4 + \left(\frac{m^*\omega}{k_F^2}\right)^2\chi_0^2 - \frac{m^*}{6\pi^2 k_F}q^2\chi_0\right] + 2\chi_0\left(\frac{m^*\omega}{q}\right)^2\frac{W_2^{(0,0)}}{1-\frac{m^*k_F^3}{3\pi^2}W_2^{(0,0)}}. \quad (16)$$

(ii) For $S = 0$, $I = 1$:

$$\frac{\chi_{\rm HF}}{\chi_{\rm RPA}^{(0,1)}} = 1 - \widetilde{\widetilde{W}}_1^{(0,1)}\chi_0 + W_2^{(0,1)}\left(\frac{q^2}{2}\chi_0 - 2k_F^2\chi_2\right) + \left[W_2^{(0,1)}k_F^2\right]^2\left[\chi_2^2 - \chi_0\chi_4 + \left(\frac{m^*\omega}{k_F^2}\right)^2\chi_0^2 - \frac{m^*}{6\pi^2 k_F}q^2\chi_0\right] + 2\chi_0\left(\frac{m^*\omega}{q}\right)^2\frac{W_2^{(0,1)}}{1-\frac{m^*k_F^3}{3\pi^2}W_2^{(0,1)}}. \quad (17)$$

(iii) For $S = 1$, $I = 0$, $M = \pm 1$:

$$\begin{aligned}\frac{\chi_{\rm HF}}{\chi_{\rm RPA}^{(1,0,\pm1)}} &= \left[1 + (t_e - 5t_o)\frac{m^*k_F^3}{2\pi^2}\right]^2 - \widetilde{\widetilde{W}}_1^{(1,0,\pm1)}\chi_0 \\ &+ \left[W_2^{(1,0)} - (t_e - 5t_o)\right] \\ &\times\left\{\frac{q^2}{2}\chi_0\left[1 + (t_e - 5t_o)\frac{m^*k_F^3}{\pi^2}\right] - 2k_F^2\chi_2 + (t_e - 5t_o)\frac{m^*k_F^5}{\pi^2}(\chi_0 - \chi_2)\right\} \\ &+ \left[W_2^{(1,0)} - (t_e - 5t_o)\right]^2 k_F^4\left[\chi_2^2 - \chi_0\chi_4 + \left(\frac{m^*\omega}{k_F^2}\right)^2\chi_0^2 - \frac{m^*}{6\pi^2k_F}q^2\chi_0\right] + 2\left(\frac{m^*\omega}{q}\right)^2 \\ &\times\frac{W_2^{(1,0)} + 2(t_e - 5t_o) + X^{(1,0,\pm1)}}{1 - \frac{m^*k_F^3}{3\pi^2}\left[W_2^{(1,0)} + 2(t_e - 5t_o) - X^{(1,0,\pm1)}\right]}\chi_0 \\ &+ \left\{2m^*\omega X^{(1,0,\pm1)} + \frac{2m^*k_F^3}{3\pi^2}[X^{(1,0,\pm1)}]^2 \times\left[\frac{q}{2} + \frac{m^*\omega}{q}\right]^2\right\} \\ &\times\frac{\chi_0}{1 - \frac{m^*k_F^3}{3\pi^2}\left[W_2^{(1,0)} + 2(t_e - 5t_o) - X^{(1,0,\pm1)}\right]}. \end{aligned} \quad (18)$$

(iv) For $S = 1$, $I = 0$, $M = 0$:

$$\begin{aligned}\frac{\chi_{\rm HF}}{\chi_{\rm RPA}^{(1,0,0)}} &= \left[1 - \frac{1}{2}(t_e - 5t_o)\frac{m^*k_F^3}{2\pi^2}\right]^2 - \widetilde{\widetilde{W}}_1^{(1,0,0)}\chi_0 \\ &+ \left[W_2^{(1,0)} + \frac{1}{2}(t_e - 5t_o)\right] \\ &\times\left\{\frac{q^2}{2}\chi_0\left[1 - (t_e - 5t_o)\frac{m^*k_F^3}{2\pi^2}\right] - 2k_F^2\chi_2 - (t_e - 5t_o)\frac{m^*k_F^5}{2\pi^2}(\chi_0 - \chi_2)\right\} \\ &+ \left[W_2^{(1,0)} + \frac{1}{2}(t_e - 5t_o)\right]^2 k_F^4\left[\chi_2^2 - \chi_0\chi_4 + \left(\frac{m^*\omega}{k_F^2}\right)^2\chi_0^2 - \frac{m^*}{6\pi^2k_F}q^2\chi_0\right] + 2\left(\frac{m^*\omega}{q}\right)^2 \\ &\times\frac{W_2^{(1,0)} - (t_e - 5t_o) + X^{(1,0,0)}}{1 - \frac{m^*k_F^3}{3\pi^2}\left[W_2^{(1,0)} - (t_e - 5t_o) - X^{(1,0,0)}\right]}\chi_0 \\ &+ \left\{-2m^*\omega X^{(1,0,0)} + \frac{2m^*k_F^3}{3\pi^2}[X^{(1,0,0)}]^2 \times\left[\frac{q}{2} - \frac{m^*\omega}{q}\right]^2\right\} \\ &\times\frac{\chi_0}{1 - \frac{m^*k_F^3}{3\pi^2}\left[W_2^{(1,0)} - (t_e - 5t_o) - X^{(1,0,0)}\right]}. \end{aligned} \quad (19)$$

(v) For $S = 1$, $I = 1$, $M = \pm 1$:

$$\begin{aligned}\frac{\chi_{\rm HF}}{\chi_{\rm RPA}^{(1,1,\pm1)}} &= \left[1 + 3(t_e - t_o)\frac{m^*k_F^3}{2\pi^2}\right]^2 - \widetilde{\widetilde{W}}_1^{(1,1,\pm1)}\chi_0 \\ &+ \left[W_2^{(1,1)} - 3(t_e - t_o)\right] \\ &\times\left\{\frac{q^2}{2}\chi_0\left[1 + 3(t_e - t_o)\frac{m^*k_F^3}{\pi^2}\right] - 2k_F^2\chi_2 + 3(t_e - t_o)\frac{m^*k_F^5}{\pi^2}(\chi_0 - \chi_2)\right\} \\ &+ \left[W_2^{(1,1)} - 3(t_e - t_o)\right]^2 k_F^4\left[\chi_2^2 - \chi_0\chi_4 + \left(\frac{m^*\omega}{k_F^2}\right)^2\chi_0^2 - \frac{m^*}{6\pi^2k_F}q^2\chi_0\right] + 2\left(\frac{m^*\omega}{q}\right)^2 \\ &\times\frac{W_2^{(1,1)} + 6(t_e - t_0) + X^{(1,1,\pm1)}}{1 - \frac{m^*k_F^3}{3\pi^2}\left[W_2^{(1,1)} + 6(t_e - t_o) - X^{(1,1,\pm1)}\right]}\chi_0 \\ &+ 2\left(\frac{m^*\omega}{q}\right)^2\frac{m^*k_F^3}{3\pi^2}[X^{(1,1,\pm1)}]^2 \\ &\times\frac{\chi_0}{1 - \frac{m^*k_F^3}{3\pi^2}\left[W_2^{(1,1)} + 6(t_e - t_o) - X^{(1,1,\pm1)}\right]}. \end{aligned} \quad (20)$$

(vi) For $S=1, I=1, M=0$:

$$\begin{aligned}
\frac{\chi_{\mathrm{HF}}}{\chi_{\mathrm{RPA}}^{(1,1,0)}} =& \left[1-\frac{3}{2}(t_e-t_o)\frac{m^*k_F^3}{2\pi^2}\right]^2 - \overset{\approx}{W}{}_1^{(1,1,0)}\chi_0 \\
&+\left[W_2^{(1,1)}+\frac{3}{2}(t_e-t_o)\right] \\
&\times\left\{\frac{q^2}{2}\chi_0\left[1-3(t_e-t_o)\frac{m^*k_F^3}{2\pi^2}\right]\right. \\
&\left. -2k_F^2\chi_2-\frac{3}{2}(t_e-t_o)\frac{m^*k_F^5}{\pi^2}(\chi_0-\chi_2)\right\} \\
&+\left[W_2^{(1,1)}+\frac{3}{2}(t_e-t_o)\right]^2 k_F^4\left[\chi_2^2-\chi_0\chi_4\right. \\
&\left.+\left(\frac{m^*\omega}{k_F^2}\right)^2\chi_0^2-\frac{m^*}{6\pi^2k_F}q^2\chi_0\right]+2\left(\frac{m^*\omega}{q}\right)^2 \\
&\times\frac{W_2^{(1,1)}-3(t_e-t_o)+X^{(1,1,0)}}{1-\frac{m^*k_F^3}{3\pi^2}\left[W_2^{(1,1)}-3(t_e-t_o)-X^{(1,1,0)}\right]}\chi_0 \\
&+2\left(\frac{m^*\omega}{q}\right)^2\frac{m^*k_F^3}{3\pi^2}[X^{(1,1,0)}]^2 \\
&\times\frac{\chi_0}{1-\frac{m^*k_F^3}{3\pi^2}\left[W_2^{(1,1)}-3(t_e-t_o)-X^{(1,1,0)}\right]}.
\end{aligned} \tag{21}$$

The coefficients $\overset{\approx}{W}{}_1^{(S,I)}$ for $S=0$ or $\overset{\approx}{W}{}_1^{(S,I,M)}$ for $S=1$ are defined as

$$\begin{aligned}
\overset{\approx}{W}{}_1^{(0,0)} =& W_1^{(0,0)}+9W_0^2q^4 \\
&\times\frac{\beta_2-\beta_3}{1+q^2(\beta_2-\beta_3)\left(W_2^{(1,0)}-(7t_e+13t_o)\right)},
\end{aligned} \tag{22}$$

$$\begin{aligned}
\overset{\approx}{W}{}_1^{(0,1)} =& W_1^{(0,1)}+W_0^2q^4 \\
&\times\frac{\beta_2-\beta_3}{1+q^2(\beta_2-\beta_3)\left[W_2^{(1,1)}+3(t_e-t_o)\right]},
\end{aligned} \tag{23}$$

$$\begin{aligned}
\overset{\approx}{W}{}_1^{(1,0,\pm1)} =& \widetilde{W}_1^{(1,0,\pm1)}+K_1+K_2-t_o^2\frac{24m^*k_F^5}{5\pi^2} \\
&+9(t_e+3t_o)^2q^4(\beta_5-2\beta_8+\beta_7) \\
&+2X^{(1,0,\pm1)}\left[\frac{q}{2}+\frac{m^*\omega}{q}\right]^2,
\end{aligned} \tag{24}$$

$$\begin{aligned}
\overset{\approx}{W}{}_1^{(1,0,0)} =& \widetilde{W}_1^{(1,0,0)}-\frac{1}{2}K_1+\frac{1}{4}K_2-t_o^2\frac{48m^*k_F^5}{5\pi^2} \\
&+2X^{(1,0,0)}\left[\frac{q}{2}-\frac{m^*\omega}{q}\right]^2,
\end{aligned} \tag{25}$$

$$\begin{aligned}
\overset{\approx}{W}{}_1^{(1,1,\pm1)} =& \widetilde{W}_1^{(1,1,\pm1)}+3K_1'+9K_2'-(t_e-t_o)^2\frac{6m^*k_F^5}{5\pi^2} \\
&+9(t_e-t_o)^2q^4(\beta_5-2\beta_8+\beta_7) \\
&+2X^{(1,1,\pm1)}\left(\frac{m^*\omega}{q}\right)^2,
\end{aligned} \tag{26}$$

$$\begin{aligned}
\overset{\approx}{W}{}_1^{(1,1,0)} =& \widetilde{W}_1^{(1,1,0)}-\frac{3}{2}K_1'+\frac{9}{4}K_2' \\
&-(t_e-t_o)^2\frac{12m^*k_F^5}{5\pi^2}+2X^{(1,1,0)}\left(\frac{m^*\omega}{q}\right)^2.
\end{aligned} \tag{27}$$

The functions β_i are given in Appendix D and the coefficients K_i, K_i', and $X^{(S,I,M)}$ are

$$K_1 = 2(t_e+5t_o)q^2+6\left(\frac{m^*\omega}{q}\right)^2(t_e-5t_o), \tag{28}$$

$$K_2 = -(t_e-5t_o)^2\frac{m^*k_F^3}{\pi^2}\left[\frac{9}{10}k_F^2+\frac{3}{8}q^2-\frac{3}{2}\left(\frac{m^*\omega}{q}\right)^2\right], \tag{29}$$

$$K_1' = 2(t_e+t_o)q^2+6\left(\frac{m^*\omega}{q}\right)^2(t_e-t_o), \tag{30}$$

$$K_2' = -(t_e-t_o)^2\frac{m^*k_F^3}{\pi^2}\left[\frac{9}{10}k_F^2+\frac{3}{8}q^2-\frac{3}{2}\left(\frac{m^*\omega}{q}\right)^2\right] \tag{31}$$

and

$$\begin{aligned}
X^{(1,0,\pm1)} =& 36t_o^2q^2 \\
&\times\frac{\beta_2-\beta_3}{1+q^2(\beta_2-\beta_3)\left(W_2^{(1,0)}+\frac{1}{2}(t_e-5t_o)\right)},
\end{aligned} \tag{32}$$

$$\begin{aligned}
X^{(1,0,0)} =& 72t_o^2q^2 \\
&\times\frac{\beta_2-\beta_3}{1+q^2(\beta_2-\beta_3)\left(W_2^{(1,0)}+5t_e+23t_o\right)},
\end{aligned} \tag{33}$$

$$\begin{aligned}
X^{(1,1,\pm1)} =& 9(t_e-t_o)^2q^2 \\
&\times\frac{\beta_2-\beta_3}{1+q^2(\beta_2-\beta_3)\left(W_2^{(1,1)}+\frac{3}{2}(t_e-t_o)\right)},
\end{aligned} \tag{34}$$

$$\begin{aligned}
X^{(1,1,0)} =& 18(t_e-t_o)^2q^2 \\
&\times\frac{\beta_2-\beta_3}{1+q^2(\beta_2-\beta_3)\left(W_2^{(1,1)}-9(t_e-t_o)\right)}.
\end{aligned} \tag{35}$$

It is important to note that the above response functions have exactly the same structure as in RPA with just central channels and with or without the spin-orbit interaction. Moreover, we can see that the response function depends on different linear combinations of the parameters t_e and t_o that will lead to nontrivial effects.

IV. DYNAMICAL STRUCTURE FUNCTION WITH TENSOR CONTRIBUTION

As an example for the effect of a zero-range tensor force in the p-h interaction, we have calculated the nuclear responses in (S, I, M) channels of symmetric infinite nuclear matter for parametrization T44 of the Skyrme interaction built by Lesinski *et al.* [17]. The parameters of this force are given in Table II. The dynamical structure functions $S^{(\alpha)}(q,\omega)$ [defined in Eq. (14)] calculated for $q=k_F$ and at the saturation density $\rho=\rho_{\mathrm{sat}}=0.16\ \mathrm{fm}^{-3}$ are shown in Fig. 2. To clearly isolate the effect of the tensor part of the force, the functions $S^{(\alpha)}(q,\omega)$ are plotted for two cases: (a) with no tensor contribution but including the spin-orbit part and (b) with the full force. The first case allows us to compare with the previous results of

TABLE II. Particle-hole matrix elements for the tensor part of the effective force. Only the vector channels are of concern in this case and a factor $2\delta_{S1}\delta_{S'1}\delta_{II'}$ is implicit for each term.

		$M'=1$	$M'=0$	$M'=-1$
$M=1$	$I=0$	$(t_e+5t_o)q^2$ $+(t_e-5t_o)[(k_{12})_0^{(1)}(k_{12})_0^{(1)}+(k_{12})_{-1}^{(1)}(k_{12})_1^{(1)}]$	$3qt_o(k_{12})_{-1}^{(1)}$ $-6t_o(k_{12})_{-1}^{(1)}(k_{12})_0^{(1)}$	$-3(t_e+3t_o)(k_{12})_{-1}^{(1)}(k_{12})_{-1}^{(1)}$
	$I=1$	$3(t_e+t_o)q^2$ $+3(t_e-t_o)[(k_{12})_0^{(1)}(k_{12})_0^{(1)}+(k_{12})_{-1}^{(1)}(k_{12})_1^{(1)}]$	$3(t_e-t_o)(k_{12})_{-1}^{(1)}(k_{12})_0^{(1)}$	$3(t_e-t_o)(k_{12})_{-1}^{(1)}(k_{12})_{-1}^{(1)}$
$M=0$	$I=0$	$3qt_o(k_{12})_1^{(1)}$ $+6t_o(k_{12})_1^{(1)}(k_{12})_0^{(1)}$	$-\frac{1}{2}(t_e+5t_o)q^2$ $-\frac{1}{2}(t_e-5t_o)[(k_{12})_0^{(1)}(k_{12})_0^{(1)}+(k_{12})_{-1}^{(1)}(k_{12})_1^{(1)}]$	$3t_oq(k_{12})_{-1}^{(1)}$ $+6t_o(k_{12})_{-1}^{(1)}(k_{12})_0^{(1)}$
	$I=1$	$-3(t_e-t_o)(k_{12})_1^{(1)}(k_{12})_0^{(1)}$	$-\frac{3}{2}(t_e+t_o)q^2$ $-\frac{3}{2}(t_e-t_o)[(k_{12})_0^{(1)}(k_{12})_0^{(1)}+(k_{12})_{-1}^{(1)}(k_{12})_1^{(1)}]$	$-3(t_e-t_o)(k_{12})_{-1}^{(1)}(k_{12})_0^{(1)}$
$M=-1$	$I=0$	$-3(t_e+3t_o)(k_{12})_1^{(1)}(k_{12})_1^{(1)}$	$3qt_o(k_{12})_1^{(1)}$ $-6t_o(k_{12})_1^{(1)}(k_{12})_0^{(1)}$	$(t_e+5t_o)q^2$ $+(t_e-5t_o)[(k_{12})_0^{(1)}(k_{12})_0^{(1)}+(k_{12})_{-1}^{(1)}(k_{12})_1^{(1)}]$
	$I=1$	$3(t_e-t_o)(k_{12})_1^{(1)}(k_{12})_1^{(1)}$	$3(t_e-t_o)(k_{12})_1^{(1)}(k_{12})_0^{(1)}$	$3(t_e+t_o)q^2$ $+3(t_e-t_o)[(k_{12})_0^{(1)}(k_{12})_0^{(1)}+(k_{12})_{-1}^{(1)}(k_{12})_1^{(1)}]$

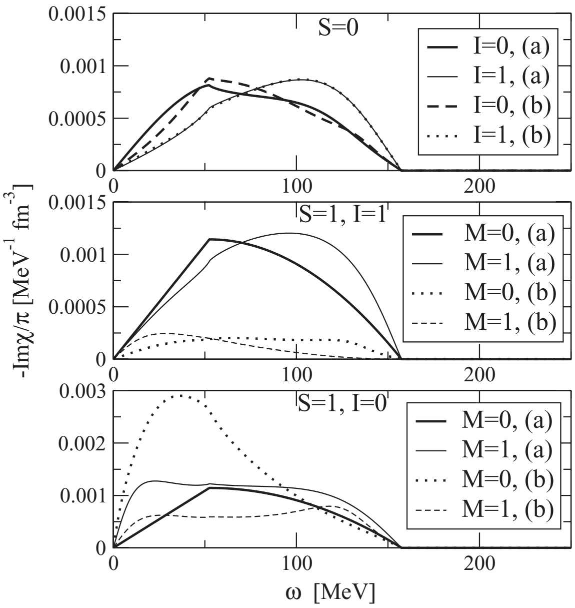

FIG. 2. Dynamical structure functions $S^{(\alpha)}(q,\omega)$ calculated for the Skyrme tensor parametrization T44 [17]. The upper, middle, and lower parts, respectively, concern the channels $S=0$, $S=1$ and $I=0$, and $S=1$ and $I=0$. For each (S,I,M) channel the responses (in $\text{MeV}^{-1}\,\text{fm}^{-3}$) are plotted as a function of ω (in MeV). All the responses are calculated for $q=k_F$ at saturation density. Cases (a) and (b) are discussed in the text.

Margueron *et al.* [34] but are presented here for the four channels of symmetric nuclear matter instead of neutron matter.

In the $S=0$ channel, the tensor terms do not affect qualitatively the response. Strictly, this comment only applies to the case of the T44 tensor parametrization but several tests performed using other Tij tensor interactions discussed in Ref. [17] exhibit the same qualitative behavior. The situation is quite different in the $S=1$ channels: The effect from the tensor terms is large whatever the value of the spin projection M is. Actually, depending on the values of the transferred momentum q and the density ρ, the response functions increase significantly and diverge at finite q for a certain critical density ρ_c. This divergence reveals the presence of instabilities observed in nuclei [21], with the appearance of domains with typical size of the order of $2\pi/q$. Even if a one-to-one correspondence between infinite matter and finite nuclei is obviously incorrect, the center of a nucleus still explores, because of fluctuations, not only the saturation density but also some larger values for which one may observe a divergence of the response functions, and then, possibly, the appearance of finite-size instabilities in the nucleus.

V. SUMMARY AND CONCLUSIONS

We have derived the contribution from the tensor terms in the Skyrme effective interaction to the RPA response function. We have shown that the formal structure of the response function is the same as without tensor terms, although with the latter, all channels are coupled in a nontrivial way. The simple example presented here, using the interaction T44, shows that the effects of the tensor contributions are strong in vector channels.

We have shown that the dynamical structure functions $S^{(\alpha)}(q,\omega)$ become large for finite values of q. This indicates the vicinity of a pole related with a finite-size instability for given values of q and ρ in infinite matter and, possibly, in finite nuclei. A systematic study of the critical densities is in progress to determine whether the link between the divergences of χ_{RPA} and the instabilities encountered in nuclei at the Hartree-Fock approximation is robust.

Another important point under study is the identification, directly from the Skyrme energy functional, of the origin of each tensorial contribution in the response functions. In the same spirit, a detailed study of sum rules can shed some light on the contribution of the tensor for various physical situations (see Ref. [39]). Finally, applications to pure neutron matter can be very important (see, e.g., Refs. [40–52]). However, the formulas presented here are no longer directly usable and have to be adapted to that specific case. Work in that direction is also in progress.

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APPENDIX A: COUPLING CONSTANTS OF THE SKYRME ENERGY FUNCTIONAL

The Skyrme energy density functional used in this article is a functional of local densities and currents (with $q=n$, p for neutrons and protons),that is, the particle densities $\rho_q(\mathbf{r})$, the kinetic densities $\tau_q(\mathbf{r})$, the current (vector) densities $\mathbf{j}_q(\mathbf{r})$, the spin (pseudovector) densities $\mathbf{s}_q(\mathbf{r})$, the spin kinetic (pseudovector) densities $\mathbf{T}_q(\mathbf{r})$, the spin-current (pseudotensor) densities $J_{q,\mu\nu}(\mathbf{r})$, and the tensor-kinetic (pseudovector) densities $\mathbf{F}_q(\mathbf{r})$ that have been defined in Lesinski *et al.* [17], where the definition of the vector spin current density $\mathbf{j}^{(1)}(\mathbf{r})\equiv\mathbf{j}(\mathbf{r})$ was also recalled. We will use also the isoscalar and isovector densities defined from the proton and neutron densities, respectively, as $\rho_0(\mathbf{r})=\rho_n(\mathbf{r})+\rho_p(\mathbf{r})$ and $\rho_1(\mathbf{r})=\rho_n(\mathbf{r})-\rho_p(\mathbf{r})$, and similarly for all other densities.

The Skyrme energy functional representing central, tensor, and spin-orbit contributions is given by

$$\begin{aligned}\mathcal{E}_{\text{Skyrme}} &= \mathcal{E}_C+\mathcal{E}_{\text{LS}}+\mathcal{E}_T\\ &= \int d^3\mathbf{r}\sum_{t=0,1}\Big\{C_t^{\rho}[\rho_0]\rho_t^2+C_t^{\Delta\rho}\rho_t\Delta\rho_t\\ &\quad+C_t^{\tau}\left(\rho_t\tau_t-\mathbf{j}_t^2\right)+C_t^{s}[\rho_0]\mathbf{s}_t^2+C_t^{\nabla s}(\nabla\cdot\mathbf{s}_t)^2\\ &\quad+C_t^{\Delta s}\mathbf{s}_t\cdot\Delta\mathbf{s}_t+C_t^{T}\Big(\mathbf{s}_t\cdot\mathbf{T}_t-\sum_{\mu,\nu=x}^{z}J_{t,\mu\nu}J_{t,\mu\nu}\Big)\end{aligned}$$

$$+ C_t^F\left[\mathbf{s}_t\cdot\mathbf{F}_t - \frac{1}{2}\left(\sum_{\mu=x}^{z} J_{t,\mu\mu}\right)^2 - \frac{1}{2}\sum_{\mu,\nu=x}^{z} J_{t,\mu\nu}J_{t,\nu\mu}\right] + C_t^{\nabla J}[\rho_t\nabla\cdot\mathbf{j}_t + \mathbf{s}_t\cdot(\nabla\times\mathbf{j}_t)]\Bigg\}. \quad \text{(A1)}$$

All the coupling constants can be expressed as function of the parameters of the Skyrme interaction given in Eqs. (2), (3), and (4). Some of the coupling constants are fully defined by the standard central part of the Skyrme force: $C_t^\rho = A_t^\rho$, $C_t^{\Delta\rho} = A_t^{\Delta\rho}$, $C_t^\tau = A_t^\tau$, and $C_t^s = A_t^s$, or by its spin-orbit part: $C_t^{\nabla J} = A_t^{\nabla J}$. Two coupling constants only depend on the tensor part of the interaction: $C_t^{\nabla s} = B_t^{\nabla s}$ and $C_t^F = B_t^F$. Finally, two coupling constants are the sum of contributions from both central and tensor forces: $C_t^T = A_t^T + B_t^T$ and $C_t^{\Delta s} = A_t^{\Delta s} + B_t^{\Delta s}$.

The coupling constants of the Skyrme EDF that come from the central and spin-orbit parts of the interaction are given in terms of the parameters by

$$\begin{aligned}
A_0^\rho &= \tfrac{3}{8}t_0 + \tfrac{3}{48}t_3\,\rho_0^\gamma(\mathbf{r}),\\
A_1^\rho &= -\tfrac{1}{4}t_0\left(\tfrac{1}{2}+x_0\right) - \tfrac{1}{24}t_3\left(\tfrac{1}{2}+x_3\right)\rho_0^\gamma(\mathbf{r}),\\
A_0^{\Delta\rho} &= -\tfrac{9}{64}t_1 + \tfrac{1}{16}t_2\left(\tfrac{5}{4}+x_2\right),\\
A_1^{\Delta\rho} &= \tfrac{3}{32}t_1\left(\tfrac{1}{2}+x_1\right) + \tfrac{1}{32}t_2\left(\tfrac{1}{2}+x_2\right),\\
A_0^\tau &= \tfrac{3}{16}t_1 + \tfrac{1}{4}t_2\,\left(\tfrac{5}{4}+x_2\right),\\
A_1^\tau &= -\tfrac{1}{8}t_1\left(\tfrac{1}{2}+x_1\right) + \tfrac{1}{8}t_2\left(\tfrac{1}{2}+x_2\right),\\
A_0^s &= -\tfrac{1}{4}t_0\left(\tfrac{1}{2}-x_0\right) - \tfrac{1}{24}t_3\left(\tfrac{1}{2}-x_3\right)\rho_0^\gamma(\mathbf{r}),\\
A_1^s &= -\tfrac{1}{8}t_0 - \tfrac{1}{48}t_3\rho_0^\gamma(\mathbf{r}),\\
A_0^{\Delta s} &= \tfrac{3}{32}t_1\left(\tfrac{1}{2}-x_1\right) + \tfrac{1}{32}t_2\left(\tfrac{1}{2}+x_2\right),\\
A_1^{\Delta s} &= \tfrac{3}{64}t_1 + \tfrac{1}{64}t_2, \quad A_0^T = -\tfrac{1}{8}t_1\left(\tfrac{1}{2}-x_1\right) + \tfrac{1}{8}t_2\left(\tfrac{1}{2}+x_2\right),\\
A_1^T &= -\tfrac{1}{16}t_1 + \tfrac{1}{16}t_2, \quad A_0^{\nabla J} = -\tfrac{3}{4}W_0, \quad A_1^{\nabla J} = -\tfrac{1}{4}W_0.
\end{aligned}$$

Because of the density dependence of some coupling constants it is also useful to define the following coefficients that occur in the definition of the $W^{(\alpha)}$ coefficients [1]:

$$A_t^\rho = A_t^{\rho,0} + A_t^{\rho,\gamma}\rho_0^\gamma, \quad A_t^s = A_t^{s,0} + A_t^{s,\gamma}\rho_0^\gamma.$$

In each case where we have considered a single density-dependent term, the generalization to more than one density-dependent term is straightforward by just adding new density-dependent terms in the corresponding coupling constants.

Finally, the coupling constants of the Skyrme EDF that come from the tensor part of the interaction are given by (Table I in Ref. [32])

$$\begin{aligned}
B_0^T &= -\tfrac{1}{8}(t_e + 3t_o), & B_1^T &= \tfrac{1}{8}(t_e - t_o),\\
B_0^F &= \tfrac{3}{8}(t_e + 3t_o), & B_1^F &= -\tfrac{3}{8}(t_e - t_o),\\
B_0^{\Delta s} &= \tfrac{3}{32}(t_e - t_o), & B_1^{\Delta s} &= -\tfrac{1}{32}(3t_e + t_o),\\
B_0^{\nabla s} &= \tfrac{9}{32}(t_e - t_o), & B_1^{\nabla s} &= -\tfrac{3}{32}(3t_e + t_o).
\end{aligned}$$

APPENDIX B: W^α COEFFICIENTS: CENTRAL PART OF THE FORCE

With the coupling constants defined in the Skyrme energy density functional and recalled in Appendix A, the $W^{(\alpha)}$ coefficients take the following expressions for symmetric nuclear matter ($\rho_n = \rho_p = \frac{1}{2}\rho_0$ and $\rho_1 = 0$):

$$\begin{aligned}
\tfrac{1}{4}W_1^{(0,0)} &= 2A_0^{\rho,0} + A_0^{\rho,\gamma}(\gamma+1)(\gamma+2)\rho_0^\gamma\\
&\quad - \left[2A_0^{\Delta\rho} + \tfrac{1}{2}A_0^\tau\right]q^2,\\
\tfrac{1}{4}W_1^{(0,1)} &= 2A_1^{\rho,0} + 2A_1^{\rho,\gamma}\rho_0^\gamma - \left[2A_1^{\Delta\rho} + \tfrac{1}{2}A_1^\tau\right]q^2,\\
\tfrac{1}{4}W_1^{(1,0)} &= 2A_0^{s,0} + A_0^{s,\gamma}\rho_0^\gamma - \left[2A_0^{\Delta s} + \tfrac{1}{2}A_0^T\right]q^2,\\
\tfrac{1}{4}W_1^{(1,1)} &= 2A_1^{s,0} + 2A_1^{s,\gamma}\rho_0^\gamma - \left[2A_1^{\Delta s} + \tfrac{1}{2}A_1^T\right]q^2,\\
\tfrac{1}{4}W_2^{(0,0)} &= A_0^\tau, \quad \tfrac{1}{4}W_2^{(0,1)} = A_1^\tau,\\
\tfrac{1}{4}W_2^{(1,0)} &= A_0^T, \quad \tfrac{1}{4}W_2^{(1,1)} = A_1^T.
\end{aligned}$$

APPENDIX C: $\widetilde{W}^{(\alpha)}$ COEFFICIENTS: SPIN-ORBIT PART OF THE FORCE

When the spin-orbit force is taken into account in the RPA formalism, one has to define with the help of the β_i moments [33] given in the Appendix D:

(i) for $S = 0$, the coefficients $\widetilde{W}_1^{(S,I)}$:

$$\begin{aligned}
\widetilde{W}_1^{(0,0)} &= W_1^{(0,0)} + 9W_0^2q^4\frac{\beta_2-\beta_3}{1+W_2^{(1,0)}q^2(\beta_2-\beta_3)},\\
\widetilde{W}_1^{(0,1)} &= W_1^{(0,1)} + W_0^2q^4\frac{\beta_2-\beta_3}{1+W_2^{(1,1)}q^2(\beta_2-\beta_3)};
\end{aligned}$$

(ii) for $S = 1$, the coefficients $\widetilde{W}_1^{(S,I,M)}$:

$$\begin{aligned}
\widetilde{W}_1^{(1,0,1)} &= W_1^{(1,0)} + \frac{9}{2}W_0^2q^4\frac{\beta_2-\beta_3}{1+W_2^{(0,0)}q^2(\beta_2-\beta_3)},\\
\widetilde{W}_1^{(1,0,0)} &= W_1^{(1,0)},\\
\widetilde{W}_1^{(1,0,-1)} &= W_1^{(1,0)} + \frac{9}{2}W_0^2q^4\frac{\beta_2-\beta_3}{1+W_2^{(0,0)}q^2(\beta_2-\beta_3)},\\
\widetilde{W}_1^{(1,1,1)} &= W_1^{(1,1)} + \frac{1}{2}W_0^2q^4\frac{\beta_2-\beta_3}{1+W_2^{(0,1)}q^2(\beta_2-\beta_3)},\\
\widetilde{W}_1^{(1,1,0)} &= W_1^{(1,1)},\\
\widetilde{W}_1^{(1,1,-1)} &= W_1^{(1,1)} + \frac{1}{2}W_0^2q^4\frac{\beta_2-\beta_3}{1+W_2^{(0,1)}q^2(\beta_2-\beta_3)}.
\end{aligned}$$

APPENDIX D: GENERALIZED LINDHARD FUNCTIONS

Following Ref. [33], we define the generalized free response functions as

$$\chi_{2i}(q) = \int \frac{d^4q_3}{2(2\pi)^4}\left[\left(\frac{q_3^2}{k_F^2}\right)^i + \left(\frac{(\mathbf{q}_3+\mathbf{q})^2}{k_F^2}\right)^i\right]G_{\rm HF}(q_3,q). \quad \text{(D1)}$$

The explicit expression can be found in Ref. [33] for $i = 0, 1, 2$. With χ_0, χ_2, and χ_4 we can compute the different moments of $G_{\rm HF}$ occurring in the calculation. As in Garcia-Recio *et al.* [33], we introduce the functions β_i as

$$[\beta_i, i=0,8] = \int \frac{d^4q_3}{(2\pi)^4} G_{\rm HF}\Big[1, \frac{\mathbf{q}\cdot\mathbf{q}_3}{q^2}, \frac{\mathbf{q}_3^2}{q^2}, \frac{(\mathbf{q}\cdot\mathbf{q}_3)^2}{q^4}, \frac{(\mathbf{q}\cdot\mathbf{q}_3)\mathbf{q}_3^2}{q^4}, \frac{\mathbf{q}_3^4}{q^4}, \frac{(\mathbf{q}\cdot\mathbf{q}_3)^3}{q^6}, \frac{(\mathbf{q}\cdot\mathbf{q}_3)^4}{q^8}, \frac{(\mathbf{q}\cdot\mathbf{q}_3)^2\mathbf{q}_3^2}{q^6}\Big],$$

with the functions β_i written as

$$\beta_0 = \chi_0, \quad 2\tilde{k}\beta_1 = (\nu-\tilde{k})\chi_0, \quad 4\tilde{k}^2\beta_2 = \chi_2 - 2\tilde{k}\nu\chi_0,$$

$$4\tilde{k}^2\beta_3 = (\nu-\tilde{k})^2\chi_0 - \frac{m^*k_F}{6\pi^2},$$

$$8\tilde{k}^3\beta_4 = 2\tilde{k}\nu(\tilde{k}-\nu)\chi_0 + (\nu-\tilde{k})\chi_2 + \frac{m^*k_F}{3\pi^2}\tilde{k},$$

$$16\tilde{k}^4\beta_5 = \chi_4 - 4\tilde{k}\nu\chi_2,$$

$$8\tilde{k}^3\beta_6 = (\nu-\tilde{k})^3\chi_0 + (3\tilde{k}-\nu)\frac{m^*k_F}{6\pi^2},$$

$$16\tilde{k}^4\beta_7 = (\nu-\tilde{k})^4\chi_0 - \frac{m^*k_F}{2\pi^2}\left[\tilde{k}^2 + \frac{1}{5} + \frac{1}{3}(2\tilde{k}-\nu)^2\right],$$

$$16\tilde{k}^4\beta_8 = (\nu-\tilde{k})^2\chi_2 - 2\tilde{k}\nu(\nu-\tilde{k})^2\chi_0 - \frac{m^*k_F}{6\pi^2}\left[1 + 2\tilde{k}(3\tilde{k}-\nu)\right],$$

where $\tilde{k} = \frac{q}{2k_F}$ and $\nu = \frac{m^*\omega}{qk_F}$.

For completeness, we now quote the different moments of $G_{\rm HF}$ encountered in the RPA equations:

$$\langle G_{\rm HF}\rangle = \beta_0, \quad \langle kY_0^{(1)}G_{\rm HF}\rangle = q\sqrt{\frac{3}{4\pi}}\beta_1,$$

$$\langle k^2G_{\rm HF}\rangle = q^2\beta_2, \quad \langle k^2|Y_0^{(1)}|^2G_{\rm HF}\rangle = q^2\frac{3}{4\pi}\beta_3,$$

$$\langle k^2|Y_1^{(1)}|^2G_{\rm HF}\rangle = q^2\frac{3}{8\pi}(\beta_2-\beta_3),$$

$$\langle k^3Y_0^{(1)}G_{\rm HF}\rangle = q^3\sqrt{\frac{3}{4\pi}}\beta_4,$$

$$\langle k^3|Y_0^{(1)}|^2Y_0^{(1)}G_{\rm HF}\rangle = q^3\left(\frac{3}{4\pi}\right)^{3/2}\beta_6,$$

$$\langle k^3|Y_1^{(1)}|^2Y_0^{(1)}G_{\rm HF}\rangle = q^3\sqrt{\frac{3}{4\pi}}\frac{3}{8\pi}(\beta_4-\beta_6),$$

$$\langle k^4\,G_{\rm HF}\rangle = q^4\beta_5,$$

$$\langle k^4|Y_0^{(1)}|^4G_{\rm HF}\rangle = q^4\left(\frac{3}{4\pi}\right)^2\beta_7,$$

$$\langle k^4|Y_1^{(1)}|^2|Y_0^{(1)}|^2G_{\rm HF}\rangle = q^4\frac{9}{32\pi^2}(\beta_8-\beta_7),$$

$$\langle k^4|Y_1^{(1)}|^4G_{\rm HF}\rangle = q^4\frac{9}{64\pi^2}(\beta_5-2\beta_8+\beta_7).$$

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