# Wigner energy and shell gaps in two-nucleon separation energies

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Two-nucleon separation energies are differences of binding energies. They provide important information on the relative stability of nuclei and, in particular, on shell gaps. In this work, the behavior of the decrements (change of slope) of two-nucleon separation energies has been studied. It has been shown that the bulk of this decrement at a shell-gap consists mainly of twice the difference of two effective single particle energies, plus a pairing correction. The decrement of the two-nucleon separation energies has a maximum (spike) for $N = Z$ nuclei. A comparison with values calculated by using a seniority binding energy formula shows that the spike is due to the Wigner energy. The evolution with nuclear mass of the isospin dependence of the Wigner energy is discussed.

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## I. INTRODUCTION

Binding energies of atomic nuclei are fundamental for studying nuclear properties, such as two-body interactions. For instance, the recent advent of radioactive isotope beams has expanded regions of nuclei for which binding energies have been measured, and magnitudes of shell gaps and of pairing energies in nuclei far from the stability line have been extensively discussed [1]. In deriving such quantities, differentiation of binding energies is often employed.

For instance, the single-neutron separation energy is the derivative with respect to the neutron number (with a discrete increment)

$$S_n(N,Z) = B(N,Z) - B(N-1,Z), \tag{1}$$

where $B(N, Z)$ is the *positive* binding energy of a nucleus with $N$ neutrons and $Z$ protons. $S_n$ expresses the energy required to remove a neutron from the nucleus. The single-proton separation energy $S_p$ is similarly defined. In an analogous way, the two-nucleon separation energies represent twice the derivatives with respect to $N$ or $Z$, denoted by $S_{2n}$ and $S_{2p}$, respectively.

Experimental values of the single separation energies $S_n$, $S_p$ and double separation energies $S_{2n}$, $S_{2p}$ can be found in [2]. If we look, for example, at the $S_{2n}(N,Z)$ curves shown there, we notice that the two-neutron separation energy generally decreases with increasing $N$, reflecting the symmetry energy and the shell filling, while at certain values of $N$ the slope of the curve changes sharply. Therefore it becomes interesting to examine the decrements of the separation energies (DSE), defined as

$$\Delta S_{2n}(N,Z) = S_{2n}(N,Z) - S_{2n}(N+2,Z), \tag{2}$$

$$\Delta S_{2p}(N,Z) = S_{2p}(N,Z) - S_{2p}(N,Z+2) \tag{3}$$

for neutrons and protons, respectively [1,3,4]. $\Delta S_{2n}(N,Z)$ and $\Delta S_{2p}(N,Z)$ represent second derivatives with respect to $N$ or $Z$, respectively. Although single-nucleon separation energies can be used for the same purpose, we will concentrate on properties of two-nucleon separation energies, which display a smoother $N$, $Z$ dependence, being relatively free of pairing effects. The DSE indicates the amount by which the last neutrons become less bound as a pair of neutrons is added to the nucleus.

If we plot $\Delta S_{2n}$ and $\Delta S_{2p}$ as functions of $N$ and $Z$, we notice sharp maxima (up to 10 MeV) at magic numbers [3]. The reduction in binding of the last neutron becomes much larger when a major shell fills, forcing further neutrons to enter higher-energy (less bound) orbitals. Moreover, if one looks at such a plot, another feature appears, namely that the values of $\Delta S_{2n}$ ($\Delta S_{2p}$) and other linear combinations of binding energies, are quite large for even-even $N = Z$ nuclei [1,5–7]. These spikes were attributed to the Wigner energy [8] or, more generally, to $T = 0$ interactions.

The main aim of the present work is to study the origin of the spikes observed for $\Delta S_{2n}$ and $\Delta S_{2p}$ in $N = Z$ nuclei. For this purpose, we will use a seniority-based mass formula for the binding energies, and we will consider only even-even nuclei. The results for the $sd$ and $f_{7/2}$-shells will be discussed.

Before concentrating on the Wigner energy, we will examine a few important properties of the separation energies, including their behavior at shell closures. A convenient definition of a shell gap will be proposed.

## II. PROPERTIES OF SEPARATION ENERGIES

### A. Relations between separation energies

The two-neutron separation energy

$$S_{2n}(N,Z) = B(N,Z) - B(N-2,Z) \tag{4}$$

can be rewritten, with the aid of Eq. (1), in the form

$$S_{2n}(N,Z) = S_n(N,Z) + S_n(N-1,Z). \tag{5}$$

Parallel results hold for protons, here and throughout. The difference $\Delta S_{2n}(N,Z)$ [see Eq. (2)] can be decomposed into four single-neutron separation energies

$$\Delta S_{2n}(N,Z) = S_n(N,Z) + S_n(N-1,Z) - S_n(N+2,Z) - S_n(N+1,Z). \quad (6)$$

Another important linear combination of binding energies is the pairing energy $P_n$. We will use the simple "three-nuclei" definition of the pairing energy, which is taken to be the difference between the gain in binding energy on adding a pair of neutrons and twice the gain in binding energy on adding a single neutron. So

$$P_n(N+2,Z) = B(N+2,Z) - B(N,Z) - 2[B(N+1,Z) - B(N,Z)], \quad (7)$$

$$P_n(N+2,Z) = B(N+2,Z) + B(N,Z) - 2B(N+1,Z) \quad (8)$$

which is equivalent to

$$P_n(N,Z) = S_n(N,Z) - S_n(N-1,Z). \quad (9)$$

The following relation can be deduced from Eqs. (5) and (9):

$$S_{2n}(N,Z) = 2S_n(N,Z) - P_n(N,Z). \quad (10)$$

In analogy to the two-neutron DSE of Eq. (2), let us introduce the difference of two single-neutron separation energies

$$\Delta S_n(N,Z) = S_n(N,Z) - S_n(N+2,Z) \quad (11)$$

and the difference of two pairing energies

$$\Delta P_n(N,Z) = P_n(N+2,Z) - P_n(N,Z). \quad (12)$$

(Note the order of the arguments in the last two equations.) The relation

$$\Delta S_{2n}(N,Z) = 2\Delta S_n(N,Z) + \Delta P_n(N,Z) \quad (13)$$

can be deduced from Eqs. (2), (11), and (12). It is a model independent identity, following directly from the definitions.

### B. Shell gaps

How can one define a shell gap? Defining it as the difference of single particle energies (SPE) delimiting the gap is not acceptable, if only because there is no unique definition of the SPE. They are often defined as the eigenvalues of a single-particle Hamiltonian which contains a realistic or just convenient one-particle potential, such as the harmonic oscillator or the Woods-Saxon potentials [9]. Single particle energies could be directly extracted from excitation energies, but only in particular cases, e.g., for a doubly magic core plus one particle, and even then it is open to argument whether or not they should be weighted by spectroscopic factors. If other valence particles are added, this method need no longer work. In many cases, SPE have been treated as free parameters, thus avoiding the requirement of a physical definition.

In recent years, the concept of effective single particle energy (ESPE) has been introduced [10,11]. In shell-model language, the ESPE contains a bare single particle energy (arising from the interaction of a valence nucleon with the closed doubly-magic core) as well as the effect of interactions with the other valence nucleons. It can be taken to be the change in the binding energy of the nucleus upon introducing a single valence nucleon, and as such *it is equal to the separation energy of a nucleon occupying the single particle level, with opposite sign.* In this way, we deal with a definition which is directly connected to an observable. It is the analog of the ionization energy of an electron but it is a negative number. The theoretical definition of the ESPE depends on both the SPE and the matrix elements of the two-body interactions, in such a way that the connection to experiment mentioned above is respected.

TABLE I. $\Delta P_n = \Delta S_{2n} - 2\Delta S_n$ for $^{48}$Ca and $^{208}$Pb in MeV.

| Nucl. | $\Delta S_{2n}$ | $\Delta S_n$ | $\Delta P_n$ |
|---|---|---|---|
| $^{48}$Ca | 5.723 | 3.593 | −1.463 |
| $^{208}$Pb | 4.984 | 2.183 | 0.618 |

Now, we can grasp the physical meaning of Eq. (13). It tells us that the DSE $\Delta S_{2n}$ is equal to twice the difference of two ESPE, plus a pairing correction. One may ask how important is the latter. As we can see in Table I, the pairing correction represents 26% of the DSE for $^{48}$Ca and 12% for $^{208}$Pb. This means that the bulk of the DSE comes from twice the difference of the two effective single particle energies.

We note that, as a major shell is filled, each addition of a pair of neutrons leads to a reduction in the neutron separation energy and hence to a higher ESPE. This change in ESPE, measured by $\Delta S_n$, is roughly constant within a shell, assuming no major changes in the pairing energy. At the magic numbers, where the major shell is filled, the next ESPE (for $N+2$ neutrons) rises above the shell gap, producing large values for $\Delta S_n$ and $\Delta S_{2n}$. This accounts for the spikes in $\Delta S_{2n}$ at magic numbers. The DSE value at a magic number is, up to pairing, twice the difference in ESPE of the levels delimiting the shell gap. Therefore, it is natural to define the width of the shell gap as the difference of these two ESPE [10]. It may be approximated by half the DSE at magic numbers.

## III. WIGNER ENERGY

### A. Seniority binding energy formula

A proton-neutron seniority formula for the binding energy of the ground state (g.s.) [12–14] will be used, namely,

$$B(j^n;\text{g.s.}) = B(n=0) + nC + \frac{n(n-1)}{2}\alpha + \beta\left[T(T+1) - \frac{3}{4}n\right] + \gamma\left\lfloor\frac{n}{2}\right\rfloor + E'_{\text{Coul}}. \quad (14)$$

$B(j^n;\text{g.s.})$ is the ground state binding energy of a nucleus with $n$ valence nucleons (both neutrons and protons) in a single-$j$ subshell, with isospin $T$ and lowest seniority. $B(n=0)$ is the binding energy of the doubly magic core, which has $N=Z$ and an isospin $T=0$. The Wigner term is $\beta T$. $\lfloor\frac{n}{2}\rfloor$ is the largest integer smaller than or equal to $\frac{n}{2}$. The term in $\gamma$ represents the pairing energy. The number of $J=0$, $T=1$ valence pairs in the ground state of an even-even nucleus is $n/2$. $E'_{\text{Coul}}$ is the

Coulomb energy, from which the Coulomb energy of the core has been subtracted. Instead of $Z$ and $N$, a nucleus is specified by $n$ and $T_z$, with $n$ the *total* number of valence nucleons and $T_z = (Z - N)/2$.

The coefficients $C, \alpha, \beta$, and $\gamma$ are free parameters which have been fitted to experimental binding energies for the $sd$-shell [13]. In order to check the exact form of the isospin dependence, $T(T+1)$ was replaced in the present work by $T(T+w)$ in the calculation of $\Delta S_{2n}$ (see Sec. III C). The free parameter $w$ has been introduced by Zeldes [14].

The initial calculation in [13] was carried out for $j = 3/2$ in the $sd$-shell. Although Eq. (14) has been derived for the single-$j$ case, and with $w = 1$, its successful application to the g.s. of nuclei between $^{28}$Si and $^{36}$Ar shows that it may be a good approximation also in the case of configuration mixing. We will assume that $T = |Z - N|/2$. In particular, in even-even $N = Z$ nuclei $T(\text{g.s.}) = 0$. A similar equation, based on the notion of generalized seniority, has been successfully used [15] for describing the binding energy of a semimagic nucleus.

The presence of a term in $|Z - N|$ leads to a singularity of the binding energy due to the fact that the derivative of this absolute value with respect to $Z$ or $N$ is not continuous at $N = Z$. This is also the origin of the "cusp" of the binding energy of $N = Z$ nuclei described in [1].

If we introduce the binding energies from Eq. (14) [with $T(T+1)$ replaced by $T(T+w)$] into Eq. (4) we obtain

$$\begin{aligned} S_{2n} &= B(n, T) - B(n-2, T') \\ &= 2C + \alpha(2n-3) + \gamma + \delta E_{\text{Coul}} \\ &\quad + \beta\left[T(T+w) - T'(T'+w) - \tfrac{3}{2}\right] \end{aligned} \qquad (15)$$

with

$$\delta E_{\text{Coul}}(2n) = -0.717 Z^2 \left(\frac{1}{A^{1/3}} - \frac{1}{(A-2)^{1/3}}\right) \qquad (16)$$

in MeV. $T$ and $T'$ are the isospins of the nuclei $(N, Z)$ and $(N-2, Z)$, respectively. The core contribution to the Coulomb energy has vanished following the subtraction. The Wigner energy (WE) is equal to $-\beta T w$. We can now use Eq. (15) for calculating $\Delta S_{2n}(N, Z)$ according to Eq. (2). $\Delta S_{2p}$ can be calculated in a similar way; only the Coulomb correction will be different, namely,

$$\delta E_{\text{Coul}}(2p) = -0.717 \left(\frac{Z^2}{A^{1/3}} - \frac{(Z-2)^2}{(A-2)^{1/3}}\right). \qquad (17)$$

The decrement of the separation energy (DSE) can be expressed in an analytical form

$$\Delta S_{2n} = -4\alpha - 2\beta(1 + w\delta_{NZ}), \qquad (18)$$

up to the Coulomb correction. The single-neutron DSE is

$$\Delta S_n = -2\alpha - \beta(1 + w\delta_{NZ}), \qquad (19)$$

again up to the Coulomb correction. If the ground states of the $(N,Z)$ and $(N+2,Z)$ nuclei arise from filling the same shell,

$$\Delta S_{2n}(N, Z) = 2\Delta S_n(N, Z). \qquad (20)$$

This result is compatible with the identity, Eq. (13), since the seniority formula, together with Eq. (12), implies $\Delta P_n(N,Z) = 0$.

### B. Role of parameters

Before seeing the results of the calculation, it will be interesting to identify the roles played by different parameters. Since $\Delta S_{2n}$ and $\Delta S_{2p}$ represent second derivatives (with finite increments), the terms linear in $n$ in Eq. (5), as well as the constant ones, will not play any role. Therefore, only the two-body and isospin terms will be relevant. Moreover, we should not forget the Coulomb corrections.

The simple analytical equation (18) shows that $\Delta S_{2n}$ or $\Delta S_{2p}$ is approximately constant in an isotope (or isotone) series, with the exception of the $N = Z$ nucleus, which displays a maximum. The height of the constant baseline, which will be clearly seen on the figures, is a linear combination of the parameters $\alpha$ and $\beta$, i.e., the baseline comes from the two-body and $T$-quadratic terms. The term linear in $T$, i.e., the WE does not play any role.

On the other hand, in the $N = Z$ case, $\Delta S_{2n}$ is higher than the baseline by the amount $-2w\beta$ ($\beta < 0$). This increase is caused only by the term linear in $T$, i.e., by the WE. Of course, the same considerations are valid for $\Delta S_{2p}$.

Moreover, the heights of the baseline and of the spike are the same for all isotope (isotone) series within the validity limits of the parameter set. Of course, there are small differences caused by the Coulomb correction.

There are more elaborate formulas for the WE such as, e.g., eq. (20) of [16]. In the case of even-even nuclei, the latter is reduced to a term proportional to $|N - Z|$. The WE has a weak mass dependence. Our results show that the mass dependence can be neglected within the limits covered by a set of parameters of Eq. (14).

The role of the Coulomb correction cannot be described in simple terms, but the calculation shows that it represents a relatively small correction, albeit not negligible, especially for $\Delta S_{2p}$.

### C. Results

In order to extract values of the Wigner energy parameter $w$ from the decrements of two-nucleon separation energies, Eq. (14) was first fitted to all measured masses (as listed in Ref. [2]) in the $sd$ shell and, separately, in the $f_{7/2}$ shell. This fit was performed with a $T(T+w)$ isospin dependence, after first correcting the binding energies for Coulomb effects and subtracting the Coulomb-corrected measured binding energy of the core nucleus $^{16}$O or $^{40}$Ca, respectively, and produced the parameter values listed in Table II. Here $w$ is dimensionless, while all other parameters are in MeV.

The associated rms deviations of the fit were 2.976 MeV for 123 data points in the $sd$-shell and 1.549 MeV for 60 data points in the $f_{7/2}$-shell. This should be compared to the fitted data, which range up to almost 300 MeV. It is noteworthy that, in these "global" fits, which included both odd and even $N$ and $Z$, $w$ differs from zero by $4 - 5\sigma$ and from $w = 1$ by about $2.5\sigma$.

Next, the two-nucleon separation energies $S_{2n}$ and $S_{2p}$ listed in Ref. [2] were corrected for Coulomb energy and used to generate Coulomb-corrected decrements of two-nucleon separation energies for even-even nuclei in the $sd$ and $f_{7/2}$-shells. For each even $N$ from $N = 10$ to $N = 18$ in the $sd$

TABLE II. Parameters of the binding energy formula.

| Shell | $C$ | $\alpha$ | $\beta$ | $\gamma$ | $w$ |
|---|---|---|---|---|---|
| $sd$ | 7.692(555) | 0.114(12) | −2.065(155) | 2.697(1055) | 1.959(364) |
| $f_{7/2}$ | 10.059(451) | 0.159(20) | −1.069(174) | 1.692(791) | 3.073(773) |

shell, $\Delta S_{2n}$ for isotope chains for even $Z$ in the same 10 to 18 range were fitted to Eq. (18), producing values for the constant background $-4\alpha - 2\beta$ and for the Wigner "spike" $-2\beta w$ (see Fig. 1). The latter was divided by $-2\beta$, using the $\beta$ value obtained in the full binding energy fit, to obtain a value of $w$ for each even $N$ involved. The corresponding procedure produced values of $w$ for each even $Z$ in the relevant range from fits of $\Delta S_{2p}$ to isotone chains for even $N$. (See Fig. 2.) The same process was repeated for the $f_{7/2}$-shell. (See Figs. 3 and 4.) The standard error in $w$ for each isotope or isotone chain included the statistical error of the fit and the fit error in $-2\beta$, in quadrature.

The ten values of $w$ obtained in the $sd$-shell have an average value of 1.46, with a standard deviation of 0.59, and individual standard errors of roughly 30%. A few individual fits have rather small $w$ values, generally with large standard errors. Those with standard errors of 40% or less differ from zero by roughly 3–6$\sigma$ and from $w = 1$ by roughly 1–2$\sigma$. When weighted with the inverse squares of their standard errors, these 10 $w$ values have a weighted mean of $\langle w \rangle = 1.65(12)$. Note that the weighted mean is larger than the unweighted mean because the smaller values of $w$ tend to have larger standard errors.

The four values of $w$ obtained in the $f_{7/2}$-shell have an average value of 1.68, with a standard deviation of 0.17, and individual standard errors of about 35%. They differ from zero by some 3$\sigma$ and from $w = 1$ by roughly 1–1.5$\sigma$. When weighted with the inverse squares of their standard errors, these four $w$ values have a weighted mean of $\langle w \rangle = 1.64(28)$.

The theoretical curves in the figures displayed here were calculated in each shell using fixed values of $w$ and of the constant background. Both $\Delta S_{2n}$ and $\Delta S_{2p}$ of nuclei in the $sd$-shell (Figs. 1 and 2) were calculated with the same value $w = 1.46$. The rms deviation of the calculated points from the experimental data is 1.26 MeV. The value $w = 1.68$ was used for the $f_{7/2}$-shell (Figs. 3 and 4), with an rms deviation of 0.34 MeV. Most experimental values of $\Delta S_{2n}$ in the $sd$-shell are higher than the calculated ones for $N = 14$. This is probably due to the closure of the $d_{5/2}$-subshell.

It may be noted that the Coulomb correction to $S_{2n}$ is about 1 MeV, the Coulomb correction to $\Delta S_{2n}$ about 0.1 MeV, in the $sd$-shell. The corresponding numbers for $S_{2p}$ and $\Delta S_{2p}$ are 11.5 MeV and 1.5 MeV, respectively. There is little variation in the Coulomb correction to $\Delta S_{2n}$ or $\Delta S_{2p}$ across the shell.

Despite the variability of the individual $w$ values and their sometimes sizable standard errors, the overall pattern seems reasonably clear. There is a systematic Wigner effect, differing from zero by several $\sigma$ but not strongly incompatible with $w = 1$, in both the $sd$- and the $f_{7/2}$-shells.

### D. Isospin dependence

In both the $sd$ and $f_{7/2}$ shells the expression $T(T+1)$ represents a rough approximation to the isospin dependence of Eq. (14). As a matter of fact, due to the appearance at $N = Z$ of a spike above the baseline, the DSE is a sensitive indicator of the Wigner energy.

The $T(T+1)$ isospin dependence in Eq. (14) reflects the isospin SU(2) symmetry. However, if we imagine a dynamic symmetry which starts from the Wigner SU(4) spin-isospin symmetry [8,13], and also includes an SU(2) isospin symmetry, we may expect a linear combination of $T(T+1)$ and $T(T+4)$ [17]. Depending on the relative sizes and signs of the coefficients of this linear combination, the result is of the form $T(T+w)$, where $w$ can take on any value, of either sign (though $1 \leqslant w \leqslant 4$ if the coefficients of $T(T+1)$ and $T(T+4)$ have the same sign). Maxima at $N = Z$ of a

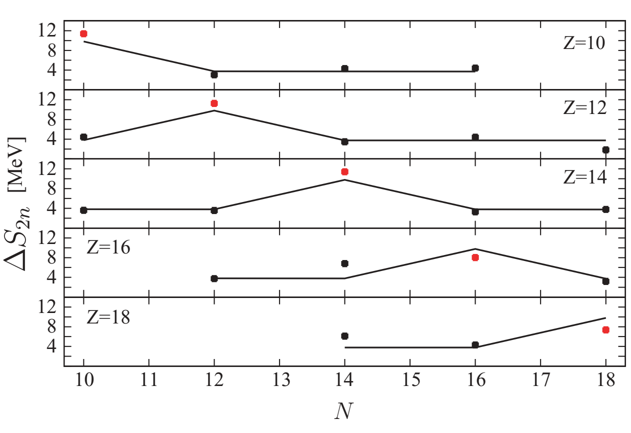


FIG. 1. (Color online) $\Delta S_{2n}$ for the $sd$-shell. Dots: experimental data; continuous line: calculation. The value $w = 1.46$ has been used for all nuclei. Data for $N = Z$ nuclei are shown in red.

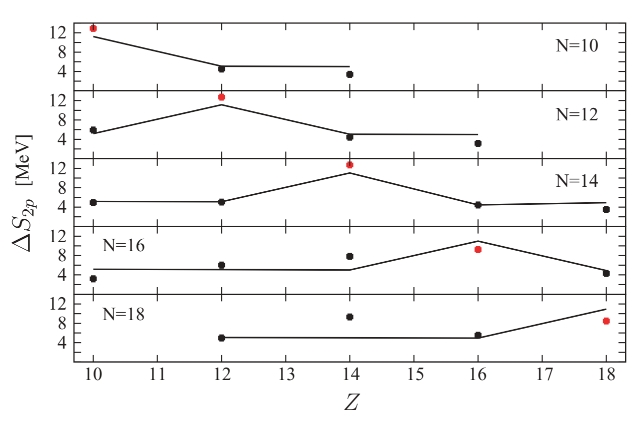


FIG. 2. (Color online) $\Delta S_{2p}$ for the $sd$-shell. Same notation and value of $w$ as in Fig. 1.

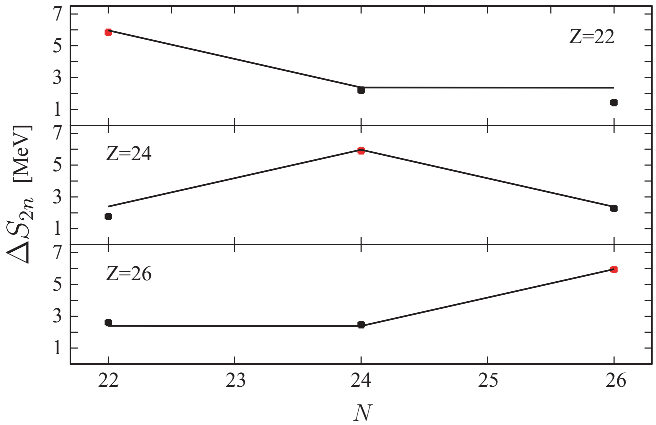


FIG. 3. (Color online) $\Delta S_{2n}$ for the $f_{7/2}$-shell. The value $w = 1.68$ has been used. Same notation as in Figs. 1 and 2.

proton-neutron double binding energy difference have been studied in [5]. A test of the SU(4) symmetry using the same double binding energy differences has been carried out by Van Isacker *et al.* [7]. A qualitative agreement with experiment has been observed for the $p$-shell.

One may consider an extended dynamic symmetry which includes the Elliott SU(3) symmetry [7,18]. A corresponding binding energy formula may be deduced, but such a project goes beyond the scope of the present work.

The form of the isospin term in the mass formulas has been discussed in [14] and, more recently in [19]. Both authors use $Q_{2\beta}$ as indicator. The former work indicates a wide range of $w$ in the mass interval $72 \leqslant A \leqslant 208$, with the $w$ parameter increasing with $A$ and reaching a maximum $w \sim 9$ at $A = 207$. Also the latter work indicates deviations from $w = 1$, suggesting a global $\langle w \rangle \sim 0.6$, across the full range of measured $A$ values, for a term of the form $T(T + w)/A$.

In [20,21] the isospin dependence is extracted from differences between isobaric analog states, with different isospins, in the same nucleus. The authors come to the conclusion that the form $T(T + 1)$ is valid for nuclei with $A \leqslant 60$. When A increases, $w$ gradually approaches the value 4. According to [21] this feature suggests quartet structures for medium mass nuclei with $N \approx Z$. The formation of quartets in $N = Z$ nuclei has been investigated in [22].

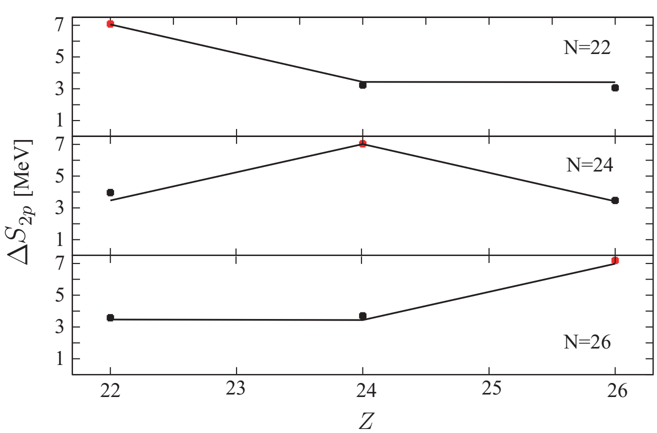


FIG. 4. (Color online) $\Delta S_{2p}$ for the $f_{7/2}$-shell. Same notation and same value of $w$ as in Fig. 3.

The values of $w$ extracted from global fits of mass formulas to measured nuclear masses are not well determined [23], but tend to be of order $w \sim 0.5$ to $w \sim 2$ (when positive). Such formulas have an explicit $A$ dependence, the Wigner term being proportional to $T/A$. A single global value of $w$ can be determined from the ratio of the coefficients of the Wigner and volume symmetry terms, but its interpretation may be an object of discussion. The approaches of [14,20] and [21] have no explicit $A$ dependence.

The present work, being limited to a small range of $A$ values, ignores the possibility of $A$ dependence within the limits of a shell. However, the Wigner parameter $w$ fitted in this work increases when going from the $sd$ to the $f_{7/2}$-shell, in qualitative agreement with [19].

The present approach is restricted to nuclei with $N, Z \leqslant 28$ where the validity of the proton-neutron seniority scheme is reasonably well established. Besides, since we study only nuclei with $N \approx Z$, this method makes sense only for diagonal shells.

## IV. CONCLUSION

The difference of two-neutron separation energies is defined as

$$\Delta S_{2n} = S_{2n}(N,Z) - S_{2n}(N + 2,Z), \tag{21}$$

with a similar expression for two-proton separation energies. $\Delta S_{2n}$ and $\Delta S_{2p}$ have maxima at magic numbers. It has been shown that $\Delta S_{2n}$ at a shell closure is dominated by the difference of two effective-single particle energies. The latter difference represents a realistic definition of the shell gap.

$\Delta S_{2n}$ and $\Delta S_{2p}$ display a spike for most known even-even $N = Z$ nuclei with $A \leqslant 70$. This spike is due to the Wigner energy. The experimental values of this observable have been compared with the results of calculations based on a seniority mass formula. The latter contains the isospin term $T(T + 1)$, which is the Casimir operator of the SU(2) isospin group. A more general expression is $T(T + w)$, where $w$ is a free parameter. Values of $w$ between 1 and 4 may be expected if we assume a dynamic symmetry of $SU(4) \supset SU(2)$ type with coefficients of the same sign. The values of $w$ fitted in this work to $N = Z$ nuclei in the $sd$- and $f_{7/2}$-shells are 1.46(59) and 1.68(17), respectively. Existing literature indicates deviations from the $T(T + 1)$ form for heavier nuclei.

This work shows that the isospin dependence of nuclear mass formulas must include a term proportional to $T$, representing the Wigner energy. A binding energy formula which contains the symmetry energy term $T^2$ but no Wigner term linear in $T$ is incomplete. The spikes of $\Delta S_{2n}$ and $\Delta S_{2p}$ at $N = Z$ constitute a sensitive indicator of the presence of the Wigner energy.

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